Mass and Locality in the Double Copy

Shruti Paranjape
University of Michigan, Ann Arbor
Leinweber Center for Theoretical Physics

Amplitudes 2021

Based on arXiv:2004.12948
L Johnson, C R T Jones, SP

Double Copy

- Theory C = Theory A $\otimes$ B
- Works for tree-level amplitudes, loop integrands, classical observables, string amplitudes...

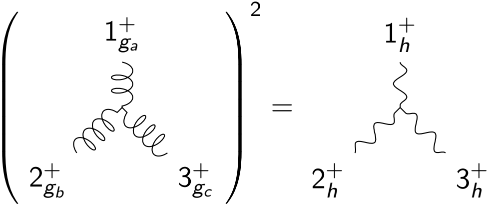
[Kawai, Lewellen, Tye][Bern, Carrasco, Johansson][Cachazo, He, Yuan]

Double Copy

- Theory C = Theory A ⊗ B
- Works for tree-level amplitudes, loop integrands, classical observables, string amplitudes...

[Kawai, Lewellen, Tye][Bern, Carrasco, Johansson][Cachazo, He, Yuan]

Original example : (Yang-Mills)^2 = Gravity + Dilaton + 2-form



Our Goals

- Develop a formalism for the double copy of massive states
- Understand unitarity, massless limits and locality in this novel double copy
- Does massive Yang-Mills double copy to massive gravity?
- Can other massive amplitudes be double-copied?

[Johansson, Ochirov, Naculich, Chiodaroli, Gunaydin, Roiban, Bautista, Guevara...]

The KLT Algebra

$$\mathcal{M}_n^{\mathsf{C}=\mathsf{A}\otimes\mathsf{B}} = \sum_{\alpha,\beta} \mathcal{A}_n^{\mathsf{A}}[\alpha] S_n^{\mathsf{KLT}}[\alpha|\beta] \mathcal{A}_n^{\mathsf{B}}[\beta]$$

[Kawai, Lewellen, Tye]

	ϕ^3	χPT	YM
ϕ^3	ϕ^3	χPT	YM
χPT	χPT	Special Galileon	Born-Infeld
YM	YM	Born-Infeld	Dilaton Gravity

[Cachazo, He, Yuan]

χPT = chiral perturbation theory or non-linear sigma model of pions
ϕ^3 = bi-adjoint scalar theory with interaction $f_{abc} f_{a'b'c'} \phi_{aa'} \phi_{bb'} \phi_{cc'}$

The KLT Algebra

$$\mathcal{M}_n^{\mathsf{C}=\mathsf{A}\otimes\mathsf{B}} = \sum_{\alpha,\beta} \mathcal{A}_n^{\mathsf{A}}[\alpha] S_n^{\mathsf{KLT}}[\alpha|\beta] \mathcal{A}_n^{\mathsf{B}}[\beta]$$

[Kawai, Lewellen, Tye]

	ϕ^3	χPT	YM
ϕ^3	ϕ^3	χPT	YM
χPT	χPT	Special Galileon	Born-Infeld
YM	YM	Born-Infeld	Dilaton Gravity

[Cachazo, He, Yuan]

χPT = chiral perturbation theory or non-linear sigma model of pions
ϕ^3 = bi-adjoint scalar theory with interaction $f_{abc} f_{a'b'c'} \phi_{aa'} \phi_{bb'} \phi_{cc'}$

Bi-adjoint scalar theory is special because it is the identity of the double copy.

Bi-adjoint Scalar Theory

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + g\ f_{abc} f_{a'b'c'} \phi^{aa'} \phi^{bb'} \phi^{cc'}$$

- Doubly-color ordered tree amplitudes

$$\mathcal{A}_n(1^{a_1 a'_1} \cdots n^{a_n a'_n}) = \sum_{\sigma,\gamma} \mathcal{A}_n[1\gamma|1\sigma] \mathrm{Tr}\left[T^{a_1} T^{\gamma(a_2} \ldots T^{a_n)}\right] \times \mathrm{Tr}\left[T^{a'_1} T^{\sigma\left(a'_2\right.} \ldots T^{\left.a'_n\right)}\right]$$

- Kleiss-Kuijf relations $\Rightarrow$ $(n-2)!$ distinct color-orderings
- BCJ relations $\Rightarrow$ $(n-2)! \times (n-2)!$ matrix of amplitudes has rank $(n-3)!$

BAS and the Double Copy

- BAS is the identity so BAS $\times$ BAS $=$ BAS i.e.

$$\sum_{\beta,\gamma} m[\alpha|\beta]S[\beta|\gamma]m[\gamma|\beta] = m[\alpha|\beta]$$

$$\Rightarrow S[\beta|\gamma] = m[\gamma|\beta]^{-1} = m^{-1}[\beta|\gamma]$$

[Cachazo, He, Yuan]

- Inverse of any full-rank $(n-3)! \times (n-3)!$ submatrix gives field theory KLT kernel
- KLT formula must be basis-independent

$$\mathcal{M}_n = \mathcal{A}_n[\alpha]m_n[\alpha|\beta]^{-1}\mathcal{A}_n[\beta] = \mathcal{A}_n[\alpha]m_n[\alpha|\gamma]^{-1}\mathcal{A}_n[\gamma]$$
$$\Rightarrow m_n[\alpha|\beta]^{-1}\mathcal{A}_n[\beta] = m_n[\alpha|\gamma]^{-1}\mathcal{A}_n[\gamma]$$

- For a theory to be double-copyable it must satisfy these BCJ relations

[Bern, Carrasco, Johansson]

The Massless Double Copy

Construct rank $(n-3)!$ matrix of bi-adjoint scalar amplitudes

↓

Take the inverse to construct a KLT kernel

↓

Check that amplitudes $\mathcal{A}_n[\alpha]$ satisfy the BCJ relations

↓

Use KLT formula to construct a local double copy

[Cachazo, He, Yuan][Mizera]

Example: 4-point Massless Double Copy

A simple 4-point example,

$$m[1234|1234] = \frac{1}{s} + \frac{1}{u} \qquad m[1234|1243] = -\frac{1}{s}$$

$$m[1243|1234] = -\frac{1}{s} \qquad m[1243|1243] = \frac{1}{s} + \frac{1}{t}$$

$m[\alpha|\beta]$ has rank 1 $\Rightarrow$ $S[1234|1243] = -s$.

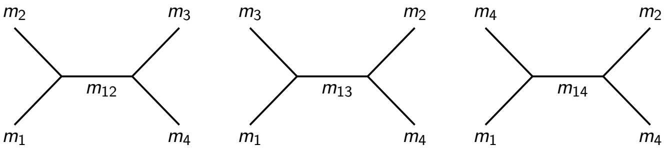
Choosing different bases of orderings gives us consistency conditions or BCJ relations on the single-copy amplitudes,

$$\mathcal{M}_4 = -\frac{us}{t}\mathcal{A}_4[1234]^2 \overset{\text{BCJ}}{=\!=} -\frac{ts}{u}\mathcal{A}_4[1243]^2 \overset{\text{BCJ}}{=\!=} -s\mathcal{A}_4[1234]\mathcal{A}_4[1243]$$

$$\Rightarrow \mathcal{A}_4[1234] \overset{\text{BCJ}}{=\!=} \frac{t}{u}\mathcal{A}_4[1243]$$

Constructing a Massive Version

At 4-point, adding masses to bi-adjoint scalar theory in the most generic fashion:



And making propagator replacements

$$/.\{\mathtt{s_{ij}} \to \mathtt{s_{ij}} + \mathtt{m_{ij}^2}\}$$

gives us a matrix of massive bi-adjoint scalar amplitudes:

$$m_4[\alpha|\beta] = \begin{bmatrix} \frac{1}{s+m_{12}^2} + \frac{1}{u+m_{14}^2} & -\frac{1}{s+m_{12}^2} \\ -\frac{1}{s+m_{12}^2} & \frac{1}{s+m_{12}^2} + \frac{1}{t+m_{13}^2} \end{bmatrix}$$

[Johnson, Jones, SP]

Massive KLT Kernel

At 4-point, rank of the matrix $m[\alpha|\beta]$ is $2 = (n-2)!$ i.e. it is full-rank.

In the $\{1234, 1243\}$ basis of orderings, $S_{\mathrm{KLT}}[\alpha|\beta] = m[\alpha|\beta]^{-1}$

$$= \frac{1}{F(m_i, m_{1i})} \begin{bmatrix} (u+m_{14}^2)(s+t+m_{12}^2+m_{13}^2) & (t+m_{13}^2)(u+m_{14}^2) \\ (t+m_{13}^2)(u+m_{14}^2) & (t+m_{13}^2)(s+u+m_{12}^2+m_{14}^2) \end{bmatrix}$$

where $F(m_i, m_{1i}) = \sum_i m_i^2 - m_{12}^2 - m_{13}^2 - m_{14}^2$.

Massive KLT Kernel

At 4-point, rank of the matrix $m[\alpha|\beta]$ is $2 = (n-2)!$ i.e. it is full-rank.

In the $\{1234, 1243\}$ basis of orderings, $S_{\text{KLT}}[\alpha|\beta] = m[\alpha|\beta]^{-1}$

$$= \frac{1}{F(m_i, m_{1i})} \begin{bmatrix} (u+m_{14}^2)(s+t+m_{12}^2+m_{13}^2) & (t+m_{13}^2)(u+m_{14}^2) \\ (t+m_{13}^2)(u+m_{14}^2) & (t+m_{13}^2)(s+u+m_{12}^2+m_{14}^2) \end{bmatrix}$$

where $F(m_i, m_{1i}) = \sum_i m_i^2 - m_{12}^2 - m_{13}^2 - m_{14}^2$.

- No new poles are generated at 4-point

Massive KLT Kernel

At 4-point, rank of the matrix $m[\alpha|\beta]$ is $2 = (n-2)!$ i.e. it is full-rank.

In the $\{1234, 1243\}$ basis of orderings, $S_{\mathrm{KLT}}[\alpha|\beta] = m[\alpha|\beta]^{-1}$

$$= \frac{1}{F(m_i, m_{1i})} \begin{bmatrix} (u+m_{14}^2)(s+t+m_{12}^2+m_{13}^2) & (t+m_{13}^2)(u+m_{14}^2) \\ (t+m_{13}^2)(u+m_{14}^2) & (t+m_{13}^2)(s+u+m_{12}^2+m_{14}^2) \end{bmatrix}$$

where $F(m_i, m_{1i}) = \sum_i m_i^2 - m_{12}^2 - m_{13}^2 - m_{14}^2$.

- No new poles are generated at 4-point
- Kernel doesn't have a smooth massless limit

Massive KLT Kernel

At 4-point, rank of the matrix $m[\alpha|\beta]$ is $2 = (n-2)!$ i.e. it is full-rank.

In the $\{1234, 1243\}$ basis of orderings, $S_{\text{KLT}}[\alpha|\beta] = m[\alpha|\beta]^{-1}$

$$= \frac{1}{F(m_i, m_{1i})} \begin{bmatrix} (u+m_{14}^2)(s+t+m_{12}^2+m_{13}^2) & (t+m_{13}^2)(u+m_{14}^2) \\ (t+m_{13}^2)(u+m_{14}^2) & (t+m_{13}^2)(s+u+m_{12}^2+m_{14}^2) \end{bmatrix}$$

where $F(m_i, m_{1i}) = \sum_i m_i^2 - m_{12}^2 - m_{13}^2 - m_{14}^2$.

- No new poles are generated at 4-point
- Kernel doesn't have a smooth massless limit
- Only one possible basis $\Rightarrow$ No BCJ relations
  $\Rightarrow$ ANY theory can be double-copied. . . suspicious!

Massive KLT Kernel

At 4-point, rank of the matrix $m[\alpha|\beta]$ is $2 = (n-2)!$ i.e. it is full-rank.

In the $\{1234, 1243\}$ basis of orderings, $S_{\text{KLT}}[\alpha|\beta] = m[\alpha|\beta]^{-1}$

$$= \frac{1}{F(m_i, m_{1i})} \begin{bmatrix} (u+m_{14}^2)(s+t+m_{12}^2+m_{13}^2) & (t+m_{13}^2)(u+m_{14}^2) \\ (t+m_{13}^2)(u+m_{14}^2) & (t+m_{13}^2)(s+u+m_{12}^2+m_{14}^2) \end{bmatrix}$$

where $F(m_i, m_{1i}) = \sum_i m_i^2 - m_{12}^2 - m_{13}^2 - m_{14}^2$.

- No new poles are generated at 4-point
- Kernel doesn't have a smooth massless limit
- Only one possible basis $\Rightarrow$ No BCJ relations
  $\Rightarrow$ ANY theory can be double-copied. . . suspicious!

One can always find unique color-kinematic numerators.

The Massive Double Copy

Construct massive bi-adjoint scalar amplitudes

↓

Take the inverse of a full-rank R_n submatrix to construct a KLT kernel

↓

Use KLT formula to construct a double copy

For generic masses, $R_n = (n-2)!$ and there are no BCJ relations so :

The Massive Double Copy

Construct massive bi-adjoint scalar amplitudes

↓

Take the inverse of a full-rank R_n submatrix to construct a KLT kernel

↓

Use KLT formula to construct a double copy

For generic masses, $R_n = (n-2)!$ and there are no BCJ relations so :

How can we ensure that this double copy is *local*?

The Massive Double Copy

Construct massive bi-adjoint scalar amplitudes

↓

Take the inverse of a full-rank R_n submatrix to construct a KLT kernel

↓

Use KLT formula to construct a double copy

For generic masses, $R_n = (n-2)!$ and there are no BCJ relations so :

How can we ensure that this double copy is *local*?

When are the unique color-kinematic numerators local?

Locality Part 1: Factorization Properties

- Tree-level amplitudes are rational functions with simple poles when intermediate momenta go on-shell.
- The residue on these poles is a product of lower-point trees.
- Unique pole structure of massive bi-adjoint scalar theory ensures

$$\begin{aligned} &\operatorname*{Res}_{s_{12}=m_{12}^2} \mathcal{A}_n(1\cdots n) = \mathcal{A}_3(12(-P_{12})) \times \mathcal{A}_n(P_{12}3\cdots n)\,, \\ \Rightarrow &\operatorname*{Res}_{s_{12}=m_{12}^2} \mathcal{M}_n(1\cdots n) = \mathcal{M}_3(12(-P_{12})) \times \mathcal{M}_n(P_{12}3\cdots n)\,, \end{aligned}$$

where $\mathcal{M}_n = \mathcal{A}_n[\alpha] m_n[\alpha|\beta]^{-1} \mathcal{A}_n[\beta]$.

[Johnson, Jones, SP]

Locality Part 2: Spurious Poles

At 5-point, the kernel has non-physical singularities:

$$m^8\Big(s_{12}^4+\big(2s_{13}+2s_{14}+2s_{23}+2s_{24}-6m^2\big)s_{12}^3+\big(s_{13}^2+2s_{13}s_{14}+2s_{13}s_{23}+4s_{13}s_{24}+s_{14}^2+4s_{14}s_{23}+2s_{14}s_{24}+s_{23}^2$$
$$+2s_{23}s_{24}+s_{24}^2-6m^2s_{13}-6m^2s_{14}-6m^2s_{23}-6m^2s_{24}+m^4\big)s_{12}^2+\big(2s_{13}^2s_{24}+2s_{13}s_{14}s_{23}+2s_{13}s_{14}s_{24}+2s_{13}s_{23}s_{24}$$
$$+2s_{13}s_{24}^2+2s_{14}^2s_{23}+2s_{14}s_{23}^2+2s_{14}s_{23}s_{24}+4m^2s_{13}s_{14}-4m^2s_{13}s_{23}-6m^2s_{13}s_{24}-6m^2s_{14}s_{23}-4m^2s_{14}s_{24}$$
$$+4m^2s_{23}s_{24}-8m^4s_{13}-8m^4s_{14}-8m^4s_{23}-8m^4s_{24}+24m^6\big)s_{12}+s_{13}^2s_{24}^2-2s_{13}s_{14}s_{23}s_{24}+s_{14}^2s_{23}^2+4m^2s_{13}^2s_{14}$$
$$+4m^2s_{13}s_{14}^2+4m^2s_{13}s_{14}s_{23}+4m^2s_{13}s_{14}s_{24}+4m^2s_{13}s_{23}s_{24}+4m^2s_{14}s_{23}s_{24}+4m^2s_{23}^2s_{24}+4m^2s_{23}s_{24}^2-8m^4s_{13}^2$$
$$-20m^4s_{13}s_{14}-8m^4s_{13}s_{23}-8m^4s_{13}s_{24}-8m^4s_{14}^2-8m^4s_{14}s_{23}-8m^4s_{14}s_{24}-8m^4s_{23}^2-20m^4s_{23}s_{24}-8m^4s_{24}^2$$
$$+24m^6s_{13}+24m^6s_{14}+24m^6s_{23}+24m^6s_{24}-20m^8\Big)\,.$$

Locality Part 2: Spurious Poles

At 5-point, the kernel has non-physical singularities:

$$m^8\Big(s_{12}^4+\left(2s_{13}+2s_{14}+2s_{23}+2s_{24}-6m^2\right)s_{12}^3+\left(s_{13}^2+2s_{13}s_{14}+2s_{13}s_{23}+4s_{13}s_{24}+s_{14}^2+4s_{14}s_{23}+2s_{14}s_{24}+s_{23}^2\right.$$
$$\left.+2s_{23}s_{24}+s_{24}^2-6m^2s_{13}-6m^2s_{14}-6m^2s_{23}-6m^2s_{24}+m^4\right)s_{12}^2+\left(2s_{13}^2s_{24}+2s_{13}s_{14}s_{23}+2s_{13}s_{14}s_{24}+2s_{13}s_{23}s_{24}\right.$$
$$+2s_{13}s_{24}^2+2s_{14}^2s_{23}+2s_{14}s_{23}^2+2s_{14}s_{23}s_{24}+4m^2s_{13}s_{14}-4m^2s_{13}s_{23}-6m^2s_{13}s_{24}-6m^2s_{14}s_{23}-4m^2s_{14}s_{24}$$
$$\left.+4m^2s_{23}s_{24}-8m^4s_{13}-8m^4s_{14}-8m^4s_{23}-8m^4s_{24}+24m^6\right)s_{12}+s_{13}^2s_{24}^2-2s_{13}s_{14}s_{23}s_{24}+s_{14}^2s_{23}^2+4m^2s_{13}^2s_{14}$$
$$+4m^2s_{13}s_{14}^2+4m^2s_{13}s_{14}s_{23}+4m^2s_{13}s_{14}s_{24}+4m^2s_{13}s_{23}s_{24}+4m^2s_{14}s_{23}s_{24}+4m^2s_{23}^2s_{24}+4m^2s_{23}s_{24}^2-8m^4s_{13}^2$$
$$-20m^4s_{13}s_{14}-8m^4s_{13}s_{23}-8m^4s_{13}s_{24}-8m^4s_{14}^2-8m^4s_{14}s_{23}-8m^4s_{14}s_{24}-8m^4s_{23}^2-20m^4s_{23}s_{24}-8m^4s_{24}^2$$
$$+24m^6s_{13}+24m^6s_{14}+24m^6s_{23}+24m^6s_{24}-20m^8\Big)\,.$$

Where did these come from?

$$S_{\mathrm{KLT}}[\alpha|\beta]=m[\alpha|\beta]^{-1}=\frac{1}{\det m}(\text{matrix of cofactors})$$

where m is a matrix of amplitudes i.e. contains no spurious singularities.

$\Rightarrow$ The only source of spurious poles is the numerator of $\det m$.

Two Options

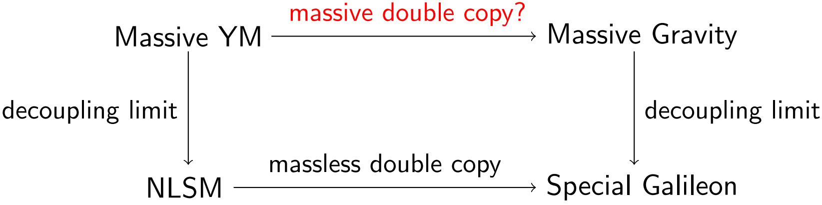
For $\mathcal{M}_n = \mathcal{A}_n[\alpha] m_n[\alpha|\beta]^{-1} \mathcal{A}_n[\beta]$ to be local,

1. KLT kernel has spurious poles but these are canceled by $\mathcal{A}_n$ in the final double-copy $\mathcal{M}_n$
2. KLT kernel has no spurious poles when the masses are carefully tuned: the "minimal rank" condition

A Case Study: Massive Yang-Mills

$$\mathcal{L}_{\text{mYM}} = -\frac{1}{4}\left(\partial_{[\mu}A^a_{\nu]}\right)^2 - \frac{1}{2}m^2 A^a_\mu A^{a\mu} - gf^{abc}A^a_\mu A^b_\nu \partial^\mu A^{c\nu} - \frac{1}{4}g^2 f^{abe}f^{cde}A^a_\mu A^{\mu c}A^b_\nu A^{\nu d}$$
$$= \mathcal{L}_{\text{YM}} - \frac{1}{2}m^2 A^a_\mu A^{a\mu}$$

Goldstone boson equivalence theorem: Longitudinal modes decouple to give NLSM pions in the high energy/massless/decoupling limit.



[Bonafacio, Hinterbichler, deRham, Gabadadze, Tolley, Cheung...]

Can we construct massive gravity from massive YM?

Option 1: Double-Copying Massive Yang-Mills at 3-Point

$$A^\mu \otimes A^\nu = 3 \otimes 3 = 5 \oplus 3 \oplus 1 = h^{\mu\nu} \oplus B^{\mu\nu} \oplus \phi.$$

$$\begin{aligned}\mathcal{A}_3 =& 2g\big((\epsilon_2 \cdot \epsilon_3)(\epsilon_1 \cdot p_2) + (\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot p_3) + (\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot p_1)\big).\\ \mathcal{M}_3 =& \frac{2}{M_p}\big((\epsilon_2 \cdot \epsilon_3)(\epsilon_1 \cdot p_2) + (\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot p_3) + (\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot p_1)\big)^2.\end{aligned}$$

where $\epsilon_i^{\mu\nu} = \epsilon_i^\mu \epsilon_i^\nu$ is replaced by projection operator for different states:

- Dilaton parity is violated so graviton amplitudes have dilaton channels.
- B-parity is preserved.

Option 1: Double-Copying Massive Yang-Mills at 4-Point

$$n_s = [(\epsilon_1 \cdot \epsilon_2) p_1^\mu + 2(\epsilon_1 \cdot p_2)\epsilon_2^\mu - (1 \leftrightarrow 2)]\left(g_{\mu\nu} + \frac{(-p_{1\mu} - p_{2\mu})(p_{3\nu} + p_{4\nu})}{m^2}\right)$$
$$\times [(\epsilon_3 \cdot \epsilon_4) p_3{}^\nu + 2(\epsilon_3 \cdot p_4)\epsilon_4{}^\nu - (3 \leftrightarrow 4)]$$
$$+ (s + m^2)[(\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4) - (\epsilon_1 \cdot \epsilon_4)(\epsilon_2 \cdot \epsilon_3)],$$

with $n_t = n_s|_{1\to3\to2\to1}$ and $n_u = n_s|_{1\to2\to3\to1}$.

$$\mathcal{M}_4 = \frac{{n_s}^2}{s + m^2} + \frac{{n_t}^2}{t + m^2} + \frac{{n_u}^2}{u + m^2}$$

- Factorizes into $\mathcal{A}_3(hh\phi)$ and $\mathcal{A}_3(hhh)$ correctly
- Has no spurious poles
- Matches well-known dRGT theory of ghost-free massive gravity.

[deRham, Gabadadze, Tolley]

[Momeni, Rumbutis, Tolley]

Pions and Special Galileons

$$\mathcal{A}_4(g_L g_L g_L g_L) \xrightarrow{\text{decouple}} \mathcal{A}_4(\pi\pi\pi\pi)$$

If massive double copy and decoupling limit commute, we expect

$$\mathcal{M}_4(h_L h_L h_L h_L) \xrightarrow{\text{decouple}} \mathcal{M}_4(\phi\phi\phi\phi)$$

to be the only non-zero contribution. We find

$$\mathcal{M}(0000) = \frac{7}{144} st(s+t) \quad \checkmark$$
$$\mathcal{M}(2^+000) = -\frac{1}{24\sqrt{6}} st(s+t).$$

$\Rightarrow$ massless limit and massive double copy do not commute.

Option 1: Double-Copying Massive Yang-Mills at 5-Point

No special cancellations between $\mathcal{A}_5^{\text{mYM}}$ and $\det(m_5)$ $\Rightarrow$ Spurious poles $\Rightarrow$ mYM cannot be double-copied to dRGT massive gravity by Option 1!

Option 1: Double-Copying Massive Yang-Mills at 5-Point

No special cancellations between $\mathcal{A}_5^{\text{mYM}}$ and $\det(m_5)$ $\Rightarrow$ Spurious poles $\Rightarrow$ mYM cannot be double-copied to dRGT massive gravity by Option 1!

... in $d = 4$.

Option 1: Double-Copying Massive Yang-Mills at 5-Point

No special cancellations between $\mathcal{A}_5^{\text{mYM}}$ and $\det(m_5)$ $\Rightarrow$ Spurious poles $\Rightarrow$ mYM cannot be double-copied to dRGT massive gravity by Option 1!

... in $d = 4$.

In $d = 3$, the rank reduces to $R_5 = (n-2)! - 1 = 5$:

- 5×5 kernel S_5^{KLT} still has spurious singularities
- Reduction of rank allows for basis choices $\Rightarrow$ BCJ relations
- Topologically massive YM satisfies the 3d BCJ relations
- Cancels non-physical poles to give topologically massive gravity in 3d at 5-point

[Moynihan][Gonzalez, Momeni, Rumbutis]

Two Options

For $\mathcal{M}_n = \mathcal{A}_n[\alpha] m_n[\alpha|\beta]^{-1} \mathcal{A}_n[\beta]$ to be local,

1. KLT kernel has spurious poles but these are canceled by $\mathcal{A}_n$ in the final double-copy $\mathcal{M}_n$ ✓
2. KLT kernel has no spurious poles when the masses are carefully tuned: the "minimal rank" condition

Option 2: Minimal Rank at 4-Point

Conjecture: The KLT prescription for double-copying models with massive states generates physical amplitudes without spurious singularities, and reduces smoothly to the massless double-copy in an appropriate $m \to 0$ decoupling limit, if the associated bi-adjoint scalar matrix has minimal rank $(n-3)!$.

For $n = 4$, this means rank 1:

$$\det m[\alpha|\beta] = \frac{\sum_i m_i^2 - m_{12}^2 - m_{13}^2 - m_{14}^2}{(s+m_{12}^2)(t+m_{12}^2)(u+m_{12}^2)} = 0$$
$$\Rightarrow m_1^2 + m_2^2 + m_3^2 + m_4^2 - m_{12}^2 - m_{13}^2 - m_{14}^2 = 0$$

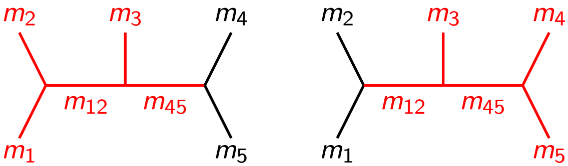
and additionally,

$$\mathcal{A}_4[1234] \overset{\text{mBCJ}}{=\!=\!=} \frac{t+m_{13}^2}{u+m_{14}^2} \mathcal{A}_4[1243]$$

[Johnson, Jones, SP]

Option 2: Minimal Rank at 5-Point

At $n = 5$, impose spectral condition on every 4-point subgraph:



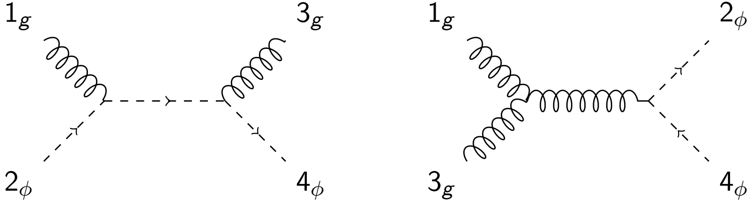
Impose spectral condition on every 4-point subgraph:

- No spurious poles
- Smooth massless limit
- Massive BCJ relations

[Johnson, Jones, SP]

Satisfying the Spectral Conditions: Special Amplitudes

Compton scattering: $g + \phi \to g + \phi$



$$m^2 + m^2 + 0^2 = m^2 + m^2 + 0^2 + 0^2$$

Satisfying the Spectral Conditions: KK Tower

- Theories that result from dimensional reduction e.g. on S^1 have vertices that conserve mass
- For a process 1+2→3+4,

$$m_{12} = m_1 + m_2 \qquad m_{13} = m_1 - m_3$$

$$m_{14} = m_1 - m_4 \qquad m_1 + m_2 = m_3 + m_4$$

$$\Rightarrow m_{12}^2 + m_{13}^2 + m_{14}^2 = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

- So any BCJ-compatible theory with a Kaluza-Klein tower of states and vertices that conserve KK number can be double-copied.

[Johnson, Jones, SP]

Color-kinematic mYM is fixed to KK compactification of 5d YM.

[Momeni, Rumbutis, Tolley]

Summary

- Massive bi-adjoint scalar theory can be used to define the KLT kernel for a massive double copy.
- Double-copy amplitudes constructed in this way factorize correctly.
- To ensure the correct pole structure i.e. locality of the double copy,

	Spectral Condition	Spurious Pole Cancellation
Local	✓	✓
Commutes with massless limit	✓	×
BCJ Relations	✓	×

- mYM does not cancel the spurious poles $\Rightarrow$ only mYM with the correct mass spectrum can lead to a local double copy
- KK towers, Compton scattering can be double-copied.

Up Next:

- Landscape exploration: What theories satisfy the spectral condition/cancel the spurious singularities?
- Multiple masses: Generalize to case of multiple masses exchanged on each channel, string kernel is a special case
- Other ranks: Fundamental matter couplings and d^{abcd} color structures
- Supersymmetry: Are there massive supersymmetric theories that double-copy?

⋮

Thank you for listening!