Generalizing the Double-Copy: the KLT Bootstrap

Henriette Elvang

University of Michigan

Leinweber Center for Theoretical Physics

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HH Chi, H.E., A. Herderschee,
C. Jones, S. Paranjape 2106.12600
and work-to-appear with Alan (Shih-Kuan) Chen

The field theory KLT form of the double-copy is

Color-ordered tree amplitudes

Everything in this talk is tree-level

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

[Kawai-Lewellen-Tye 1985]

2 choices of (n-3)! color orderings

KLT kernel

a function of Mandelstams

For example

gravity$^+$ = (Yang-Mills) x (Yang Mills)

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For example

gravity$^+$ = (Yang-Mills) x (Yang Mills)

4pt: $$A_4^{\text{gravity}} = A_4^{YM}[1234]\, S_4[1234|1243]\, A_4^{YM}[1243]$$

with $S_4[1234|1243] = -s$

It remarkable that this works!!

Color-ordered YM gluon amplitudes: $A_4[1234]$ has simple poles in s and u, but not t.
$A_4[1243]$ has simple poles in s and t, but not u.

$$s = (p_1 + p_2)^2$$
$$t = (p_1 + p_3)^2$$
$$u = (p_1 + p_4)^2$$

Graviton amplitudes have no color-structure, so $M_4(1234)$ has simple poles in the s, t and u channels.

How can a product of A_4's possibly get even the pole structure of M_4 right??? *And* avoid double-poles?

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Answer: DOUBLE-COPY **KERNEL**

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Because

The double-copy kernel:
1) Eliminates double-poles from A_4 * A_4
2) Provides "missing" poles

Bonus properties:
ensures enhancement of soft behaviors and in certain cases enhances unbroken global symmetries

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2) Provides "missing" poles

Changing the kernel may well destroy these desirable properties!

Another important aspect of field theory KLT: KKBCJ relations

$$M_4 = -s A_4[1234] A_4[1243] \qquad \textit{and} \qquad M_4 = -\frac{su}{t} A_4[1234] A_4[1234]$$

then their difference must be zero, i.e.

$$0 = A_4[1243] - \frac{u}{t} A_4[1234]$$

And this **is** true for YM amplitudes.

This is an example of a **BCJ** (Bern-Carrasco-Johansson) **relation** at 4-point.

Kleiss-Kuijf:

$$\text{Trace-reversal:} \quad \mathcal{A}_4[1432] = \mathcal{A}_4[1234]\,, \quad etc$$

$$U(1)\text{-decoupling:} \quad \mathcal{A}_4[1234] + \mathcal{A}_4[1243] + \mathcal{A}_4[1423] = 0\,,$$

$$\text{BCJ:} \quad \mathcal{A}_4[1234] - \frac{t}{u}\mathcal{A}_4[1243] = 0\,.$$

"KKBCJ relations"

A ***n*-point**

$$A_n^{L\otimes R} = \sum_{a,b} A_n^{L}[a]\, S_n[a|b]\, A_n^{R}[b]$$

choices of (n-3)! color orderings

KLT kernel

and associated **KKBCJ relations** that ensure that the result of the double-copy is *independent* of the choice of (n-3)! color-orders out of the (n-1)! possible in the KLT sum.

Field theory double-copy `selection criterium'

In order to be "double-copyable", a theory's tree amplitudes must obey the KK and BCJ relations.

reduces the number of color-orderings from (n-1)! to (n-2)!

reduces the number of color-orderings from (n-2)! to (n-3)!

A new way to explore the landscape of field theories: *which theories can be input/output of the double-copy?*

Which theories obey the field theory KK&BCJ relations?

YM theory ✓

Chiral perturbation theory ✓

Super YM theory ✓

Bi-adjoint scalar model ✓

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What about higher-derivative operators in EFTs?

YM: $\mathrm{tr}\,F^2$ ✓ $\mathrm{tr}\,F^3$ ✓ $\mathrm{tr}\,F^4$ 1✗ $\mathrm{tr}\,D^2F^4$ 1✓1✗ $\mathrm{tr}\,D^4F^4$ 1✓2✗ ...

Here just MHV counting

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Here only MHV counting

χPT: $\mathrm{tr}\,\partial^2\phi^n$ ✓ $\mathrm{tr}\,\partial^4\phi^4$ 2✗ $\mathrm{tr}\,\partial^6\phi^4$ 1✓1✗ $\mathrm{tr}\,\partial^8\phi^4$ 1✓2✗ $\mathrm{tr}\,\partial^{10}\phi^4$ 1✓2✗ . . .

Why are some operators allowed and not others? ***Is this the most general story?***

YM + h.d.

YM + h.d.

Gravity^{+} + h.d.

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

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Include **higher-derivative corrections in the double-copy kernel**

String theory does that!

String theory KLT

KLT originally came from closed string = (open string)^2 at tree-level

$$A_n^{L\otimes R} = \sum_{a,b} A_n^L[a]\, S_n[a|b]\, A_n^R[b]$$

string KLT kernel

The KLT kernel is deeply linked with the open string amplitudes to ensure correct pole structure in the closed string amps.

Upon expansion in alpha', this translates to very particular higher-derivative corrections of the kernel:
not the most general options and tuned exactly to the alpha' corrections in the open string.

Example: $$S_4[1234|1243] = -\sin(\pi\alpha' s) = -\pi\alpha' s + \frac{1}{6}(\pi\alpha' s)^3 + \ldots$$

Only *s*-dependence, no *t* or *u*; why?

Only odd powers in *s*; why?

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

What are the rules for generalizing the KLT kernel?

The generalized double-copy kernel should

1) eliminate double-poles
2) provide "missing" poles
3) not introduce spurious poles

We propose a new framework for systematically analyzing generalizations of the double-copy kernel: the KLT bootstrap

The proposal is based on the **KLT algebra** which I'll now introduce

The double-copy is a **map** FT x FT -> FT

The kernel defines the **multiplication rule** of this map

Multiplication table for FT x FT -> FT

FT $\otimes$ FT	YM	$\mathcal{N}=4$ SYM	χPT	BAS
YM	gravity+	$\mathcal{N}=4$ SG	BI	YM
$\mathcal{N}=4$ SYM	$\mathcal{N}=4$ SG	$\mathcal{N}=8$ SG	$\mathcal{N}=4$ sDBI	$\mathcal{N}=4$ SYM
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Cubic Bi-Adjoint Scalar model (BAS)

$$\mathcal{L}_{\text{BAS}} = -\frac{1}{2}\left(\partial_\mu \phi^{aa'}\right)^2 - g\, f^{abc}\, \tilde{f}^{a'b'c'}\, \phi^{aa'}\, \phi^{bb'}\, \phi^{cc'}$$

structure constants

Scalar in adjoint of (say) U(N) and U(N')

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This map has an ***identity element*** **1**: the **bi-adjoint scalar model** (BAS)

String KLT *also* has an identity element:
BAS + very specific higher-derivative operators.

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KLT algebra

$$\mathrm{L}=\mathrm{L}\otimes\mathbf{1}\,,\qquad \mathrm{R}=\mathbf{1}\otimes\mathrm{R}\,,\qquad \mathbf{1}=\mathbf{1}\otimes\mathbf{1}\,.$$

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KLT Bootstrap Equation

When the multiplication rule is changed,
the identity element is changed, and vice versa:
The kernel and the identity model are uniquely linked!

KLT algebra

$$\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \quad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,, \quad \mathbf{1} = \mathbf{1} \otimes \mathbf{1}\,.$$

Generalize the **KKBCJ / monodromy** relations

(will not derive here; pls see 2106.12600)

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Next

1) Relation between **BAS** tree amplitudes and the field theory **KLT kernel**

2) How to **generalize the KLT kernel** by generalizing BAS

3) Examples at 4-point

4) Higher-point

5) Summary & outlook

Bi-adjoint model

$$\mathcal{L}_{\mathrm{BAS}} = -\frac{1}{2}\left(\partial_\mu \phi^{aa'}\right)^2 - g f^{abc} \tilde{f}^{a'b'c'} \phi^{aa'} \phi^{bb'} \phi^{cc'}$$

(Who ordered that?)

Another formulation of the double-copy is BCJ (Bern-Carrasco-Johansson): write any color-ordered amplitude as

$$A_n = \sum \frac{c_i n_i}{\prod_I P_I^2}$$

e.g. at 4-pt $c_s = f^{abx} f^{cdx}$ etc

Color-kinematic duality

$$c_s + c_t + c_u = 0 \quad \Longrightarrow \quad n_s + n_t + n_u = 0$$

and then

$$A_4^{\mathrm{L}\otimes\mathrm{R}} = \sum \frac{n_i^{\mathrm{L}} n_i^{\mathrm{R}}}{\prod_I P_I^2}$$

is the double-copy tree amplitude

If instead we replace the numerator factor with another color-factor

$$A_4^{\mathrm{BAS}} = \sum \frac{c_i \tilde{c}_i}{\prod_I P_I^2}$$

then we get precisely the tree amplitudes of the bi-adjoint scalar model (BAS).

Bi-Adjoint Scalar model (BAS)

$$\mathcal{L}_{\rm BAS} = -\frac{1}{2}\left(\partial_\mu \phi^{aa'}\right)^2 - g f^{abc} \tilde{f}^{a'b'c'} \phi^{aa'} \phi^{bb'} \phi^{cc'}$$

Statement BAS = BAS x BAS --- or $\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$ can be written as

$$m_n[\gamma|\delta] = \sum_{\alpha,\beta} m_n[\gamma|\alpha]\, S_n[\alpha|\beta]\, m_n[\beta|\delta]$$

or in **matrix form** $\quad m_n = m_n . S_n . m_n$

(n-3)! x *(n-3)! submatrices*

Double-sum over *(n-3)!* color orderings

Bi-Adjoint Scalar model (BAS)

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So multiplying from both the left and right with inverses of matrices of BAS amplitudes gives

$$S_n = (m_n)^{-1}$$

[Cachazo et al]

The field theory KLT kernel is the inverse of an *(n-3)! x (n-3)!* submatrix of BAS amplitudes!

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4-point case

Tree amplitudes color-ordered wrt both color-factors, e.g.

$$m_4[1234|1234] = \frac{g^2}{s} + \frac{g^2}{u}\,, \qquad m_4[1234|1243] = -\frac{g^2}{s}$$

=>

$$S_4[1234|1234] = \left(m_4[1234|1234]\right)^{-1} = -\frac{su}{tg^2}\,,$$

$$S_4[1234|1243] = \left(m_4[1243|1234]\right)^{-1} = -\frac{s}{g^2}\,.$$

Strings KLT kernel

The string theory KLT kernel is the inverse of an *(n-3)!* x *(n-3)!* submatrix of BAS+ (*specific* h.d.) amplitudes!

[Mizera]

$$m_4^{(\alpha')}[1234|1243] = -\frac{1}{\sin(\pi\alpha' s)} \quad = -\frac{1}{\pi\alpha' s} - \frac{1}{6}\pi\alpha' s - \frac{7}{360}(\alpha'\pi s)^3 + \ldots .$$

BAS

How to generalize the double-copy kernel?

Which terms are allowed in BAS+h.d.?

KLT bootstrap

n=4 => (n-1)! = 6 single-trace color-orderings: 1234, 1243, 1324, 1342, 1423, 1432

Recall that $\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$ means $m_n = m_n.S_n.m_n$ and this implies $S_n = (m_n)^{-1}$

Written out for rank (4-3)!=1 at 4-point means, *for example*:

$$m_n[1234|1234] = m_n[1234|1243]\frac{1}{m_4[1243|1243]}m_n[1243|1234]$$

KLT bootstrap equation

$$\implies m_n[1234|1234]m_4[1243|1243] - m_n[1234|1243]m_n[1243|1234] = 0$$

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$$\implies m_n[1234|1234]m_4[1243|1243] - m_n[1234|1243]m_n[1243|1234] = 0$$

So this condition is that a 2x2 minor of the 6x6 **matrix of m_4[a|b] amplitudes** have to vanish:

$$\begin{pmatrix} m_4[1234|1234] & m_4[1234|1243] & \cdots \\ m_4[1243|1234] & m_4[1243|1243] & \cdots \\ \cdots & & \end{pmatrix}$$

Similarly, all 2x2 minors must vanish! But that’s just saying that we must have a rank 1 system. **Aha!**

KLT bootstrap at 4-pt

$$m_4[1234|1234] = f_1(s,t) \quad \text{with} \quad f_1(s,t) = f_1(-s-t,t)\,,$$
$$m_4[1234|1243] = f_2(s,t)\,,$$
$$m_4[1234|1324] = f_3(s,t) = f_2(-s-t,t)\,,$$
$$m_4[1234|1342] = f_4(s,t) = f_2(s,t)\,,$$
$$m_4[1234|1423] = f_5(s,t) = f_2(-s-t,t)\,,$$
$$m_4[1234|1432] = f_6(s,t) \quad \text{with} \quad f_6(s,t) = f_6(-s-t,t)\,,$$

Cyclic symmetry & momentum relabeling

6 x 6 matrix for these amplitudes has rank 6.

Imposing the vanishing of all 2x2 minors =>

4-point KLT bootstrap equations

$$f_1(s,t) = \frac{f_2(s,t)f_2(-s-t,s)}{f_2(t,s)}\,, \qquad f_6(s,t) = f_1(s,t)\,.$$

$$f_2(s,t)f_2(-s-t,s)f_2(t,-s-t) = f_2(t,s)f_2(-s-t,t)f_2(s,-s-t)\,.$$

Solved by BAS and the strings BAS+h.d. amplitudes.

What else solves it?

Most general rank (n-3)! kernel at 4-point

Write the most general ansatz for f_2: $$f_2(s,t) = -\frac{g^2\Lambda^2}{s} + \sum_{k=0}^{N}\sum_{r=0,k} \frac{a_{k,r}}{\Lambda^{2k}} s^r t^{k-r}$$

Solve the **KLT bootstrap equations** order by order. Impose **locality**. Result:

$$f_2(s,t) = -\frac{g^2\Lambda^2}{s} + \frac{1}{\Lambda^2}(a_{1,0}t + a_{1,1}s) + \frac{a_{2,0}}{\Lambda^4}t(s+t) + \frac{1}{\Lambda^6}\left[a_{3,0}t^3 + a_{3,1}st^2 + a_{3,2}s^2t + a_{3,3}s^3\right] + \mathcal{O}\left(\frac{1}{\Lambda^8}\right)$$

Strings result recovered for

$$a_{1,1} = -\frac{1}{6}, \qquad a_{3,3} = -\frac{7}{360}, \ldots$$

and all other $a_{i,j} = 0$

New double-copy kernel much more general.

4-point result as BAS + h.d. Lagrangian

$$\begin{aligned}\mathcal{L} \;&= -\frac{1}{2}\left(\partial_\mu \phi^{aa'}\right)^2 - \frac{g}{6} f^{abc}\tilde{f}^{a'b'c'}\phi^{aa'}\phi^{bb'}\phi^{cc'} \\ &-\frac{a_{\rm L}+a_{\rm R}}{2\Lambda^4} f^{abx}f^{cdx}f^{a'b'x'}f^{c'd'x'}(\partial_\mu\phi^{aa'})(\partial^\mu\phi^{bb'})\phi^{cc'}\phi^{dd'} \\ &+\frac{a_{\rm L}}{\Lambda^4} f^{abx}f^{cdx}d^{a'b'x'}d^{c'd'x'}(\partial_\mu\phi^{aa'})\phi^{bb'}(\partial^\mu\phi^{cc'})\phi^{dd'} \\ &+\frac{a_{\rm R}}{\Lambda^4} d^{abx}d^{cdx}f^{a'b'x'}f^{c'd'x'}(\partial_\mu\phi^{aa'})\phi^{bb'}(\partial^\mu\phi^{cc'})\phi^{dd'} + \ldots\end{aligned}$$

$$d^{abc} = \mathrm{Tr}\left[T^a\{T^b, T^c\}\right]$$

Observations

- There is no $d^{abc}d^{a'b'c'}\phi_{aa'}\phi_{bb'}\phi_{cc'}$ term: it does not solve the rank 1 bootstrap equations.
- There is no ϕ^4 term; does not solve the rank 1 bootstrap equations
- The d^{abc} terms modify the U(1) decoupling identities that are part of the field theory KK relations and generalize the strings monodromy relations.
- Known strings kernel has $a_L=a_R$. The generalization allows “heterotic”-type double-copy.

Double-copy of YM + h.d.

Impose generalized KKBCJ relations <=> $\mathbf{1} \otimes \mathbf{R} = \mathbf{R} \quad \mathbf{L} \otimes \mathbf{1} = \mathbf{L}$

on a general ansatz for MHV 4-pt YM + h.d. to find

$$\mathcal{A}_4^L[1^+2^+3^-4^-] = [12]^2\langle 34\rangle^2 \left[\frac{(g_{\mathrm{YM}}^L)^2}{su} - \frac{1}{\Lambda^4}\left(\frac{(g_{\mathrm{YM}}^L)^2}{g^2} a_L + (g_{F^3}^L)^2 \frac{t}{s} \right) - \frac{e_{3,1}^L}{\Lambda^6} t + \mathcal{O}\left(\frac{1}{\Lambda^8}\right) \right]$$

Usual YM → $\frac{(g_{\mathrm{YM}}^L)^2}{su}$

tr F^4 → $\frac{(g_{\mathrm{YM}}^L)^2}{g^2} a_L$

Pole term w/ two tr F^3 vertices → $(g_{F^3}^L)^2 \frac{t}{s}$

tr D^2F^4 → $\frac{e_{3,1}^L}{\Lambda^6} t$

Its coefficient (of tr F^4) is controlled by the generalized KLT kernel

And similarly for the R sector.

Summarizing the difference between admissible operators in ordinary field theory KLT vs. the new generalized KLT:

For YM + higher-derivatives

FT KLT YM: $\text{tr}\,F^2$ ✓ $\text{tr}\,F^3$ ✓ $\text{tr}\,F^4$ 1✗ $\text{tr}\,D^2F^4$ 1✓1✗ $\text{tr}\,D^4F^4$ 1✓2✗ ...

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Green checkmark: operator allowed with arbitrary coefficient.
Blue checkmark: operator allowed with coefficient fixed by the parameters in the KLT kernel.

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Gen. KLT YM: $\mathrm{tr}\,F^2$ ✓ $\mathrm{tr}\,F^3$ ✓ $\mathrm{tr}\,F^4$ 1✓ $\mathrm{tr}\,D^2F^4$ 1✓1✗ $\mathrm{tr}\,D^4F^4$ 1✓2✓ . . .

For chiPT + higher-derivatives

FT KLT χPT: $\mathrm{tr}\,\partial^2\phi^n$ ✓ $\mathrm{tr}\,\partial^4\phi^4$ 2✗ $\mathrm{tr}\,\partial^6\phi^4$ 1✓1✗ $\mathrm{tr}\,\partial^8\phi^4$ 1✓2✗ $\mathrm{tr}\,\partial^{10}\phi^4$ 1✓2✗

Gen. KLT χPT: $\mathrm{tr}\,\partial^2\phi^n$ ✓ $\mathrm{tr}\,\partial^4\phi^4$ 2✗ $\mathrm{tr}\,\partial^6\phi^4$ 1✓1✓ $\mathrm{tr}\,\partial^8\phi^4$ 1✓2✗ $\mathrm{tr}\,\partial^{10}\phi^4$ 1✓2✓

Green checkmark: operator allowed with arbitrary coefficient.
Blue checkmark: operator allowed with coefficient fixed by the parameters in the KLT kernel.

Summarizing the difference between admissible operators in ordinary field theory KLT vs. the new generalized KLT:

For YM + higher-derivatives

FT KLT YM: $\mathrm{tr}\,F^2$ ✓ $\mathrm{tr}\,F^3$ ✓ $\mathrm{tr}\,F^4$ 1✗ $\mathrm{tr}\,D^2F^4$ 1✓1✗ $\mathrm{tr}\,D^4F^4$ 1✓2✗ . . .

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Green checkmark: operator allowed with arbitrary coefficient.
Blue checkmark: operator allowed with coefficient fixed by the parameters in the KLT kernel.

For FIXED choice of kernel, this LINKS the coefficients of $\mathrm{tr}\,F^4$ with that of one of the $\mathrm{tr}\,\partial^6\phi^4$ operators.

Double-copy of YM + h.d. -> Gravity$^+$ + h.d.

Usual Einstein gravity

Pole term from exchanges of dilaton *and* axion!

$$\mathcal{M}_4(1^+2^+3^-4^-) = [12]^4\langle 34\rangle^4 \times\Big[-\frac{(g^L_{\text{YM}})^2(g^R_{\text{YM}})^2}{g^2\Lambda^2}\frac{1}{stu} + \frac{((g^L_{\text{YM}})^2(g^R_{F^3})^2+(g^R_{\text{YM}})^2(g^L_{F^3})^2)}{g^2\Lambda^6}\frac{1}{s} + \frac{1}{\Lambda^8}\left(\frac{(g^L_{\text{YM}})^2(g^R_{\text{YM}})^2}{g^4}a_{2,0} + \frac{1}{g^2}((g^R_{\text{YM}})^2e^L_{3,1} + (g^L_{\text{YM}})^2e^R_{3,1})\right) + \mathcal{O}\left(\frac{1}{\Lambda^{10}}\right)\Big]$$

vanishes in string theory

local R^4 contribution

In the field theory or strings double copy, there is less freedom in the coefficient of R^4.

The result of the double-copy: in all cases checked, *same operators produced but with shifts of their coefficients.*

Higher-point

Necessary to test consistency by going to higher point:

What if the KLT bootstrap at 5-point further fixed some of the 4-point kernel coefficients $a_{i,j}$?
(Then we'd be in trouble!)

For n=5 => (n-1)! = 4! = 24 distinct orderings.

Cyclic symmetry + momentum relabelings => parameterized by 8 functions $g_i(s,t)$, i=1,2,…,8.

We impose the rank (n-3)! = 2 conditions equivalent to $\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$ on this 24x24 system and solve.

Found consistent solution for the bootstrapped 5pt (BAS+h.d.) amplitudes; **no** constraints placed on 4-pt coefficients; in fact, up to quadratic order in Mandelstams, the amplitudes are completely fixed by 4-pt input.

Tested for 5pt +++++ YM+h.d.

Summary

- We have investigated the algebraic structure of the KLT multiplication rule.
- The KLT algebra gives a systematic way to generalize the double-copy in the KLT form: the **double-copy bootstrap.**
- Solved as BAS + most general h.d. terms for minimal rank (n-3)! at 4- and 5-point.
- Tested in examples with YM and chiPT.

$$\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \quad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,, \quad \mathbf{1} = \mathbf{1} \otimes \mathbf{1}\,.$$

Generalize the **KKBCJ / monodromy** relations

KLT Bootstrap Equation

Outlook

1) To the orders checked, the generalized double-copy produces the same h.d. operators in the double-copy LxR amplitude, but with some shifted Wilson coefficients: why?

   small multiplicity / low-enough dim effect? or something more fundamental?

   => Currently studying similarity transformations from ``hybrid" double-copy kernels, finding interesting algebraic structures. [Alan Chen & H.E., work in progress].

2) The method is more than BAS+hd. It is a *framework for exploring more general forms of the double-copy:*

   - Does there exist other form of the double-copy without the cubic BAS interaction?
   - Is minimal rank $(n-3)!$ fundamental? (See also Paranjape's talk on massive double-copy).
   - Initiated study of non-minimal rank examples in our paper, more to do.

Non-minimal rank?

We change the identity theory at cubic order: $d^{abc}\tilde{d}^{a'b'c'}\phi^{aa'}\phi^{bb'}\phi^{cc'}$

3pt rank 1 => 4-pt rank 3 (no problems) => 5-pt rank 11 (problem: inverse has spurious poles!)

Actually OK with $\mathrm{tr}\,\phi^3$ ✓

but not with $\mathrm{tr}\,\phi F^2$ ✗

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3pt rank 1 => 4-pt rank 3 (no problems) => 5-pt rank 11 (problem: inverse has spurious poles!)

Actually OK with $\mathrm{tr}\,\phi^3$ ✓

but not with $\mathrm{tr}\,ZF^2 + \mathrm{h.c.}$ ✗

We drop cubic orders and start at 4-pt with leading ϕ^4 **?** $d^{abcd}\tilde{d}^{a'b'c'd'}\phi^{aa'}\phi^{bb'}\phi^{cc'}\phi^{dd'}$

4-pt rank 1 (no problems) => 6-pt rank 10 (OK!) => 8-pt rank 273 (spurious poles in the inverse!)

Actually OK with $\mathrm{tr}\,\phi^4$ ✓

Two no-go results, but…

Are there new exact solutions?

Are there new combinations of operators that can give rise to a new form of the double-copy?

Outlook

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3) Also, recent work on higher-derivative terms in the color-factors in the BCJ formulation

   [Carrasco, Rodina, Zekioglu, Z.Yin (2019+2021)]

   => their BCJ-form => BAS + h.d. also with rank (n-3)! (in the examples we have checked)

   => have translated a few examples to their form to ours

   The relationship should be studied more.

Example of exact kernel solution

$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu \phi^{aa'}\right)^2 - \frac{g}{6} f^{abc}\tilde{f}^{a'b'c'}\phi^{aa'}\phi^{bb'}\phi^{cc'}$$
$$-\frac{a_L + a_R}{2\Lambda^4} f^{abx} f^{cdx} f^{a'b'x'} f^{c'd'x'} (\partial_\mu \phi^{aa'})(\partial^\mu \phi^{bb'})\phi^{cc'}\phi^{dd'}$$
$$+\frac{a_L}{\Lambda^4} f^{abx} f^{cdx} d^{a'b'x'} d^{c'd'x'} (\partial_\mu \phi^{aa'})\phi^{bb'}(\partial^\mu \phi^{cc'})\phi^{dd'}$$
$$+\frac{a_R}{\Lambda^4} d^{abx} d^{cdx} f^{a'b'x'} f^{c'd'x'} (\partial_\mu \phi^{aa'})\phi^{bb'}(\partial^\mu \phi^{cc'})\phi^{dd'} + \ldots$$

Exact minimal rank solution at 4pt and 5pt

- L sector unmodified BAS KKBCJ relations
- R sector has modified KKBCJ relations

Kernel

$$S_4[1234|1234] = (m_4[1234|1234])^{-1} = -\frac{g^2 t}{su} - 4\frac{a_R}{\Lambda^4}t$$

YM + h.d.

$$\mathcal{A}_4^L[1^+2^+3^-4^-] = [12]^2\langle 34\rangle^2 \frac{(g_{YM}^L)^2}{su}$$

$$\mathcal{A}_4^R[1^+2^+3^-4^-] = (g_{YM}^R)^2[12]^2\langle 34\rangle^2\left[\frac{1}{su} + \frac{4a_R}{g^2\Lambda^4}\right]$$

Double-copy:

$$\mathcal{M}_4(1^+2^+3^-4^-) = \kappa^2\frac{[12]^4\langle 34\rangle^4}{stu}$$

So: GR = YM x (YM + F^4) !!!

Outlook

1) To the orders checked, the generalized double-copy produces the same h.d. operators in the double-copy LxR amplitude, but with some shifted Wilson coefficients: why?

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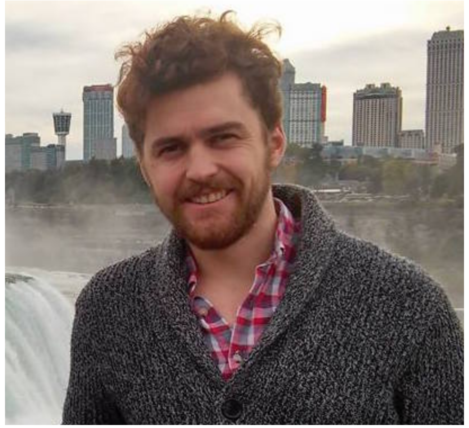
    The relationship should be studied more.

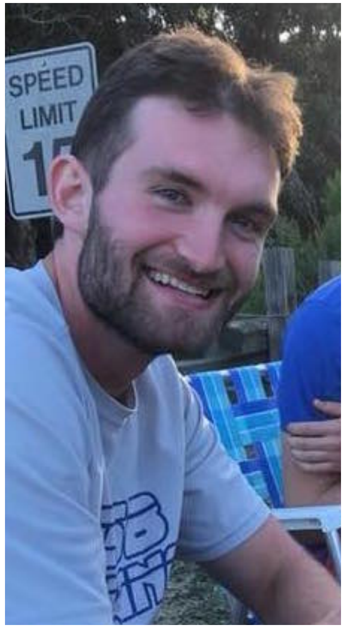
4) Positivity constraints? EFT-hedron? UV completability? What makes the strings kernel special?

Collaborators

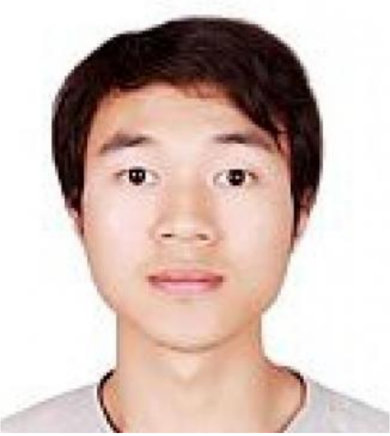


Callum Jones
Graduated April 2020
Postdoc at UCLA

Shruti Paranjape
5th year graduate student
Graduated Spring 2021
-> UC Davis



Aidan Herderschee
3rd year graduate student



HuanHang Chi
postdoc

Thank you

EXTRA: Basis Indep & KKBCJ

What ensures independence of choice of *(n-3)!* basis?

For example, compare $M_n = A^{\mathsf{L}}_n . S_n . A^{\mathsf{R}}_n$, $M_n = A^{\mathsf{L}}_n . S'_n . A^{\mathsf{R}'}_n$

Basis indep. if $0 = S_n . A^{\mathsf{R}}_n - S'_n . A^{\mathsf{R}'}_n$ => $m'_n . S_n . A^{\mathsf{R}}_n = A^{\mathsf{R}'}_n$ => BAS x R = R => $\mathbf{1} \otimes \mathbf{R} = \mathbf{R}$

Similarly, independence of the L sector basis choice is ensured by $\mathbf{L} \otimes \mathbf{1} = \mathbf{L}$

The relations $\mathbf{L} \otimes \mathbf{1} = \mathbf{L}$ and $\mathbf{1} \otimes \mathbf{R} = \mathbf{R}$ combine the Kleiss-Kuijf (KK) and BCJ relations.

The field theory landscape is incredibly rich.

The double-copy is a map among theories that are extremely different:

- **Yang-Mills**: renormalizable theory, part of the Standard Model
- **N=4 SYM**: a conformal field theory, widely used in high energy theory
- **gravity**: non-renormalizable, …. but a phenomenologically amazing EFT!
- **chiral perturbation theory**: low-energy EFT of pions
- **BI** or **sDBI**: low-energy EFT on D-branes
- **special Galileon**: used in cosmology, but by itself a swampland model
- **BAS**: phi^3 theory, potential unbounded from below.

Connected by the "KLT algebra".

The double-copy is part of exploring the space of field theories.

This work is a systematic study of generalizations of the KLT double-copy kernel.