Hadrons in gravitational field, shear forces and viscosity in relativistic Heavy Ion collisions

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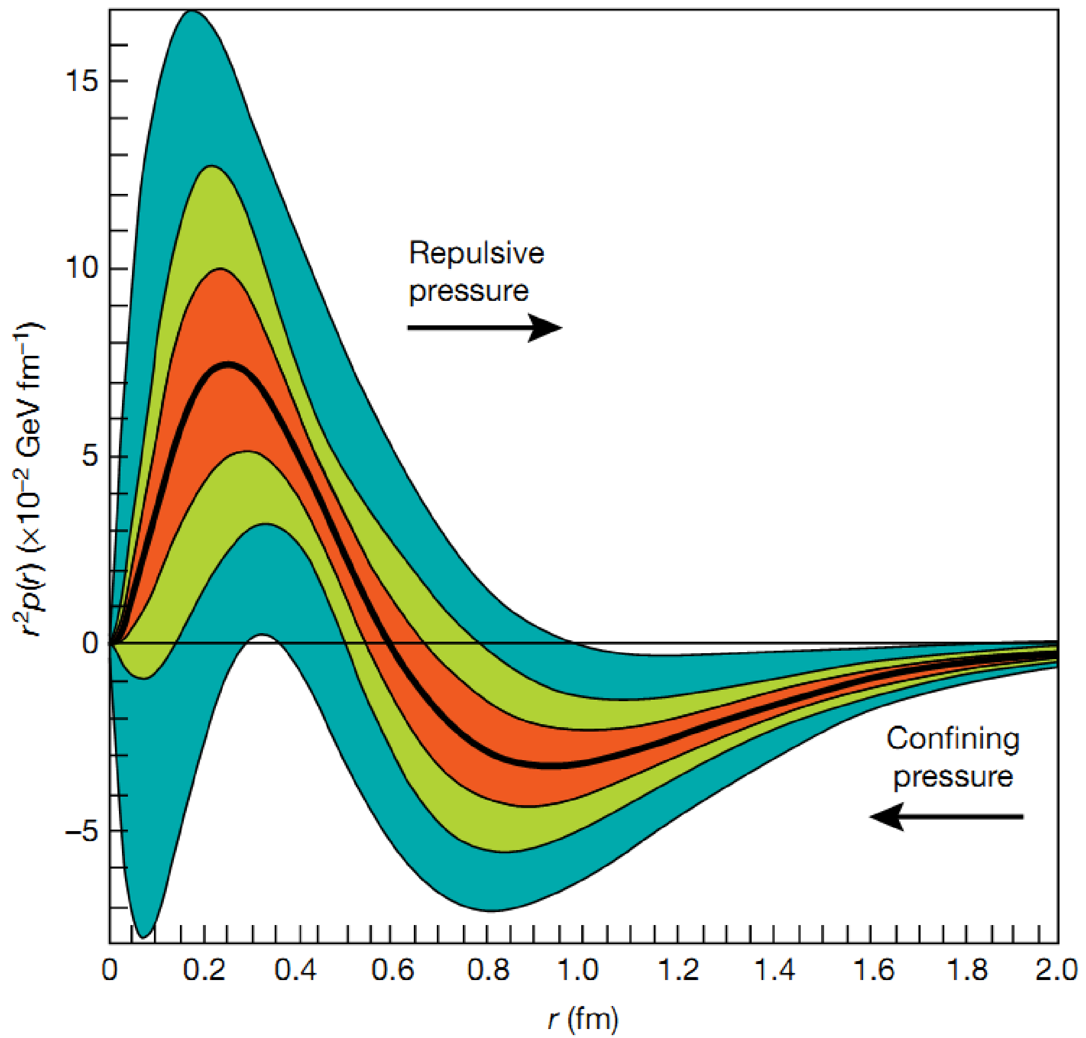
Main Topics: various manifestations of quark/gluons Energy-Momentum Tensors in hadrons

- EMT as a mechanical probe (pressure, shear)
- EMT as a gravity coupling
- EMT in Hydro: hadrons/heavy ions interface

NATURE | www.nature.com/nature
https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]



Gravitational Formfactors (spin ½)

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\Big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction of quarks and gluons (separately!) with both classical and TeV gravity

Gravity and hadron structure

- Interaction – field vs metric deviation

$$M = \langle P'|J_q^\mu|P\rangle A_\mu(q)$$

$$M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P|J_q^\mu|P\rangle = 2e_q P^\mu$$

$$\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P|J_q^\mu|P\rangle A_\mu = 2e_q M\phi(q)$$

$$M_0 = \frac{1}{2}\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle h_{\mu\nu} = 2M\cdot M\phi(q)$$

- Mass as charge – equivalence principle

Equivalence Principle and hadron structure

"Microscopic" EP (coupling of gravity to EMT)

+

Conservation law

(Momentum SR to get local from LC: $\int dx\, x\, (\Sigma\, q(x) + G(x))=1$)

=

"Macroscopic" EP (universal falling) :

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves: complementary probe?!) – action on (quantum) spin

$$M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$$

$$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$$

½ - spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$: Larmor frequency same as EM $\vec{H}_L = rot\vec{g}$

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L$$

- Orbital and Spin momenta dragging – the same - Equivalence principle

Equivalence principle

- Newtonian – “Falling elevator” – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on quantum SPIN – known since 1962 (Kobzarev and Okun’; rederived from conservarion laws - Kobzarev and V.I. Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way
- Dirac eq; also for STRONG fields (Obukhov, Silenko, OT) : More general constraint ?

Experimental test of PNEP

Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW LETTERS

VOLUME 68 13 JANUARY 1992 NUMBER 2

Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson
Physics Department, FM-15, University of Washington, Seattle, Washington 98195
(Received 25 September 1991)

If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \quad \zeta = 1 + \chi$$

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

Quantum measurement and EP

- If spin is just a (pseudo) vector : EP due to Earth rotation is trivial
- Crucial if measured by a device in rotating frame
- Quantum measurement problem becomes practical
- Cf Unruh effect in HIC (Prokhorov, OT, Zakharov’19) : non-inertial frames simulate extremely large gravity

Gravity vs accelerated frame for spin and helicity

Spin precession – well known factor 3

Helicity flip – the same!

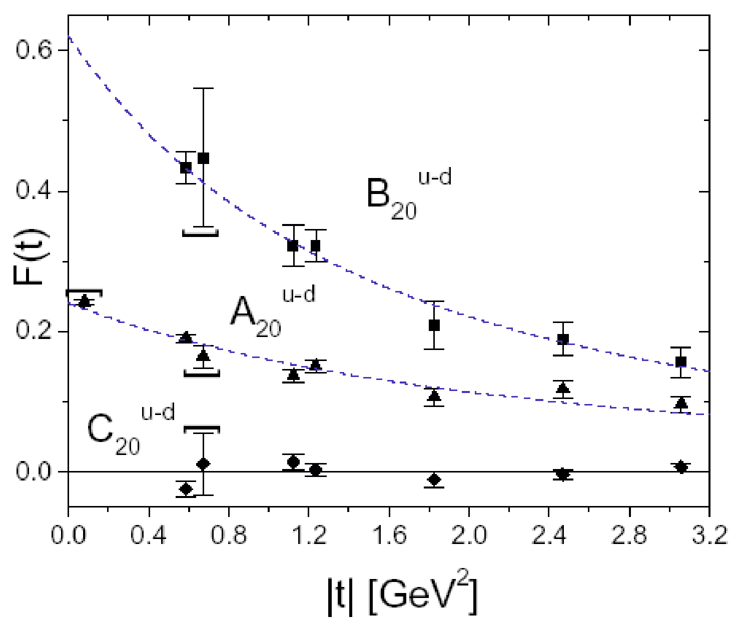
No helicity flip in gravitomagnetic field – another formulation of PNEP (OT'99) and

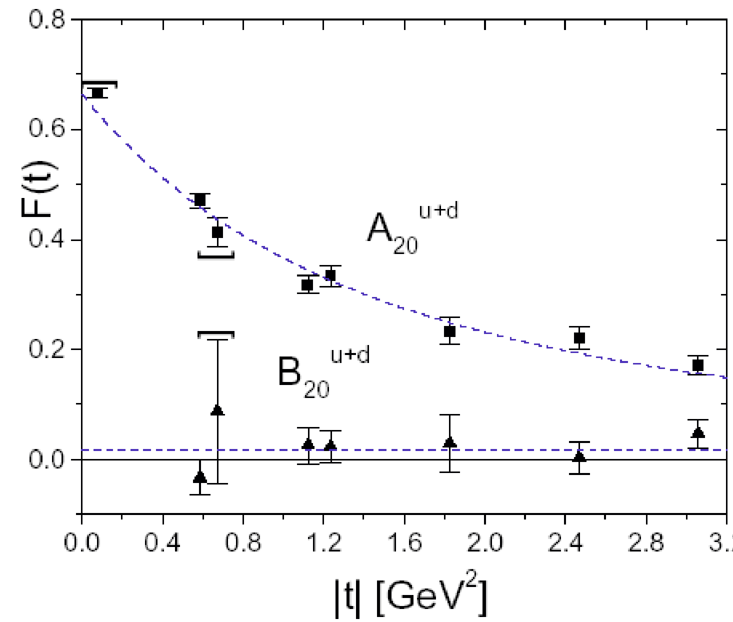
Flip by "gravitoelectric" field: relic neutrino? Black hole?

$$\frac{d\sigma_{+-}}{d\sigma_{++}} = \frac{tg^2(\frac{\phi}{2})}{(2\gamma - \gamma^{-1})^2}$$

Generalization of Equivalence principle

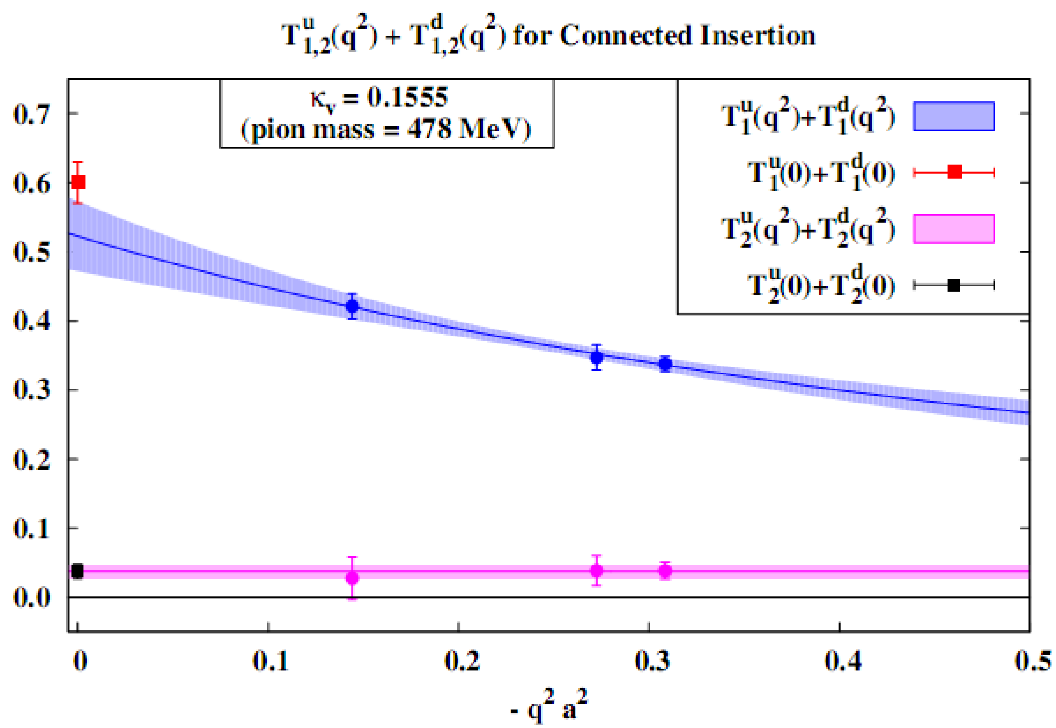
- Various arguments: AGM ≈0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





New lattice study (M. Deka et al. Phys.Rev. D91 (2015) no.1, 014505)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In QED, pQCD – violated (Brodsky et al)
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement? Nucleons do not break even by black holes?
- Support by recent observation of smalness of (nucleon "cosmological constant") Cbar

One more gravitational formfactor

- *Quadrupole*

$$\langle P+q/2|T^{\mu\nu}|P-q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

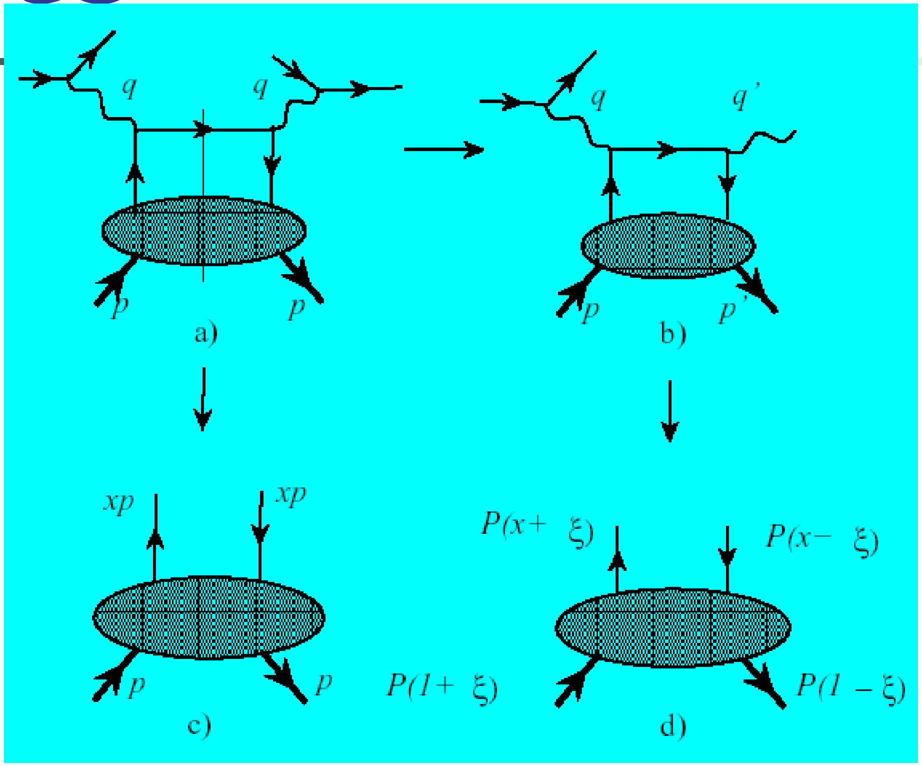
- *Cf vacuum matrix element – cosmological constant*

$$\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$$

$$\Lambda = C(q^2)q^2$$

- *Inflation ~ annihilation (q^2>0)*
- *Excluded by EP (also approximately valid separately for quarks and gluons)*
- *How to measure experimentally? – DVCS*

QCD Factorization for DIS and DVCS



- *Manifestly spectral*

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}$$

- *Extra dependence on* ξ

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon},$$

Holographic property (OT'05)

- *Factorization Formula ->*

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x-\xi+i\epsilon}$$

- *Analyticity -> Imaginary part -> Dispersion relation:*

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,x)}{x-\xi+i\epsilon}$$

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x)-H(x,\xi)}{x-\xi+i\epsilon}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n!}\frac{\partial^n}{\partial\xi^n}\int_{-1}^{1} H(x,\xi)dx(x-\xi)^{n-1} = const$$

- *"Holographic" equation*
- *Subtraction related to pressure*

Road to pressure

- *FF – moment of pressure: C~<p r^4> (<p r^2> =0)* *M.Polyakov'03*

$$T^Q_{\mu\nu}(\vec{r},\vec{s}) = \frac{1}{2E}\int \frac{d^3\Delta}{(2\pi)^3}\, e^{i\vec{r}\cdot\vec{\Delta}}\, \langle p', S'|\hat{T}^Q_{\mu\nu}(0)|p, S\rangle$$

$$T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\delta_{ij}$$

- *Possible justification: Born gravitational scattering*
- *Stable equilibrium C>0: Holds for all known cases and also quarks (or leptons) in photon*

Spin 1 EMT and inclusive processes

- Forward matrix element ->density matrix
- Contains P-even term: tensor polarization
- Symmetric and traceless: correspond to (average) shear forces
- For spin ½: P-odd vector polarization requires another vector (q) to form vector product

SUM RULEs

- Efremov,OT’81 : zero sum rules:
- 1^{st} moment: also in parton model by Close and Kumano (90)
- 2^{nd} moment (EMT)
- Average shear force (compensated between quarks and gluons)
- Gravity and (Ex)EP (zero average shear separately for quarks and gluons) – OT’09

Manifestation of post-Newtonian (Ex)EP for spin 1 hadrons

- Tensor polarization - coupling of EMT to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$A_T = \frac{\sigma_+ + \sigma_- - 2\sigma_0}{3\bar{\sigma}}$$

$$\int_0^1 C_i^T(x)dx = 0$$

$$\langle P,S|\bar{\psi}(0)\gamma^\nu D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2 S^{\nu\nu_1}P^{\nu_2}...P\nu_n \int_0^1 C_q^T(x)x^n dx$$

(AVE,OT'91,93)

$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu(1-\delta(\mu^2)) + 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

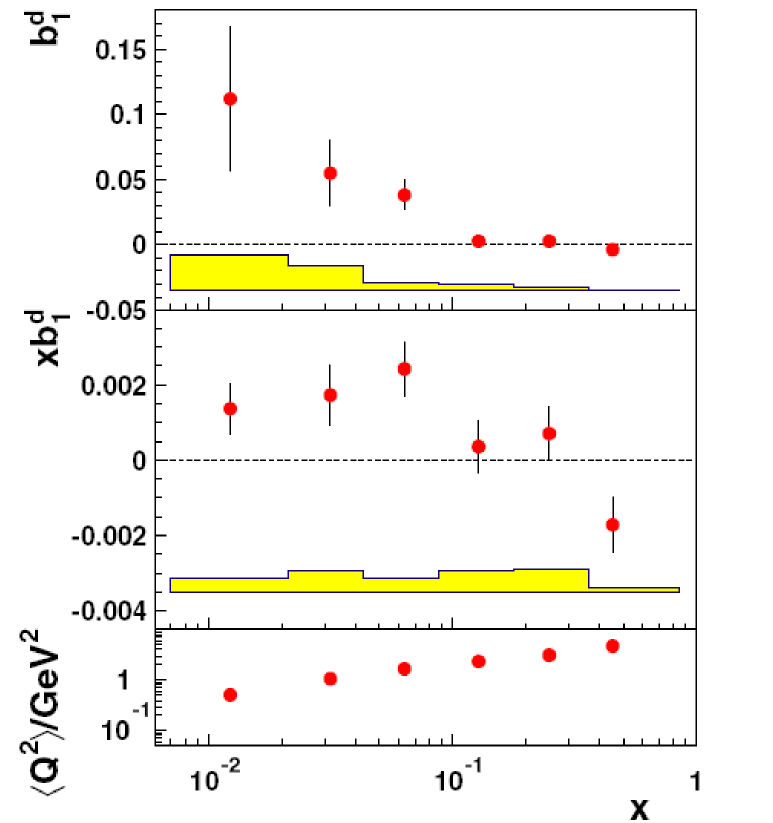
$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu \delta(\mu^2) - 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

$$\sum_q \int_0^1 C_i^T(x)x dx = \delta_1(\mu^2) = 0$$ for ExEP

HERMES – data on tensor spin structure function

PRL **95**, 242001 (2005)

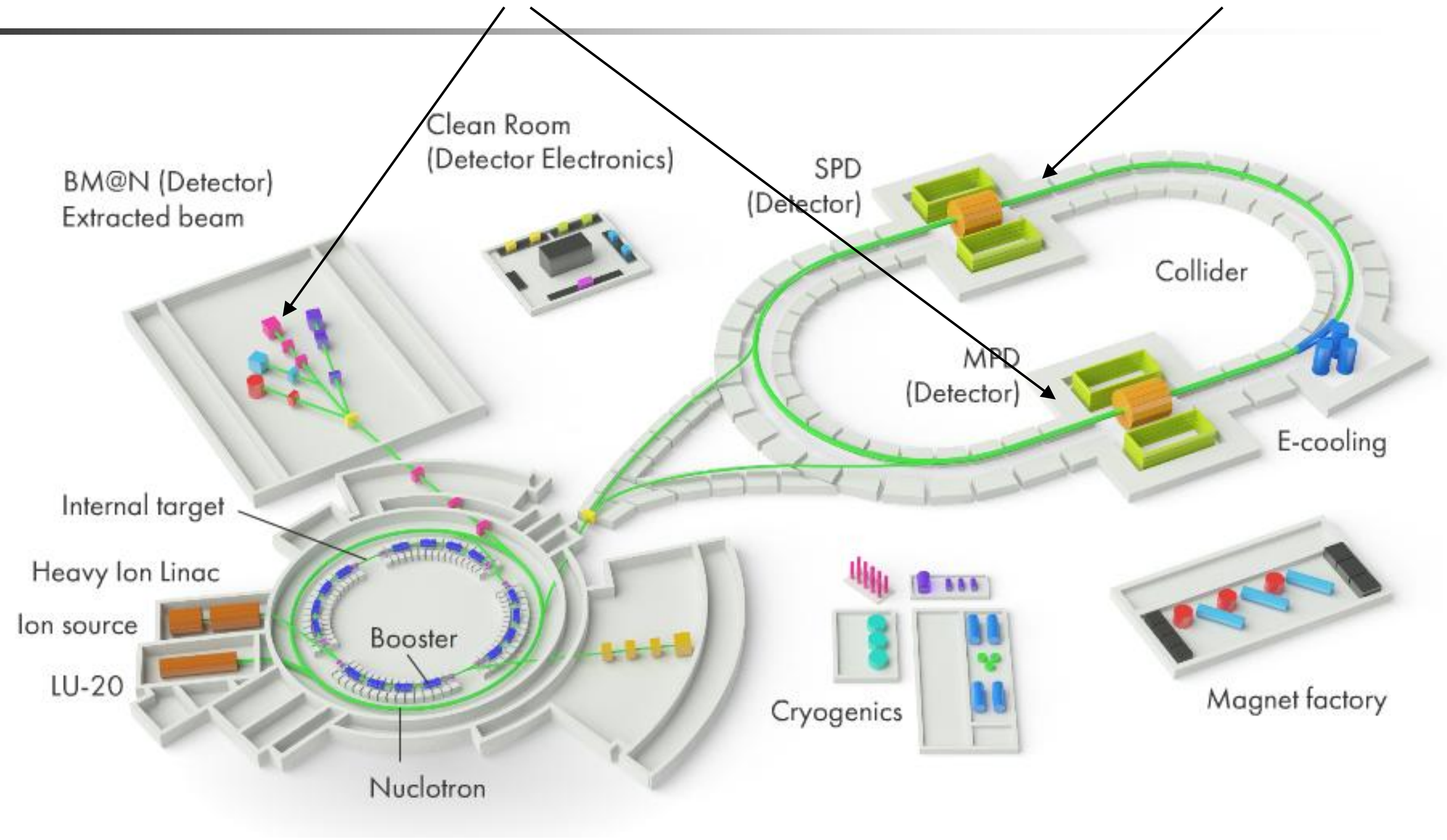
- Isoscalar target – proportional to the sum of u and d quarks – combination required by (Ex)EP
- Second moments – compatible to zero better than the first one (collective glue << sea)



Where else to test?

- COMPASS
- EIC
- DY@J-PARC: (Song,Kumano:1902.04712)
- ET'81-any hard process
- Suggestion: hadronic tensor SSA

NICA: heavy ions and hadrons



Tensor polarized beams

- Opportunity: NICA@JINR with polarized hadronic beams

- Polarized deuterons is easier to accelerate: no depolarizing resonances

- DY, J/Ψ (+hadronic SSA)

Vector vs Tensor SSA

- Vector: $A = (\sigma(+)-\sigma(-))/(\sigma(+)+\sigma(-))$

  Tensor:

  $A = (\sigma(+)+\sigma(-)-2\sigma(0)) / (\sigma(+)+\sigma(-)+\sigma(0))$

- Inclusive pion production: (T-odd) vector SSA may be also excluded by summing $\sigma(L)+ \sigma(R)$

Shear: viscosity (OT'19)?!

From spherically symmetric object to fluid (EoS!)

$T^{\mu\lambda} = (e+p)\, v^{\mu} v^{\lambda} - p\, g^{\mu\lambda}$

$V^{\mu} = P^{\mu}/M$: correct normalization but no coordinate dependence

Another suggestion:

$V^{\mu} = (P^{\mu} + a(t)\, k_T^{\ \mu}) / (M^2 + a^2(t)\, k_T^{\ 2})^{1/2}$

Viscosity: $\sim \eta\ \ p^{[\mu} \Delta^{\lambda]}$
Naïve T-oddness: phases

Viscosity in crossed channel

Possibility to study gravitational FFs in time-like region by meson pair production in real and virtual photons collisions (Kumano, Song, OT'18)

Viscosity (new!): will correspond to

Exotic $J^{PC}=1^{-+}$ meson (already studied earlier : Anikin, Pire, Szymanowski,OT, Wallon'06)

$\pi\eta$ pairs observation instead of $\pi\,\pi$ required

Smallness of viscosity in HIC: related to smallness of T-odd GPDs and exotic GDAs ?!

Conclusions

- EMT: link between QCD, gravity and HIC
- Gravitational Ffs; probes of coupling of quarks and gluons to gravity separately: EP/ExEP (at least in 3 cases: spin, "inflation" shear): gravity-proof confinement?!
- Spin precession in gravitational field: neutrino helicity flip; complementary probe (in coincidence with standard method) of GW?
- Tensor polarization: way to test of gravitational coupling of quarks and gluons in inclusive processes: shear (NICA@JINR)
- Shear viscosity: smallness in HIC related to exotic hybrid mesons production in real and virtual photons collisions (BELLE) ?

BACKUP

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta\mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x,y)}{1-y}$$

$$= \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$$

Another appearance of T-oddness in EMT: Burkardt SR

T-invariance : antisymmetry of twist 3 gluonic pole matrix element nullifies its contribution to EMT

BUT Pole prescription (dynamics!) provides ("T-odd") symmetric part (OT'14)!

SR:

$$\sum \int \int dx_1 dx_2 \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon} = 0$$

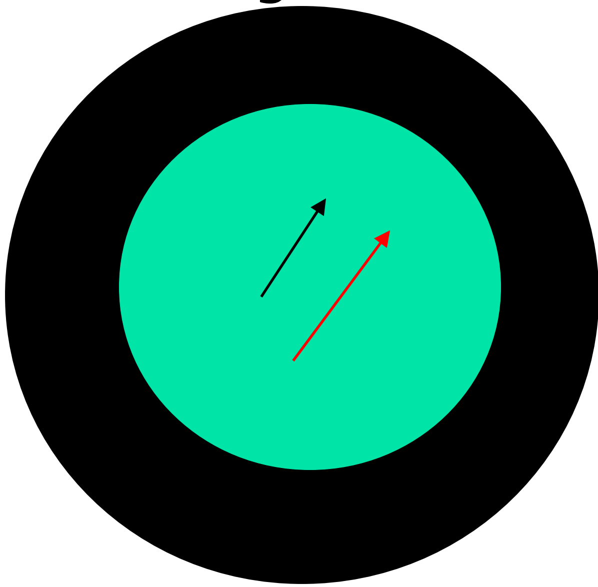
$$\sum \int dx T(x, x) = 0$$

Also EP!

ExEP: approximate validity separately for quarks and gluons: smallness of deuteron Sivers function

Cosmological implications of PNEP

- Necessary condition for Mach's Principle (in the spirit of Weinberg's and MTW textbooks) Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For "flat Universe" - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and quantum rotators – PNEP!
- More elaborate models - Tests for cosmology ?!



D-term interpretation: Inflation and annihilation

- Quadrupole gravitational FF

$$\langle P+q/2|T^{\mu\nu}|P-q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

- Moment of D-term – positive
- Vacuum – Cosmological Constant $\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$

- 2D effective CC – negative in scattering, positive in annihilation

$$\Lambda = C(q^2)q^2$$

- Similarity of inflation and Schwinger pair production – Starobisnky, Zel'dovich
- Was OUR Big Bang resulting from one graviton annihilation at extra dimensions??! Version of "ekpyrotic" ("pyrotechnic") universe

C vs Cbar

- Cancellations of Cbars – negative pressure (cf Chaplygin gas)
- Cancellation in vacuum; Pauli (divergent), Zel'dovich (finite)
- Flavour structure of pressure: DVMP!

Unphysical regions

- DIS : Analytical function – polynomial in $1/x_B$ if $1 \leq |x_B|$

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x,\xi)$
- Solved by using of Double Distributions Radon transform

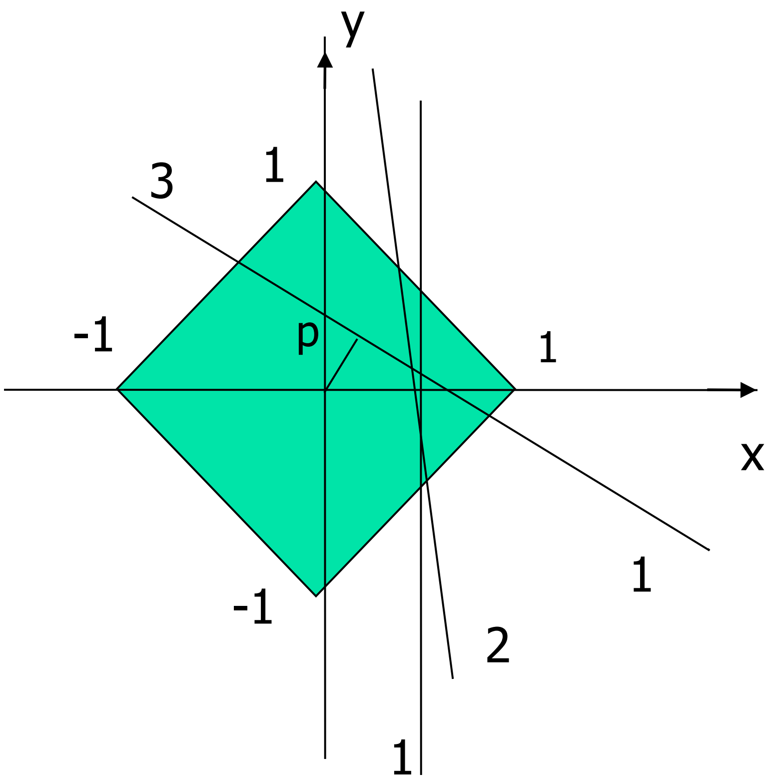
$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

Double distributions and their integration

- Slope of the integration line-skewness
- Kinematics of DIS: $\xi = 0$

  ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$

  - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

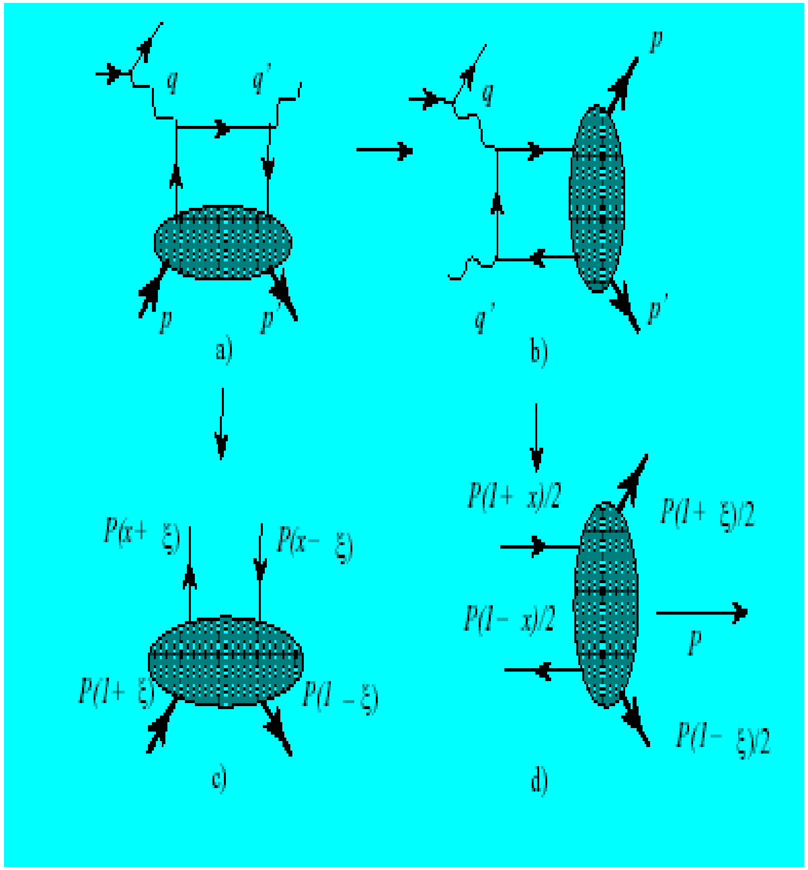


$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty \frac{dp}{p^2}\int_0^{2\pi} d\phi|cos\phi|(H(p/cos\phi + x + ytg\phi, tg\phi) - H(x + ytg\phi, tg\phi)) =$$

$$= -\frac{1}{2\pi^2}\int_{-\infty}^\infty \frac{dz}{z^2}\int_{-\infty}^\infty d\xi(H(z + x + y\xi, \xi) - H(x + y\xi, \xi))$$

Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes
- Duality between s and t channels (Polyakov,Shuvaev, Guzey, Vanderhaeghen)



GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms): moments - integrals in x weighted with x^n - are polynomials in $1/\xi$ of power n+1
- As a result, analyticity is preserved: only non-positive powers of ξ appear

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi)=\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dy(F(x,y)+\xi G(x,y))\delta(z-x-\xi y)$$

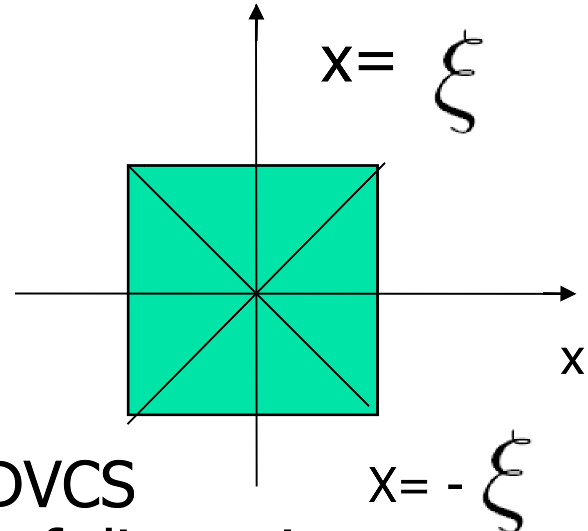
- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta\mathcal{H}(\xi)=\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dy\frac{G(x,y)}{1-y}$$

$$= \int_{-\xi}^{\xi}dx\frac{D(x/\xi)}{x-\xi+i\epsilon}=\int_{-1}^{1}dz\frac{D(z)}{z-1}=const$$

Holographic property - III

- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space ?!



- Strategy (now adopted) of GPD’s studies: start at diagonals (through SSA due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants

Pressure in hadron pairs production

- Back to GDA region
- -> moments of H(x,x) - define the coefficients of powers of cosine!– 1/ξ
- Higher powers of cosine in t-channel – threshold in s -channel
- Larger for pion than for nucleon pairs because of less fast decrease at x ->1
- Stability defines the sign of GDA and (via soft pion theorem) DA: work in progress

$$\mathscr{H}(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,\xi)\frac{x^n}{\xi^{n+1}}$$

$$= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,x)\frac{x^n}{\xi^{n+1}} + \Delta\mathscr{H}.$$

Analyticity of Compton amplitudes in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\mathcal{A}(\nu, Q^2) = \frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu', Q^2)}{(\nu'^2-\nu^2)} + \Delta \qquad \Delta = 2\int_{-1}^{1} d\beta \frac{D(\beta)}{\beta - 1}$$

$$\Delta^p_{\mathrm{CQM}}(2) = \Delta^n_{\mathrm{CQM}}(2) \approx 4.4, \qquad \Delta^p_{\mathrm{latt}} \approx \Delta^n_{\mathrm{latt}} \approx 1.1$$

- Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!

- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!

Loss of stability?

- D=0 -> extra node required (cf tensor distribution - Efremov,OT- mechanical analogy – c.m. and c.i.)
- Smooth decrease – two extra nodes
- ++++------------------
- +++++++-------+++++--------------
- J=2 (Talk of Barbara Pasquini, comment by Maxim – zeros of Bessel functs?!)

Is D-term independent?

- Fast enough decrease at large energy ->

$$\mathrm{Re}\,\mathcal{A}(\nu) = \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2 - \nu^2} + \mathrm{C}_0$$

$$\mathrm{C}_0 = \Delta - \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2}$$

$$= \Delta + \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,x)}{x}.$$

- FORWARD limit of Holographic equation

$$\Delta = \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,0) - H^{(+)}(x,x)}{x}$$

$$= 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0) - H(x,x)}{x},$$

$$\mathrm{C}_0(t) = 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0,t)}{x}$$

"D – term" 30 years before…

- Cf Brodsky, Close, Gunion'72 (seagull ~ pressure) – but NOT DVMP
- D-term – a sort of renormalization constant
- May be calculated in effective theory if we know fundamental one
- OR
- Recover through special regularization procedure (D. Mueller)?

Vector mesons and EEP

- J=1/2 -> J=1. QCD SR calculation of Rho's AMM gives g close to 2.

- Maybe because of similarity of moments
- g-2=<E(x)>; B=<xE(x)>
- Directly for charged Rho (combinations like p+n for nucleons unnecessary!). Not reduced to non-extended EP:

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible

EEP and Sivers function

- Sivers function – process dependent (effective) one
- T-odd effect in T-conserving theory- phase
- FSI – Brodsky-Hwang-Schmidt model
- Unsuppressed by M/Q twist 3
- Process dependence- colour factors
- After Extraction of phase – relation to universal (T-even) matrix elements

EEP and Sivers function -II

- Qualitatively similar to OAM and Anomalous Magnetic Moment (talk of S. Brodsky)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (OT'07, **hep-ph/0612205**): $x f_T(x) \sim xE(x)$
- Burkardt SR for Sivers functions is then related to Ji's SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dx x f_T(x) = \sum_{q,G} \int dx x E(x) = 0$$

EEP and Sivers function for deuteron

- EEP - smallness of deuteron Sivers function
- Cancellation of Sivers functions – separately for quarks (before inclusion gluons)
- Equipartition + small gluon spin – large longitudinal orbital momenta (BUT small transverse ones –Brodsky, Gardner)

Another relation of Gravitational FF and NP QCD (first reported at 1992: **hep-ph/9303228**)

- BELINFANTE (relocalization) invariance :

decreasing in coordinate –

$$M^{\mu,\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J^5_{S\sigma} + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}$$

smoothness in momentum space

$$M^{\mu,\nu\rho} = x^\nu T^{\mu\rho}_B - x^\rho T^{\mu\nu}_B$$

- Leads to absence of massless pole in singlet channel – U_A(1)

$$\epsilon_{\mu\nu\rho\alpha} M^{\mu,\nu\rho} = 0.$$

- Delicate effect of NP QCD

$$(g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu})\partial^\rho (J^\alpha_{5S} x^\nu) = 0$$

- Equipartition – deeply related to relocalization invariance by QCD evolution

$$q^2 \frac{\partial}{\partial q^\alpha}\langle P|J^\alpha_{5S}|P+q\rangle = (q^\beta \frac{\partial}{\partial q^\beta} - 1) q_\gamma \langle P|J^\gamma_{5S}|P+q\rangle$$

$$\langle P,S|J^5_\mu(0)|P+q,S\rangle = 2MS_\mu G_1 + q_\mu (Sq) G_2,$$

$$q^2 G_2|_0 = 0$$