Early equilibration, phase diagram and freeze-out in relativistic heavy-ion collisions

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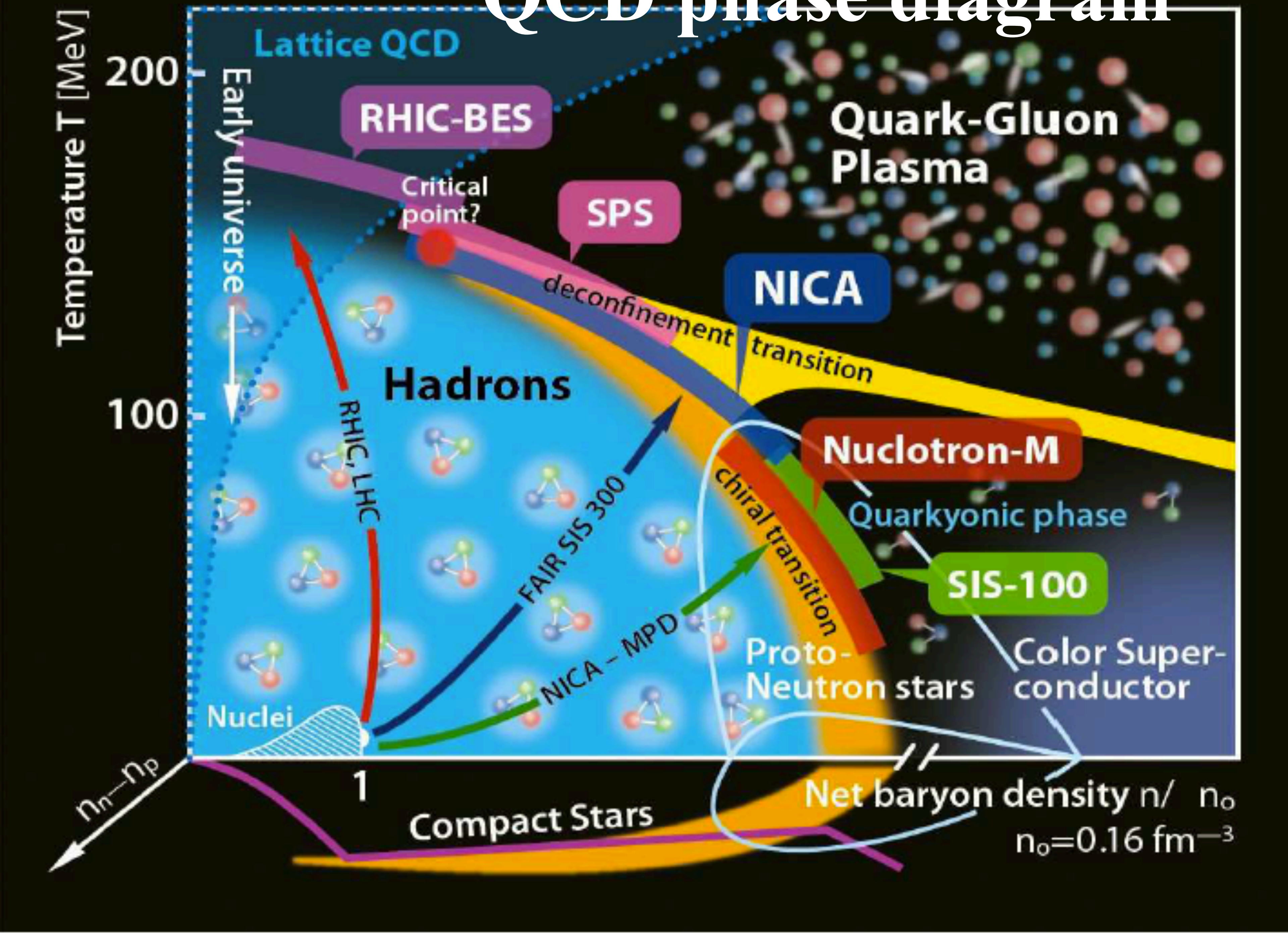
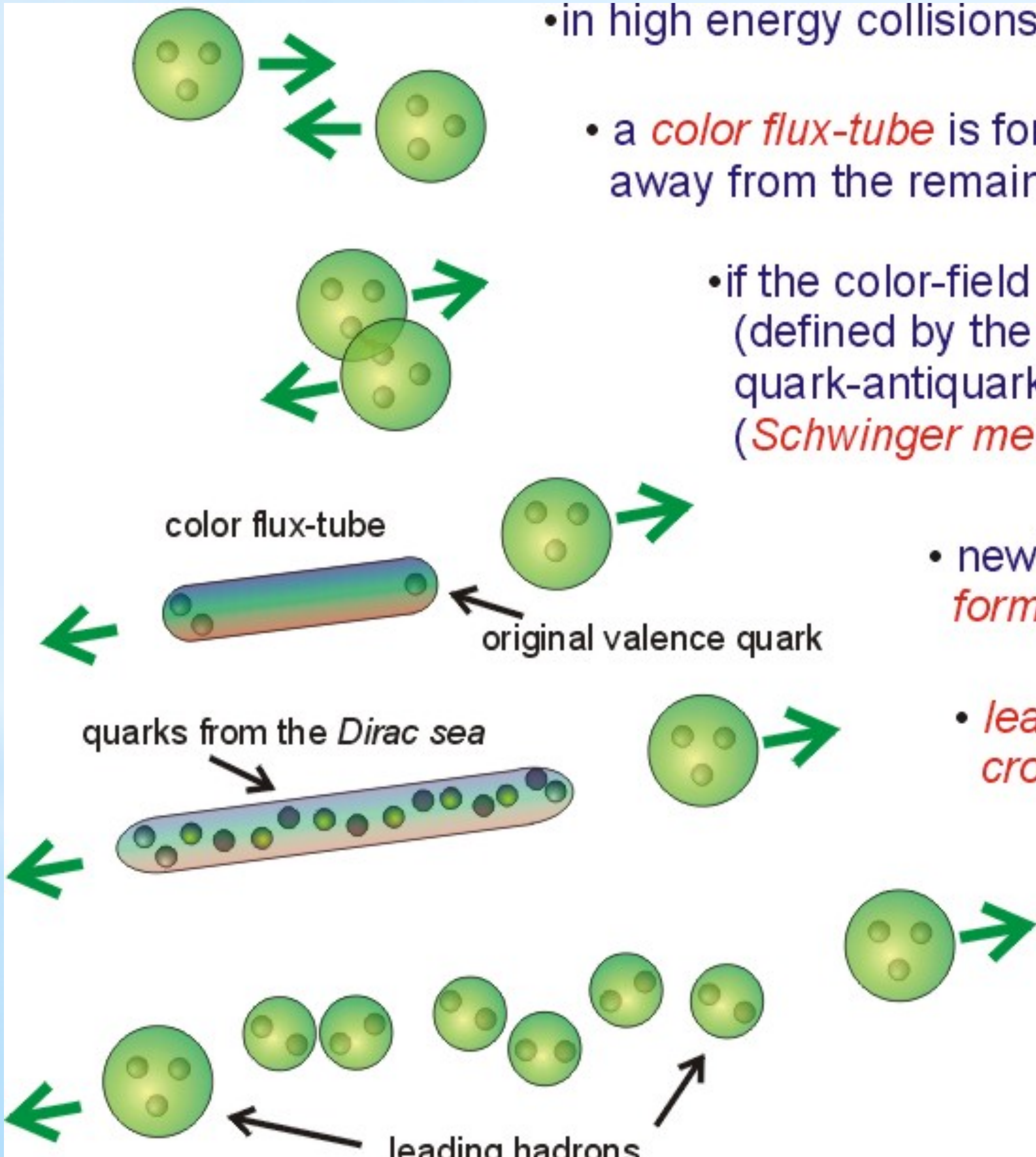
QCD phase diagram



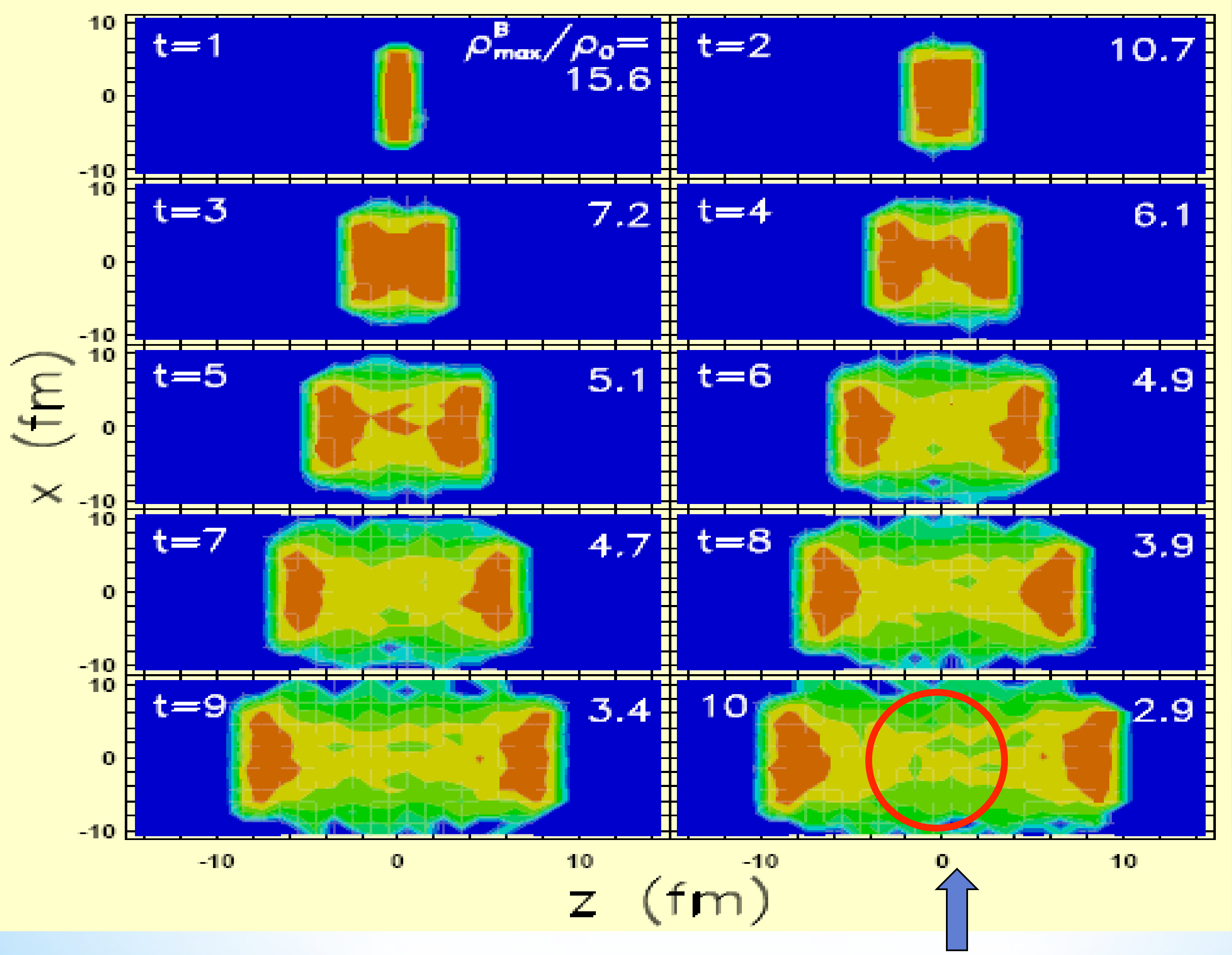
Particle production in string models



- in high energy collisions hadrons can be excited into *strings*
- a *color flux-tube* is formed by pulling one valence quark away from the remaining ones in the hadron
- if the color-field increases beyond a critical value (defined by the *string-tension*), spontaneous quark-antiquark creation from the *Dirac sea* occurs (*Schwinger mechanism*)
- newly created (anti-)quarks require a *formation time* to form hadrons
- *leading hadrons* interact with *reduced cross sections* during their formation time
- *newly created hadrons* have *zero cross section* during their formation time

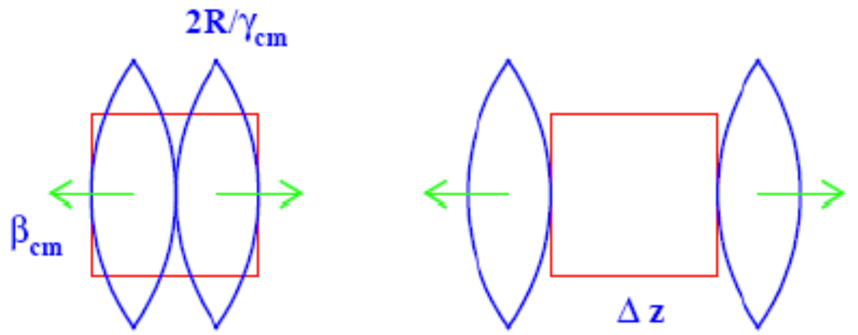
Pre-equilibrium: Homogeneity of baryon matter

L.Bravina et al., PRC 60 (1999) 024904



The local equilibrium in the central zone is quite possible

Equilibration in the Central Cell



$t^{cross} = 2R/(\gamma_{cm}\,\beta_{cm})$

$t^{eq} \geq t^{cross} + \Delta z/(2\beta_{cm})$

Kinetic equilibrium:

Isotropy of velocity distributions

Isotropy of pressure

Thermal equilibrium:

Energy spectra of particles are described by Boltzmann distribution

L.Bravina et al., PLB 434 (1998) 379; JPG 25 (1999) 351

$$\frac{dN_i}{4\pi p E dE} = \frac{V g_i}{(2\pi\hbar)^3} \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Chemical equlibrium:

Particle yields are reproduced by SM with the same values of $(T,\ \mu_B,\ \mu_S)$:

$$N_i = \frac{V g_i}{2\pi^2\hbar^3} \int_0^\infty p^2 dp \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Statistical model of ideal hadron gas

input values

output values

$$\varepsilon^{\rm mic} = \frac{1}{V}\sum_i E_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm B}^{\rm mic} = \frac{1}{V}\sum_i B_i\cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm S}^{\rm mic} = \frac{1}{V}\sum_i S_i\cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}).$$

Multiplicity

Energy

Pressure

Entropy density

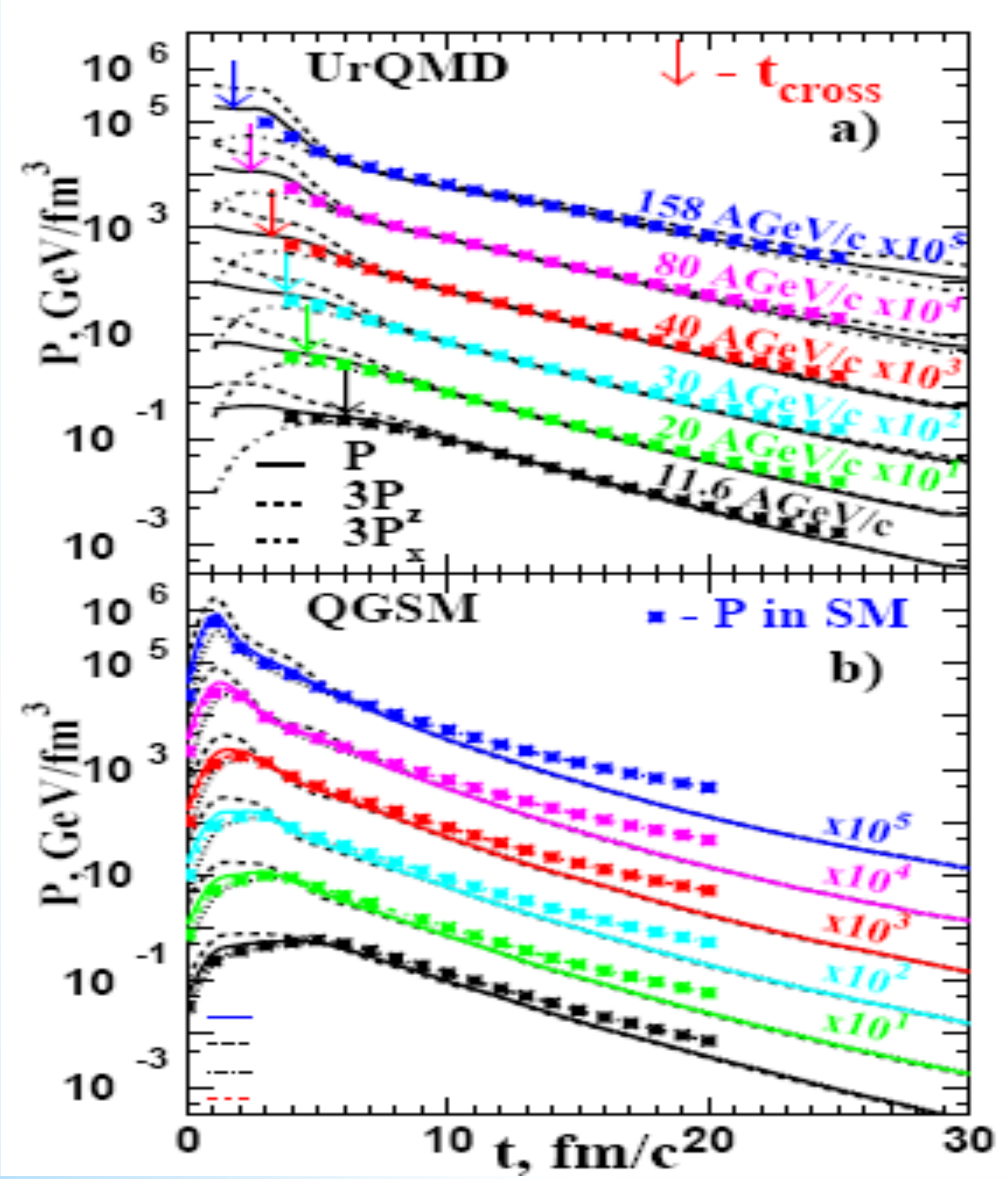
$$N_i^{\rm SM} = \frac{V g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 f(p,m_i)dp,$$

$$E_i^{\rm SM} = \frac{V g_i}{2\pi^2\hbar^3}\int_0^\infty p^2\sqrt{p^2+m_i^2}\, f(p,m_i)dp$$

$$P^{\rm SM} = \sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 \frac{p^2}{3(p^2+m_i^2)^{1/2}}\, f(p,m_i)dp$$

$$s^{\rm SM} = -\sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty f(p,m_i)\,\left[\ln f(p,m_i)-1\right]\, p^2 dp$$

Kinetic Equilibrium



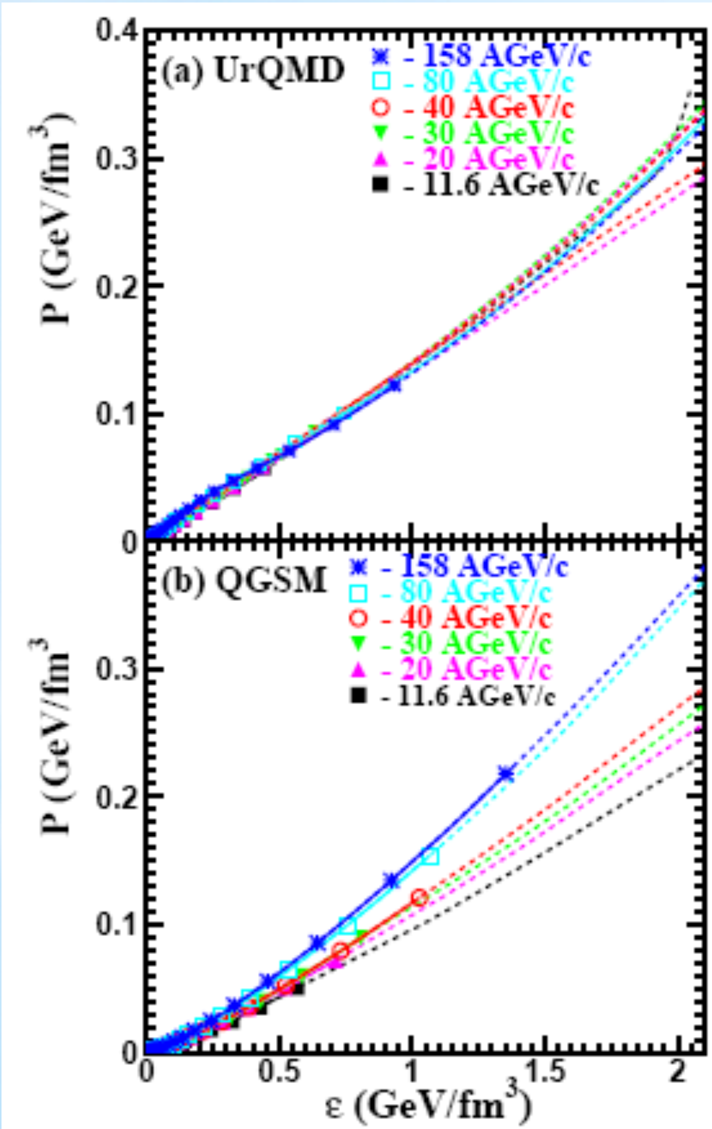
Isotropy of pressure

L.Bravina et al., PRC 78 (2008) 014907

Pressure becomes isotropic for all energies from 11.6 AGeV to 158 AGeV

Equation of State in the cell

pressure vs. energy



MC models favor early pre-equilibration of hot and dense nuclear matter already at t ≈ 2 fm/c

- ***After that the expansion of matter in the central cell proceeds isentropically with constant s/ρ_B (hydro!)***
- ***The EOS has a simple form: P/ε = const (hydro!) even at far-from-equilibrium stage***

P/ε = const very early (!)

$P/\varepsilon = 0.13(\mathrm{AGS}),\ 0.14(40),\ 0.146(\mathrm{SPS}),\ 0.15(\mathrm{RHIC})$

Space-time freeze-out of hadrons

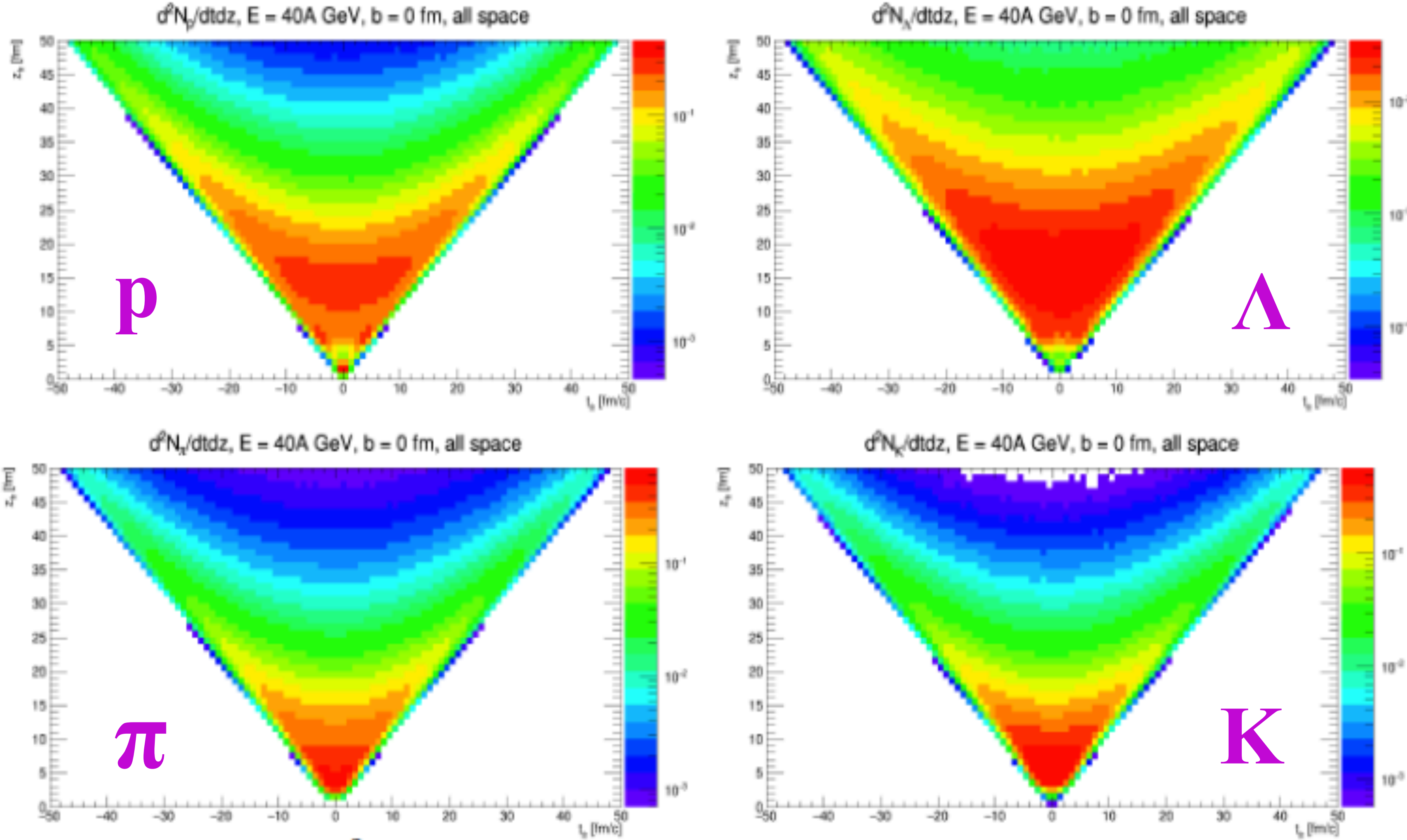


Figure 4: $d^2N/dtdz$ for protons, lambdas, pions and kaons.

Mesons are frozen earlier than baryons

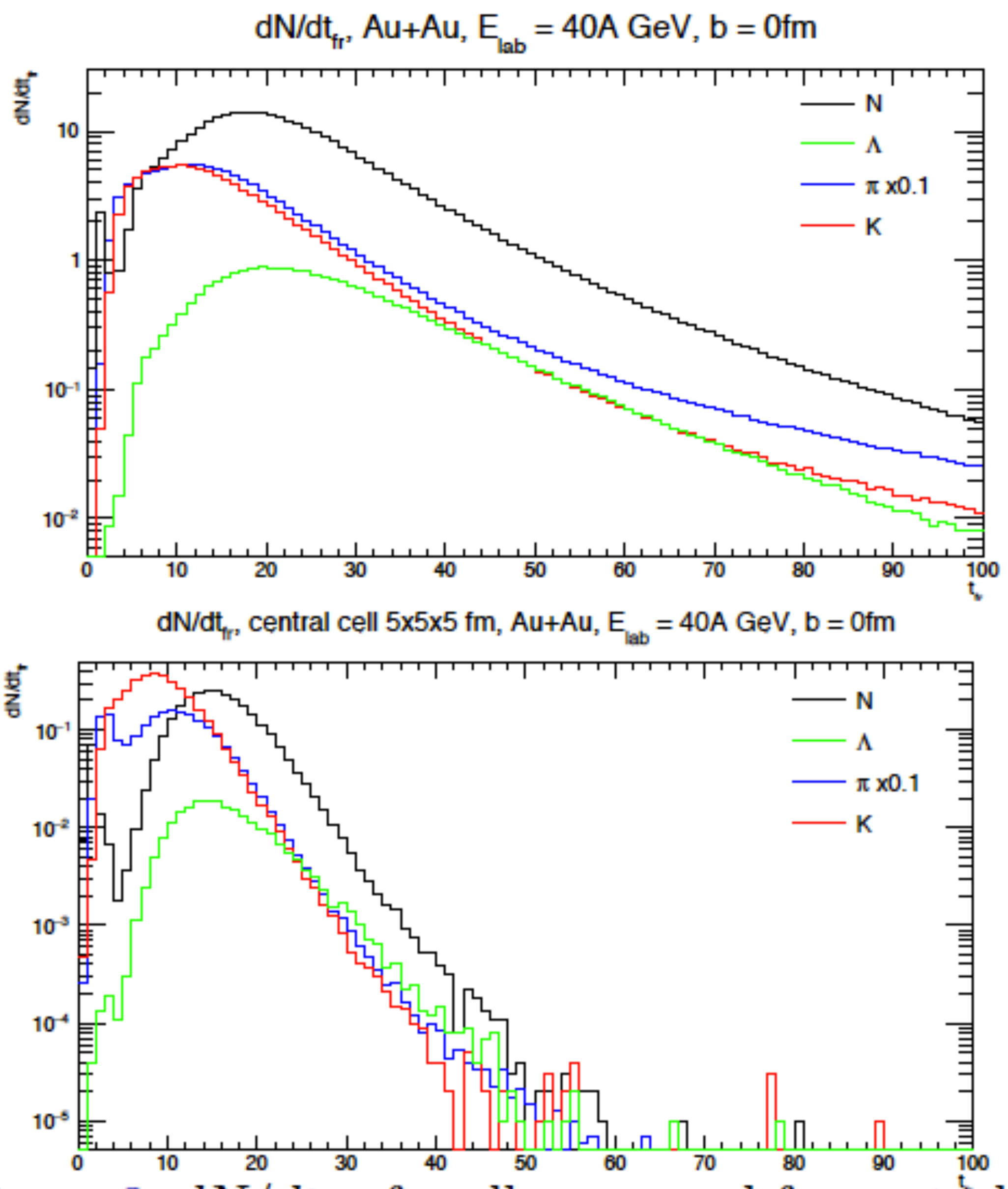


Figure 3: dN/dt_{fr} for all space and for central cell.

Different particles are frozen at different space – times with different values of T-μ_B-μ_S

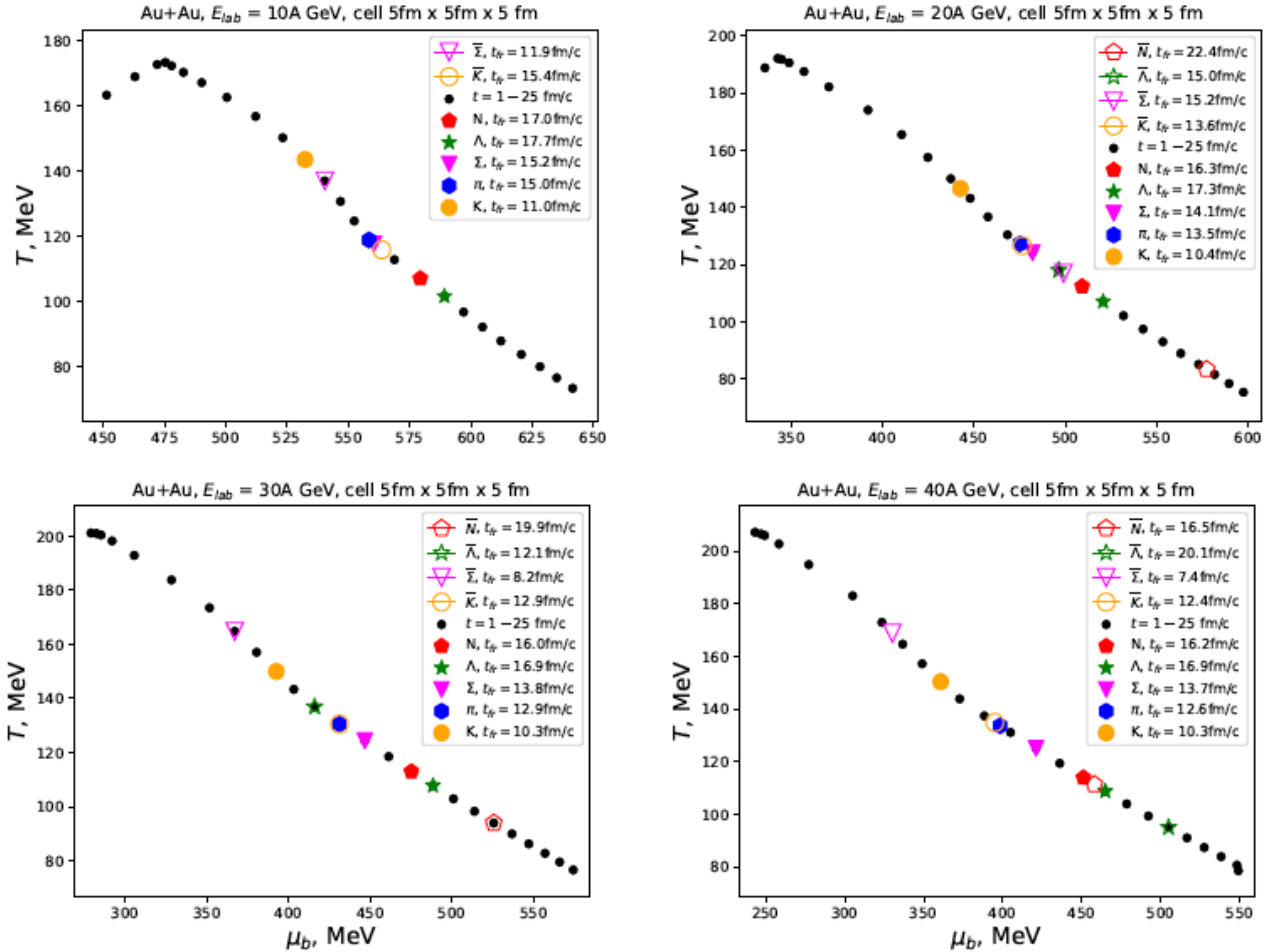


Figure 2: $T(\mu_B)$ in the central cell. Average freezeout times of different particles in the central cell are marked by colored markers.

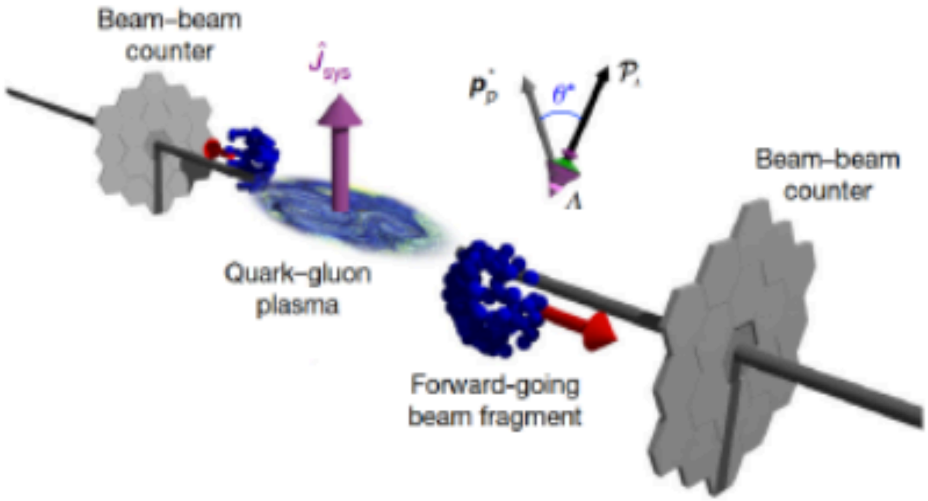
Consequences of the different space-time freeze-out:

- Difference in **Polarization** for lambdas and antilambdas

arXiv: 1910.06292

Why Λ?

Λ and $\bar{\Lambda}$ hyperons are "self-analyzing". That is, in the weak decay $\Lambda \to p + \pi^-$, the proton tends to be emitted along the spin direction of the parent Λ.



If θ^* is the angle between the daughter proton momentum Λ polarization vector in the hyperon rest frame, then:

$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}(1 + \alpha_H |\vec{P}_H| \cos\theta^*) \quad \to \quad P_H = \frac{8}{\pi\alpha_H} \sin(\phi_p^* - \Psi_{RP})$$

[Nature 548 (2017) 62]

Thermal Approach

In local thermal equilibrium, the ensemble average of the spin vector for spin-1/2 fermions with four-momentum p at space-time point x is obtained from the statistical-hydrodynamical model as well as the Wigner function approach and reads

$$S^\mu(x,p) = -\frac{1}{8m}(1-n_F)\,\epsilon^{\mu\nu\rho\sigma}p_\nu\varpi_{\rho\sigma}(x),$$

where the thermal vorticity tensor is given by

$$\varpi_{\mu\nu} = \frac{1}{2}\left(\partial_\nu\beta_\mu - \partial_\mu\beta_\nu\right),$$

with $\beta^\mu = u^\mu/T$ being the inverse-temperature four-velocity. The number density of Λ's is very small so that we can make the approximation $1-n_F \simeq 1$ Therefore:

$$S^\mu(x,p) = -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\nu\varpi_{\rho\sigma}(x).$$

By decomposing the thermal vorticity into the following components,

$$\varpi_T = (\varpi_{0x}, \varpi_{0y}, \varpi_{0z}) = \frac{1}{2}\left[\nabla\left(\frac{\gamma}{T}\right) + \partial_t\left(\frac{\gamma \mathbf{v}}{T}\right)\right],$$

$$\varpi_S = (\varpi_{yz}, \varpi_{zx}, \varpi_{xy}) = \frac{1}{2}\nabla \times \left(\frac{\gamma \mathbf{v}}{T}\right),$$

Equation can be rewritten as

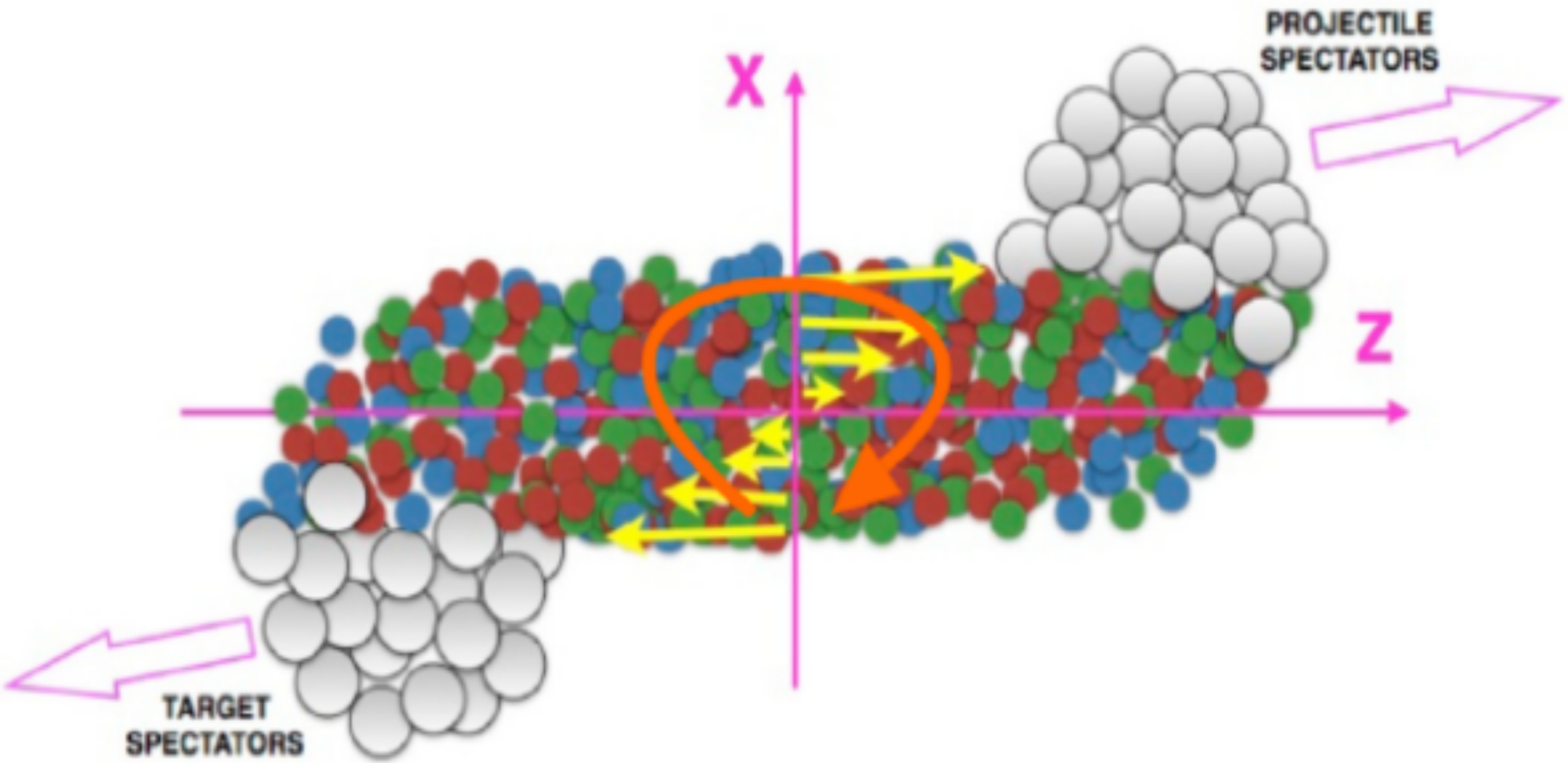
$$S^0(x,p) = \frac{1}{4m}\mathbf{p}\cdot\varpi_S, \quad \mathbf{S}(x,p) = \frac{1}{4m}\left(E_p\varpi_S + \mathbf{p}\times\varpi_T\right),$$

where E_p, $\mathbf{p}$, m are the Λ's energy, momentum, and mass, respectively.The spin vector of Λ in its rest frame is denoted as $S^{*\mu} = (0, \mathbf{S}^*)$ and is related to the same quantity in the c.m. frame by a Lorentz boost. Finally:

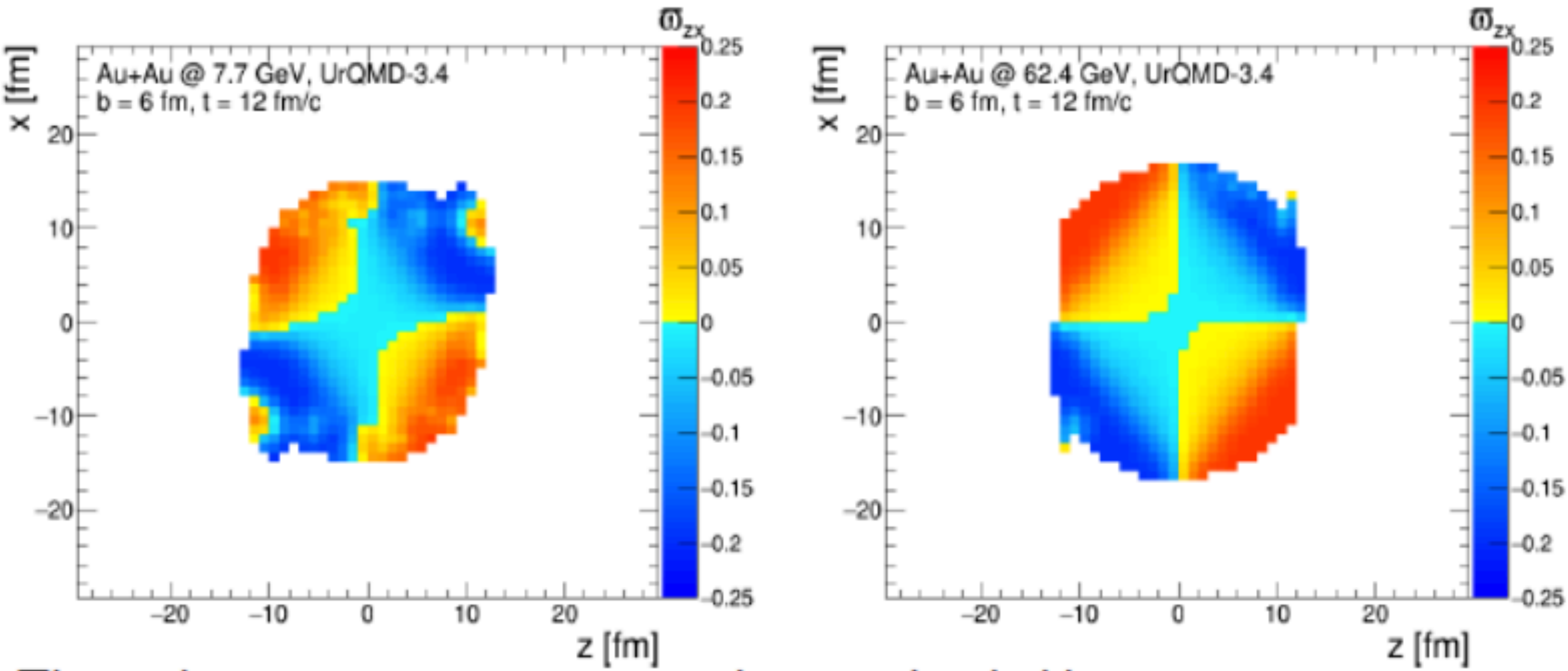
$$P = \frac{\langle \mathbf{S}^* \rangle \cdot \mathbf{J}}{S \cdot J},$$

[F. Becattini et al, Phys. Rev. C 95, 054902 (2017)]

Vorticity in microscopic model

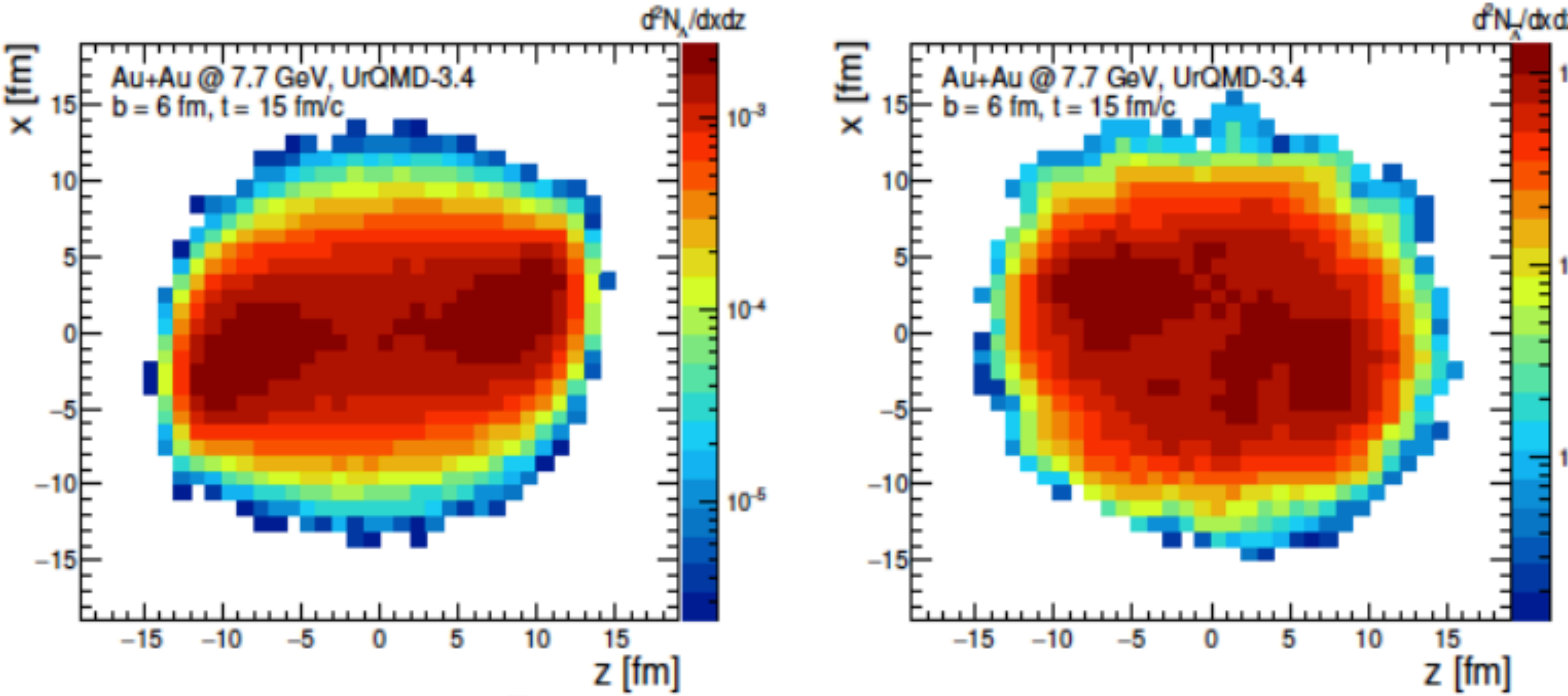


Thermal vorticity in reaction plane



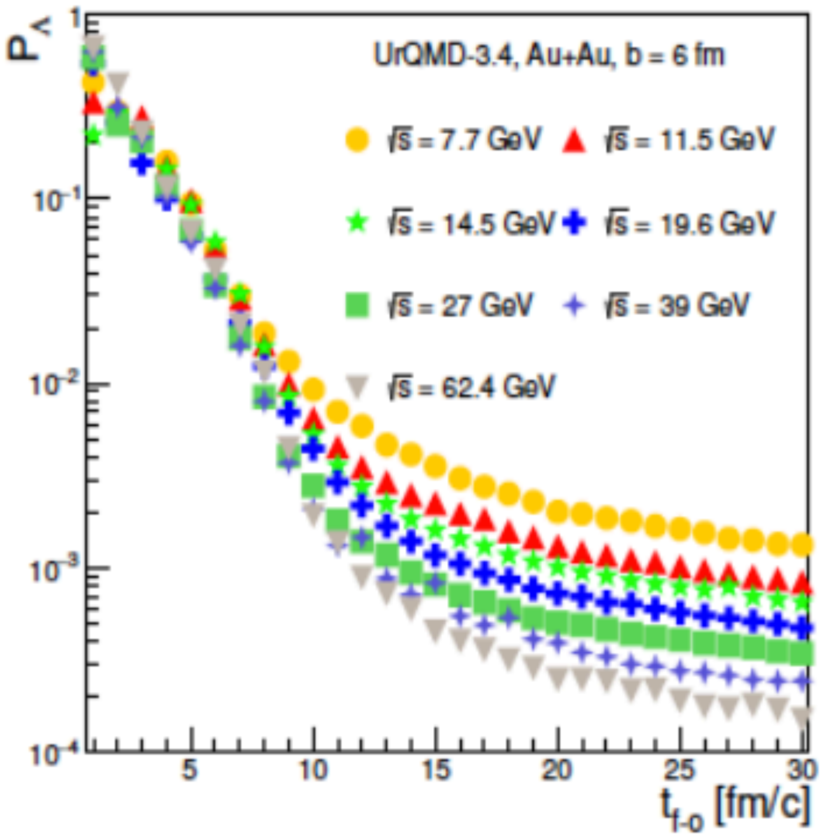
Thermal vorticity component ϖ_{zx} has quadruple-like structure in reaction plane which is stable in time but magnitude decreases due to system expansion. First and third quadrant are connected with central region which has small negative vorticity. This connection part becomes smaller when energy increases.

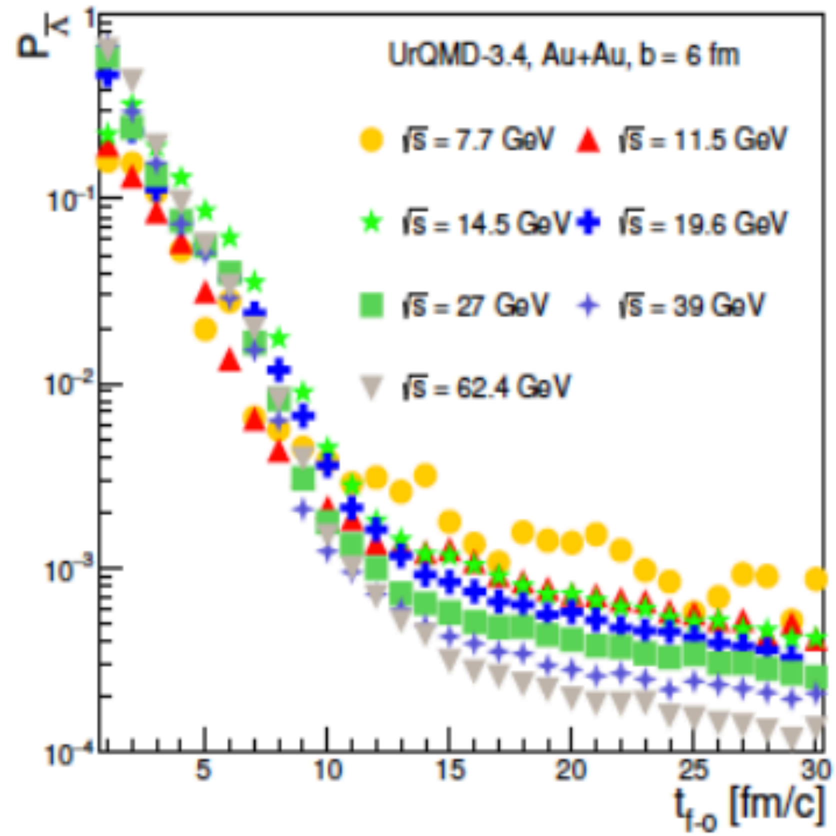
Space distribution of Lambdas



At low energies Λ and $\bar{\Lambda}$ are produced and emitted from the same regions as protons and antiprotons respectively. Λ's are concentrated also near hot and dense spectators, whereas $\bar{\Lambda}$'s are mostly produced in central region.

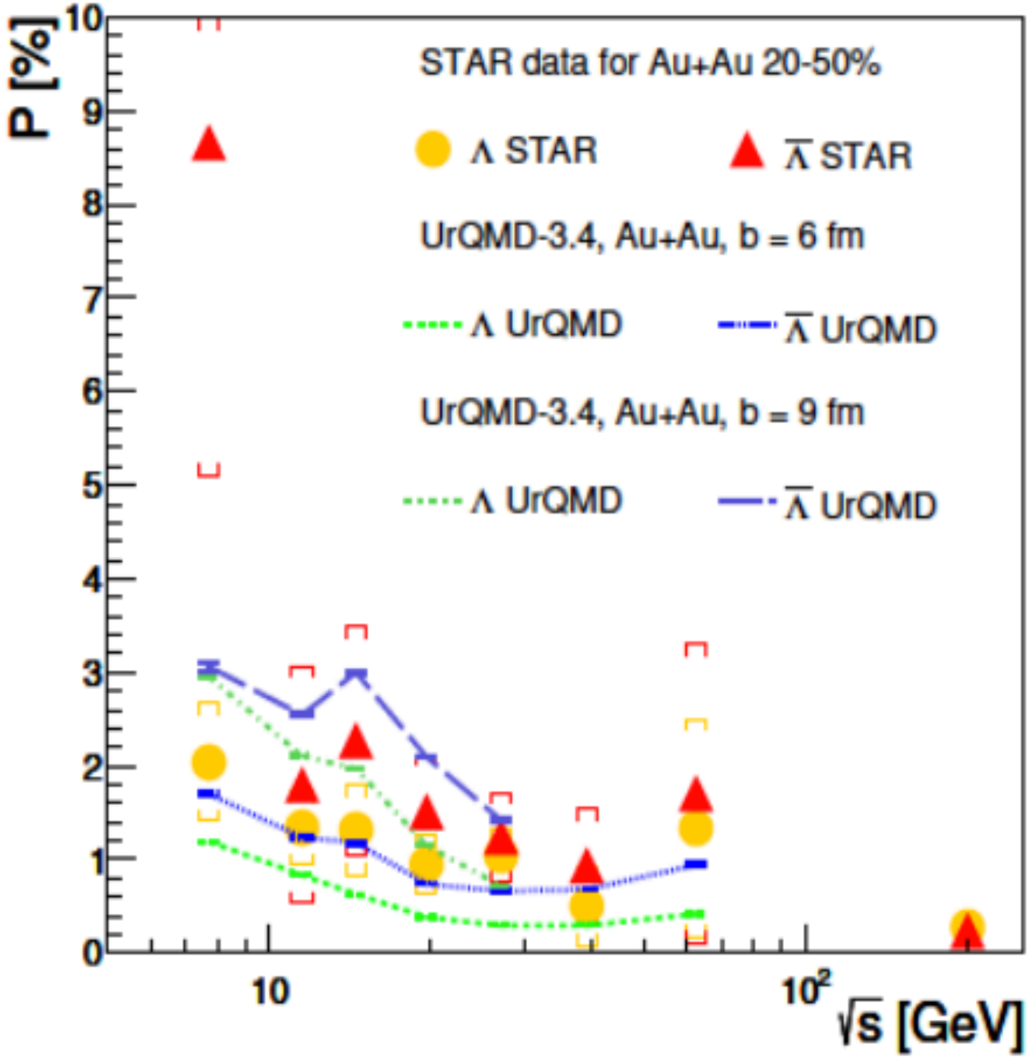
Polarization time evolution





Polarization of Λ's and $\bar{\Lambda}$'s hyperons decreases with time. At the beginning of collision particles are preferably formed in hot and dense regions with high polarization. But later lambdas and antilambdas are formed uniformly in fireball and average polarization is almost zero.

Polarization energy dependency



Polarization of Λ and $\bar{\Lambda}$ decreases with energy as in the experiment.
Λ and $\bar{\Lambda}$ global polarization agrees well with experimental data (except of point $\sqrt{s} = 7.7$ GeV).
Correct difference between Λ and $\bar{\Lambda}$ global polarization is obtained.

STAR data from [Phys. Rev. C 98 (2018) 14910]

Conclusions

- ***MC models favor early pre-equilibration of hot and dense nuclear matter already at $t \approx 2$ fm/c***
- ***After that the expansion of matter in the central cell proceeds isentropically with constant s/ρ_B***
- ***The EOS has a simple form: $P/\varepsilon = const$ even at far-from-equilibrium stage***
- ***There is no sharp freeze-out of hadrons in microscopic models. Mesons are frozen earlier than baryons***
- ***Freeze-out of Λ and anti-Λ is different in space and time. Emission takes place from areas with different vorticities***
- ***Good agreement with the experimental data***

Backup slides

Relativistic Hydrodynamics

Basic Equations

Energy-momentum tensor

$$\mathbf{T}^{\mu\nu} = \underbrace{(\varepsilon + \mathbf{P})\mathbf{u}^{\mu}\mathbf{u}^{\nu} - \mathbf{P}\mathbf{g}^{\mu\nu}}_{\text{inertial}} + \underbrace{\eta^{\mu\nu}}_{\text{dissipative}}$$

The space-time evolution of relativistic fluid is described by the set of differential equations

$$\partial_{\mu}\mathbf{N}^{\mu}(\mathbf{x}) = \mathbf{0}$$

$$\partial_{\mu}\mathbf{T}^{\mu\nu} = \mathbf{0};\ \ \mu,\nu = \mathbf{0},\mathbf{1},\mathbf{2},\mathbf{3}$$

For perfect fluid (i.e. $\eta^{\mu\nu} = 0$) these equations take the familiar form

$$\begin{aligned}
(\partial_t + \vec{\mathbf{v}} \cdot grad)\mathcal{N} &= -\mathcal{N}\, div\vec{\mathbf{v}} & \mathcal{N} &\equiv \gamma\, \mathbf{N}^{\mu}\mathbf{u}_{\mu} \\
(\partial_t + \vec{\mathbf{v}} \cdot grad)\vec{\mathcal{M}} &= -\vec{\mathcal{M}} \cdot div\vec{\mathbf{v}} - grad\,\mathbf{P} & \vec{\mathcal{M}} &\equiv \mathbf{T}^{\mathbf{0i}} = (\varepsilon + \mathbf{P})\gamma^2\vec{\mathbf{v}} \\
(\partial_t + \vec{\mathbf{v}} \cdot grad)\mathcal{E} &= -\mathcal{E} div\vec{\mathbf{v}} - div(\mathbf{P}\vec{\mathbf{v}}) & \mathcal{E} &\equiv \mathbf{T}^{\mathbf{00}} = (\varepsilon + \mathbf{P}\vec{\mathbf{v}}^{\mathbf{2}})\gamma^2
\end{aligned}$$

$$\partial_\mu N^\mu(x) = 0$$

$$\partial_\mu T^{\mu\nu} = 0; \quad \mu, \nu = 0, 1, 2, 3$$

$$T^{\mu\nu} = \underbrace{(\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu}}$$

Number of variables – 6

Number of equations – 4

Missing equations:

(1) EOS, that links energy density and pressure

- **Four-velocity**

$$u^\mu = (\gamma, \gamma\vec{v}); \quad \vec{v} \equiv \frac{\vec{p}}{p^0}; \quad \gamma = \frac{1}{\sqrt{1 - (\vec{v})^2}}$$

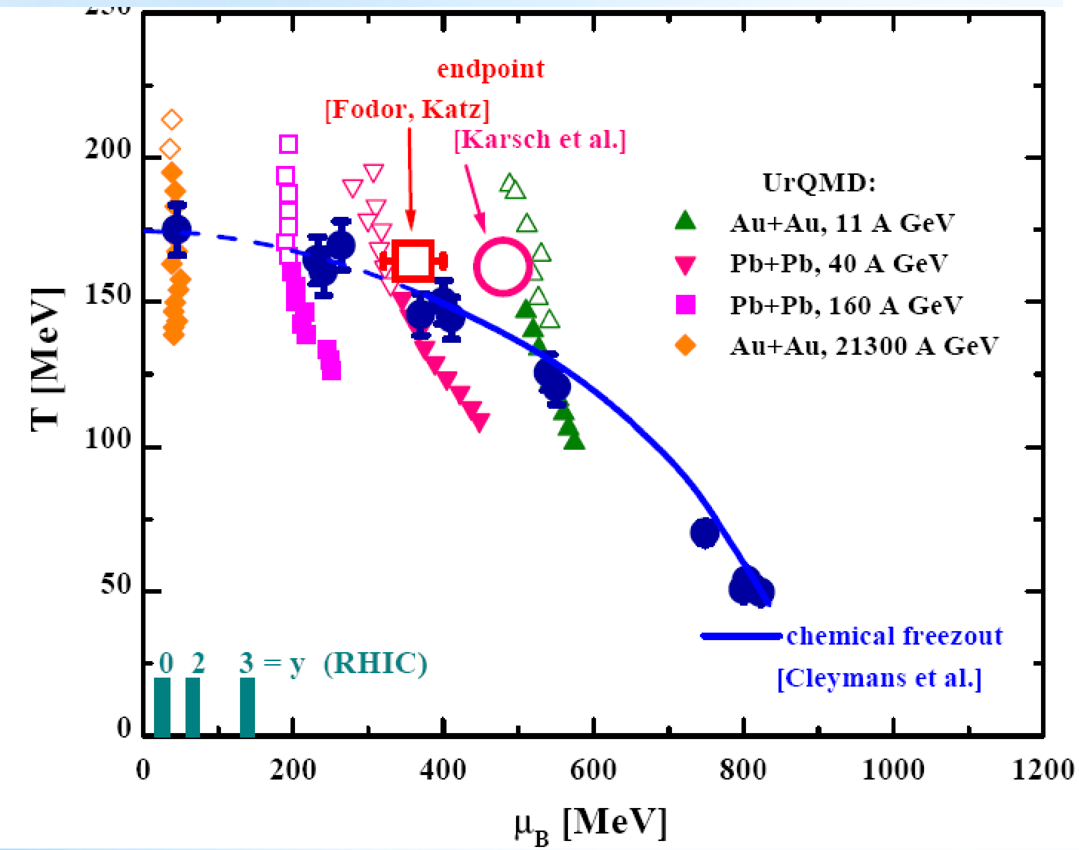
thus

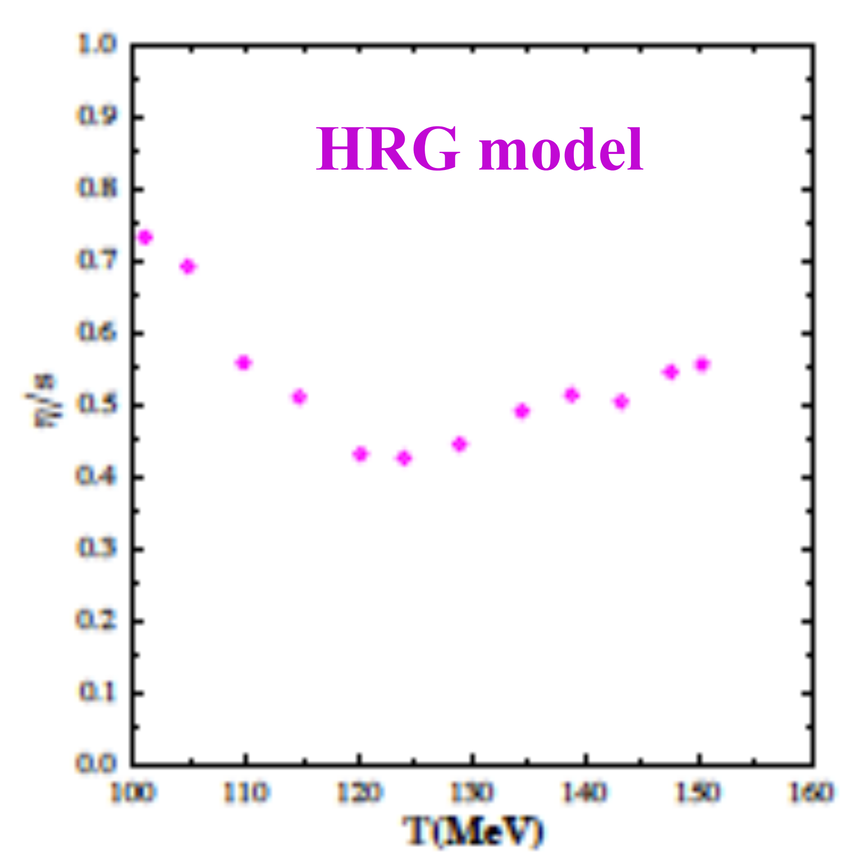
(2)

$$u^\mu u_\mu = 1$$

Reliability of obtained results

Central cell: comparison to chemical freeze-out conditions





G.Kadam, S.Pawar, arXiv:1802.01942 [hep-ph]