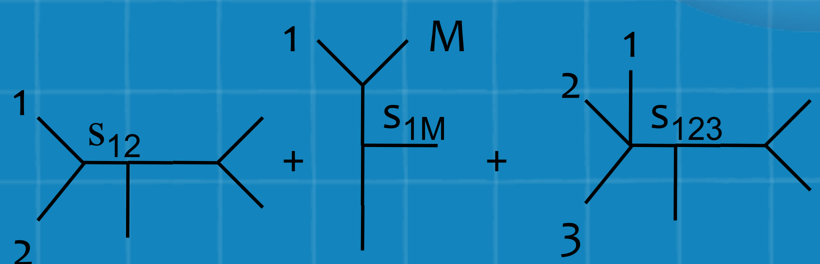
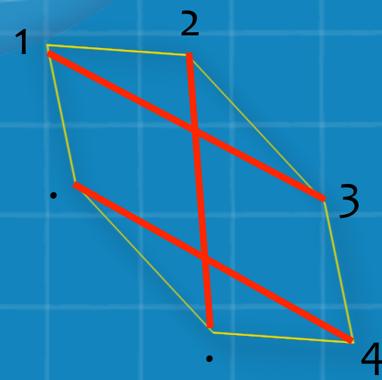
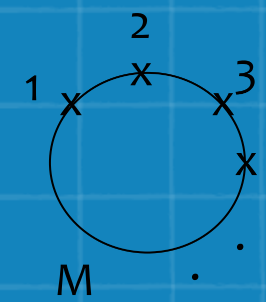
Amplitudes and Particle Physics

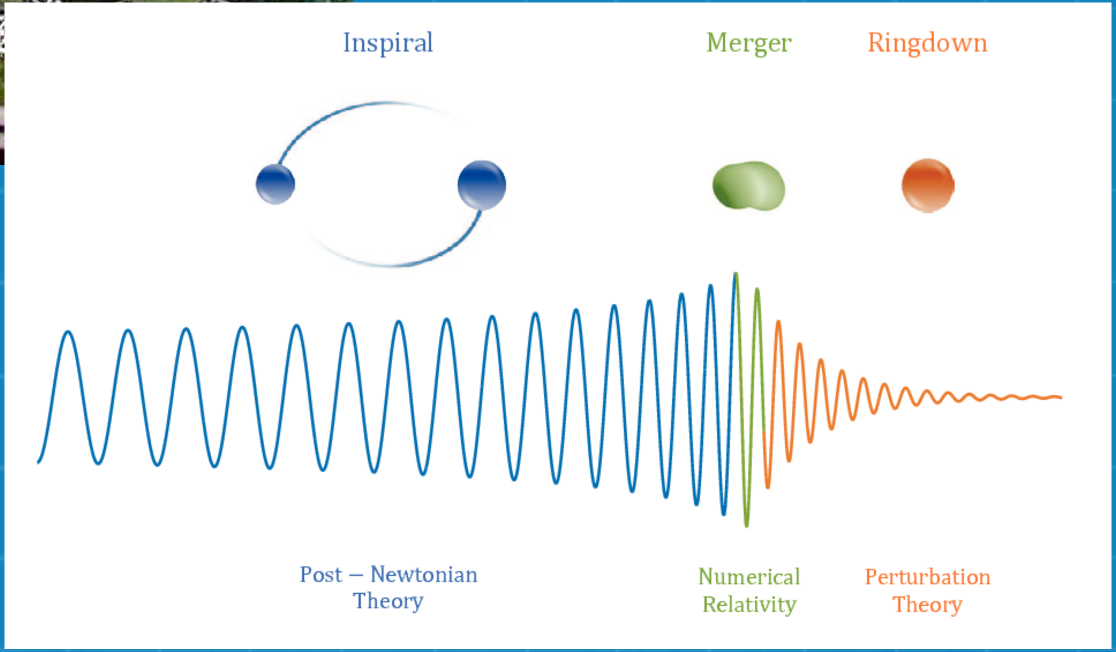






$$\sim \frac{\langle jk\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle M1\rangle}$$

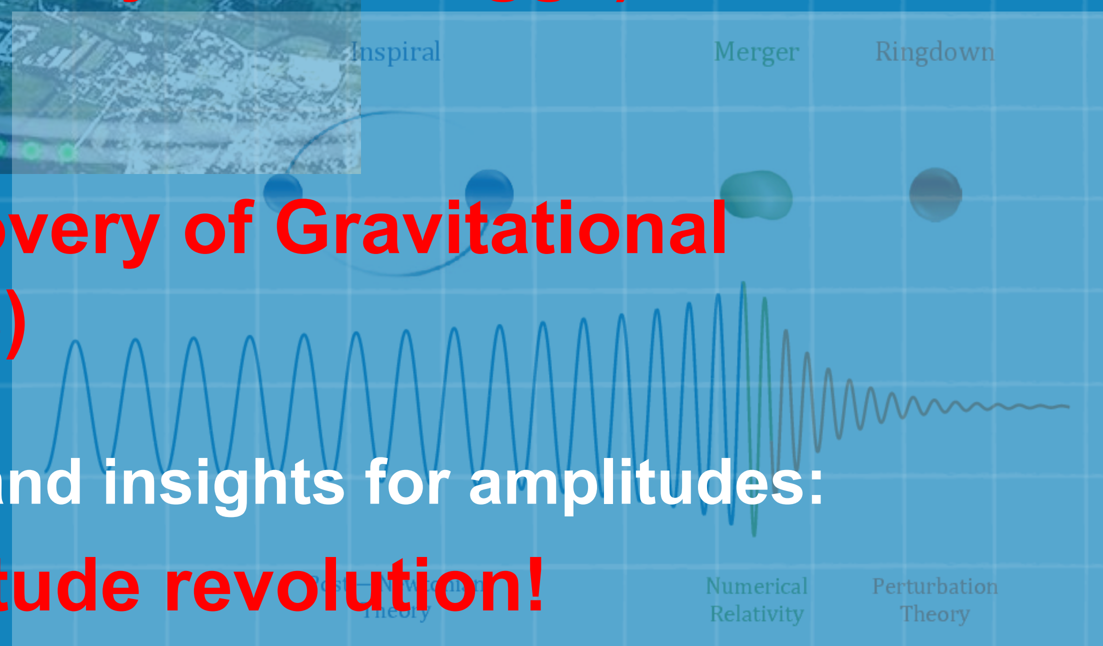
Exciting times in Physics



Exciting times in physics

New experimental facilities:

Large-Hadron Collider (Discovery of the Higgs)

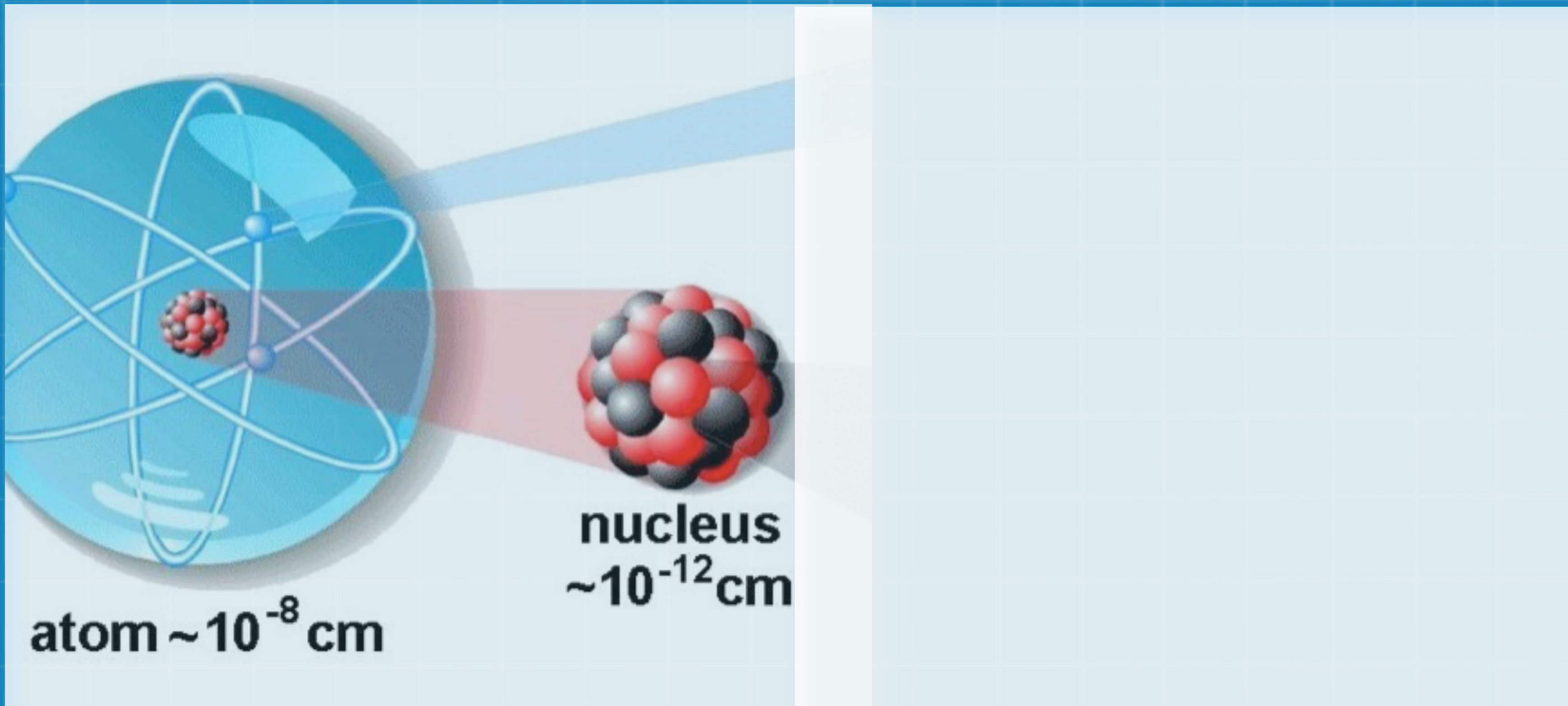


LIGO (Discovery of Gravitational waves)

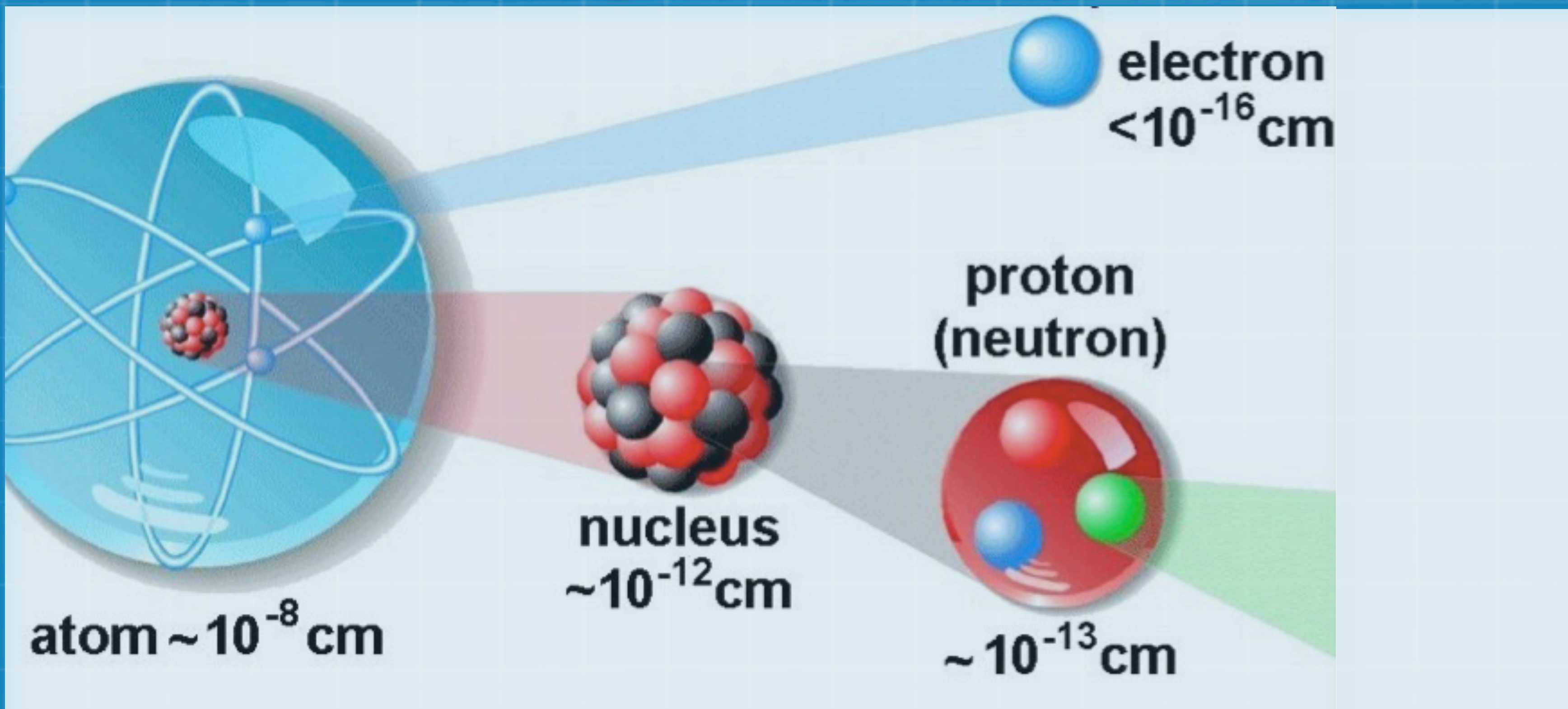
+ New discoveries and insights for amplitudes:

Amplitude revolution!

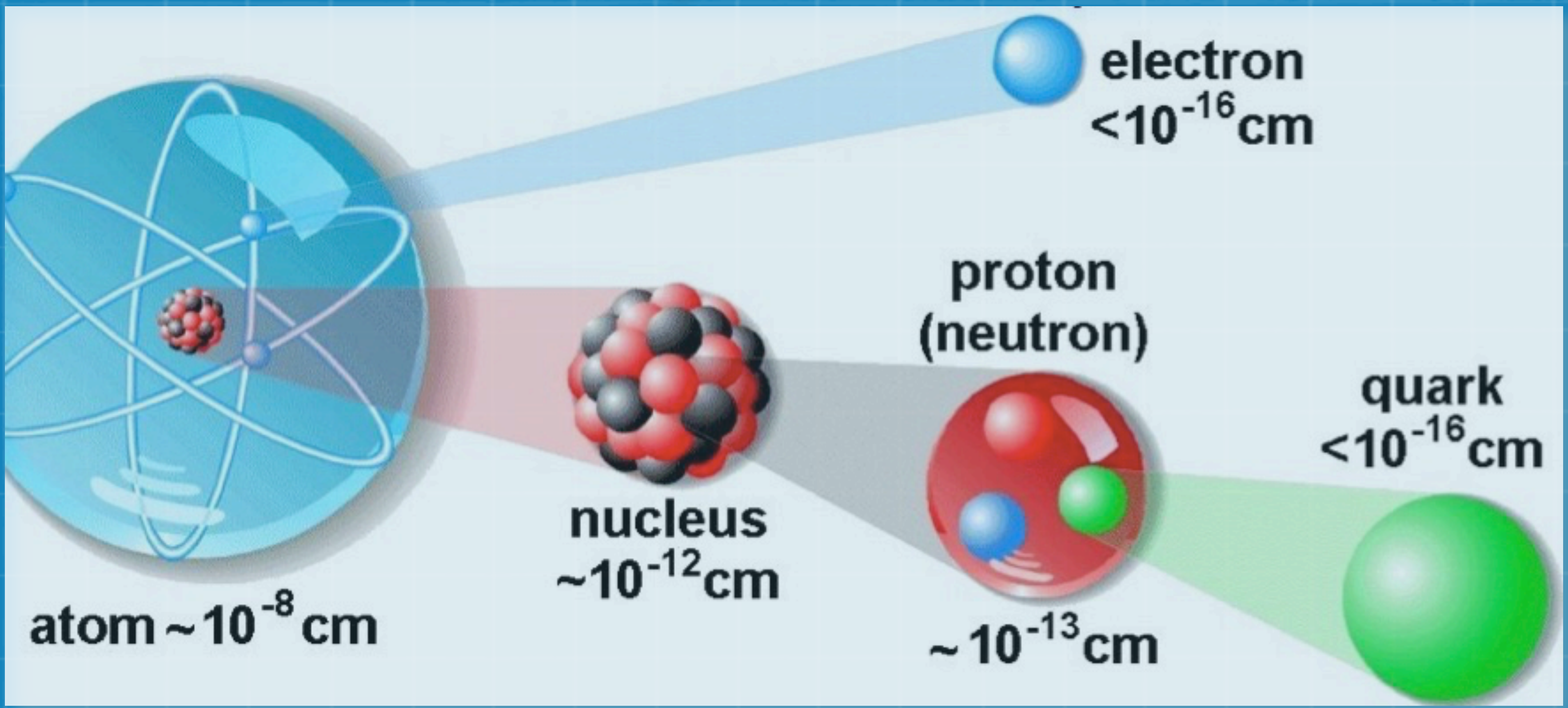
From the atom to the nucleus



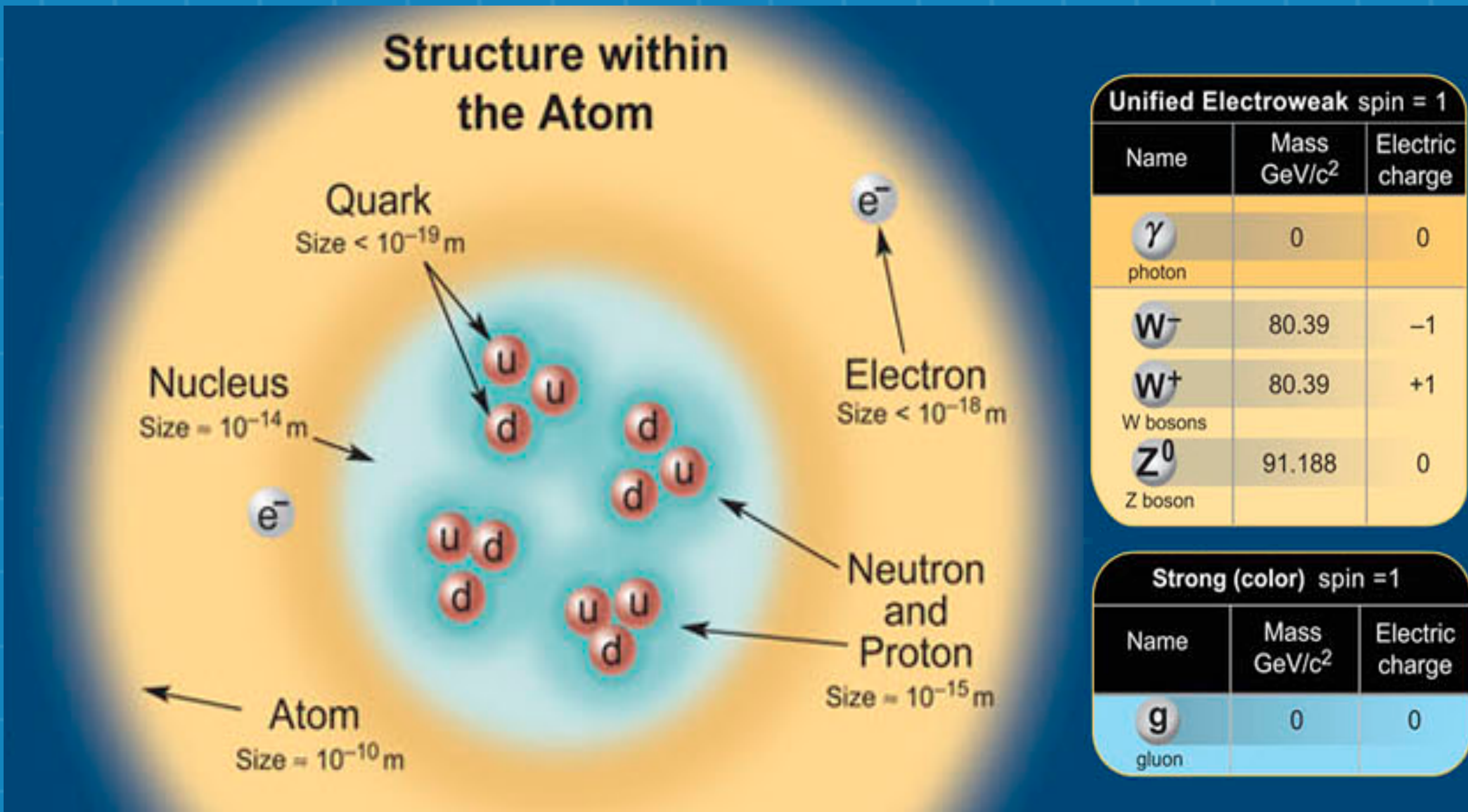
From the atom to the proton



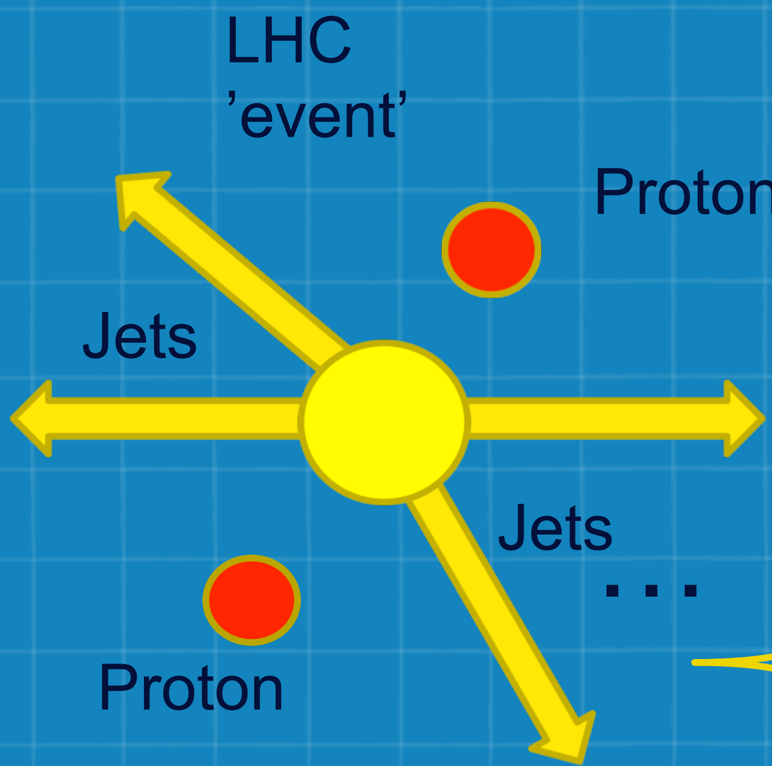
From the atom to the quark



Standard Model



Experiments at LHC



Probability for events

Jets:
Particle events
Reconstruction complicated..

Calculations necessary:
Amplitudes needed!

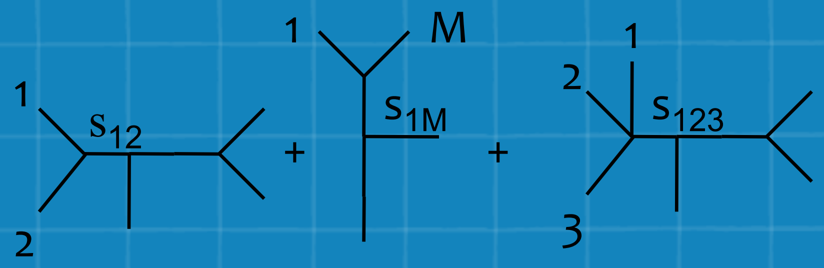
Amplitudes and probability

Quantum Mechanics:

Via solutions to the Schrödinger equation.

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V(x,y,z)\psi$$

Particle Physics:



Computed via Feynman diagrams.

Amplitudes and Feynman diagrams

- Feynman's method not flawless
- Diagrammatic expansion : huge permutational problem!
  - Scalar field theory : constant vertex (~1 term)
  - Gluons : momentum dependent vertex (~3 terms)
  - Gravitons : momentum dependent vertex (~100 terms)
- Naïve basic 4pt diagram count (graviton exchange) :

100 x 100 ~ 10^4 terms + index contractions (~ 36 pr diagram)
Number of diagrams: (~ 4 !) ~ 10^5 terms ~ 10^6 index contractions
n-point: (~ n !) ~ more atoms in your brain!

Too much off-shell (gauge dependent) clutter… ..

How do we proceed

Feynman diagrams:
Factorial Growth!

Sum over topological different diagrams

Generic Feynman amplitude

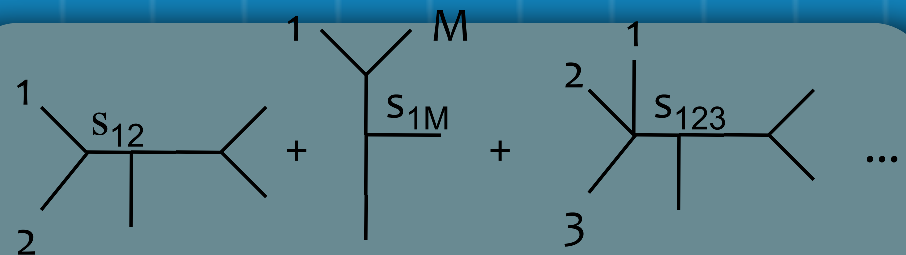
Complex expressions involving e.g.

$(p_i \cdot p_j)$

$(p_i \cdot \varepsilon_j)\ (\varepsilon_l \cdot \varepsilon_j)$

(no manifest symmetry or simplifications)

Tree amplitude revolution!



Factorial growth ~ billions of terms!!

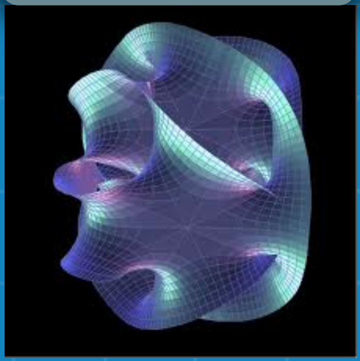
Rich hidden structure

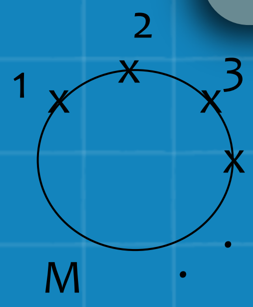
On-shell recursion
MHV only one term!

$$\sim \frac{\langle jk \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle M1 \rangle}$$

Inspiration across fields

String Theory





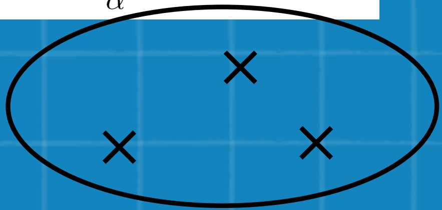
New relations

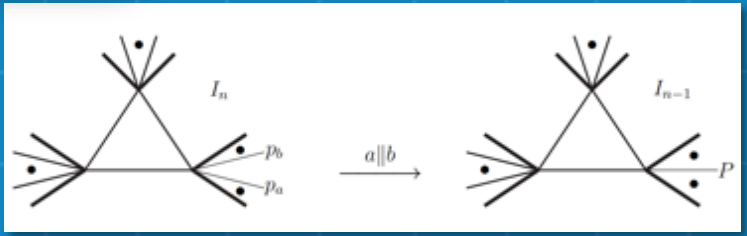
Amplitude cookbook!

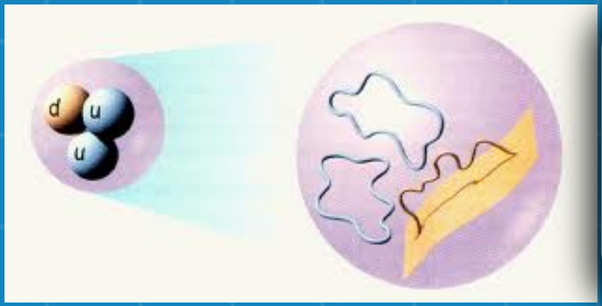
1 Unitarity: Fuse tree amplitudes into loops

2 Recursion: Extend trees and loops into more complicated amplitudes

$$A(0) = -\sum_{\alpha} \mathrm{Res}_{z=z_\alpha} \frac{A(z)}{z}$$







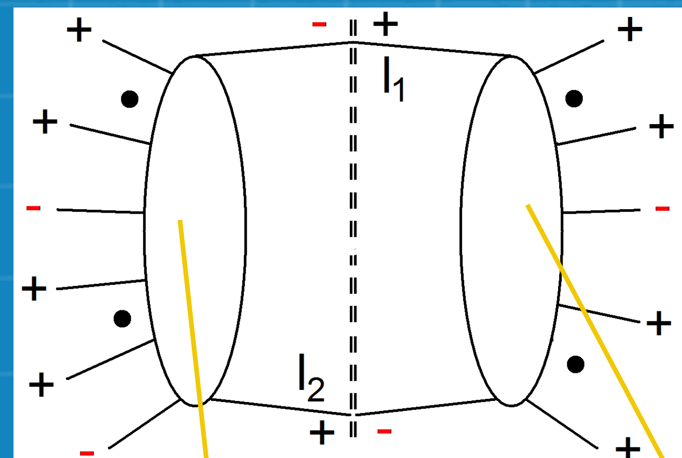
3 String theory: Complete the picture and link concepts

Key: Unitarity

Loop amplitudes

Simpler expressions for amplitudes

Unitarity



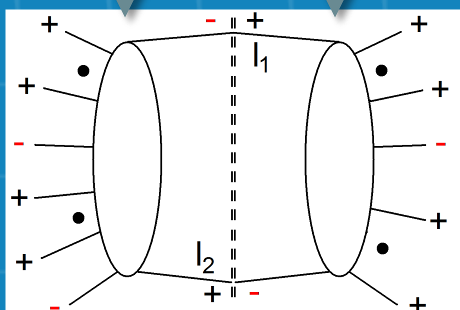
Amplitudes N=4, N=1, QCD at NLO, Gravity..

Key: Simple trees

Hidden structure!

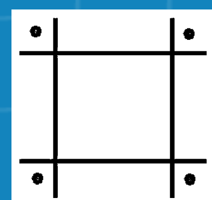
....from compact trees to loops

Compact, on-shell tree Amplitudes

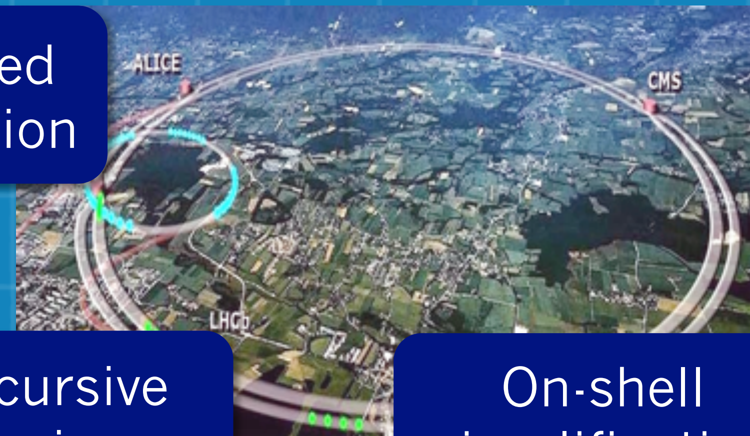


Integral basis

Quadruple cuts



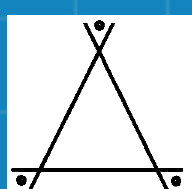
Automated computation



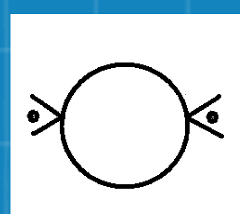
Recursive techniques

On-shell simplification

Rational polynomials



Triple cuts



Powerful computational methods
Impossible by Feynman diagrams
Revolution in doable computations

Types of projects

- Traditional recipe:
  - Find an interesting problem
  - Do a non-trivial and (hard) computation!
  - Elucidate the interesting aspects
  - Write up and defend your thesis
  - (Potential: opportunity to write your first research paper if <u>your result is truly non-trivial</u> and therefore publishable material)
  - Cook well at 180* ☺

Examples of themes

- The Cachazo-He-Yuan formalism for scattering amplitudes
  - Many interesting applications: tree and loop amplitudes
  - Relations to string theory
  - New physics
- Double-copy relations / gravity
  - Gravity from Standard Model amplitudes (Yang-Mills theory)
  - Double-copy numerators

Examples of themes

- Classical contributions from the Path integral:
  - Novel ways to compute observables in General Relativity
  - Bending of light – a new take on Quantum Gravity and potential quantum corrections in General Relativity?
  - Applications for the physics behind LIGO and observations of gravitational waves (more about this in *Michele's talk*)

Example: The scattering equations

It was suggested recently by Cachazo, He and Yuan that one can compute amplitudes via

$$\mathcal{A}_n = \int \frac{d^n\sigma}{\mathrm{volSL}(2,\mathrm{c})} \prod_a{}' \delta\left(\sum_{a\neq b} \frac{k_a \cdot k_b}{z_a - z_b}\right) \left(\frac{\mathrm{Tr}(T^{a_1}T^{a_2}T^{a_3}\dots T^{a_n})}{(z_1-z_2)(z_2-z_3)\cdots(z_n-z_1)} + \cdots\right)^{2-s} (\mathrm{Pf}'\Psi)^s$$

Exciting new framework for amplitudes

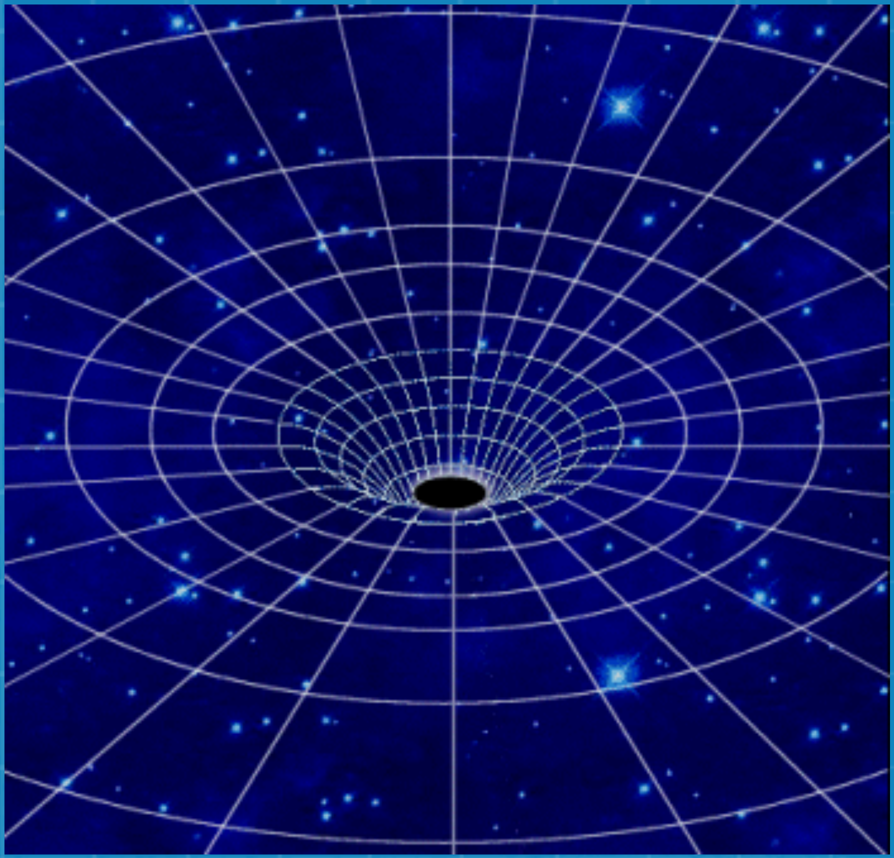
Algebraic solutions

Color trace

Pfaffian (dependent on polarisations and momenta)

Example: General Relativity

- Einstein's theory presents us with a beautiful theory for gravity.
- However geometrical description that does not fit well with a generic (flat space) formulation of quantum mechanics.
- Quantum mechanical extension of General Relativity?



Traditional quantization of gravity

- Known since the 1960ties that a particle version of General Relativity can be derived from the Einstein Hilbert Lagrangian (Feynman, DeWitt)
- Expand Einstein-Hilbert Lagrangian :

$$\mathcal{L}_{\mathrm{E}H} = \int d^4x \left[\sqrt{-g}R\right]$$

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

- Derive vertices as in a particle theory - compute amplitudes as Feynman diagrams!

Quantum theory for gravity

- Gravity as a theory with self-interactions
- Non-renormalisable theory! ('t Hooft and Veltman)

Dimensionful coupling:
$G_N = 1/M^2_{planck}$

- Traditional belief : – no known symmetry can remove all UV-divergences

String theory <u>can</u> by introducing new length scales

Quantum gravity as an effective field theory

- (Weinberg) proposed to view the quantization of general relativity from the viewpoint of effective field theory

$$\mathcal{L} = \sqrt{-g}\left[\frac{2R}{\kappa^2} + \mathcal{L}_{\mathrm{matter}}\right]$$

↓

$$\mathcal{L} = \sqrt{-g}\left\{\frac{2R}{\kappa^2} + c_1 R^2 + c_2 R^{\mu\nu} R_{\mu\nu} + \ldots\right\}$$

Effective field theory for gravity

- Consistent quantization
  - Working low energy version of quantum gravity

- New point of view:
  - General relativity hbar-> 0 limit of multi-loop expansion
  - Classical pieces comes from loop diagrams!
  - Explanation: contributions appear in loop diagrams feature a cancellation of the loop diagram hbar factor
    - (mass/hbar) expansion.

Key: String theory inspiration

Different form for amplitude

String theory adds channels up..

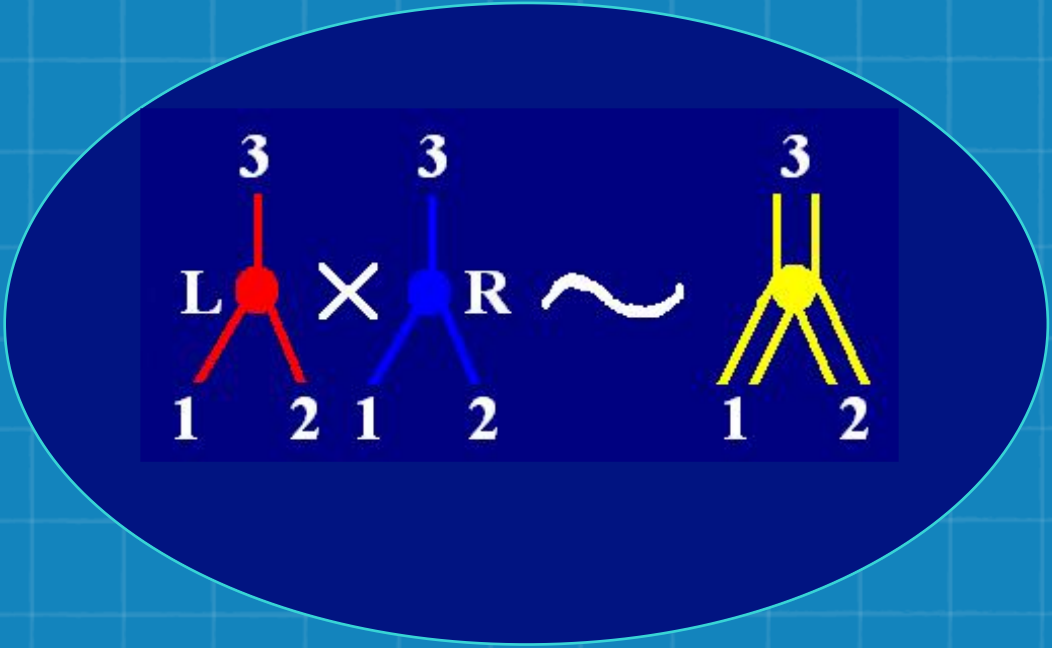
<->

Feynman diagrams sums separate kinematic poles

Key: Squaring relation for gravity

Gravity from (Yang-Mills)^2 (Kawai, Lewellen, Tye)

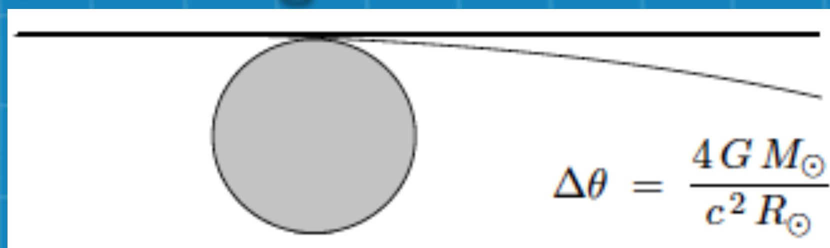
Natural from the decomposition of closed strings into open.



Gives a smart way to recycle Yang-Mills results into gravity results..

Example: Bending of massless matter

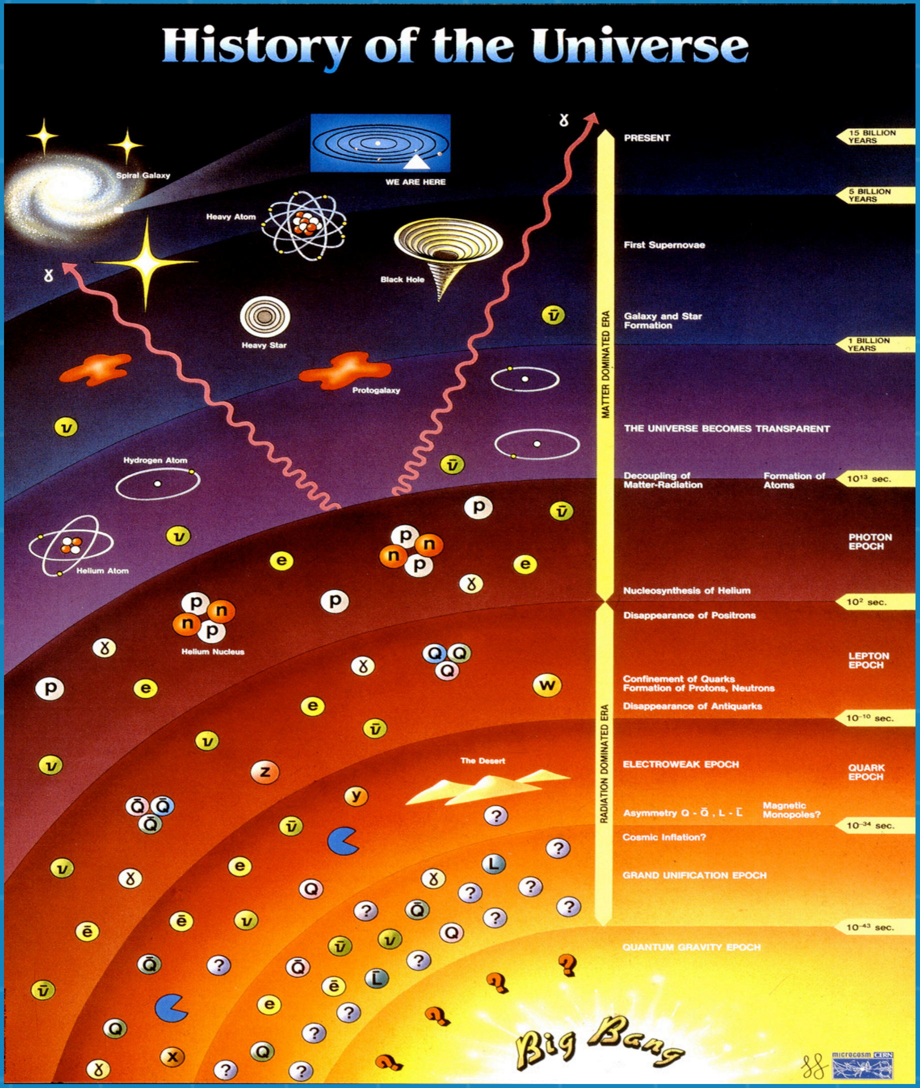
- Scattering of massless matter



- Einstein's original test: Bending of light/ massless matter around the Sun
- Features: mass-less external fields ~> IR singularities
- Features: Connection to GR/Universality of matter

High Energy Physics

Concerned with a large number of topics ranging from the sub-nuclear scales to the cosmological



Cristian Vergu

Research interests:

- scattering amplitudes in N=4 theory
- super-Wilson loops
- symmetries and integrability of N=4 scattering amplitudes
- twistor spaces
- evaluation of multiple zeta values and multiple polylogarithms
- quantization of the N=4 self-dual theory

Paolo Benincasa

Research interests:

- Crossroads between the fields of scattering amplitudes and cosmology
- Analytic structure of the wavefunction of the universe for certain models (e.g. spin-1 states)
  - Observables in cosmology.
- Relation to an underlying combinatorial-geometrical structure (the cosmological polytopes)

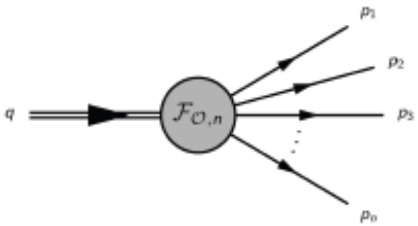
Matthias Wilhelm

Assistant Professor, Fc10 and Cc3

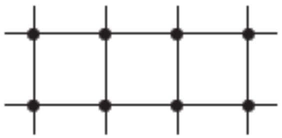
Goal Analytic and preferably non-perturbative understanding of the structure of gauge theories
$\rightarrow \mathcal{N}=4$ supersymmetric Yang-Mills theory and beyond

Scattering amplitudes and Integrability

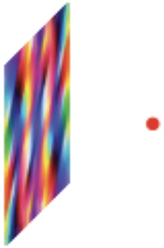
- On-shell methods beyond scattering amplitudes: form factors, anomalous dimensions, beta functions
  $\rightarrow$ Possible project



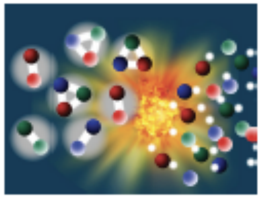
- Number theory of Feynman integrals



- Defect CFTs: One-point functions, particle-interface potential



- Thermodynamics of gauge theories: Hagedorn temperature, towards finite N
  $\rightarrow$ Possible project



Andrew McLeod

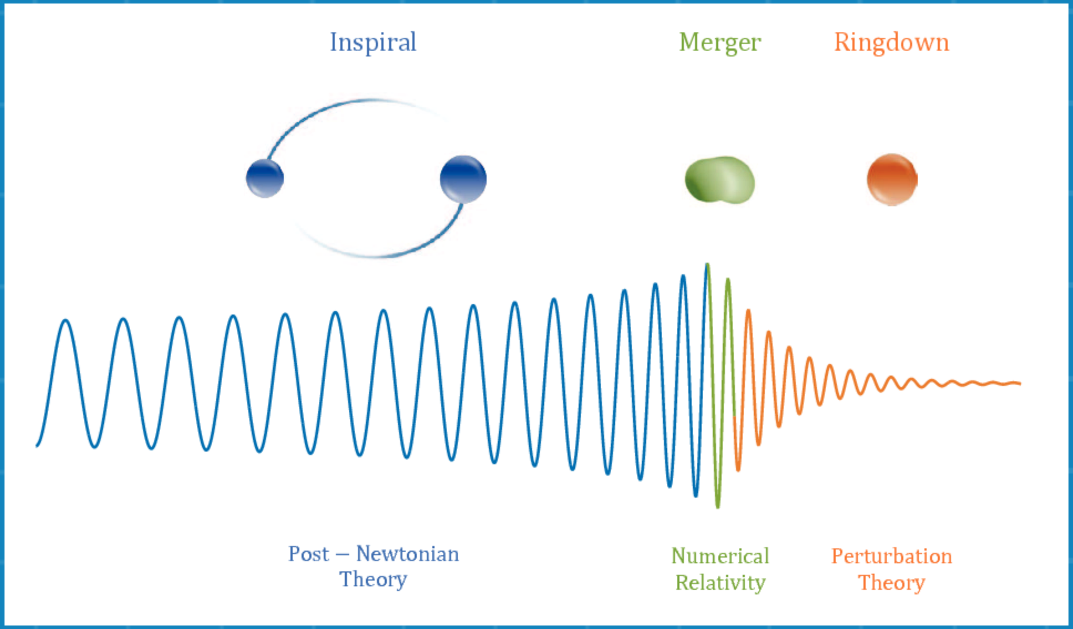
- **Graphical Color Decompositions** — There is no (known) good basis of color structures for scattering amplitudes in gauge theory involving large numbers of particles; in practice, working out such a basis at each loop order and particle multiplicity is computationally taxing. Thus, it would be nice to have a basis of such structures graphically, in terms of structure constant graphs. Such a basis is known at tree level and one loop, but not at two loops (even for small numbers of particles). This project would involve coding up general color structures (for instance, in Mathematica) to look for sets of graphs that are linearly independent.

- **Connecting Cluster Algebras and Hyperbolic Geometry in One-Loop Amplitudes** — It has been discovered over the last few years that cluster algebras appear when one studies the branch cut structure of scattering amplitudes in planar N=4 super-Yang-Mills theory, where they prove useful for understanding physical constraints such as the Steinmann relations. It is unknown whether similar cluster-algebraic structure exists in the amplitudes of more general (non-supersymmetric) quantum field theories. One place to look for this structure is at one loop, by exploring the connection between cluster algebras and the hyperbolic geometry of one-loop Feynman integrals. This project would involve generating the relevant cluster algebras (for example, in Mathematica) and analyzing the branch cut structure of one-loop integrals.

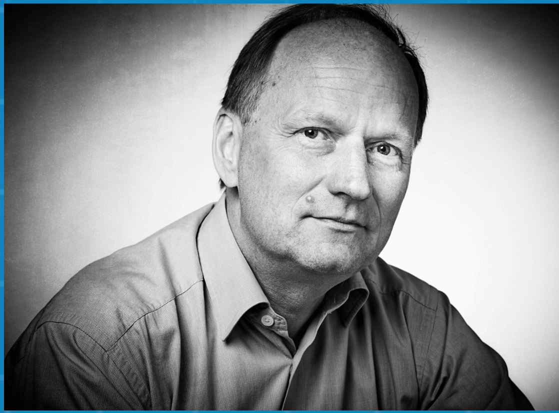
Theory projects at NBIA

- Examples of current advisors and topics
  - Poul Henrik Damgaard and Emil Bjerrum-Bohr: Classical gravity, scattering amplitudes
  - Christian Vergu: Supersymmetric scattering amplitudes, integrability
  - Paolo Benincasa: Scattering amplitudes, geometric descriptions of cosmology
  - Matt von Hippel and Andrew McLeod: Scattering amplitudes, loop integrals
  - Matthias Wilhelm: SUSY, Scattering amplitudes, non-perturbative physics

If you are interested in projects in gravity – stay tuned for Michele's talk!!







And don’t be afraid to come by and see us!!