Loop-Level Double Copy for Massive Quantum Particles

Ingrid A. Vazquez-Holm

Based on work with John Joseph Carrasco

IPhT SACLAY

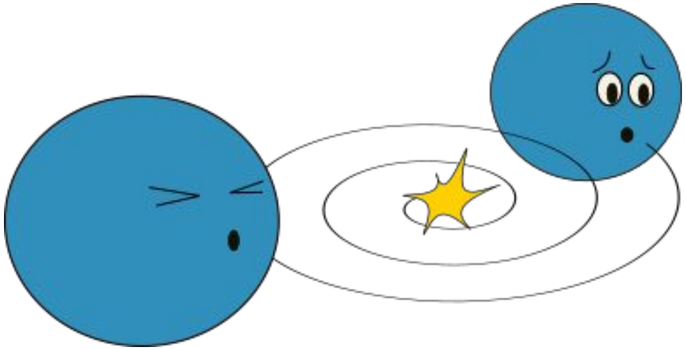
SAGEX

Scattering Amplitudes: from Geometry to Experiment

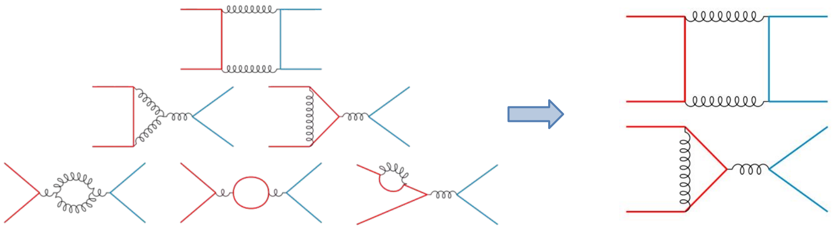
This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 764850

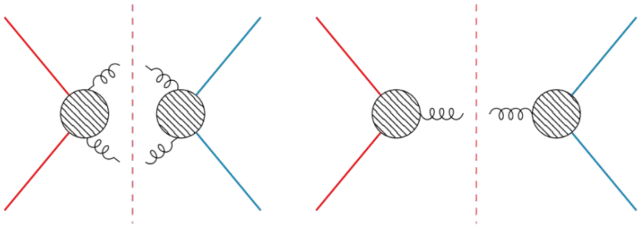
Amplitudes, 17 August 2021

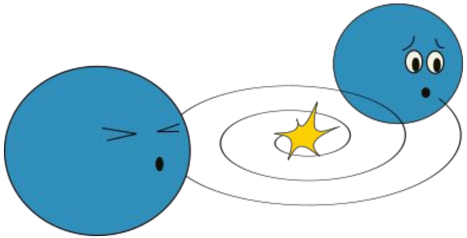
Why is gravity hard with traditional methods?



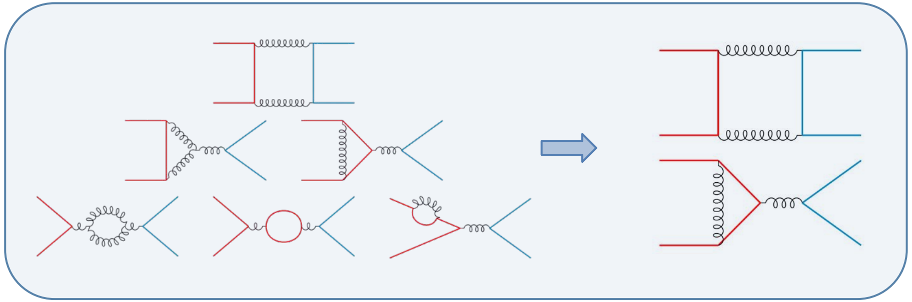
Use the color-kinematics duality and unitarity methods to find massive amplitudes

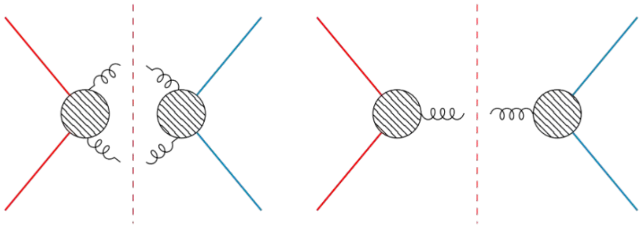


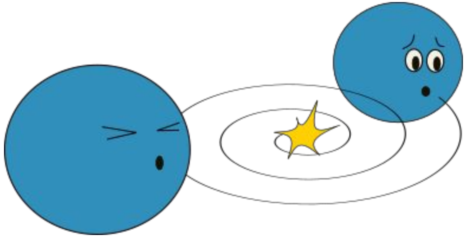




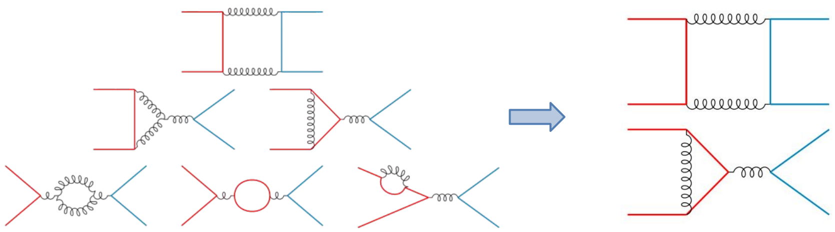
Use the color-kinematics duality and unitarity methods to find massive amplitudes

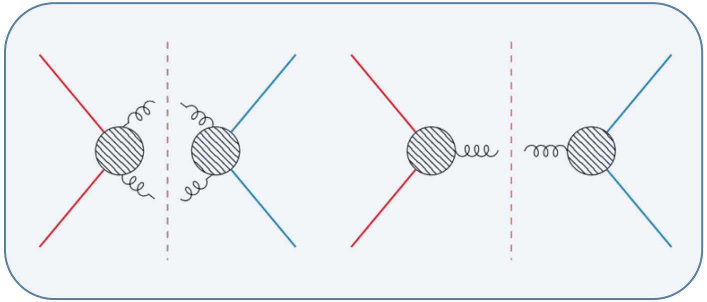


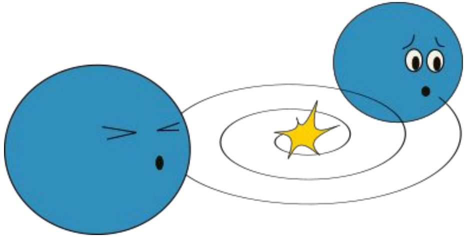




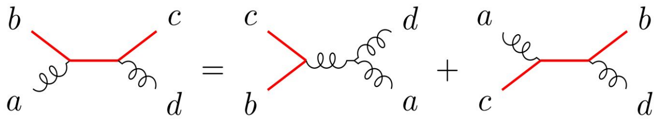
Use the color-kinematics duality and unitarity methods to find massive amplitudes



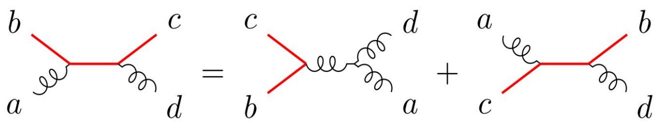


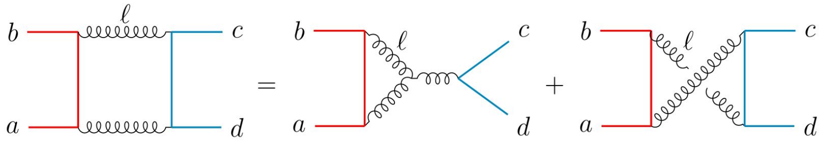


Color-kinematics and factorization are enough to bootstrap massive scalar amplitudes



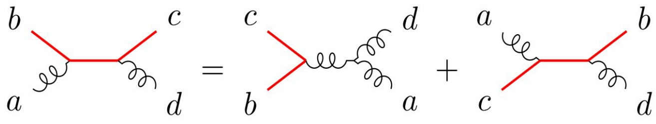
Color-kinematics and factorization are enough to bootstrap massive scalar amplitudes

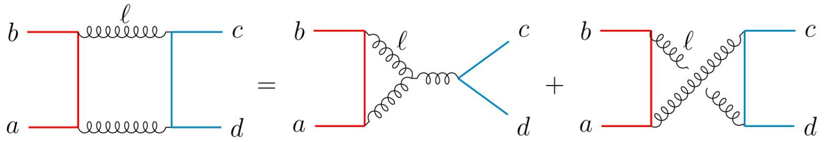




Color-kinematics duality for massive amplitudes can be manifest at loop level

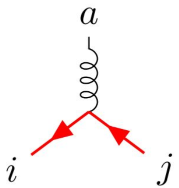
Color-kinematics and factorization are enough to bootstrap massive scalar amplitudes



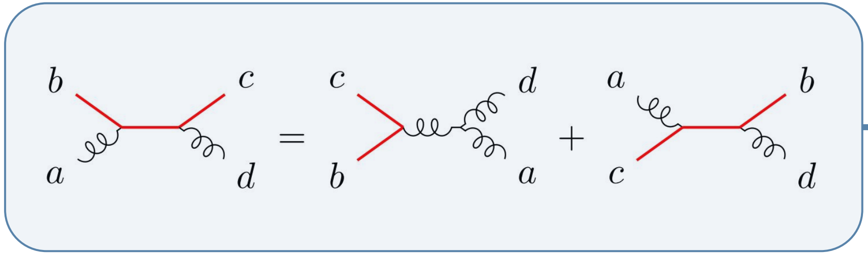


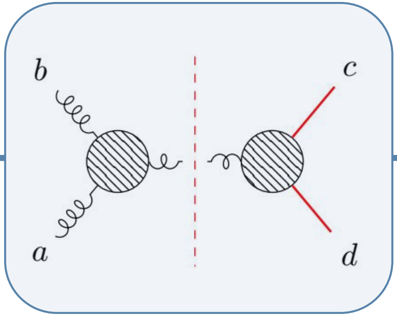
Color-kinematics duality for massive amplitudes can be manifest at loop level

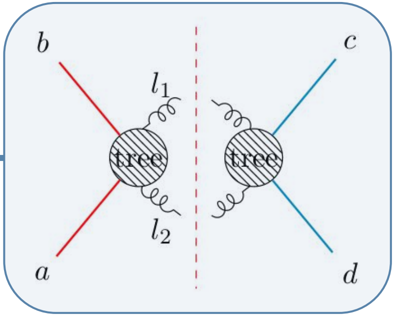
Can use the same kinematic building blocks for massive scalars charged in fundamental and adjoint



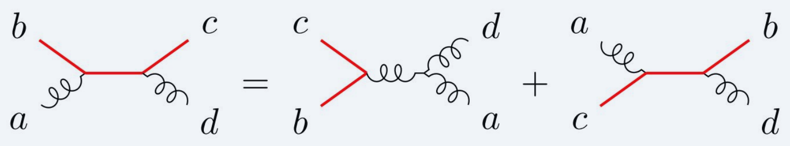
Bootstrapping tree-level amplitudes and loop-level using the color-kinematics duality







The color-kinematics duality



Color-kinematics duality

Color-dual representation: kinematic weights ~ color weights

Bern, Carrasco, Johansson '08,
Bern, Carrasco, Johansson '10

Only need small number of basis graphs (Solves combinatorics problem!)

Weaves a web of theories (Recycling is good)

See review: Bern, Carrasco, Chiodaroli, Johansson, Roiban '19

Color-kinematics for many representations (adjoint, three-algebras, arbitrary)

Bargheer, He, McLoughlin '12

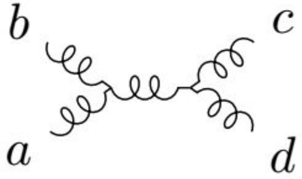
Massive matter in the fundamental

Johansson, Ochirov '16, Plefka, Shi, Wang '19,
Bjerrum-Bohr, Cristofoli, Damgaard, Gomez '19, Luna, Nicholson, O'Connell, White '17
Haddad, Helset '20

Color-kinematics duality relates kinematic weights of graphs

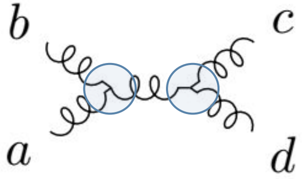
$$\mathcal{A}_m^{(L)} = i^L g^{m-2+2L} \sum_{i\in\Gamma} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i C_i}{\prod_{\alpha_i}(p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

Color-kinematics duality relates kinematic weights of graphs



$$\frac{n_i C_i}{\prod_{\alpha_i}(p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

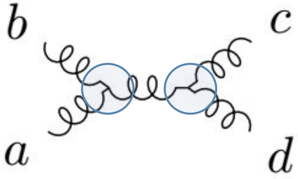
Color-kinematics duality relates kinematic weights of graphs



$$\frac{n_i \boxed{C_i}}{\prod_{\alpha_i}(p^2_{\alpha_i} - m^2_{\alpha_i})}$$

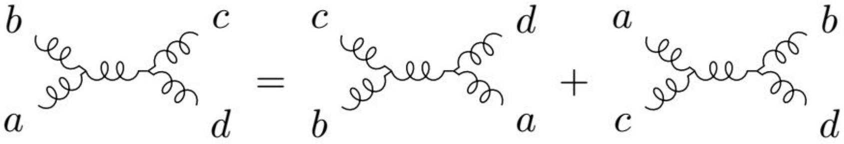
$$f^{abl} f^{lcd}$$

Color-kinematics duality relates kinematic weights of graphs

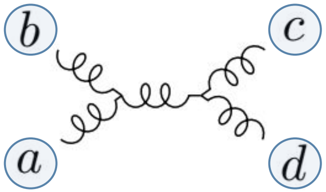


$$\frac{n_i \boxed{C_i}}{\prod_{\alpha_i}(p^2_{\alpha_i} - m^2_{\alpha_i})}$$

$$f^{abl} f^{lcd} = f^{bcl} f^{lda} + f^{cal} f^{lbd}$$

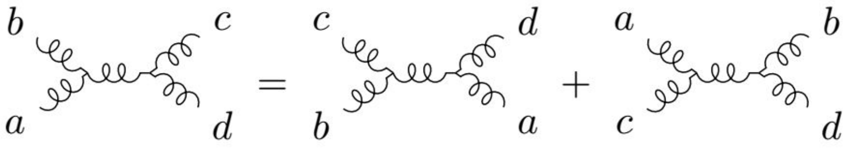


Color-kinematics duality relates kinematic weights of graphs

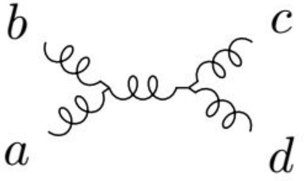


$$\frac{\boxed{n_i} C_i}{\prod_{\alpha_i}(p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

$n(a, b, c, d)$

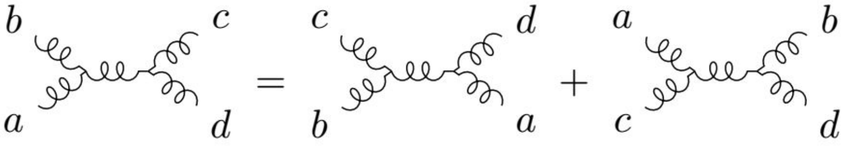


Color-kinematics duality relates kinematic weights of graphs

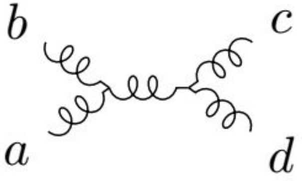


$$\frac{\boxed{n_i} C_i}{\prod_{\alpha_i}(p^2_{\alpha_i} - m^2_{\alpha_i})}$$

$$n(a,b,c,d) \; = \; n(b,c,d,a) + n(c,a,b,d)$$

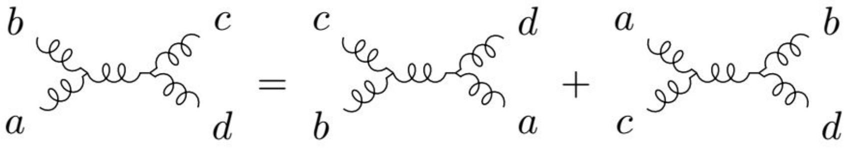


Color-kinematics duality relates kinematic weights of graphs

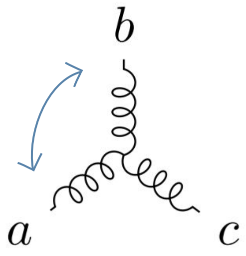


$$\frac{\boxed{n_i} C_i}{\prod_{\alpha_i}(p^2_{\alpha_i} - m^2_{\alpha_i})}$$

$$\boxed{n(a,b,c,d) = n(b,c,d,a) + n(c,a,b,d)}$$



Antisymmetries of graph weights



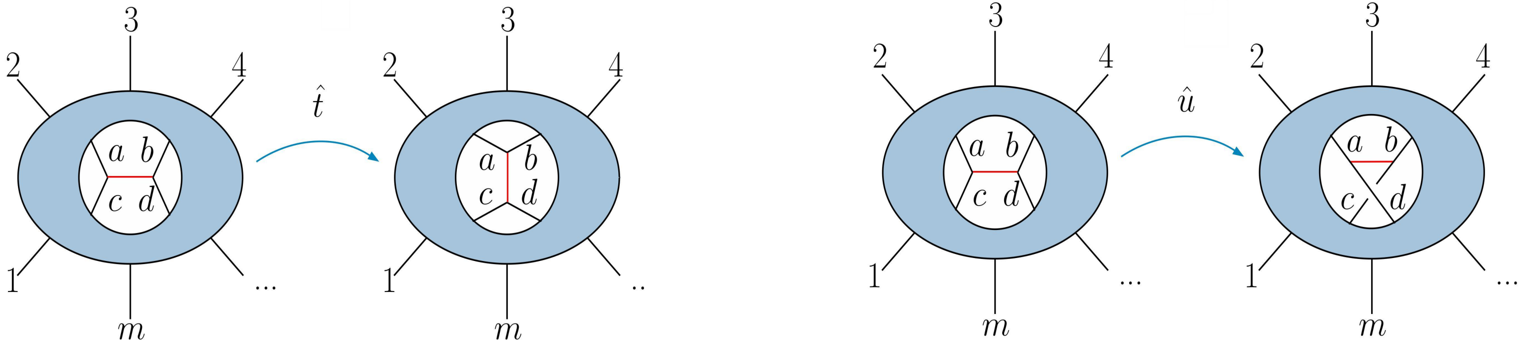
$$f^{abc} = -f^{bac}$$

$$n(a,b,c) = -n(b,a,c)$$

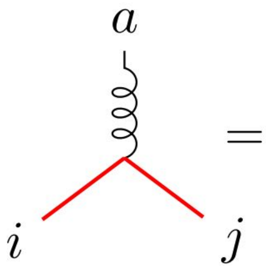
Color-kinematics duality relates kinematic weights of graphs

$$n_i - n_j = n_k \qquad \Leftrightarrow \qquad c_i - c_j = c_k,$$

$$n_i \to -n_i \qquad \Leftrightarrow \qquad c_i \to -c_i$$

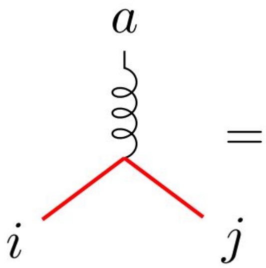


What is different about massive particles?



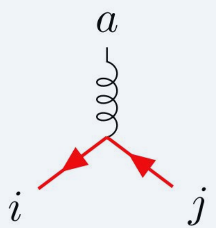
$$= T^a_{i\bar{j}}$$

What is different about massive particles?



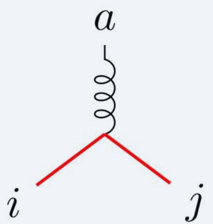
$$= T^a_{i\bar{j}}$$

Fundamental-type



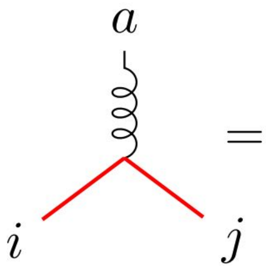
$$= \hat{T}^a_{i\bar{j}}$$

Adjoint-type



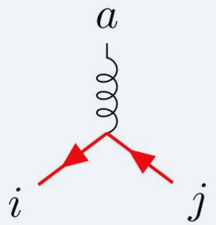
$$= \hat{T}^a_{ij}$$

What is different about massive particles?



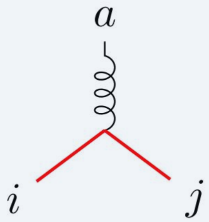
$$= T^{a}_{i\bar{j}}$$

Fundamental-type



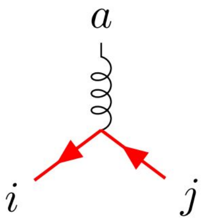
$$= \hat{T}^{a}_{i\bar{j}}$$

Adjoint-type

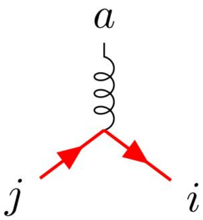


$$= \hat{T}^{a}_{ij}$$

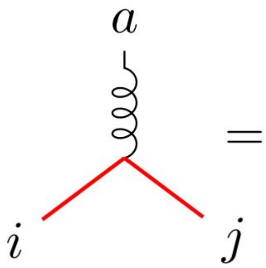
Want antisymmetry:



$$= -$$

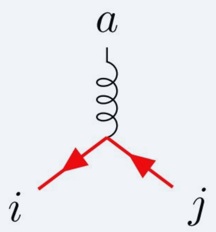


What is different about massive particles?



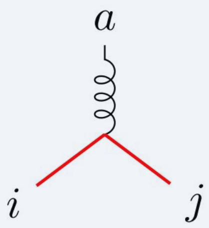
$$= T^a_{i\bar{j}}$$

Fundamental-type



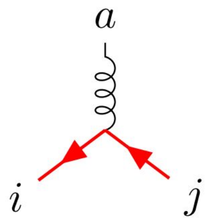
$$= \hat{T}^a_{i\bar{j}}$$

Adjoint-type

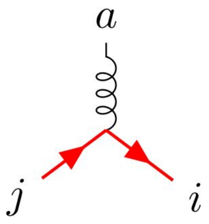


$$= \hat{T}^a_{ij}$$

Want antisymmetry:



$$= -$$



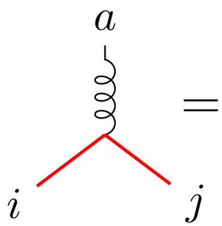
Introduce:

$$T^a_{i\bar{j}} = \hat{T}^a_{i\bar{j}}$$

$$T^a_{\bar{j}i} = -T^a_{i\bar{j}}$$

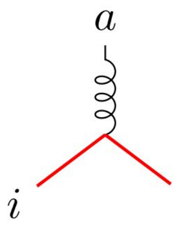
c(graph) x n(graph)

What is different about massive particles?



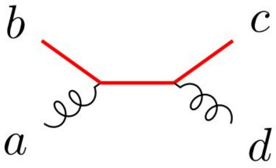
$$= T^{a}_{i\bar{j}}$$

What is different about massive particles?

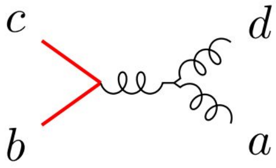


$$= T^a_{i\bar{j}}$$

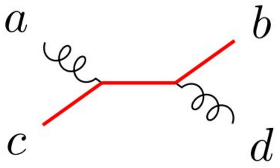
Jacobi-like relations



$=$

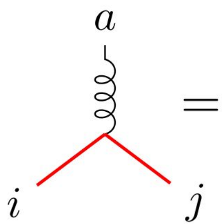


$+$



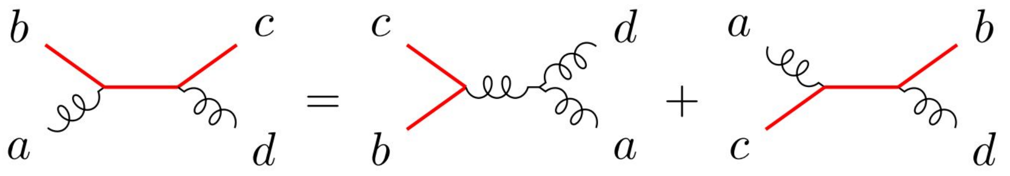
$$T^a_{b\bar{j}} T^d_{j\bar{c}} \quad = \quad f^{dal} T^l_{b\bar{c}} \quad + \quad T^a_{j\bar{c}} T^d_{b\bar{j}}$$

What is different about massive particles?



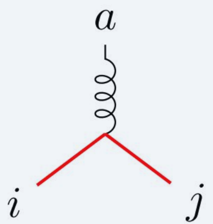
$$= T^a_{i\bar{j}}$$

Jacobi-like relations



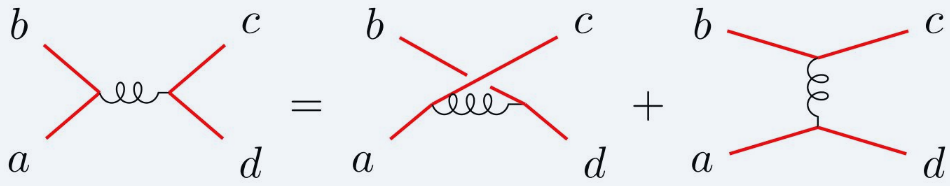
$$T^a_{b\bar{j}}T^d_{j\bar{c}} = f^{dal}T^l_{b\bar{c}} + T^a_{j\bar{c}}T^d_{b\bar{j}}$$

Adjoint-type



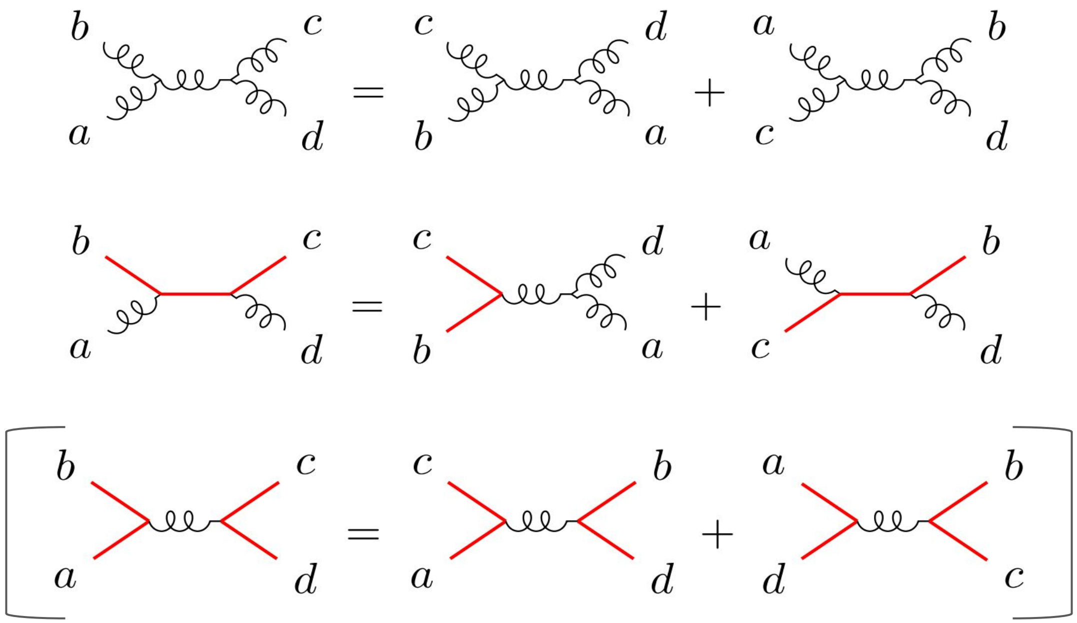
$$= \hat{T}^a_{ij}$$

Optionally



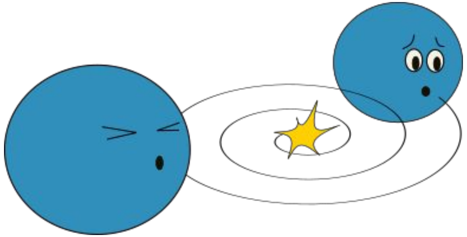
$$T^g_{ab}T^g_{cd} = T^g_{ac}T^g_{bd} + T^g_{ad}T^g_{bc}$$

Color-kinematics duality relates kinematic weights of graphs

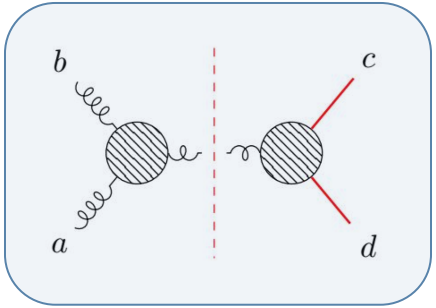


Double copy gives gravity

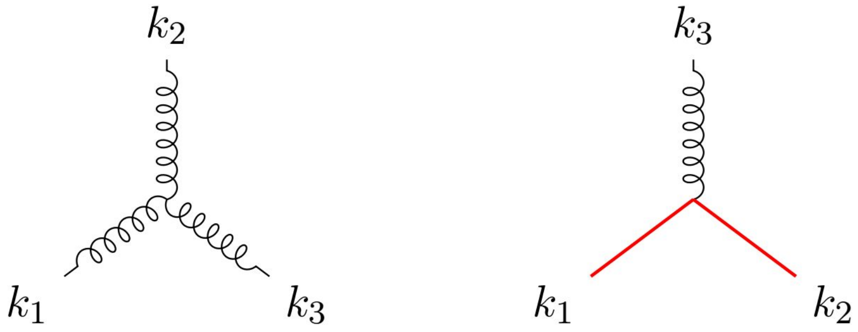
$$\mathcal{M}_m^{(L)} = i^{L+1}\left(\frac{\kappa}{2}\right)^{m-2+2L} \sum_{i\in\Gamma} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{\boxed{n_i \tilde{n}_i}}{\prod_{\alpha_i}(p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$



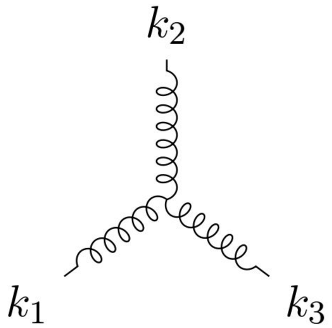
Bootstrap

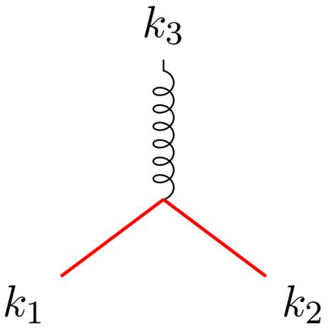


3-point amplitudes are completely determined by color-kinematics and symmetries



3-point amplitudes are completely determined by color-kinematics and symmetries

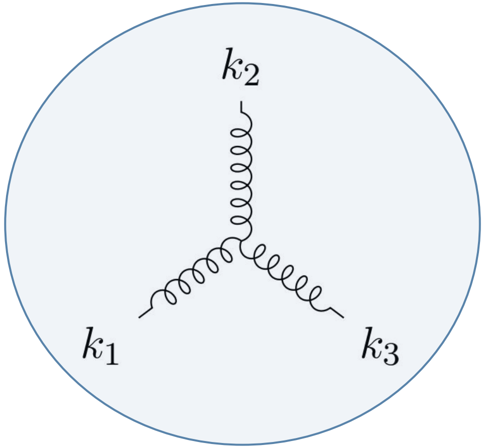


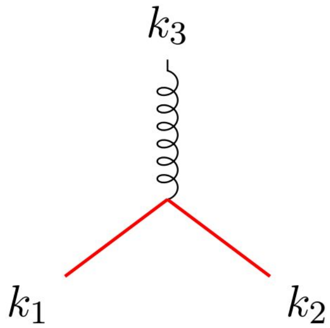


$$n_3(k_1, k_2, k_3)$$

$$n_{3,2}(k_1^m, k_2^m, k_3)$$

3-point amplitudes are completely determined by color-kinematics and symmetries

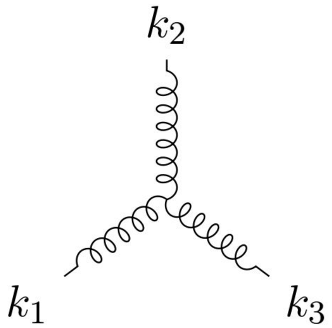


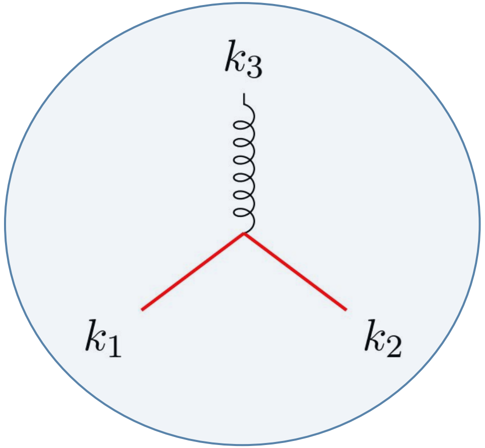


$$n_3(k_1, k_2, k_3) = \alpha_1(k_3 \cdot \epsilon_1)(\varepsilon_2 \cdot \varepsilon_3) + \alpha_2(k_3 \cdot \varepsilon_2)(\varepsilon_1 \cdot \varepsilon_3) + \alpha_3(k_2 \cdot \varepsilon_3)(\varepsilon_1 \cdot \varepsilon_2)$$

$$n_{3,2}(k_1^m, k_2^m, k_3)$$

3-point amplitudes are completely determined by color-kinematics and symmetries

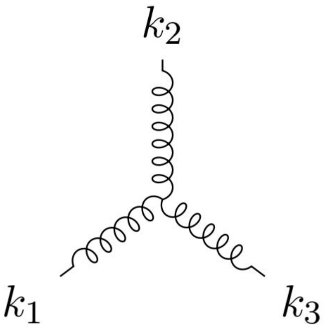


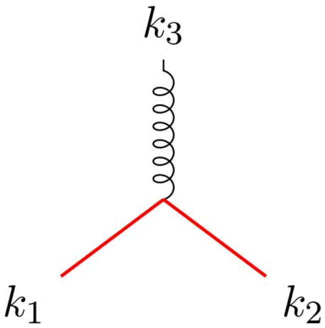


$$n_3(k_1, k_2, k_3) = \alpha_1(k_3 \cdot \epsilon_1)(\varepsilon_2 \cdot \varepsilon_3) + \alpha_2(k_3 \cdot \varepsilon_2)(\varepsilon_1 \cdot \varepsilon_3) + \alpha_3(k_2 \cdot \varepsilon_3)(\varepsilon_1 \cdot \varepsilon_2)$$

$$n_{3,2}(k_1^m, k_2^m, k_3) = \alpha_1(k_1 \cdot \varepsilon_3)$$

3-point amplitudes are completely determined by color-kinematics and symmetries

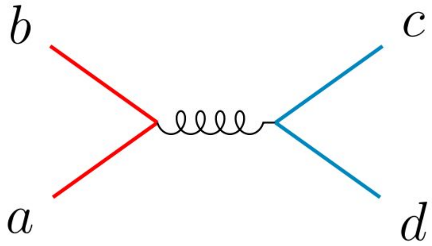




$$n_3(k_1, k_2, k_3) = \alpha_1\Big((k_3 \cdot \epsilon_1)(\varepsilon_2 \cdot \varepsilon_3) - (k_3 \cdot \varepsilon_2)(\varepsilon_1 \cdot \varepsilon_3) + (k_2 \cdot \varepsilon_3)(\varepsilon_1 \cdot \varepsilon_2)\Big)$$

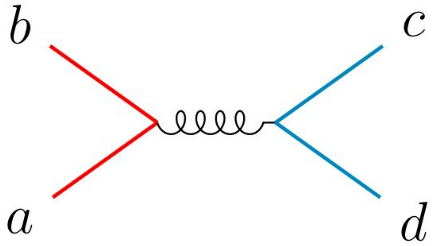
$$n_{3,2}(k_1^m, k_2^m, k_3) = \alpha_1(k_1 \cdot \varepsilon_3)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization



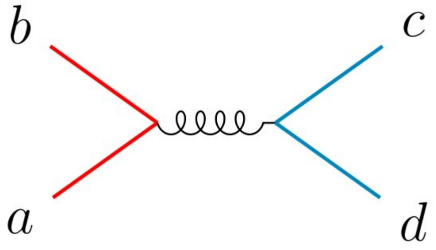
$$n_{4,2}(a,b,c,d) = \alpha_1(a\cdot b) + \alpha_2(b\cdot b) + \alpha_3(b\cdot c) + \alpha_4(c\cdot c)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

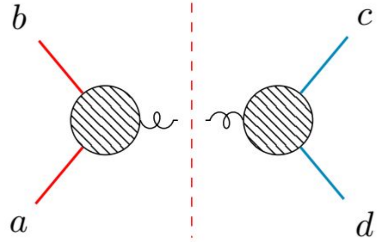


$$n_{4,2}(a,b,c,d) = \frac{\alpha_3}{2}\big((a\cdot b) + (b\cdot b) + 2(b\cdot c)\big)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

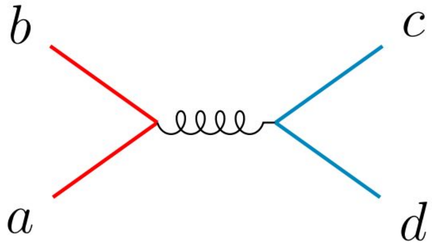


$$n_{4,2}(a,b,c,d) = \frac{\alpha_3}{2}\Big((a\cdot b) + (b\cdot b) + 2(b\cdot c)\Big)$$



$$\sum_{s\in\text{states}} A_{3,2}(a,b,l^s)A_{3,2}(-l^{\bar{s}},c,d) = n_{4,2}(a,b,c,d)|_{(a+b)^2=0}$$

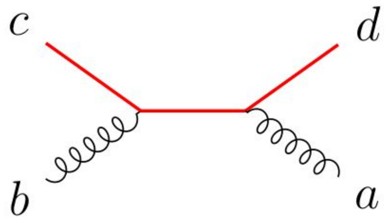
Tree-level amplitudes are completely determined by color-kinematics and factorization

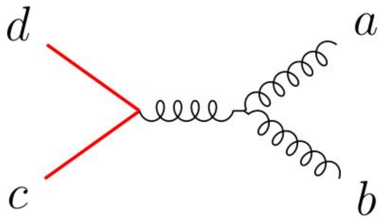


$$n_{4,2}(a,b,c,d) = \frac{\alpha_3}{2}\big((a\cdot b) + (b\cdot b) + 2(b\cdot c)\big)$$

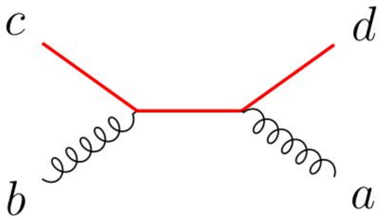
$$A^{\text{tree}}_{4,2}(a,b,c,d) = -\frac{(a\cdot b) + (b\cdot b) + 2(b\cdot c)}{2\big[(a\cdot b) + (b\cdot b)\big]}$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

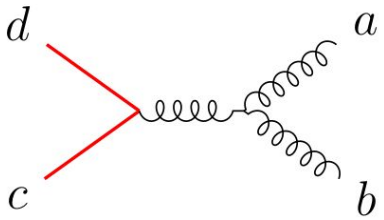




Tree-level amplitudes are completely determined by color-kinematics and factorization

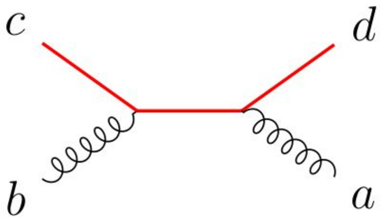


$$= n_{m}(a, b, c, d),$$

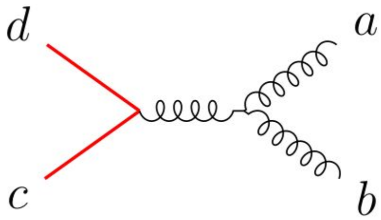


$$= n_{\overline{m}}(a, b, c, d)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

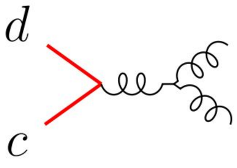


$= n_m(a,b,c,d),$

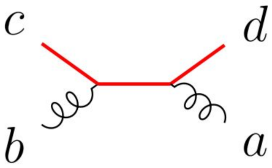


$= n_{\overline{m}}(a,b,c,d)$

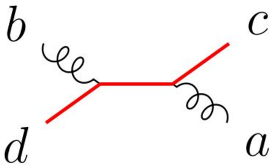
Color-kinematics



$=$

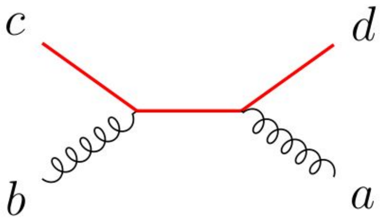


$-$

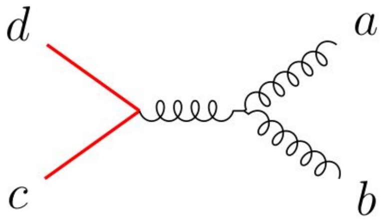


$$n_{\overline{m}}(a,b,c,d) = n_m(a,b,c,d) - n_m(b,a,c,d)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization



$$= n_{m}(a,b,c,d),$$

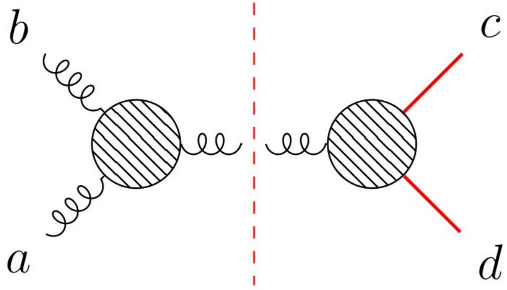


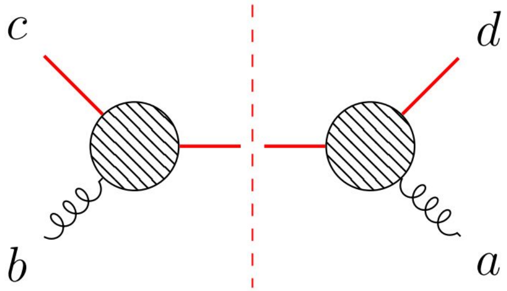
$$= n_{\overline{m}}(a,b,c,d)$$

$$n_{m}(a,b,c,d) = \left(\alpha_1(a\cdot b) + \alpha_2(b\cdot c) + \alpha_3(c\cdot c)\right)(\varepsilon_a\cdot\varepsilon_b) + \alpha_4(b\cdot\varepsilon_a)(a\cdot\varepsilon_b)$$
$$+ \alpha_5(c\cdot\varepsilon_a)(a\cdot\varepsilon_b) + \alpha_6(b\cdot\varepsilon_a)(c\cdot\varepsilon_b) + \alpha_7(c\cdot\varepsilon_a)(c\cdot\varepsilon_b)$$

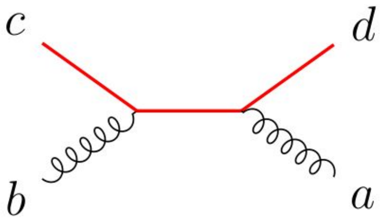
Tree-level amplitudes are completely determined by color-kinematics and factorization

Factorization

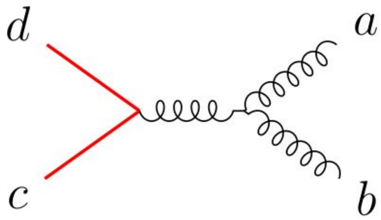




Tree-level amplitudes are completely determined by color-kinematics and factorization



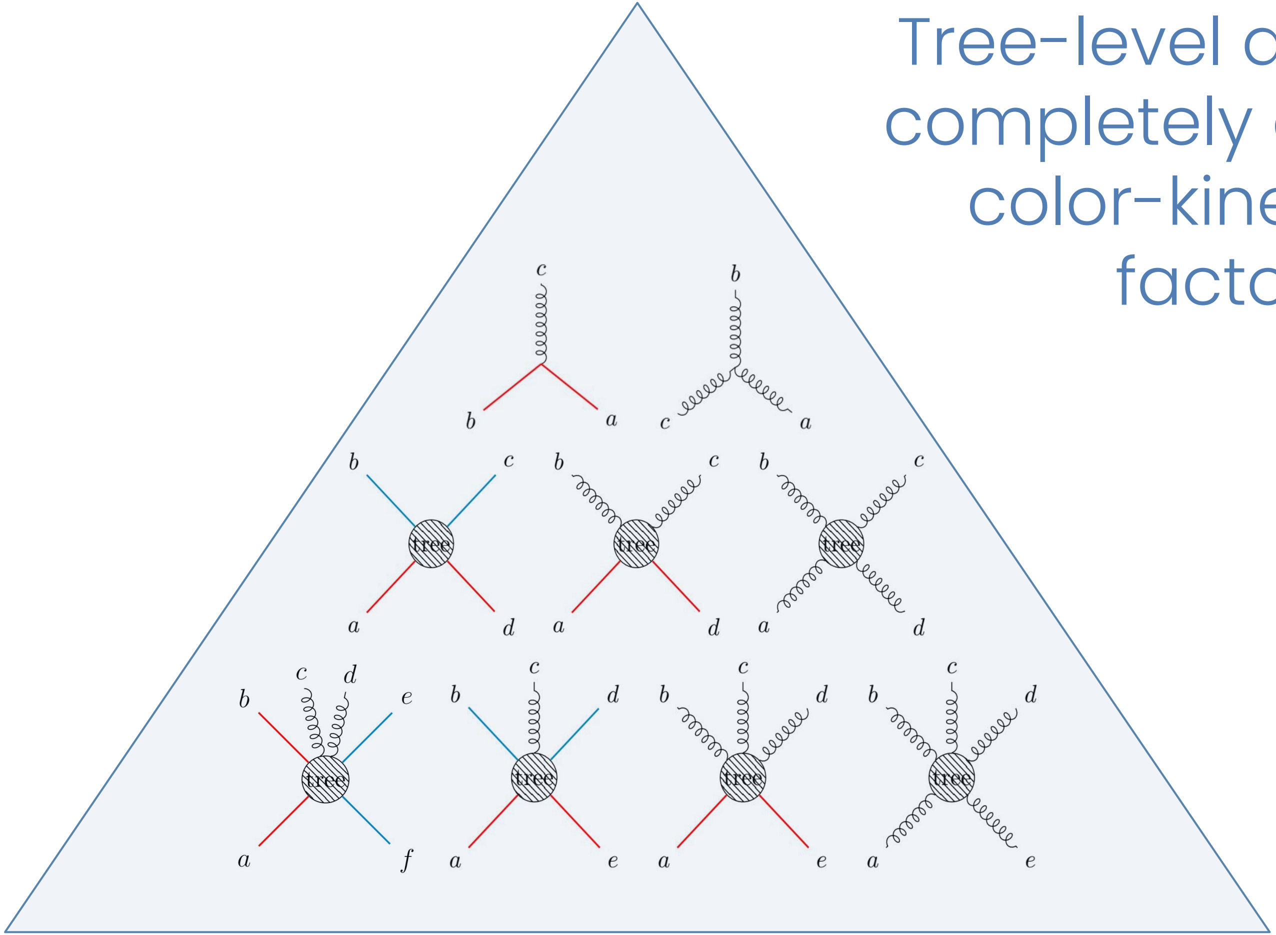
$$= n_{\color{red}m}(a,b,c,d),$$



$$= n_{\overline{m}}(a,b,c,d)$$

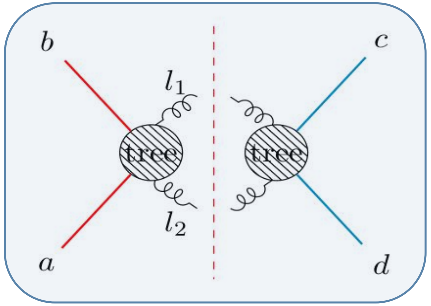
$$n_{\color{red}m}(a,b,c,d) = (c\cdot\varepsilon_b)\left((b\cdot\varepsilon_a)+(c\cdot\varepsilon_a)\right) - \frac{1}{2}(b\cdot c)(\varepsilon_a\cdot\varepsilon_b)$$

$$n_{\overline{m}}(a,b,c,d) = n_{\color{red}m}(a,b,c,d) - n_{\color{red}m}(b,a,c,d)$$



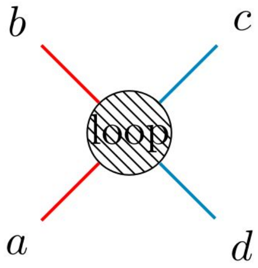
Tree-level amplitudes are completely determined by color-kinematics and factorization

Loop-level



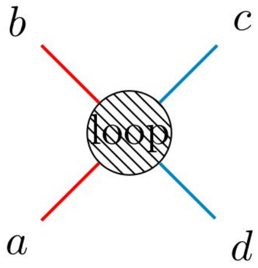
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

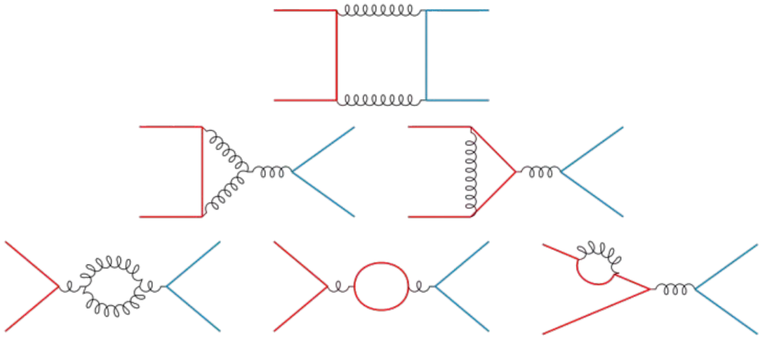
4-point one-loop



Loop-level amplitudes are determined by color-kinematics and unitarity cuts

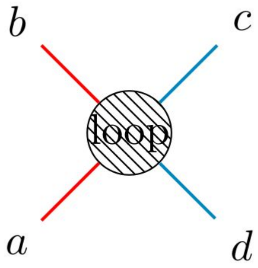
4-point one-loop

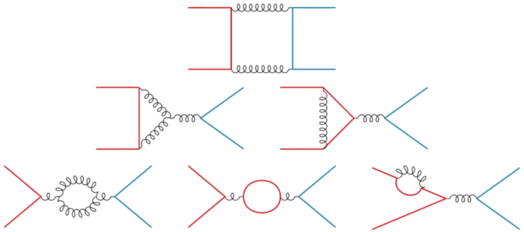




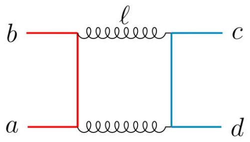
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



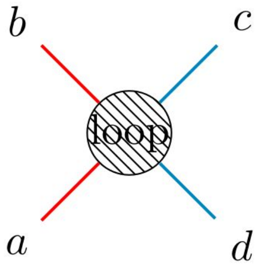


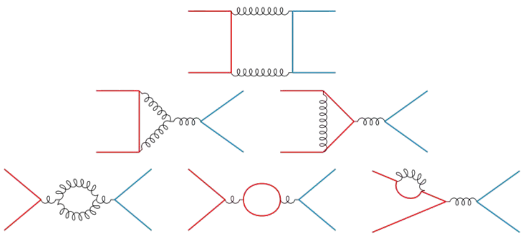
Loop level color-kinematics



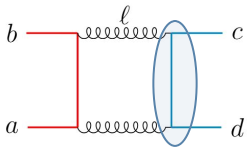
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



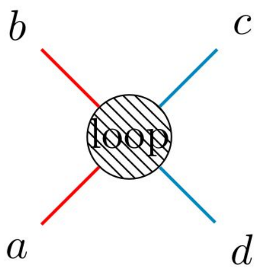


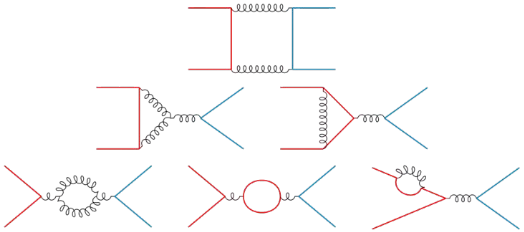
Loop level color-kinematics



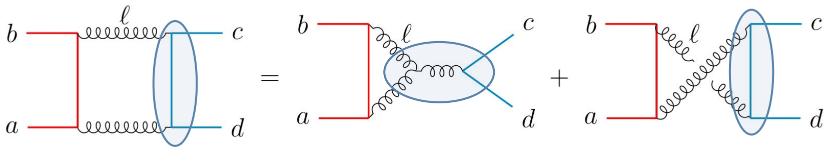
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



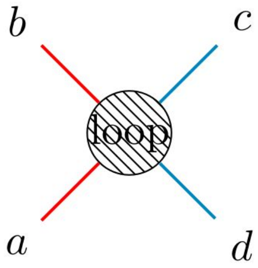


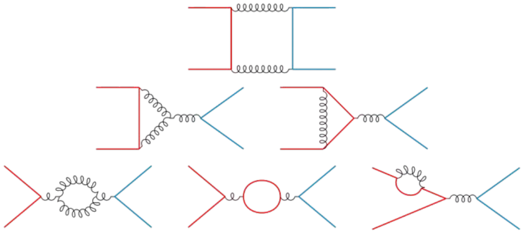
Loop level color-kinematics



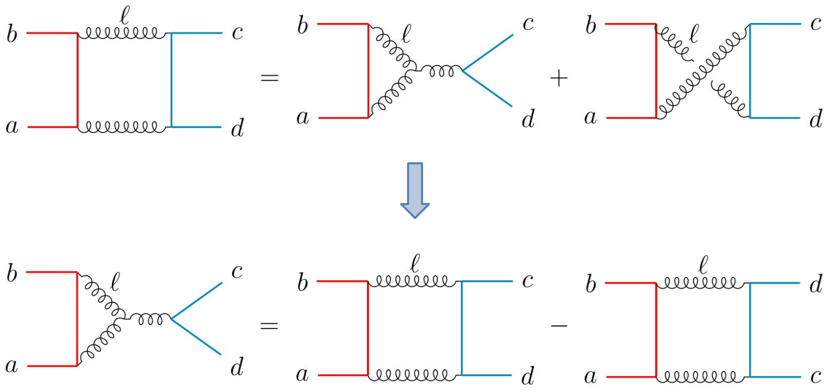
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



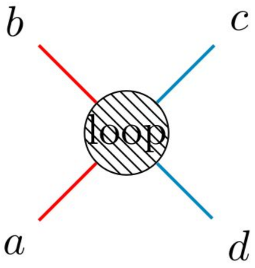


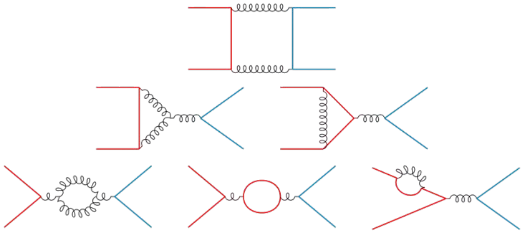
Loop level color-kinematics



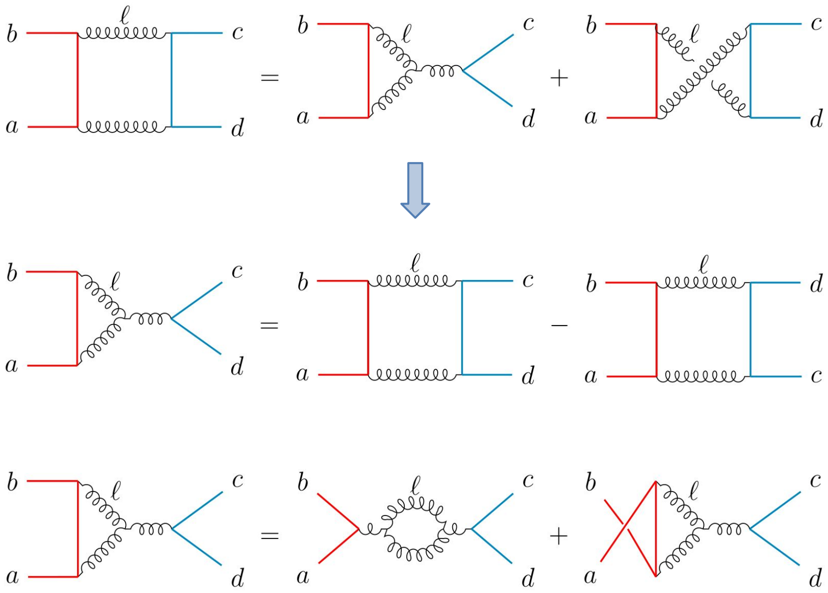
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



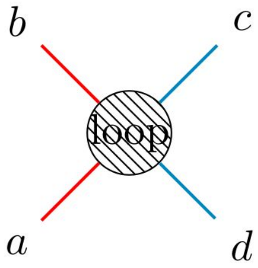


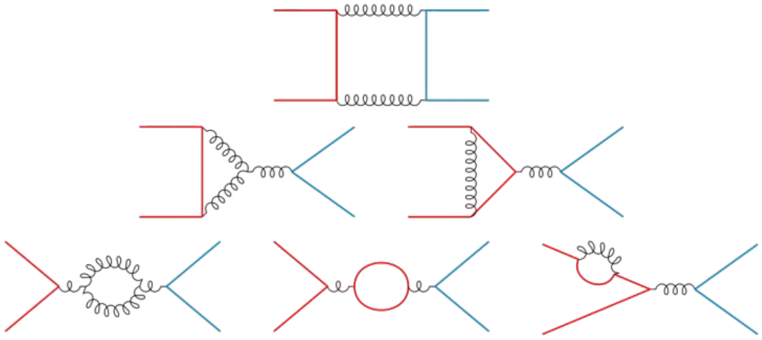
Loop level color-kinematics

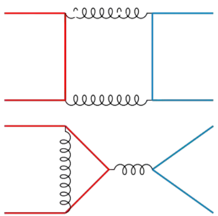


Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



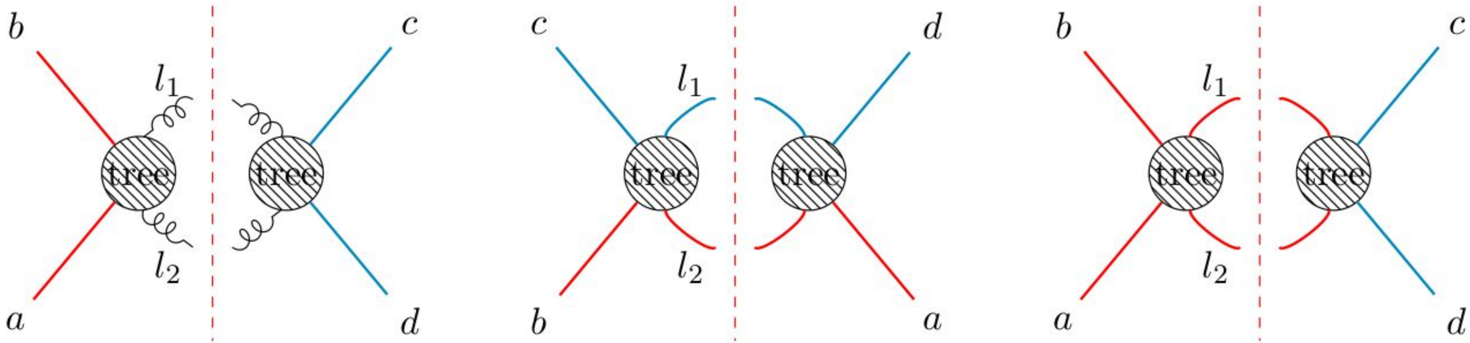




Basis graphs

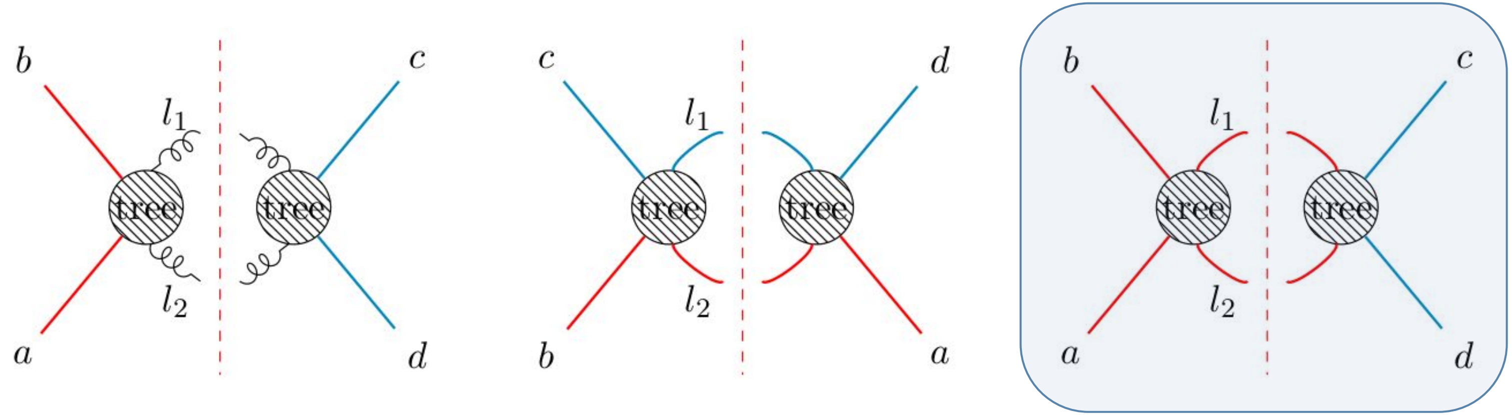
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



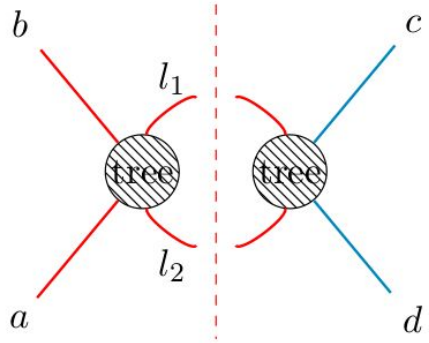
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop

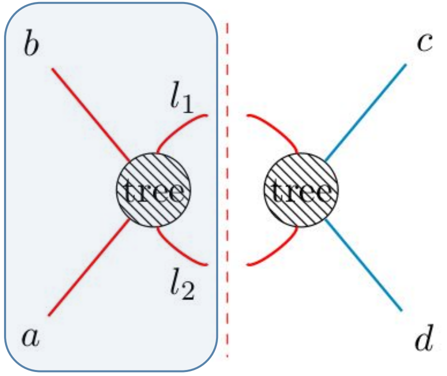


Ordered cut?

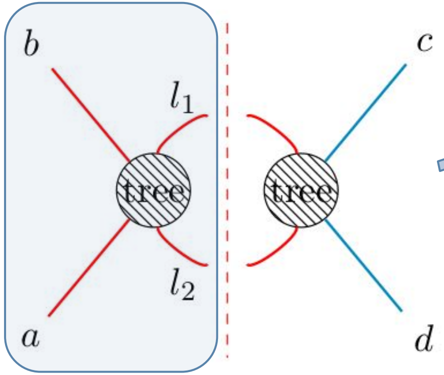
What does *ordered cut* mean?



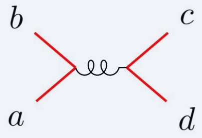
What does *ordered cut* mean?

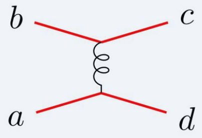


What does *ordered cut* mean?



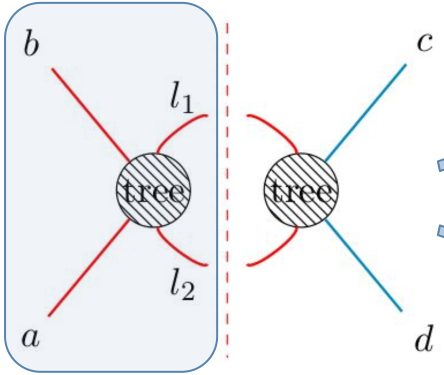
Adjoint-type ordered cut

$A^{\text{tree}}(a,b,c,d) =$ 

$+$ 

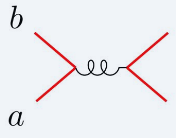
$= \dfrac{n_s}{s} + \dfrac{n_t}{t}$

What does *ordered cut* mean?

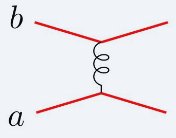


Adjoint-type ordered cut

$$A^{\text{tree}}(a, b, c, d) =$$



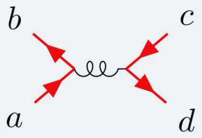
$+$



$$= \frac{n_s}{s} + \frac{n_t}{t}$$

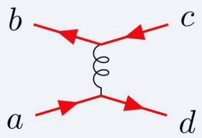
Fundamental-type ordered cut

$$A^{\text{tree}}(\overline{a}, b, \overline{c}, d) =$$



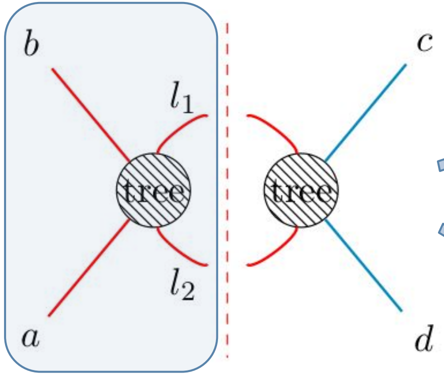
$$= \frac{n_s}{s}$$

$$A^{\text{tree}}(\overline{a}, d, \overline{c}, b) =$$

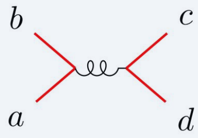
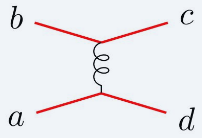


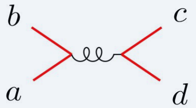
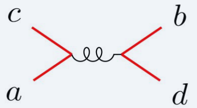
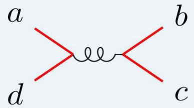
$$= \frac{n_t}{t}$$

What does *ordered cut* mean?

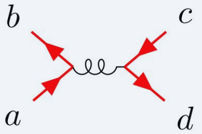


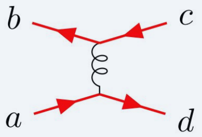
Adjoint-type ordered cut

$A^{\text{tree}}(a,b,c,d) =$  $+$  $= \dfrac{n_s}{s} + \dfrac{n_t}{t}$

 $=$  $+$ 

Fundamental-type ordered cut

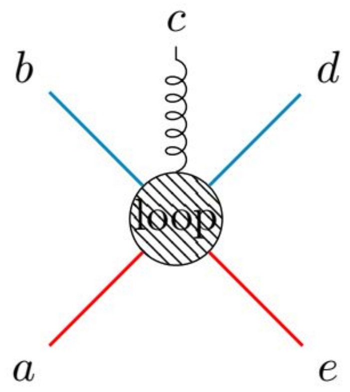
$A^{\text{tree}}(\overline{a},b,\overline{c},d) =$  $= \dfrac{n_s}{s}$

$A^{\text{tree}}(\overline{a},d,\overline{c},b) =$  $= \dfrac{n_t}{t}$

Loop-level amplitudes are determined by color-kinematics and unitarity cuts

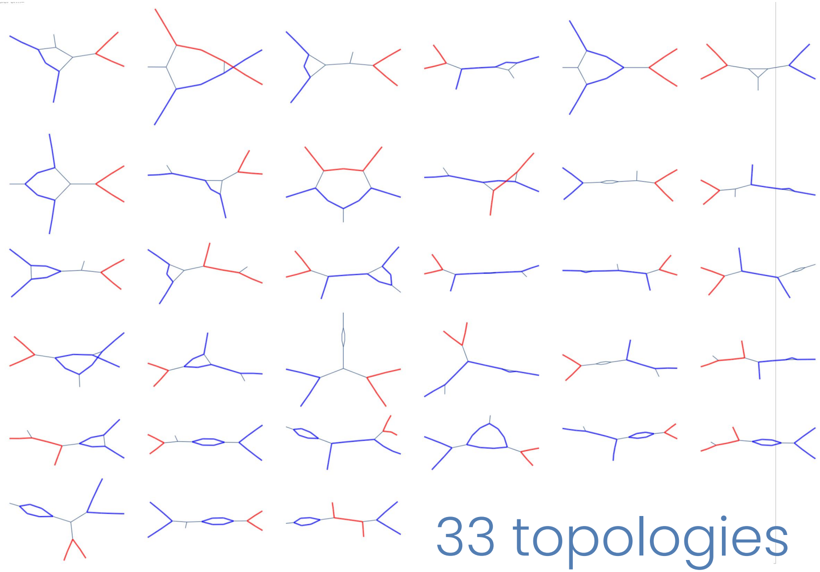
5-point one-loop

Encodes first correction to radiation

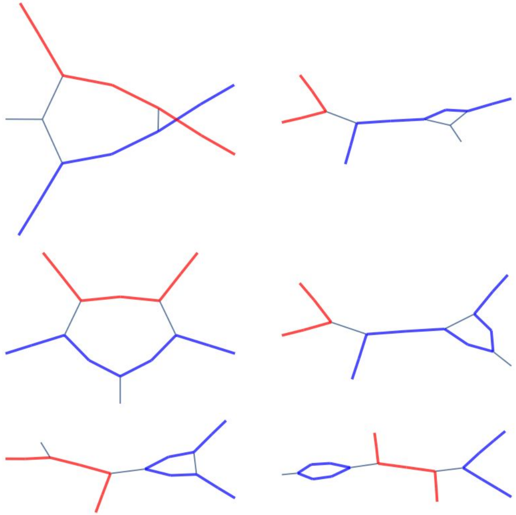


Loop-level amplitudes are determined by color-kinematics and unitarity cuts

5-point one-loop



33 topologies



6 basis graphs

Loop-level amplitudes are determined by color-kinematics and unitarity cuts

5-point one-loop

33 topologies

10 296
parameters

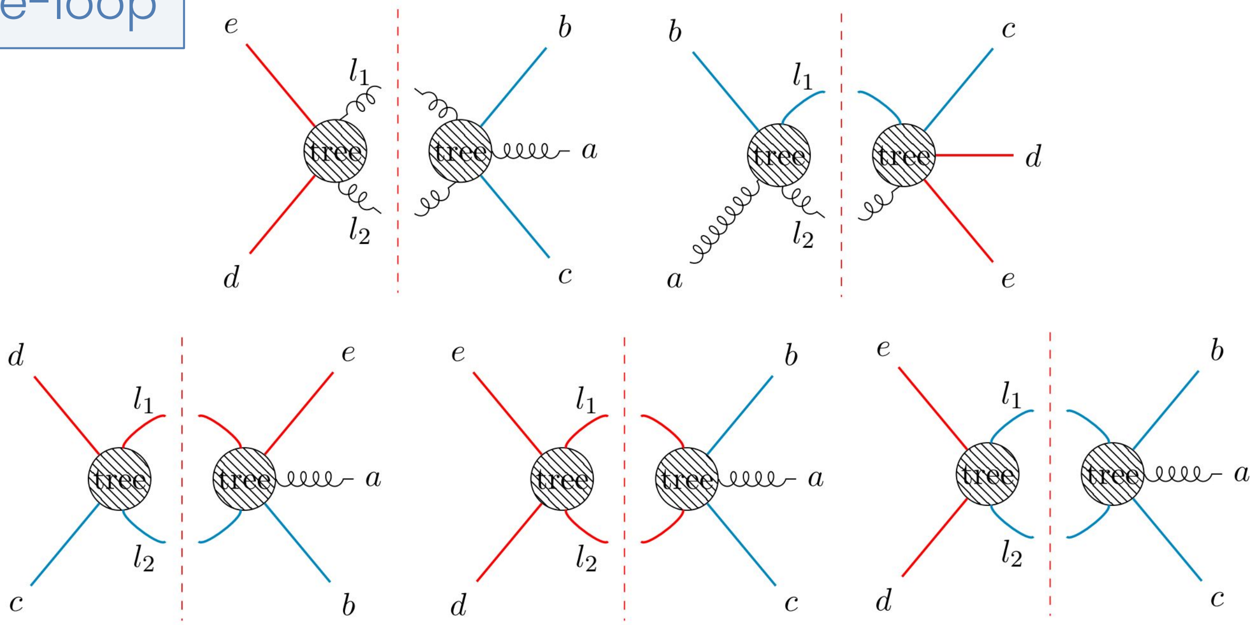
→

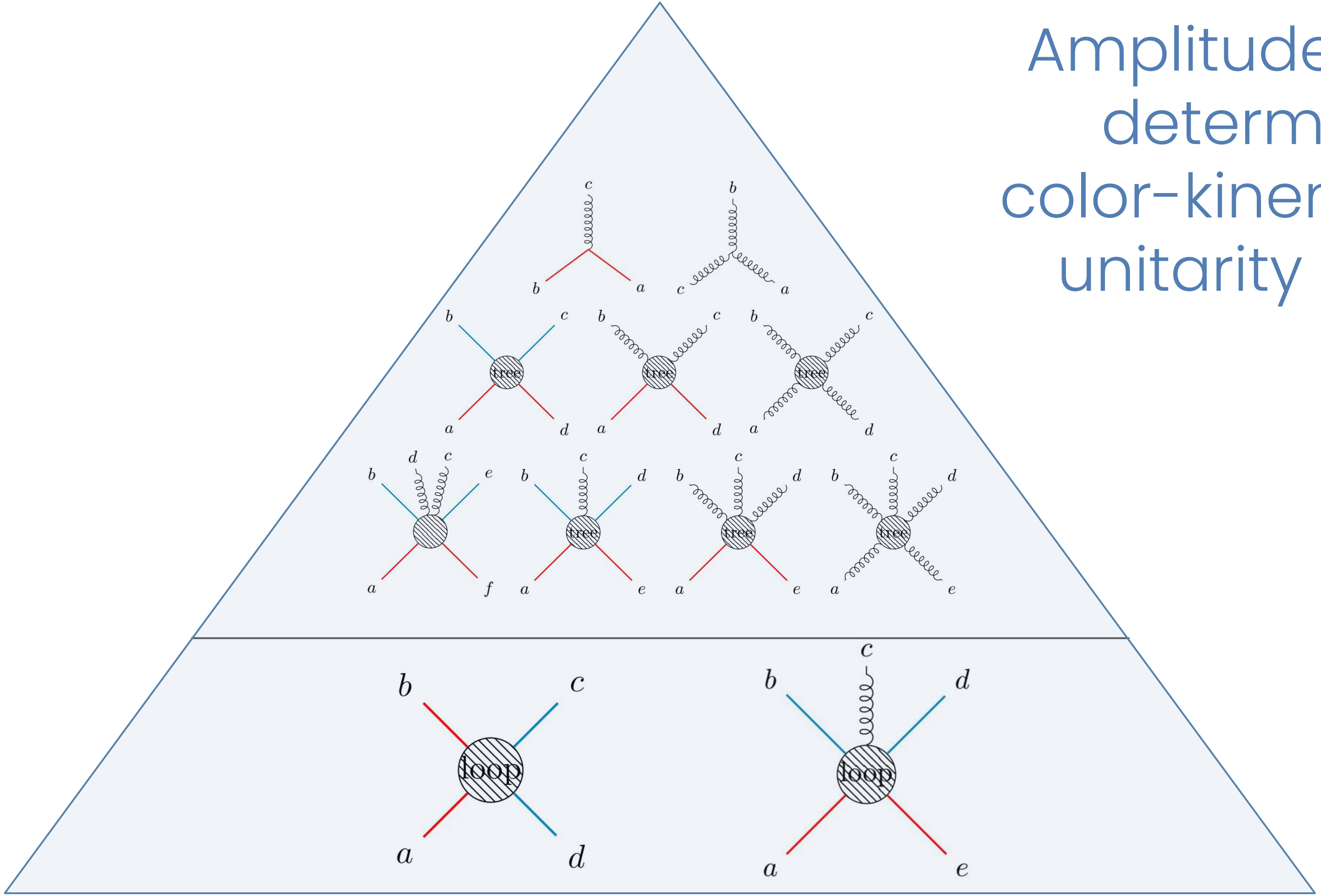
6 basis graphs

1872
parameters

Loop-level amplitudes are determined by color-kinematics and unitarity cuts

5-point one-loop





Amplitudes are fully determined by color-kinematics and unitarity methods

1 loop

The double copy gives extra massless states

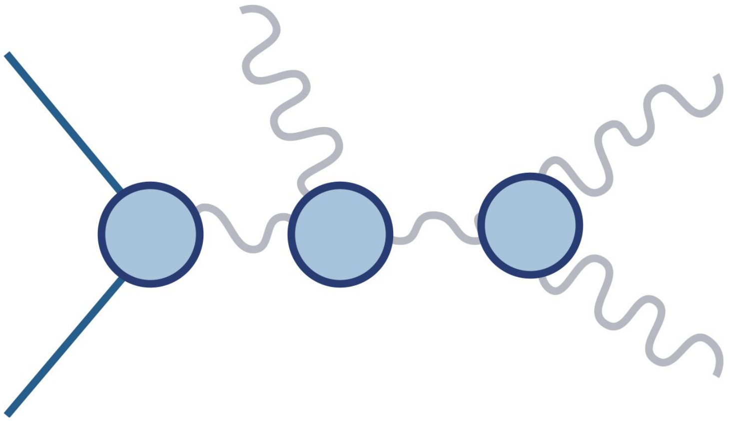
$$S = \int d^D x \sqrt{-g} \left[-\frac{1}{2} R + \frac{1}{2(D-2)} \partial^\mu \phi \partial_\mu \phi + \frac{1}{6} e^{-4\phi/(D-2)} H^{\lambda\mu\nu} H_{\lambda\mu\nu} \right]$$

Scherk, Schwarz '74
Gross, Sloan '87

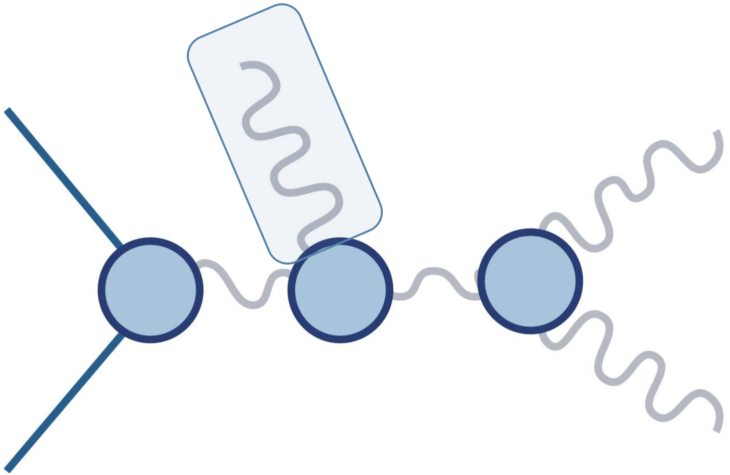
$+$ Matter terms

Johansson, Ochirov '16,'19
Bautista, Guevara '19
Plefka, Shi, Wang '19

The double copy gives extra massless states

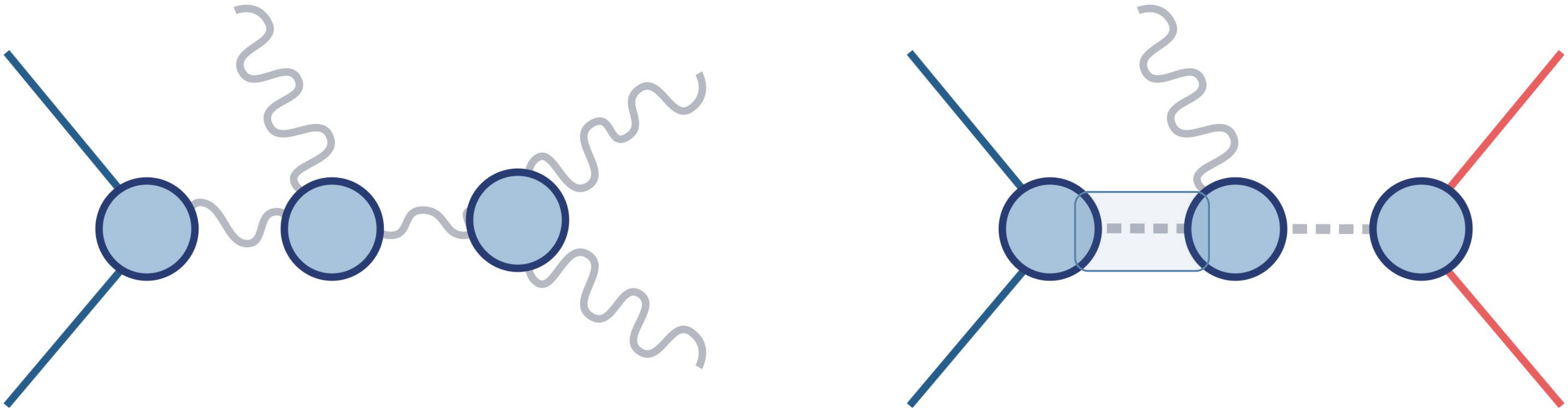


The double copy gives extra massless states

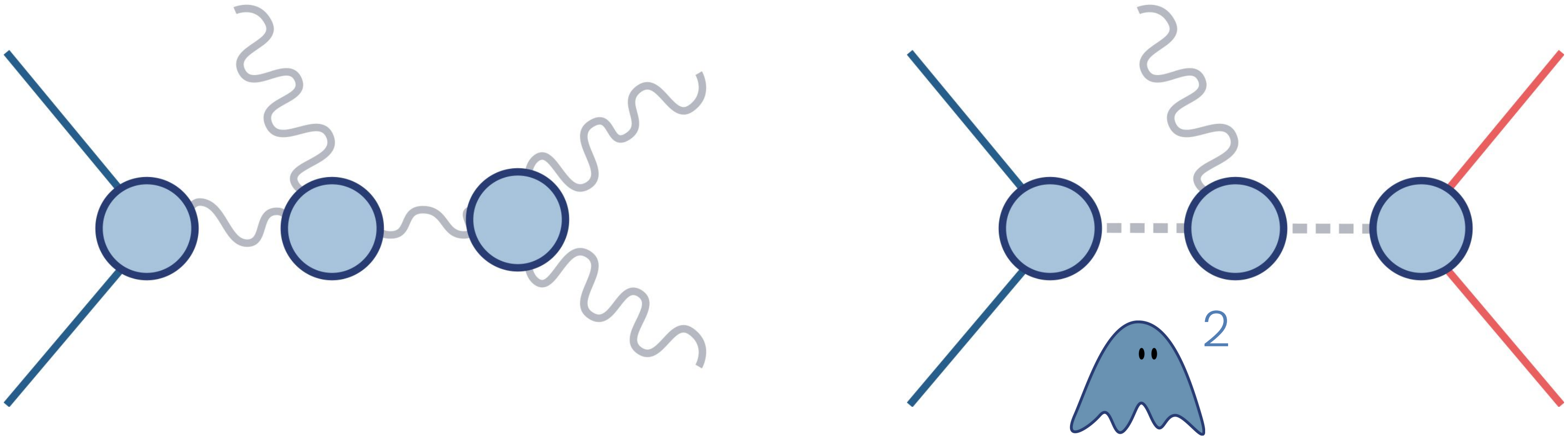


$$\epsilon^{\mu}\tilde{\epsilon}^{\nu} \rightarrow e^{\mu\nu}$$

The double copy gives extra massless states



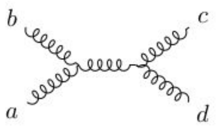
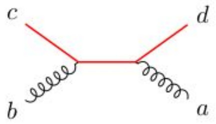
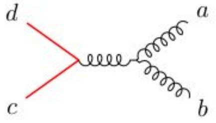
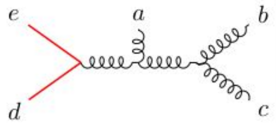
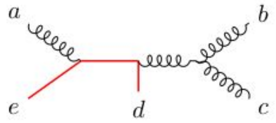
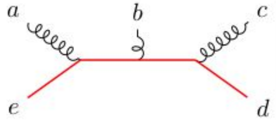
The double copy gives extra massless states



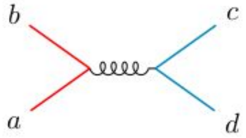
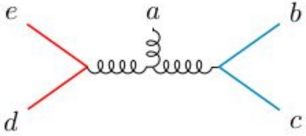
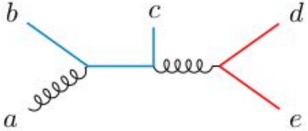
Is there a double copy approach?

Johansson, Ochirov '14
Luna, Nicholson, O'Connell, White '17

Is there a constructive approach?
Yes!

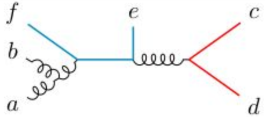
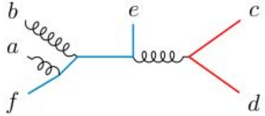
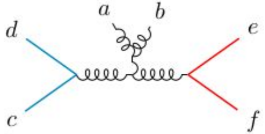
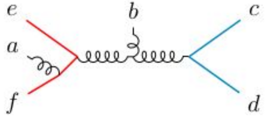
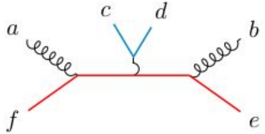
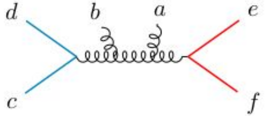
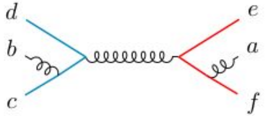
Graph topology	$\mathcal{N}=0$ supergravity numerator	Δ
	$\frac{1}{4}\Big[2(\varepsilon_a\cdot\varepsilon_b)\big((k_b\cdot\varepsilon_c)(k_c\cdot\varepsilon_d)-(k_b\cdot\varepsilon_d)(k_d\cdot\varepsilon_c)\big) - (\varepsilon_a\cdot\varepsilon_d)\big(2(k_d\cdot\varepsilon_c)(k_a\cdot\varepsilon_b)+(k_a\cdot k_b)(\varepsilon_b\cdot\varepsilon_c)\big) - 2(k_c\cdot\varepsilon_d)((\varepsilon_b\cdot\varepsilon_c)(k_b\cdot\varepsilon_a)-(\varepsilon_a\cdot\varepsilon_c)(k_a\cdot\varepsilon_b)) + (\varepsilon_b\cdot\varepsilon_d)\big((k_a\cdot k_b)(\varepsilon_a\cdot\varepsilon_c)+2(k_d\cdot\varepsilon_c)(k_b\cdot\varepsilon_a)\big) - (\varepsilon_c\cdot\varepsilon_d)(2(k_d\cdot\varepsilon_a)(k_c\cdot\varepsilon_b)-2(k_c\cdot\varepsilon_a)(k_d\cdot\varepsilon_b) + ((k_a\cdot k_b)+2(k_b\cdot k_c))(\varepsilon_a\cdot\varepsilon_b))\Big]^2$	0
	$\left[(k_c\cdot\varepsilon_b)\,(k_d\cdot\varepsilon_a)+\tfrac{1}{2}(k_b\cdot k_c)(\varepsilon_a\cdot\varepsilon_b)\right]^2$	0
	$\left[(k_a\cdot\varepsilon_b)(k_c\cdot\varepsilon_a)-(k_b\cdot\varepsilon_a)(k_c\cdot\varepsilon_b)-\frac{1}{2}\left(k^{-}_{ab}\cdot k_c\right)(\varepsilon_a\cdot\varepsilon_b)\right]^2$	0
	$(k_b\cdot\varepsilon_a)^2(k_b\cdot\varepsilon_c)^2(k_d\cdot\varepsilon_b)^2+\ldots+\frac{1}{4}(k_c\cdot k_e)^2(k_e\cdot\varepsilon_a)^2(\varepsilon_b\cdot\varepsilon_c)^2$	0
	$(k_b\cdot\varepsilon_a)^2(k_b\cdot\varepsilon_c)^2(k_d\cdot\varepsilon_b)^2+\ldots+\frac{1}{4}(k_c\cdot k_d)^2(k_d\cdot\varepsilon_a)^2(\varepsilon_b\cdot\varepsilon_c)^2$	0
	$(k_b\cdot\varepsilon_a)^2(k_c\cdot\varepsilon_b)^2(k_d\cdot\varepsilon_c)^2+\ldots+\frac{1}{4}(k_c\cdot k_d)^2(k_d\cdot\varepsilon_a)^2(\varepsilon_b\cdot\varepsilon_c)^2$	0

One massive scalar

Graph topology	$\mathcal{N}=0$ supergravity numerator	Δ
	$\frac{1}{4}(k_a \cdot k_b + 2\ k_b \cdot k_c + m_1^2)^2$	$\frac{m_1^2 m_2^2}{D_s - 2}$
	$\frac{1}{16}\Big[(k^-_{bc} \cdot k_1)(k^-_{de} \cdot \varepsilon_1) + 2(k_c \cdot k^-_{de})(k_b \cdot \varepsilon_a) - 2(k_b \cdot k^-_{de})(k_c \cdot \varepsilon_a)\Big]^2$	$\frac{m_1^2 m_2^2}{D_s - 2}(k_{de} \cdot \varepsilon_a)^2$
	$\frac{1}{16}\Big[(k_{ce} \cdot k_{de})\,(k^-_{de} \cdot \varepsilon_a) + 2\ (k_c \cdot k^-_{de})(k_b \cdot \varepsilon_a)\Big]^2$	$\frac{m_1^2 m_2^2}{D_s - 2}(k_b \cdot \varepsilon_a)^2$

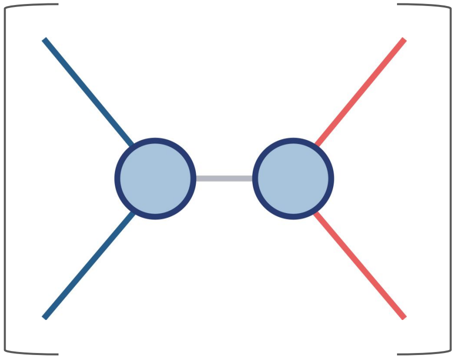
Two massive scalars

Two massive scalars

Graph topology	Δ
	$\frac{1}{4}\frac{m_1^2 m_2^2}{D_s-2}\Big[(k_b\cdot k_f - k_a\cdot k_f)\,(\varepsilon_a\cdot\varepsilon_b) - 2\,((k_{cde}\cdot\varepsilon_b)\,k_f\cdot\varepsilon_a - (k_{cde}\cdot\varepsilon_a)\,k_f\cdot\varepsilon_b)\Big]^2$
	$\frac{1}{64}\frac{m_1^2 m_2^2}{D_s-2}\Big[(k_{cd}\cdot k_{de} - 4k_a\cdot k_f)\,(\varepsilon_a\cdot\varepsilon_b) - 8(k_f\cdot\varepsilon_a)(k_{cde}\cdot\varepsilon_b)\Big]^2$
	$\frac{1}{4}\frac{m_1^2 m_2^2}{D_s-2}\Big[(2\,k_{be}\cdot k_{ef} + k_{cd}\cdot k_{ef})\,(\varepsilon_a\cdot\varepsilon_b) - 2(k_{ef}\cdot\varepsilon_a)(k_{cd}\cdot\varepsilon_b) + 2(k_{cd}\cdot\varepsilon_a)(k_{ef}\cdot\varepsilon_b)\Big]^2$
	$\frac{1}{64}\frac{m_1^2 m_2^2}{D_s-2}\Big[(k_{bcd}\cdot k_e + k_{cd}\cdot k_d - 3k_a\cdot k_f)\,(\varepsilon_a\cdot\varepsilon_b) - 8(k_f\cdot\varepsilon_a)(k_{cd}\cdot\varepsilon_b)\Big]^2$
	$\frac{1}{64}\frac{m_1^2 m_2^2}{D_s-2}\Big[(k_b\cdot k_e + k_a\cdot k_f)\,(\varepsilon_a\cdot\varepsilon_b) + 8(k_f\cdot\varepsilon_a)(k_e\cdot\varepsilon_b)\Big]^2$
	$\frac{1}{16}\frac{m_1^2 m_2^2}{D_s-2}\Big[(2k_{bcd}\cdot k_{ef} + k_c\cdot k_{cd} + 3k_e\cdot k_{ef})\,(\varepsilon_a\cdot\varepsilon_b) - 4(k_{ef}\cdot\varepsilon_a)(k_{cd}\cdot\varepsilon_b)\Big]^2$
	$\frac{1}{64}\frac{m_1^2 m_2^2}{D_s-2}\Big[(k_{bcd}\cdot k_{de} - 2k_a\cdot k_f)\,(\varepsilon_a\cdot\varepsilon_b) - 8(k_f\cdot\varepsilon_a)(k_c\cdot\varepsilon_b)\Big]^2$

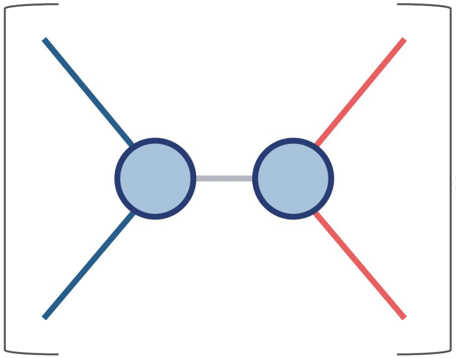
Projective double copy

$$\boxed{\mathcal{N} = 0}$$

n  $= \frac{1}{4}\left(k_b \cdot k_c - k_b \cdot k_d\right)^2$

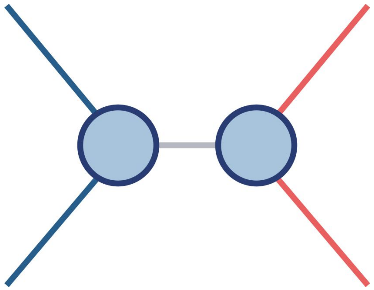
Projective double copy

$$\boxed{\mathcal{N} = 0}$$

n  $= (k_b \cdot k_d)^2$

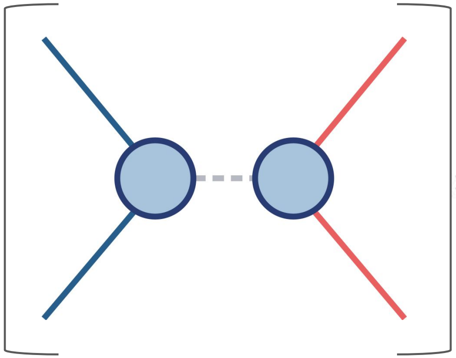
Projective double copy

$$\sum_s \mathcal{M}_3^{\text{trees}}(a, b, q^s)\mathcal{M}_3^{\text{trees}}(-q^{\bar{s}}, c, d)$$

n []

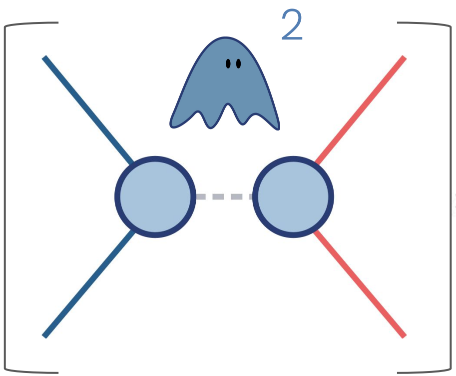
$$= (k_b \cdot k_d)^2 - \left((k_b \cdot k_d)^2 - \frac{m_1^2 m_2^2}{(D_s - 2)} \right)$$

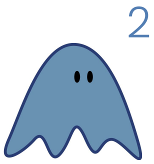
Projective double copy

n 

$$= (k_b \cdot k_d)^2 - \left((k_b \cdot k_d)^2 - \boxed{\frac{m_1^2 m_2^2}{(D_s - 2)}} \right)$$

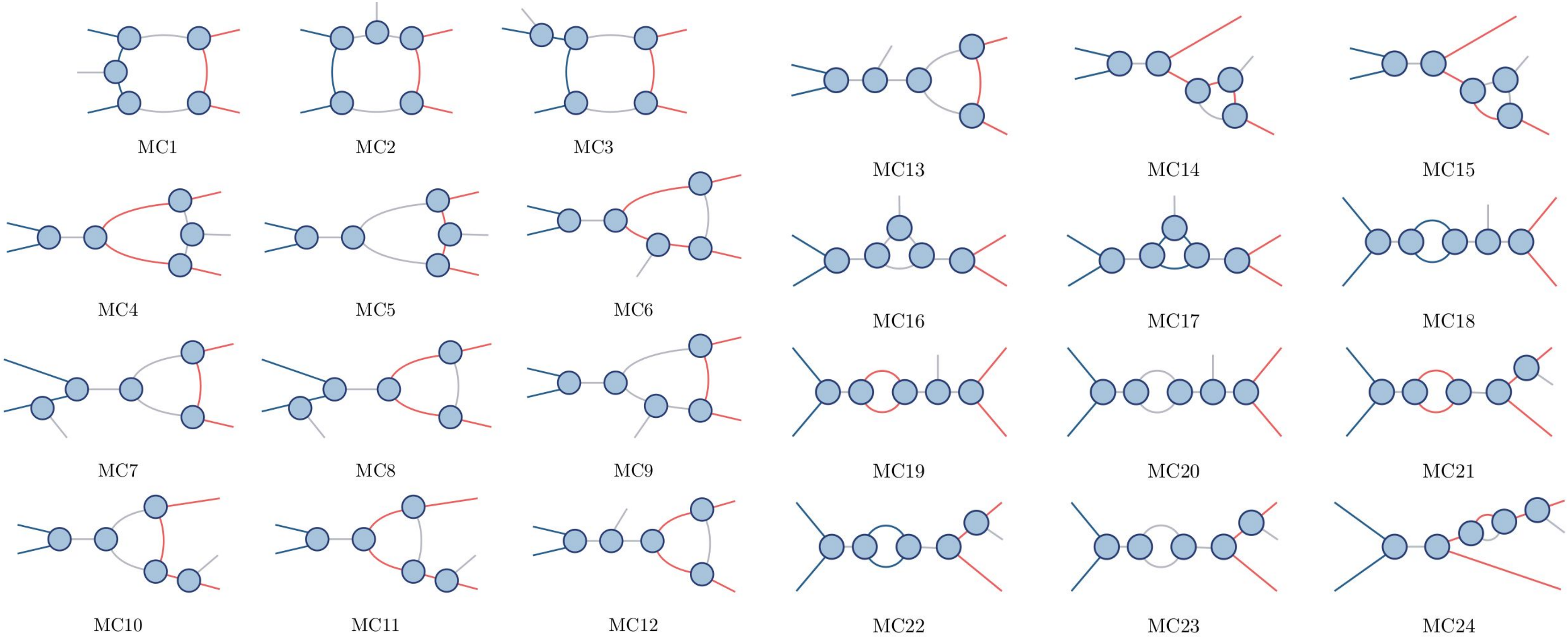
Projective double copy

n 

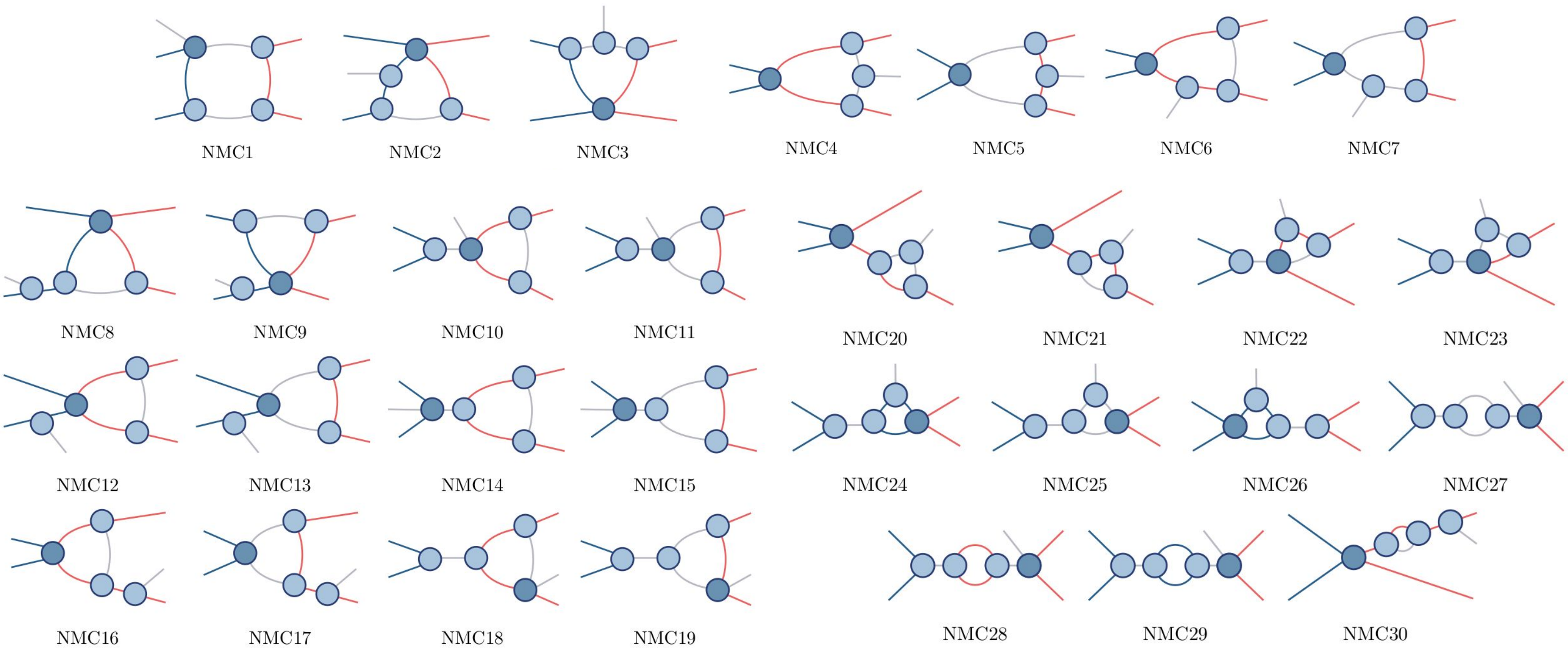


$$= (k_b \cdot k_d)^2 - \left((k_b \cdot k_d)^2 - \boxed{\frac{m_1^2 m_2^2}{(D_s - 2)}} \right)$$

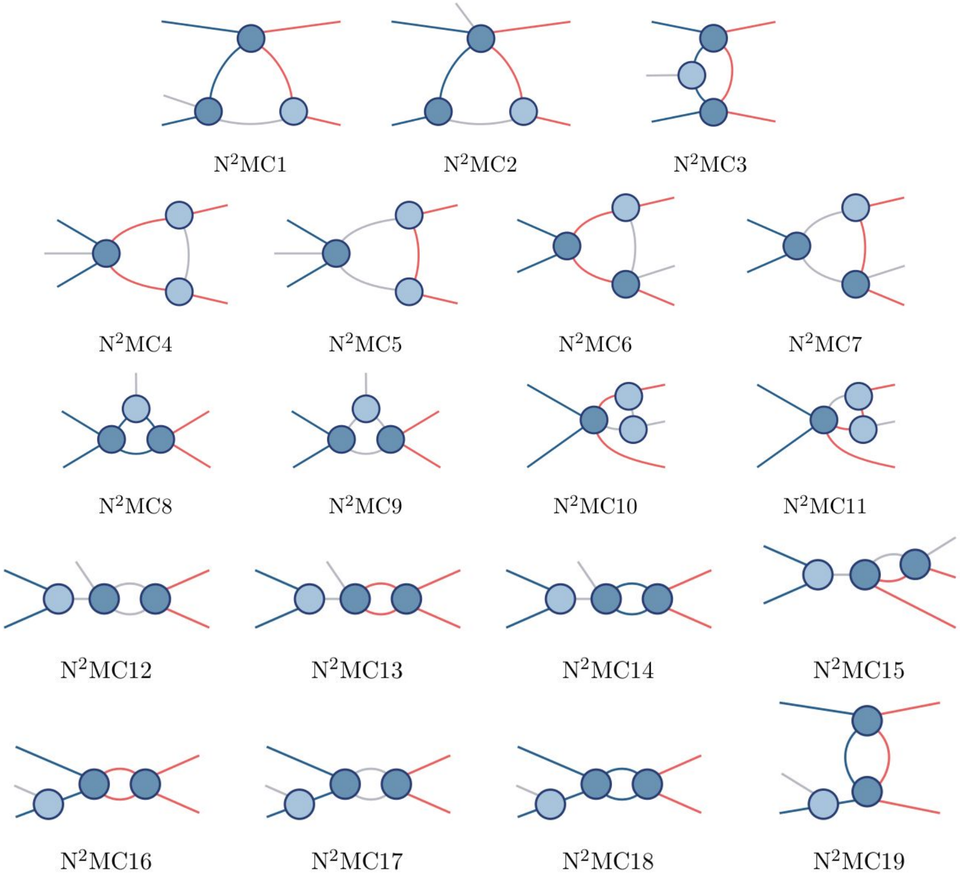
Projective double copy at loop-level



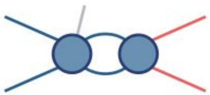
Projective double copy at loop-level



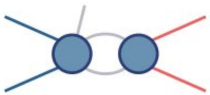
Projective double copy at loop-level



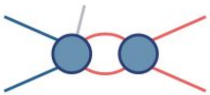
Projective double copy at loop-level



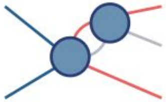
N^3MC1



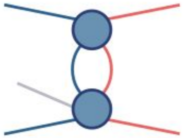
N^3MC2



N^3MC3



N^3MC4



N^3MC5

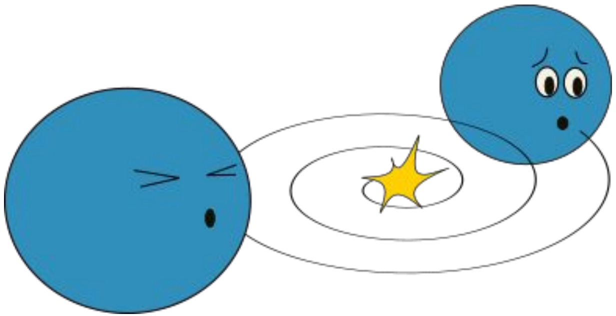
Future work includes

exploring double copies directly to pure gravity predictions,

massive higher-spins in arbitrary rep,

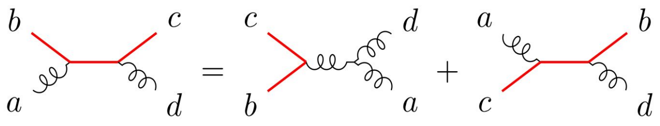
generating classical observables,

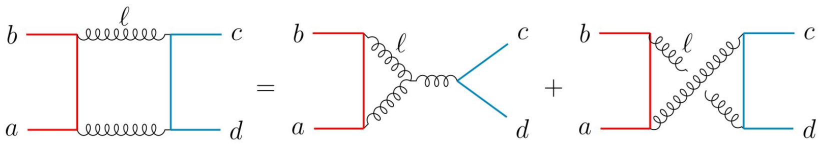
extending to higher loop order (2-loops and more)!



Summary

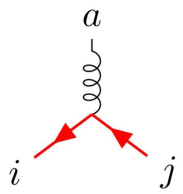
Color-kinematics and factorization are enough to build massive amplitudes





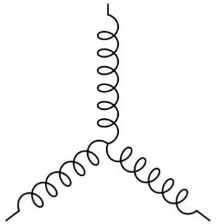
Color-kinematics duality for massive amplitudes can be manifest at loop level

Can use the same kinematic building blocks for massive scalars charged in fundamental and adjoint



Extra

3-point amplitudes are completely determined by color-kinematics and symmetries



Build the ansatz from a kinematic basis

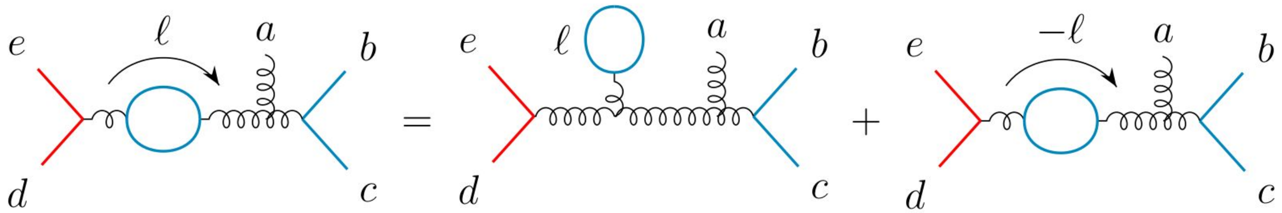
$$n_3(k_1, k_2, k_3) = \alpha_1(k_3 \cdot \epsilon_1)(\varepsilon_2 \cdot \varepsilon_3) + \alpha_2(k_3 \cdot \varepsilon_2)(\varepsilon_1 \cdot \varepsilon_3) + \alpha_3(k_2 \cdot \varepsilon_3)(\varepsilon_1 \cdot \varepsilon_2)$$

Momentum conservation $\qquad k_1 + k_2 + k_3 = 0$

Transversality $\qquad \varepsilon_i \cdot k_i = 0$

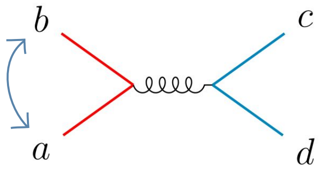
$$k_2 \cdot \varepsilon_1 = (-k_1 - k_3) \cdot \varepsilon_1 = -k_3 \cdot \varepsilon$$

Tadpoles can be reached using color-kinematics



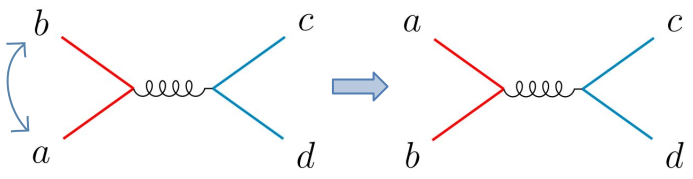
Tree-level amplitudes are completely determined by color-kinematics and factorization

What are symmetries of the graph?



Tree-level amplitudes are completely determined by color-kinematics and factorization

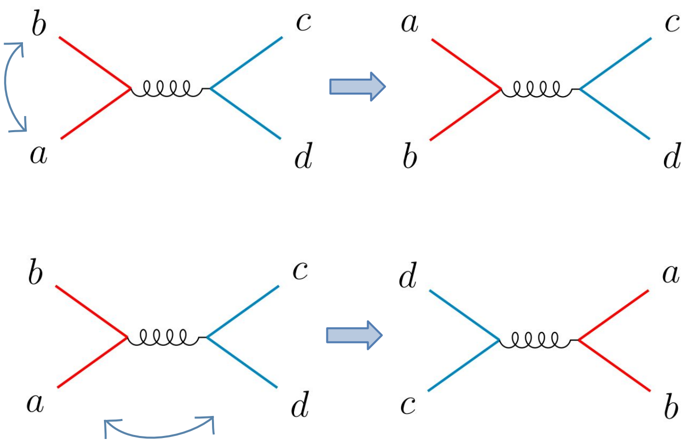
What are symmetries of the graph?



$$n_{4,2}(b,a,c,d) = -n_{4,2}(a,b,c,d)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

What are symmetries of the graph?



$$n_{4,2}(b,a,c,d) = -n_{4,2}(a,b,c,d)$$

$$n_{4,2}(c,d,a,b) = n_{4,2}(a,b,c,d)$$

$$\vdots$$