QCD at the high-energy frontier

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Outline

Topic of this talk

▶ What does a proton/nucleus look like probed at $\sqrt{s} \gg p_T \gg \Lambda_{QCD}$?
  
  (In practice $x \sim \frac{p_T}{\sqrt{s}}$, so small-$x = \text{high } \sqrt{s}$)

▶ Will argue: this is a weak coupling, but nonperturbative regime

▶ So how does one calculate cross sections?
  
  (For processes where a simple probe scatters off this complicated object)

Setup:

▶ High energy limit of Quantum Chromodynamics: gluon saturation

▶ Strong color fields & Wilson line

▶ Dilute-dense processes: orders in $\alpha_s$ and $\ln \frac{1}{x} \sim \ln \sqrt{s}$

Details, recent work:

▶ Inclusive Deep inelastic scattering, at NLO

▶ Forward rapidity in proton-nucleus at NLO

▶ Exclusive processes, UPC
Parton distributions

Content of proton in DIS, as measured by HERA

- "Valence": quantum numbers, energy
- "Sea" $q\bar{q}$ pairs
- Gluons: most (in numbers) $\sim x^{-0.3}$

(Recall: $x$ = fraction of proton longitudinal momentum carried by parton)
Parton distributions

Content of proton in DIS, as measured by HERA

- "Valence": quantum numbers, energy
- "Sea" $q\bar{q}$ pairs
- Gluons: most (in numbers) $\sim x^{-0.3}$

Gluons dominate. Why?

(Recall: $x =$ fraction of proton longitudinal momentum carried by parton)
Gluon saturation
What does a high energy proton look like? Parton model perspective

- Evolution with $Q^2$ (⊥) or $x$ (∥): cascade of gluons
- Small $x$: phase space density of gluons large
  ➞ nonlinear interactions, depending on
  - Size of one gluon $\sim 1/Q^2$
  - Transverse space available
  - Coupling

Gluon mergings matter when

$$\pi R_p^2 \sim \alpha_s x G(x, Q_s^2)/Q_s^2$$

Saturation “phase diagram”:
Eikonal scattering off target of glue

Instead of counting gluons, look at scattering amplitudes

- Dilute probe through target color field
- At high energy interaction is eikonal, relevant degree of freedom is Wilson line (= scattering amplitude of colored parton)

\[ V = \mathbb{P} \exp \left\{ -ig \int_{x^+}^{x^+} dy^+ A^- (y^+, x^-, x) \right\} \approx V(x) \in SU(N_c) \quad \text{coordinate space!} \]

- Amplitude for color dipole

\[ \mathcal{N}(r = |x - y|) = 1 - \left\langle \frac{1}{N_c} \text{Tr} V^\dagger(x) V(y) \right\rangle \]

- In perturbative limit \( \mathcal{N}(r) \sim \alpha_s [xG] r^2 \) (\( xG \) = usual gluon pdf)
- Saturation = unitarity requirement for amplitude (built in as group theory constraint for SU(\( N_c \)))

- \( 1/Q_s \) is Wilson line \perp \text{correlation length}
Power counting at small $x$
Dilute-dense process at LO

**DIS**

- $\gamma^* \rightarrow q\bar{q}$ dipole interacts with target color field
- Total cross section $2 \times \text{Im-part of amplitude}$
- Exclusive & inclusive processes

“Dipole model”: Nikolaev, Zakharov 1991
Fits to HERA data: e.g. Golec-Biernat, Wüsthoff 1998

**Forward hadrons in pp/pA**

- High $x q/g$ from probe: collinear pdf
- $|\text{quark amplitude}|^2 \sim \text{dipole}$
- Indep. fragmentation

“Hybrid formalism” Dumitru, Jalilian-Marian 2002

Universality: both involve same dipole amplitude $N = 1 - S$
Dilute-dense process at LL

Add one soft gluon: large logarithm of energy, i.e. $1/x$

DIS

- Soft gluon: large logarithm

$$\alpha_s \int_{x_{Bj}} \frac{d k_g^+}{k_g^+} \sim \alpha_s \ln \frac{1}{x_{Bj}}$$

Absorb large $\ln 1/x$ into renormalization of Wilson line:

**JIMWLK** equation, or **BK equation** for dipole Balitsky 1995, Kovchegov 1999

$$\implies$$ RG evolution for amplitudes in energy ($x$ or $\sqrt{s}$; cf DGLAP which is for pdf’s in $Q^2$ or $p_T$)

Forward hadrons

- Soft gluon $k^+ \to 0$: same large $\ln 1/x$
- Collinear gluon $k_T \to 0$:
  also DGLAP evolution of pdf, FF

Dumitru et al 2005
Dilute-dense process at NLO

Add one gluon, but not necessarily soft

DIS

\[ q^+ \rightarrow q^+ + g/2q^+ \]

- DIS impact factor
  Balitsky & Chirilli 2010, Beuf 2017

Forward hadrons

- NLO single inclusive
  Chirilli et al 2011

- Leading small-\(k^+\) gluon already in BK-evolved target
- Need to \textbf{subtract} leading log from cross section, (high energy) \textbf{factorization}

Schematically

\[
\sigma_{NLO} = \int dz \left[ \frac{\sigma_{\text{sub}}}{\sigma(z) - \sigma(z = 0) + \sigma(z = 0)} + \text{absorb in BK} \right] z = \frac{k_g}{q^+}
\]
NLO to NLL

NLO evolution equation:
- Consider NNLO DIS
- Extract leading soft logarithm
- Lengthy calculation:
  Balitsky & Chirilli 2007
  $\Rightarrow$ NLO BK/JIMWLK equation
- But additional resumations needed for practical phenomenology

- $\alpha_s^2 \ln^2 (1/x)$: two iterations of LO BK
- $\alpha_s^2 \ln 1/x$: NLO evolution
- $\alpha_s^2$: part of NNLO impact factor (not calculated)
Summary: power counting in $\alpha_s$ & $\ln 1/x$; state of the art

$\sigma \sim \mathcal{O}(1) + \mathcal{O}(\alpha_s \ln 1/x) + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2 \ln 1/x)$

Calculated at NLO/NLL

- **JIMWLK/BK evolution** Balitsky, Chirilli 2008, Grabovsky, Lublinsky, Mulian 2012
- **Total DIS cross section** $m_q = 0$ Balitsky, Chirilli 2010, Beuf 2011-2017
- **Single inclusive particles in fwd rapidity hh-collisions** Chirilli, Xiao, Yuan + others 2011 –
- **Diffractive dijets in DIS** Boussarie et al 2014
- **Exclusive light vector mesons (with PDA’s)** Boussarie et al 2016

So far only LO/LL, but NLO/NLL under way:

- **Forward rapidity dijets in pA** Partial results: Mulian & Ianuc, Ayala et al
- **Diffractive structure functions**
- **Total DIS cross section with massive quarks**
- **Exclusive quarkonium in DIS/UPC (with NRQCD)** T.L. Escobedo 2019
Benchmark processes at NLO:
  DIS and fwd hadrons in pA
At NLO need
- Real: $q\bar{q}g$ state in dipole
- Virtual: 1-loop corrections to the $\gamma^* q\bar{q}$-vertex

Divergences cancel between real and virtual

Massive quarks: in progress Beuf, T.L., Paatelainen

Soft log factorized into BK evolution of target
rest is NLO "$\gamma^*$ impact factor"

$$
\sigma_{L,T}^{qg,\text{sub.}} \sim \alpha_s C_F \int_{Z_1, x_0, x_1, x_2}^{1} \frac{dz_2}{z_2} \left[ \mathcal{K}_{L,T}^{\text{NLO}}(z_2, X(z_2)) - \mathcal{K}_{L,T}^{\text{NLO}}(0, X(z_2)) \right].
$$

$k_g^+ \sim z_2$
NLO corrections of reasonable magnitude, after major cancellation between different terms

Factorization procedure (still) somewhat naive, not good at large $Q^2$

Starting point for comparison with HERA data, to replace existing LO dipole picture fits
NLO calculations: Particle production in forward pA

Particle production in forward pA: “hybrid formalism”

▶ Quark/gluon from collinear pdf \((\text{large-}x)\)
▶ LO: deflected by target field
▶ NLO: 1-loop virtual and radiative corrections
▶ 1-loop factorization formulae
  Chirilli, Xiao, Yuan 2011
    ▶ Soft divergence: target BK
    ▶ Collinear: DGLAP for pdf, FF
    ▶ Rest: “hard function”
▶ 1st result Stasto et al 2013: NLO cross section negative at large \(p_T\).
▶ This now understood as a problem with the “naive” factorization procedure: exactly as for DIS
Exclusive DIS & UPC’s
Exclusive processes in dipole picture

Total cross section

\[ \sigma_{\text{tot}} \sim \text{forward elastic amplitude} \]

Diffractive DIS

Exclusive \sim |\text{same amplitude}|^2

Same dipole amplitude describes both

Unified description is a major advantage of the dipole picture

In hard scattering limit \( N \sim xg(x, Q^2) \) \( \implies \) often quoted formula

\[
\frac{d\sigma^{\gamma^*H\rightarrow VH}}{dt} = \frac{16\pi^3\alpha_s^2\Gamma_{ee}}{3\alpha_{em}M_V^5} \left[xg(x, Q^2)\right]^2
\]
Exclusive DIS at NLO

Many possible final states
- Jets
- Vector mesons
- Any hadrons with fixed $M_X$ . . .

Known at NLO
- Dijets Boussarie et al 2014
- Exclusive light vector mesons PDA for meson Boussarie et al 2016

NLO calculations in progress
- Diffractive structure functions: fixed $M_X$
- Quarkonium, with NRQCD for meson
  Heavy quarks are important:
  allow $Q^2 = 0$ in weak coupling
How to measure transverse geometry of gluons

Diffractive DIS gives Fourier transform of gluon distribution

\[ N(\Delta) = \int d^2b e^{ib \cdot \Delta} N(b) \]

**Coherent** target intact; measure **average** gluon distribution

\[ -t \sim \frac{1}{R_A^2} \sim 0.01\text{GeV}^2 \] (nucleus)

**Incoherent** target breaks without color exchange: **fluctuations**

\[ -t \sim \frac{1}{R_p^2} \sim 1\text{GeV}^2 \] (nucleus → nucleons)

Both very important for QGP physics
UPC results from LHC

- To understand $b$ distribution need average and fluctuations: coherent and incoherent

- This is equally true for $\gamma^{(*)}A$ and $\gamma^{(*)}p$

Highest energy $\gamma$-$p/A$ data so far: ultraperipheral collisions at LHC:

\[
\gamma A \rightarrow J/\Psi + A \quad \text{Eur. Phys. J. C 73 (2013) 2617}
\]

\[
\mathcal{Q}^2 = 0 \rightarrow J/\Psi \text{ only at one scale: heavy quark mass.}
\]

\[
\mathcal{Q}_s \text{ is } p_T\text{-scale: to study saturation dynamics need } \mathcal{Q}^2\text{-dependence: EIC}
\]
Conclusions

QCD at the HE frontier

- Access to gluon saturation
- Resummation of large logs of energy into JIMWKL/BK evolution
- Moving to NLO
  - Loop calculations done for many processes
  - In $1/x$-evolution requires resummations
    & factorization needs to be consistent with these
    $\Rightarrow$ challenges in implementation still being worked out
- Inclusive and exclusive processes in consistent framework
  - Small-$x$ physics at EIC
  - Dilute-dense processes at LHC
  - Initial conditions of heavy ion collisions.