On-shell Higgsing and the SM EFT

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Technion

YS, Yaniv Weiss 1809.09644
Gauthier Durieux, Teppei Kitahara, YS, Yaniv Weiss 1909.10551
Gauthier Durieux, Teppei Kitahara, Camila Machado, YS, Yaniv Weiss 2008.09652
Reuven Balkin, Gauthier Durieux, Teppei Kitahara, YS, Yaniv Weiss, in progress

August 2021

main goal:

constructing massive EFT amplitudes
= set of independent n-pt massive contact terms

discuss two approaches:

- bottom-up   briefly
- top-down: via (on-shell) Higgsing of EFT amplitudes of *unbroken* theory (=set of independent massless contact terms)

EFfective SM Theories: quantifying our ignorance

where are we?

ad-hoc parametrization of electroweak symmetry breaking in terms of the SM Higgs

- why $\mu^2 < 0$?
- where does scale come from and how is it stabilized?
- is naturalness (& natural extensions of the SM) dead? $(10^{-3} \neq 10^{-30}$ tuning)

turn to bottom-up approach:

- precision measurements of electroweak sector
- EFTs: a well defined-framework for interpreting these measurements (model independently*)

* different possible SM EFTs:
- SM only? SM+X?
- SU(2)$\times$U(1) realized linearly (SMEFT)? Higgs as part of an SU(2) doublet?
SU(2)$\times$U(1) realized non-linearly (HEFT)?

on-shell EFTs: motivation

Effective Field Theory

input:

- SM fields [+ possibly: some BSM fields]
- SM symmetry (global, gauge)
- + Lorentz, locality

$\rightarrow$ most general Lagrangian

$$\mathcal{L}_{\text{EFT}} = \sum_i \frac{c_i}{\Lambda^{n_i}} \mathcal{O}_i$$

on-shell EFTs: motivation

$\rightarrow$ **bottom-up** (bootstrap) on-shell approach is very natural:

YS Weiss 2018; Durieux Kitahara YS Weiss 2019

input:

- SM particles [+ possibly: some BSM particles]
- symmetry (global, gauge)
- + Lorentz, locality

$\rightarrow$ 3-points + $n > 3$ contact-terms $\leftarrow$ finding these is subject of this talk

$\rightarrow$ “glue” to get higher-point/higher-order amplitudes

on-shell EFTs: motivation

1st step of EFT construction: identify basis of operators $\mathcal{O}_i$ (complete, independent)

modulo field redefinitions, gauge redundancies, EOMs

beautifully solved using Hilbert Series Jenkins Manohar; Lehman Martin; Henning Lu Melia Murayama

on-shell: only deal with physical quantities: no fields redefinitions, gauge redundancies

problem translates to finding bases of independent contact terms: independent Lorentz structures (manifestly local): much simpler

working with physical observables only: comparison to experiment should be more transparent: EFT operators $\mathcal{O}_i$ shift masses, SM couplings in broken electroweak theory: input parameters need to be redefined

HE vs LE

on-shell SM EFT(s):
can work in High-Energy (HE) theory: above Higgs VEV v

unbroken SU(2)$\times$U(1): massless amplitudes

or in Low-Energy (LE) theory: below v: broken SU(2)$\times$U(1):

massive amplitudes: W, Z, h, ..

HE amplitudes:

- good approximation at high-energies
- all you care about for running, anomalous dimensions, [positivity]

many results on SMEFT + :

classification of operators

heavy spin-0/1 + gluons dim≤ 13 Shadmi Weiss 2018

SMEFT dim-6 Ma Shu Xiao 2019

dim-8,9 Remmen Rodd 2019; Li Ren Shu Xiao Yu Zheng 2020; Li Ren Xiao Yu Zheng 2020

LEFT dim-8,9 [Li Ren Xiao Yu Zheng 2020]

GR-SMEFT dim-9 Durieux Machado 2020

(non) interference of SM and SMEFT Azatov Contino Machado Riva 2016

anomalous dimensions, operator (non)-mixing, non-renomalization Chueng Shen 2015; Bern Parra-Martinez Sawyer 2020; Miró Ingoldby Riembau 2020; Baratella Fernandez von Harling Pomarol 2020; Jiang Ma Shu 2020

- 2-loop Bern Parra-Martinez Sawyer 2020
- full matrix up to dim-8 + "on-shell (re)derivation of SM" Accettulli Huber, De Angelis 2021

positivity Remmen Rodd 2019, 2020; Gu Wang Zhang 2020

LE theory

can take a purely bottom-up approach — Durieux Kitahara YS Weiss 2019

working with SM spectrum massless+massive:
construct most general 3-pt, 4-pt .. contact terms consistent with baryon and lepton number, EM charge conservation
to recover SM: require perturbative unitarity: $E^{n>0}$ energy growth suppressed by Λ^n

can capture general EFTs with the SM particle content

amplitudes capture all orders in v/Λ expansion

or: bottom-up assuming SU(2)$\times$U(1) — Aoude Machado 2019; Bachu Yelleshpur 2019 either way

need: basis of independent n-point contact terms

contact term basis: massless amplitudes

different strategies:

- strip and append: YS Weiss 2018; Durieux Machado 2019
  for a given helicity amplitude:
  - separate structures into (spinor structure) $\times$ (P: polynomial in Lorentz invariants)
  - [easy] find set of independent spinor structures (no poles allowed in relations)
    $[13][42] = s_{13}/s_{12}[12][34]$ but cannot be eliminated
  - [trivial] append back polynomials: s_{ij}/Λ^2 expansion $\leftrightarrow$ derivative expansion

  set of independent spinor structures: solved systematically for any n point

  Durieux Machado 2019

- harmonics Henning Melia 2019
- twistors Falkowski 2019

contact term basis: massive amplitudes

use little-group covariant massive spinor formalism Arkani-Hmed Huang Huang

massive particle i of momentum $\boldsymbol{p}$ and spin s described by pair of massless spinors

$\boldsymbol{p}]^I, \quad I = 1, 2$ or $\boldsymbol{p}\rangle^I$ $\quad \boldsymbol{p}|\boldsymbol{p}\rangle^I = m|\boldsymbol{p}]^I$

amplitude features $2s$ factors $\boldsymbol{i})\boldsymbol{i})\cdots\boldsymbol{i})$ where $) =]$ or $\rangle$

bold: LG indices symmetrized use to denote massive momenta

contact term basis: massive amplitudes

simple high-energy limit ($E \gg m$) $\leftrightarrow$ unbolding   use heavily

write $p = k + q$ $\boldsymbol{p}]^{I=1} = k]$ $\boldsymbol{p}]^{I=2} = q]$

high-energy limit: $E \gg m$: take $q \sim m^2/E$; $k \sim p$

high-energy limit of a spinor structure obtained by simply unbolding: *e.g.* $\boldsymbol{p}]\boldsymbol{p}] \to k]k]$

note: $\hat{q}$ can be chosen arbitrarily $\leftrightarrow$ spin quantization axis

contact term basis: massive amplitudes

different strategies:

- strip and append: Durieux Kitahara Machado YS Weiss 2020
  - separate structures into (spinor structure) $\times$ (P: polynomial in Lorentz invariants)
  - [hard] find set of independent spinor structures (no poles allowed in relations): Stripped Contact Terms
  - [trivial] append back polynomials: s_{ij}/Λ^2 expansion $\leftrightarrow$ derivative expansion

to find independent set: heavily rely on massless limit: massive relations map to massless relations (unbolding):

$$s_{12}[13][42] - s_{13}[12][34] = 0 \quad \rightarrow \quad s_{12}[\mathbf{13}][\mathbf{42}] - s_{13}[\mathbf{12}][\mathbf{34}] = \mathcal{O}(m)$$

mass completing the relations is not easy

+ masses obscure dimensions of corresponding operators: solved by

$$\boldsymbol{p}]\langle\boldsymbol{p} \rightarrow \boldsymbol{p}]\langle\boldsymbol{p}/\mathbf{m}$$ as appropriate for massive polarization

contact term basis: massive amplitudes

- with this:
  - explicit bases for $n = 3$ any spin
  - explicit bases for $n = 4$ for spins ≤ 1
  - recipe for all n – but not easy to implement (hard part is $\mathcal{O}(m)$ corrections to mass relations; can do without for some purposes)
  - byproduct: simple basis of spinor-structures for all-n *generic* amplitudes (non-local relations allowed): for each combination of contributing external helicities: take (unique) massless spinor-structure and simply bold!

- harmonics: write amplitudes just in terms of $\boldsymbol{i}]^I$ and momenta, then generalize massless construction Dong Ma Shu 2021
  complete solution; easy to automate; but use of EOMs obscures mass dimension; get complicated combinations of structures

4-pt Stripped Contact Term Bases

Durieux Kitahara Machado YS Weiss 2008.09652

spins	n_{SCT}	n_s	hel. cat.	spinor structures	n_{perm}	$\min\{d_{\text{op}}\}$
$ssss$	1	1	(0000)	constant	1	4
$vsss$	$4 \to 3$	3	(0000)	$[121\rangle, [131\rangle$	1	5
			(+000)	$[1231] \to [1231] - \langle 1231\rangle$	$\not{2} \to 1$	7
$ffss$	4	4	(++00)	$[12]$	2	5
			(+−00)	$[132\rangle$	2	6
$vvss$	$10 \to 9$	9	(0000)	$[12]\langle 12\rangle, [131\rangle[232\rangle$	1	4,6
			(+000)	$[12][132\rangle$	4	6
			(++00)	$[12]^2$	2	6
			(+−00)	$[132\rangle^2 \to [132\rangle^2 - \langle 132]^2$	$\not{2} \to 1$	8
$ffvs$	$14 \to 12$	12	(++00)	$[12]\{[313\rangle, [323\rangle\}$	2	6
			(+−00)	$[13]\langle 23\rangle$	2	5
			(+++0)	$[13][23]$	2	6
			(++−0)	$[12]\langle 3123\rangle \to \varnothing$	$\not{2} \to 0$	8
			(+−+0)	$[13][312\rangle$	4	7
$ffff$	18	16	(++++)	$[12][34], [13][24]$	2	6
			(++−−)	$[12]\langle 34\rangle$	6	6
			(+++−)	$[12][324\rangle$	8	7
$vvvs$	$35 \to 29$	27	(0000)	$[12][343\rangle\langle 12\rangle, [13][242\rangle\langle 13\rangle, [23][141\rangle\langle 23\rangle$	1	5
			(+000)	$[12][13]\langle 23\rangle$	6	5
			(++00)	$[12]^2\{[313\rangle, [323\rangle\}$	6	7
			(+−00)	$[13][132\rangle\langle 23\rangle$	6	7
			(+++0)	$[12][13][23]$	2	7
			(++−0)	$[12]^2\langle 3123\rangle \to \varnothing$	$\not{6} \to 0$	9
$vvff$	$46 \to 38$	36	(00++)	$\langle 12\rangle \times \{[12][34], [13][24]\}$	2	5
			(00+−)	$\langle 14\rangle\langle 231][23], \langle 24\rangle\langle 132][13]$	2	6
			(0−++)	$\langle 12\rangle[34]\langle 241] \to \langle 12\rangle[34](\langle 241]/m_1 - \langle 142]/m_2)$	$\not{4} \to 2$	7
			(0+++)	$\langle 132] \times \{[12][34], [13][24]\}$	4	7
			(0++−)	$\langle 14\rangle[12][23]$	8	6
			(+++−)	$[12]^2[314\rangle$	4	8
			(++++)	$[12] \times \{[12][34], [13][24]\}$	2	7
			(−+++)	$\langle 1231\rangle[23][24] \to \varnothing$	$\not{4} \to 0$	9
			(++−−)	$[12]^2\langle 34\rangle$	2	7
			(+−−+)	$[14][132\rangle\langle 23\rangle \to [14][132\rangle\langle 23\rangle - [24][231\rangle\langle 13\rangle$	$\not{4} \to 2$	8
$vvvv$	$116 \to 85$	81	(0000)	$\{[12][34], [13][24]\} \times \{\langle 12\rangle\langle 34\rangle, \langle 13\rangle\langle 24\rangle\}$	1	4
			(+000)	$\{[12][34], [13][24]\} \times [142\rangle\langle 34\rangle \to \cdots$	$\not{8} \to 6$	6
			(++00)	$\{[12][34], [13][24]\} \times [12]\langle 34\rangle$	12	6
			(+−00)	$[13][14]\langle 23\rangle\langle 24\rangle$	12	6
			(+++0)	$\{[12][34], [13][24]\} \times [23][134\rangle$	8	8
			(++−0)	$[12]^2\langle 34\rangle\langle 324] \to [12]^2\langle 34\rangle(\langle 324]/m_4 - \langle 423]/m_3) \to \cdots$	$\not{24} \to \not{12} \to 5$	8
			(++++)	$[12]^2[34]^2, [12][13][24][34], [13]^2[24]^2$	2	8
			(+++−)	$[12][13][23]\langle 4124\rangle \to \varnothing$	$\not{8} \to 0$	10
			(++−−)	$[12]^2\langle 34\rangle^2$	6	8

used massless $\leftrightarrow$ massive relations heavily

let's take it another step for Higgsed EFTs:

start with massless contact terms $\rightarrow$ massive contact terms

Higgs mechanism (amplitudes way)

rich history:

(1) $\mathcal{N}=4$ Coulomb branch amplitudes — Craig Elvang Kiermaier Slatyer 2011

derive massive Coulomb branch amplitudes via soft Higgs limits

(2) IR unification of UV amplitudes — Arkani-Hamed Huang Huang 2017

massive IR amplitude: for different polarizations of massive external particles

$\leftrightarrow$ different massless UV amplitudes

Higgs mechanism (amplitudes way)

rich history:

(1) $\mathcal{N}=4$ Coulomb branch amplitudes — Craig Elvang Kiermaier Slatyer 2011

derive massive Coulomb branch amplitudes via soft Higgs limits

generalize to non-supersymmetric theories, and in particular SM
little-group covariant massive spinor formalism of AHHH vs Kleiss-Striling; Dittmaier

(2) IR unification of UV amplitudes — Arkani-Hamed Huang Huang 2017

massive IR amplitude: for different polarizations of massive external particles
$\leftrightarrow$ different massless UV amplitudes

detailed understanding of (1) $\leftrightarrow$ (2)

EFT amplitudes (contact terms) are simple: easy starting point

Massive EFTs from on-shell Higgsing (SMEFT)

Balkin Durieux Kitahara YS Weiss to appear

assume: masses originate from Higgsing $(m \lesssim v \ll \Lambda)$ [as in SMEFT]

low-E: an n-point contact-term $\leftrightarrow$ an independent Wilson coefficient
[not related to lower-point couplings]

# of independent LE Wilson coefficients = # of independent HE Wilson coefficients (possibly dressed by powers of v/Λ)

$\rightarrow$ start with HE theory ($v < E < \Lambda$)
write down all contact terms (easy)
each of these gives LE (massive) contact term

how?

- bold to get correct massive LG (=covariantize wrt to SU(2) LG)
- freeze Higgs momentum q: $q + k = \mathbf{p}$ to get lower-point amplitude(s)

essentially match HE amplitude and LE amplitude at scale v:

HE: factorizable + non-factorizable

$$\mathcal{A}_n^{\text{nf}}(1^{h_1}, \ldots, n^{h_n}) = \sum_r a_r S_r^{\{h\}}$$

a_r = Wilson coefficient

$S_r^{\{h\}}$ = spinor structure structure ([massless] little group(s) weight)

$\times$ polynomial in Lorentz invariants, *e.g.* s_{ij}/Λ^2 (little group neutral)

bold = covariantize wrt massive little group(s)

$$\rightarrow \quad \sum_r a_r \boldsymbol{S}_r^{\{I\}}$$

match leading component (in E/m) of $\boldsymbol{S}_r^{\{I\}}$ to HE structure $S_r^{\{h\}}$

e.g. massless ($h = +1/2$) fermion: $i] \quad \leftrightarrow \quad \boldsymbol{i}]^{I=1}$

$\rightarrow$ $i]$ bolds to $\boldsymbol{i}]$

e.g. massless ($h = +1$) vector: $i]i] \quad \leftrightarrow \quad \boldsymbol{i}]^{I=1}\boldsymbol{i}]^{I=1}$

$\rightarrow$ $i]i]$ bolds to $\boldsymbol{i}]\boldsymbol{i}]$

subleading components (*e.g.* $\boldsymbol{i}]^{I=2}$) come from Higgsing: factorizable amplitudes with extra Higgs legs (see soon)

longitudinal vectors

longitudinal vectors $\leftrightarrow$ scalars in HE theory (zero little group weight)

to consistently bold HE amplitudes: scalar momentum must appear in HE amplitude

e.g. HE amplitude: $p_i = i]\langle i$

$\rightarrow$ $i]\langle i$ bolds to $\boldsymbol{i}]\langle\boldsymbol{i}$

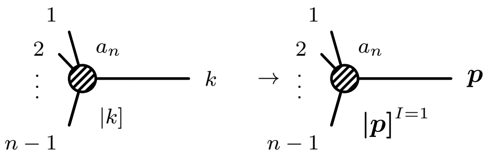
not surprising: LE longitudinal vector $\leftrightarrow$ HE Goldstone; derivatvely coupled so HE amplitude $\propto p_i$ (Adler zero)

The effects of extra Higgs legs

bolding as higgsing

above relied on matching leading components

blob = coupling (no propagators)



recall

$$p = k + q \qquad (k+q)^2 = m^2 \quad \langle kq \rangle = [qk] = m$$

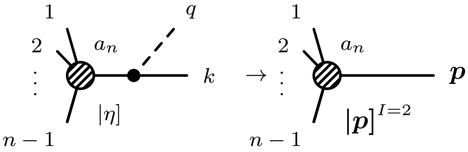
$$\boldsymbol{p}]^{I=1} = k] \qquad \boldsymbol{p}]^{I=2} = q]$$

$$\left[\text{choose:} \quad E \gg m : \quad k \sim p \quad q = \mathcal{O}(m^2/E)\right]$$

bolding as higgsing: fermion leg

subleading components: from HE amplitudes with extra Higgs legs (and Higgs momentum "frozen"):

- *freeze* Higgs momentum q s.t. $(k+q)^2 = m^2$
- isolate $1/(k+q)^2$ pole
- multiply by v (matching $n+1$-pt to n-pt at v)



with $\eta = k + q$

$$v \times \frac{1}{(k+q)^2} \times y\,\langle k\eta\rangle \times a_n[\eta\cdots) = \frac{(yv)}{m^2}\,a_n\,\langle k(k+q)\cdots)$$

$$= \frac{1}{m}\,a_n\,\langle kq\rangle[q\cdots) = a_n\,[\boldsymbol{p}^{I=2}\cdots)$$

bolding as higgsing

in principle: to get a LE n-pt contact term $(\boldsymbol{p}_1, \ldots, \boldsymbol{p}_n)$ (all components):

start with the HE $2n$-pt contact term with n extra Higgs legs: $(k_1, \ldots, k_n; q_1, \ldots, q_n)$

with Higgs momenta frozen $(k_i + q_i)^2 = m_i^2$
and proceed as above

momentum conservation holds all along: $\sum k_i + \sum q_i = 0$

in practice: just need to match leading component
and: can be more economical: reuse the Higgs momentum as "the q" for multiple massive momenta: $\hat{p}_{Higgs} = \hat{q}_i$

as in Craig et al 2010

with this choice, momentum conservation does not hold all along

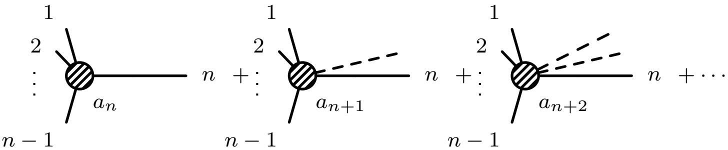
will have more to say about different choices of q_i later so far: all the Higgsing we saw $\leftrightarrow$ Lorentz (= little group; covariantization wrt massive little group)

now to actual Higgsing (which is theory dependent):
diagrams with extra Higgs legs correct Wilson coefficients

in particular, generate "forbidden" Wilson coefficients $\propto v$

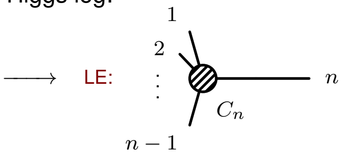
Extra Higgs legs: corrections to the Wilson coefficients

HE:



freeze Higgs momenta (*e.g.* identify with q_1 such that $(k_1 + q_1)^2 = m_1^2$)
and multiply by v for each Higgs leg:

equivalent to taking soft Higgs limit



$$C_n = a_n + v a_{n+1} + \cdots$$

not all contributions are non-zero (gauge symmetry): get v^2/Λ^2 expansion

Extra Higgs legs: corrections to the Wilson coefficients

in SM:

if $a_n \neq 0$ then $a_{n+1} = 0$ etc

if $a_n = 0$ it is typically generated from a_{n+1} at $\mathcal{O}(v)$

just as EFT Lagrangian is a function of $v + h$:
an amplitude with an external Higgs leg and Wilson coefficient c is accompanied by an amplitude with one less Higgs leg and coefficient vc

the amplitude "knows" the distinction between physical Higgs and Goldstone: an external Goldstone leg produces at least one factor of the momentum q: goes to zero in this limit; only h legs contribute

Massive EFTs from on-shell Higgsing (SMEFT)

example: want LE: $A_4(\bar{u}dWh)$ amplitude $[(f, f, V, S)]$

need HE massless amplitudes:

- $M_4(ffVS)$
[4-pts: same spins as desired amplitude]

- $M_4(ffSS)$
[4-pts: longitudinal vector from scalar]

- $M_5(ffVSS)$
[5-pts: same spins; extra Higgs required by gauge symmetry]

- $M_5(ffSSS)$
[5-pts: longitudinal vector from scalar; extra Higgs required by gauge symmetry]

helicities implicit; 4-pts & 5-pts exhaust list of independent spinor structures

$A_4(\bar{u}dWh)$

(i) HE amplitude features same spin as LE amplitude

$$M_4(ffVS): \quad \text{independent contact terms}: [13][23], \quad [12]\langle 3123\rangle$$

just need to bold to get massive contact-terms:

$$[\mathbf{13}][\mathbf{23}], \quad [\mathbf{12}]\langle \mathbf{3123}\rangle$$

showing “half” of fermion, vector helicities: opposite helicities by exchanging squares and brackets

$A_4(\bar{u}dWh)$

(ii) LE longitudinal vector $\leftrightarrow$ HE scalar:

$$M_4(ffSS):\;[132\rangle$$

bold:

$$[132\rangle = [13]\langle 32\rangle \to [\mathbf{13}]\langle\mathbf{32}\rangle$$

3-scalar $\rightarrow$ 3-vector

$A_4(\bar{u}dWh)$

(iii) LE amplitude $\leftrightarrow$ HE amplitude with same spin content plus additional scalar leg(s) required to restore gauge invariance.

$$M_5(ffVSS) : [13]\langle 243] \;\; [13]\langle 253] \, , \;\; [1243]\langle 243]$$

set Higgs (say 5) to its VEV $\rightarrow$ freeze momentum:
identify: $5 \rightarrow q_3$, $3 \rightarrow k_3$: $\rightarrow 3+5 \rightarrow \mathbf{5}$

$$[13]\langle 243] \rightarrow \mathbf{[13]\langle 243]}$$

others redundant (using momentum conservation $p_4 = -p_1 - p_2 - p_3$)

$A_4(\bar{u}dWh)$

(iv) LE amplitude: longitudinal vector $\leftrightarrow$ HE amplitude: scalar
plus additional scalar leg(s) required to restore gauge invariance

$$M_5(ffSSS) : [1342], \quad [12]$$

$$[1342] \to [13]\langle 342] \to \mathbf{[13]\langle 342]}$$

$$[12]s_{13} \to \mathbf{[12]\langle 313]} \, ..$$

- just used Lorentz (little group): general
- here 4, 5-pts suffice; more generally: list of spinor structures exhausted at some point (more work)

- specifying to SMEFT: can also match couplings: get LE broken phase couplings in terms of SMEFT (unbroken) Wilson coefficients

$A_4(\bar{u}dWh)$

Q = SU(2) doublets, U, D = SU(2) singlets

$$|2\rangle \leftrightarrow Q_2 = d\,, \quad |2] \leftrightarrow D\,, \quad |1] \leftrightarrow Q^{c,1} = u\,, \quad |1\rangle \leftrightarrow U^c\,,$$

LE amplitude arises (up to dim-8) from:

the 4-pt amplitudes:

$$\mathcal{A}_4(1_{Q^c}, 2_D, 3_W^{h_3}, 4_H)\,, \quad \mathcal{A}_4(1_{U^c}, 2_Q, 3_W^{h_3}, 4_H)\,,$$
$$\mathcal{A}_4(1_{Q^c}, 2_Q, 3_H, 4_{H^\dagger})\,, \quad \mathcal{A}_4(1_{U^c}, 2_D, 3_H, 4_H)$$

and the five-point amplitudes:

$$\mathcal{A}_5(1_{Q^c}, 2_Q, 3_W^{h_3}, 4_H, 5_{H^\dagger})\,, \quad \mathcal{A}_5(1_{Q^c}, 2_D, 3_H, 4_H, 5_{H^\dagger})\,,$$
$$\mathcal{A}_5(1_{U^c}, 2_Q, 3_H, 4_H, 5_{H^\dagger})\,, \quad \mathcal{A}_5(1_{U^c}, 2_D, 3_W^{h_3}, 4_H, 5_H)$$

Example: bolding

$$\mathcal{A}_5(1_{Q^c}, 2_D, 3_H, 4_H, 5_{H^\dagger}) = [12]\, P_- + ([1342] - [1432])\ P_+$$

[for an antisymmetric combination of $H(3) - H(4)$ SU(2) indices: $P_\pm$, are polynomials in the Lorentz invariants, (anti)symmetric in $3-4$ exchange]

want to match to LE $\bar{u}dWh$ amplitude: 3 $\rightarrow$ vector leg

work up to dim-8:

to bold first term need p_3 in P_-:

$$P_- : \frac{a_1}{\Lambda^4}(s_{13} - s_{14}) + \frac{a_2}{\Lambda^4}(s_{23} - s_{24}) = \frac{a_1}{\Lambda^4}(s_{13} - s_{23}) + \frac{a_2}{\Lambda^4}(s_{23} - s_{13})$$
$$\rightarrow \frac{a_1 - a_2}{\Lambda^4}[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle$$

$\rightarrow$ LE $\bar{u}dWh$ contact term: $\dfrac{v(a_1 - a_2)}{\Lambda^4}[\mathbf{12}][\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle$

Example: bolding

2nd term: $[1342] - [1432] \rightarrow \langle \mathbf{3(2-1)3}][\mathbf{12}]$

so up to dim-8: P_+ : $\dfrac{a_3}{\Lambda^4}\, s_{34}$

$\rightarrow$ LE $\bar{u}dWh$ contact term $\dfrac{v\, a_3}{\Lambda^4}\, s_{34}\, \langle \mathbf{3(2-1)3}][\mathbf{12}]$

so: get LE contact terms in terms of HE Wilson coefficients

easy to extend to higher orders

SM contact term

SM: $WWhh$ contact term suppressed by M_W^2

can get via bottom-up bootstrap:

- construct the relevant LE 3-pt amplitudes
- construct the factorizable 4-pt LE amplitude from these
- fix the contact term coefficient by requiring perturbative unitarity Bachu Yelleshpur 2019

but can also get it top-down:

SM contact term

the HE SM amplitude: [omitting group theory & numerical factors]

$$\mathcal{A}_4(W^+, W^-, H^\dagger, H) = g_2^2 \frac{[1_k 3_k]\langle 2_k 3_k\rangle}{[2_k 3_k]\langle 1_k 3_k\rangle}$$

to match to massive LE amplitude choose $1_q \propto 2_q \propto 3_k$:

$$\mathcal{A} = g_2^2 \frac{[1_k 2_q]\langle 2_k 1_q\rangle}{[2_k 2_q]\langle 1_k 1_q\rangle} = g_2^2 \frac{[1_k 2_q]\langle 2_k 1_q\rangle}{m_W^2} \to g_2^2 \frac{[\mathbf{12}]\langle \mathbf{12}\rangle}{m_W^2}$$

with numerical factors get correct contact term!

SM contact term

full LE $WWhh$ amplitude (for transverse Ws) matches full HE $WWH^\dagger H$ amplitude

full LE amplitude = factorizable LE amplitude + contact term

HE amplitude = $s-$, $t-$, $u-$ exchange diagrams + $WWH^\dagger H$ vtx

LE q_1/m_W, q_2/m_W $\leftrightarrow$ HE reference momenta r_1, r_2

for some choice of $q_i(r_i)$ can get rid of “contact term” piece well-known exercise in Elvang & Huang..
for the choice above do opposite: keep just “contact term”

LE contact term is needed for gauge invariance of HE amplitude

SM contact term

above looked at transverse Ws

LE $WWhh$ with longitudinal Ws:
contact term & LE factorizable amplitude each scale as s_{12}/M_W^2: contact term restores perturbative unitarity

clearly see HE gauge symmetry $\leftrightarrow$ perturbative unitarity

Conclusions

EFTs are taking center-stage
reflecting our ignorance (very healthy)

on-shell approach is natural for constructing EFT extensions of low-energy SM
= all possible couplings of SM particles consistent with Lorentz, global symmetries, locality and unitarity

Conclusions

saw methods for constructing massive amplitude bases: either directly, or by "Higgsing" massless EFT amplitudes

? landscape of massive EFTs obtain *any* massive theory from *some* pattern of Higgsing

? beyond EFTs: develop understanding of "on-shell Higgsing" in non-supersymmetric theories: massive recursion relations? Franken Schwinn 2019; Falkowski Machado 2020

see talk by Camila Machado

? connection to soft bootstrap: bottom-up constructions: Cheung Moss 2021

? calculate: framework for EFT computations for LHC processes directly based on physical observables

THANK YOU!

Durieux Kitahara Machado YS Weiss 2008.09652

spins	n_{SCT}	n_s	hel. cat.	spinor structures	n_{perm}	$\min\{d_{op}\}$
$ssss$	1	1	(0000)	constant	1	4
$vsss$	$4\to 3$	3	(0000)	$[121\rangle, [131\rangle$	1	5
			(+000)	$[1231] \to [1231] - \langle 1231\rangle$	$\not{2}\to 1$	7
$ffss$	4	4	(++00)	$[12]$	2	5
			(+−00)	$[132\rangle$	2	6
$vvss$	$10\to 9$	9	(0000)	$[12]\langle 12\rangle, [131\rangle[232\rangle$	1	4,6
			(+000)	$[12][132\rangle$	4	6
			(++00)	$[12]^2$	2	6
			(+−00)	$[132\rangle^2 \to [132\rangle^2 - \langle 132]^2$	$\not{2}\to 1$	8
$ffvs$	$14\to 12$	12	(++00)	$[12]\{[313\rangle, [323\rangle\}$	2	6
			(+−00)	$[13]\langle 23\rangle$	2	5
			(+++0)	$[13][23]$	2	6
			(++−0)	$[12]\langle 3123\rangle \to \varnothing$	$\not{2}\to 0$	8
			(+−+0)	$[13][312\rangle$	4	7
$ffff$	18	16	(++++)	$[12][34], [13][24]$	2	6
			(++−−)	$[12]\langle 34\rangle$	6	6
			(+++−)	$[12][324\rangle$	8	7
$vvvs$	$35\to 29$	27	(0000)	$[12][343\rangle\langle 12\rangle, [13][242\rangle\langle 13\rangle, [23][141\rangle\langle 23\rangle$	1	5
			(+000)	$[12][13]\langle 23\rangle$	6	5
			(++00)	$[12]^2\{[313\rangle, [323\rangle\}$	6	7
			(+−00)	$[13][132\rangle\langle 23\rangle$	6	7
			(+++0)	$[12][13][23]$	2	7
			(++−0)	$[12]^2\langle 3123\rangle \to \varnothing$	$\not{6}\to 0$	9
$vvff$	$46\to 38$	36	(00++)	$\langle 12\rangle \times \{[12][34], [13][24]\}$	2	5
			(00+−)	$\langle 14\rangle\langle 231][23], \langle 24\rangle\langle 132][13]$	2	6
			(0−++)	$\langle 12\rangle[34]\langle 241] \to \langle 12\rangle[34](\langle 241]/m_1 - \langle 142]/m_2)$	$\not{4}\to 2$	7
			(0+++)	$\langle 132] \times \{[12][34], [13][24]\}$	4	7
			(0++−)	$\langle 14\rangle[12][23]$	8	6
			(+++−)	$[12]^2[314\rangle$	4	8
			(++++)	$[12] \times \{[12][34], [13][24]\}$	2	7
			(−+++)	$\langle 1231\rangle[23][24] \to \varnothing$	$\not{4}\to 0$	9
			(++−−)	$[12]^2\langle 34\rangle$	2	7
			(+−−+)	$[14][132\rangle\langle 23\rangle \to [14][132\rangle\langle 23\rangle - [24][231\rangle\langle 13\rangle$	$\not{4}\to 2$	8
$vvvv$	$116\to 85$	81	(0000)	$\{[12][34], [13][24]\} \times \{\langle 12\rangle\langle 34\rangle, \langle 13\rangle\langle 24\rangle\}$	1	4
			(+000)	$\{[12][34], [13][24]\} \times [142\rangle\langle 34\rangle \to \cdots$	$\not{8}\to 6$	6
			(++00)	$\{[12][34], [13][24]\} \times [12]\langle 34\rangle$	12	6
			(+−00)	$[13][14]\langle 23\rangle\langle 24\rangle$	12	6
			(+++0)	$\{[12][34], [13][24]\} \times [23][134\rangle$	8	8
			(++−0)	$[12]^2\langle 34\rangle\langle 324] \to [12]^2\langle 34\rangle(\langle 324]/m_4 - \langle 423]/m_3) \to \cdots$	$\not{24}\to\not{12}\to 5$	8
			(++++)	$[12]^2[34]^2, [12][13][24][34], [13]^2[24]^2$	2	8
			(+++−)	$[12][13][23]\langle 4124\rangle \to \varnothing$	$\not{8}\to 0$	10
			(++−−)	$[12]^2\langle 34\rangle^2$	6	8