Causality constraints on gravity EFTs

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1. Motivations

2. Framework
   - Axioms for $2 \to 2$
   - EFTs: what we bound
   - Making positive sum rules

3. Results:
   - bounds with graviton pole
   - relation with near-forward bounds
   - unsolved problems

SCH + V. Duong 2011.02957

SCH + Mazáč + Rastelli + Simmons-Duffin 2008.04931

2102.08951 ←

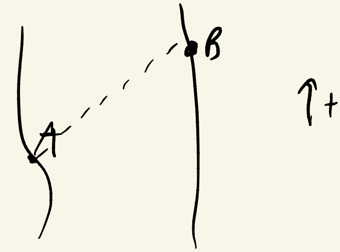
2106.10274

+ many related!

Causality

- why waves, fields
- why particles
- why anti-particles
- why gravity attractive

↳ why EFT logic has to work.



There exists a rich perturbative S-matrix bootstrap
(generalized unitarity, rigidity of function spaces, ...)

The nonperturbative conformal bootstrap can quantitatively 'solve' theories that lie at the boundaries of theory space. (ie. Ising CFT)

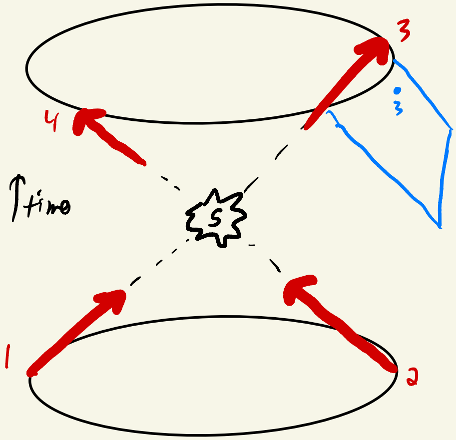
↳ Is QCD at a boundary? under which assumptions? Hard.

Today: nonperturbative constraints on $2\to2$ S-matrices with a 'heavy/light' separation.

↳ SMEFT parameter range?

↳ Gravity: how far from Einstein's GR can Nature be (causally)?

2→2 scattering



Comments:

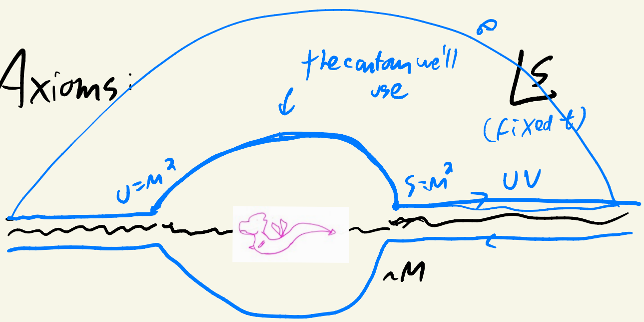
i) Fixed angle scattering can show time advances
[Giddings + Porto '09]

↳ causality probes Regge limit ($s\to\infty$, t or b fixed)

ii) strongest statements involve crossing:

Particle $1\to 3$ $\simeq$ antiparticle $3\to 1$

Axioms:



Analyticity (causality)
+ boundedness (unitarity)
= dispersion relation

i) Analyticity of $M(s,t)$ outside $(-M^2 \le t \le 0) \times$ (real axis with $s>M^2$ a $u>M^2$ + a crossing path from $s=M^2$ to $u=M^2$)

ii) Boundedness: $|M_\psi(s)/s| \le \text{const}$ as $|s|\to\infty$

where $\psi(p)$ = wavepacket { compact support in mom., fast decay in impact param

($M \sim s^{2+\alpha' t}$ in s.t.)

(this is stronger yet easier to prove than "Froissart bound" $|M(s,t)/s^2| \to 0$

Rigorously valid in AdS/CFT!
[SCH, Mazac, Rastelli, Simmons-Duffin '21])

what do we bound?

Ex: EFT of a single real scalar, below cutoff scale M

$$\mathcal{L}_{\text{low}} = \frac{R}{16\pi G} + \frac{1}{2}(D\phi)^2 - \frac{g}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4 + \frac{g_2}{2}\left[(D_\mu\phi)^2\right]^2 + \frac{g_3}{3}(D_\mu D_\nu\phi)^2(D_\sigma\phi)^2 + \frac{g_4}{4}\left[(D_\mu D_\nu\phi)^2\right]^2 + \dots$$

← higher derivatives

Problem: Lagrangian isn't a physical observable! Rather, we focus on $\mathcal{M}$

$$\mathcal{M}_{\text{low}}(s,t) = 8\pi G\left[\frac{tu}{s} + \frac{su}{t} + \frac{st}{u}\right] - g^2\left[\frac{1}{s} + \frac{1}{t} + \frac{1}{u}\right] - \lambda + g_2(s^2+t^2+u^2) + g_3\, stu + g_4(s^2+t^2+u^2)^2 + \dots + O(\text{loops})$$

Naively: EFT parameters ⇔ Taylor coefficients around $s,t \to 0$.

Problems: - Taylor series ill-defined since loops are non-analytic

- Contradicts EFT spirit: parameters should be matched through experiments at the scale $u \sim M$, NOT $u = 0$.

↳ Our approach: bound observables that are:

i) Linear in S-matrix

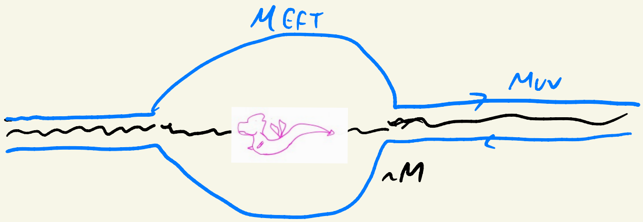
ii) Dominated by $|s|, |t| \sim M^2$

iii) Reduce to g_k if S-matrix → tree-level EFT

These observables will be bounded nonperturbatively, but harder to interpret if EFT is strongly interacting

(physics above M could be strongly coupled)

Dispersive sum rules



("twice-subtracted")

B_2: $0 = \oint_{|s|=\infty} \frac{ds}{s} \frac{M(s,t)}{s(s+t)}$ $(t<0)$

$$\oint M_{EFT} = \int \mathrm{Im}\, M_{UV}$$

$0 \le \mathrm{Im} \le 2$

$$= \sum_J \#_J \int_{M^2}^{\infty} \frac{dm^2}{m^2} m^{4-d}\, \underline{\mathrm{Im}\, a_J(s)}\, P_J\left(1+\frac{2t}{m^2}\right)$$

$= \Sigma$ Legendres with pos. coefficients.

$$\frac{8\pi G}{-t} + 2g_2 - g_3 t + 8 g_4 t^2 + \ldots = \left\langle \frac{(2m^2+t)\mathcal{P}_J(1+\frac{2t}{m^2})}{m^2(m^2+t)^2} \right\rangle_{m \ge M} \qquad -M^2 \le t \le 0$$

[Pham + Truong '85]
[Adams et al '05]
...

<u>If</u> $G_N = 0$: could single out g's by expanding around forward limit.

$\Rightarrow g_2 = \langle \frac{1}{m^4} \rangle > 0$, etc.

Today: keep $G_N \neq 0$

Clearly, $\langle \cdots \rangle \ge 0 \Rightarrow G_N \ge 0$, Gravity is attractive!

How to bound other couplings? Laurent series around forward limit yields divergent sums! $\cancel{1+2+3+4+\cdots = -\frac{1}{12}}$

$$\frac{8\pi G}{-t} + 2g_2 - g_3 t + 8g_4 t^2 + \ldots = \left\langle \frac{(2m^2+t)\mathcal{P}_J(1+\frac{2t}{m^2})}{m^2(m^2+t)^2} \right\rangle$$

Method to bound other terms:

i) Use higher-subtracted sum rules to eliminate all couplings with $s^{\geq 4}$ Regge growth.

Ex:
$$B_4: \quad 4g_4 + \ldots \qquad = \left\langle \frac{(2m^2+t)\,\mathcal{P}_J\left(1+\frac{2t}{m^2}\right)}{m^4\left(m^2+t\right)^3} \right\rangle$$

"Null constraints": IR crossing relates the coefficients of $s^2 t^2$ and s^4.

$\Rightarrow$ Improved sum rules

$$\underbrace{\frac{8\pi G}{-t} + 2g_2 - g_3 t}_{\text{Finite sum!!}} = \left\langle C^{\text{improved}}_{2,t}[m^2, J] \right\rangle$$

[SCH, Mazac, Rastelli + Simmons-Duffin '21]

ii) Construct wavefunctions $\psi(p)$ with positive heavy action:

If: $\int_0^M \psi(p)\, C^{\text{improved}}_{2,-p^2}[m, J] \geq 0 \quad \forall\ m \geq M,\ J$

$\Downarrow$

Then, $\int_0^M \psi(p) \left(\frac{8\pi G}{-p^2} + 2g_2 + g_3 p^2 \right) \geq 0$.

bound on light interactions

Linear programming: optimize $\psi(p)$ to get optimal constraint on g_2, g_3, etc.

Such $\psi(p)$ exist!

Among other properties:

- compact support in p
- positive in b

$$\left(\text{ex: } \int_0^1 (1-p)\,dp\,\cos(pb) = \frac{1-\cos(b)}{b^2} \geq 0\right)$$

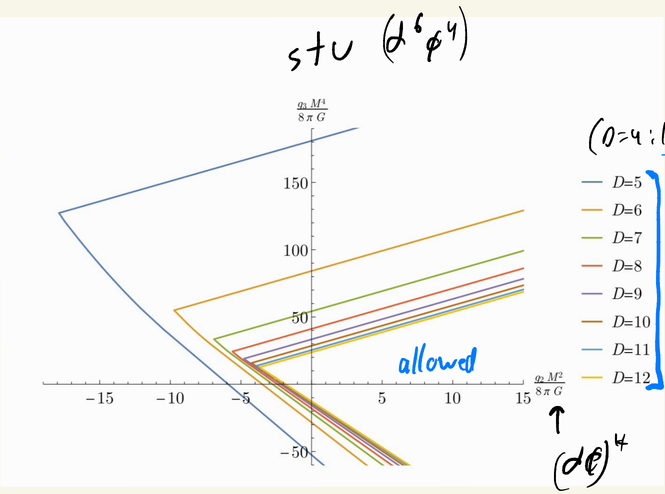
We do not know a general basis for generic d.

In practice, we make a polynomial ansatz and use linear programming.

For the scalar EFT, we find an allowed cone:

$$-8.15\frac{g_2}{M^2} - 28.8\frac{8\pi G}{M^4} \leq g_3 \leq 3\frac{g_2}{M^2} + 93.0\frac{8\pi G}{M^4} \qquad (D=6),$$

with $g_2 \geq -\# \frac{8\pi G}{M^2}$



$$g_2 \geq -\frac{G_N}{M^2}\log\left(\frac{M^2}{\mu_{IR}^2}\right)$$

negative g_2 allowed by gravity.

resolves puzzle in QED+gravity?

How about $G \to 0$? $\Rightarrow$ Near-forward sumrules

Nothing prevents $g_2 \neq 0$ with $G = 0$.

All other couplings will satisfy two-sided bounds:

$$-\# \frac{g_2}{M^2} \leq g_3 \leq +\# \frac{g_2}{M^2}$$

~~Massive Galileons~~

The #'s can be found equivalently using near-forward sum rules:

$$g_2 = \left\langle \frac{1}{m^4} \right\rangle_{m \geq M}, \qquad g_3 = \left\langle \frac{3 - \frac{4}{d-2}\mathcal{J}^2}{m^6} \right\rangle_{m \geq M}, \qquad 0 = \left\langle \frac{\mathcal{J}^2(2\mathcal{J}^2 - 5d + 4)}{m^8} \right\rangle .$$

[Tolley, Wang + Zhou '20]
[SCH + van Duong '20]
[Huang^2 + Arkani-Hamed '20]

clearly (with $G=0$): $g_2 \geq 0$, $g_3 \leq \frac{3 g_2}{M^2}$.

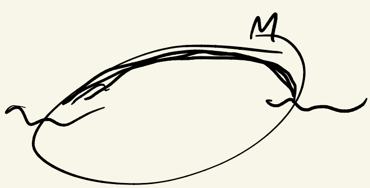
for lower-bound?

IR-crossing: high-spin states can't couple too strongly!

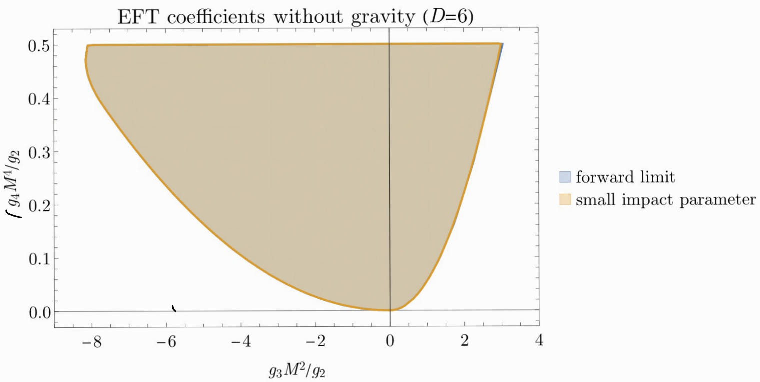
$$\left\langle \frac{1}{m^4} \frac{\overset{\sim b^2}{\mathcal{J}^2}}{m^2} \right\rangle \leq \frac{\#}{M^2} \left\langle \frac{1}{m^4} \right\rangle$$

As far as sum rules care,
all heavy states have size $b \lesssim \frac{1}{M}$.

(ie. black holes, long strings, etc. can't couple strongly enough to be significant)



The $G \to 0$ limit of the "wavepacket" bounds give the same:



wavepacket sum rules → Seamlessly deal with graviton pole (perhaps also loops?)

→ Much easier to justify physically $\left(\left| \frac{M_f(s)}{s} \right| < C \text{ vs } \frac{M(s,t)}{s^2} \to 0 \right)$

Two-sided bounds for generic coefficients:

$\frac{g_k M^{\#}}{g_2}$

EFT coefficient	Lower bound	Upper bound
$\tilde{g}_3$	-10.346	3
$\tilde{g}_4$	0	0.5
$\tilde{g}_5$	-4.096	2.5
$\tilde{g}_6$	0	0.25
$\tilde{g}'_6$	-12.83	3
$\tilde{g}_7$	-1.548	1.75
$\tilde{g}_8$	0	0.125
$\tilde{g}'_8$	-10.03	4
$\tilde{g}_9$	-0.524	1.125
$\tilde{g}'_9$	-13.60	3
$\tilde{g}_{10}$	0	0.0625
$\tilde{g}'_{10}$	-6.32	3.75

Compatible with geometric series!

$$\frac{1}{M^2 - s} \to \frac{1}{M^2} + \frac{s}{M^4} + \cdots$$

More on non-gravitational scalar EFT

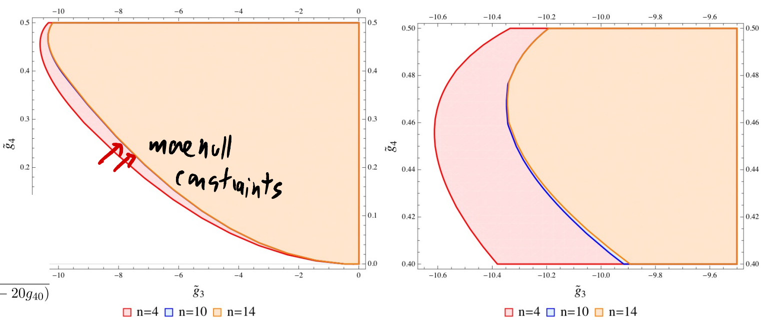
The $k=5$ EFT ahedon gives the outer curve

Region I: $g_{31}^{\min} = -\frac{3}{2}\sqrt{g_{40}}$,

Region II: $g_{31}^{\max} = \frac{1}{2}\sqrt{\frac{427}{3}}\, g_{40}$,

Region III: $g_{31}^{\max} = \frac{30}{7} g_{40} + \frac{37}{42}\sqrt{g_{40}\,(21 - 20 g_{40})}$

(see Huang's talk)



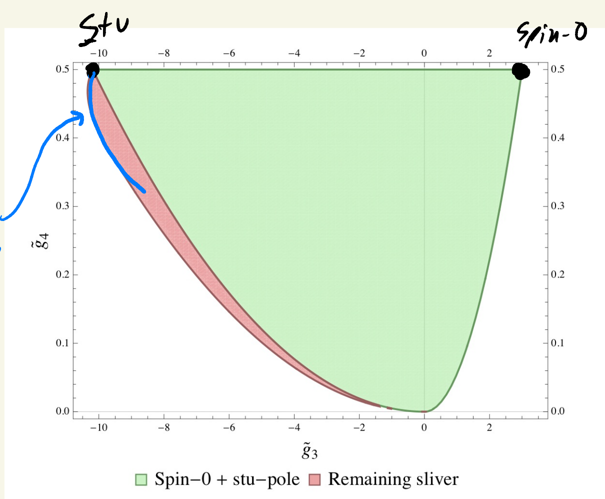
(a) Convergence is rapidly achieved by adding null constraints.

(b) Close-up near the left kink at $(-10.19, 0.5)$.

Convergence with adding null constraints ($\simeq$ larger Hankel matrices) is fast in this example.

The kinks are simple S-matrices:

$$
\begin{aligned}
\mathcal{M}_{\text{spin-0}} &= \frac{1}{m^2 - s} + \frac{1}{m^2 - t} + \frac{1}{m^2 - u}\,, \\
\mathcal{M}_{stu\text{-pole}} &= \frac{m^4}{(m^2 - s)\,(m^2 - t)\,(m^2 - u)} - \gamma(d)\mathcal{M}_{\text{spin-0}}\,.
\end{aligned}
$$



Unknown family of extremal theories

$S = 1 + iT$

$|S| \leq 1 \Rightarrow 0 \leq \underline{\mathrm{Im}}\, T \leq 2$

$\mathrm{disc}\, G = \sum \underbrace{2\sin^2(\pi\ \ )}\cdot$

What to expect for graviton scattering?

preliminary: 10D max SUSY: $\mathcal{M}_{low} = \delta^{16}(Q)\cdot\left(\frac{8\pi G_N}{stu} + \overset{R^4}{g_0} + \cdots\right)$

"anti-subtracted" sum-rules exist: $|\mathcal{M}_4 s^3| \leq \text{const.}$ ⇒ leading sum rules measure G only!!

i) g_0 is subleading in Regge limit ⇒ expect upper bound on $\frac{g_0}{G_N}$.

ii) UV spectral density can't vanish ⇒ lower bound (use $0 \leq \mathrm{Im}\, a_J \leq 2$)

Indeed: $0 < 0.14\,\frac{8\pi G}{M_1^6} \leq g_0 \leq 3{,}000\,\frac{8\pi G}{M^6}$

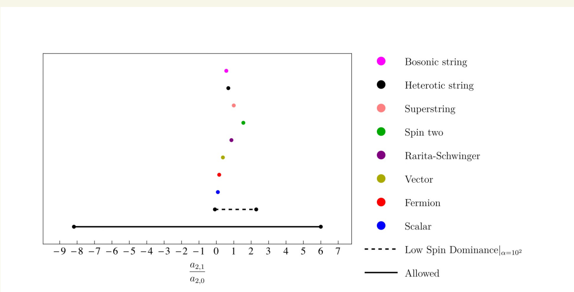
[Guerrieri, Penedones + Vieira '21]

[SCH, Mazac, Rastelli + Simmons-Duffin '21]

The lower bound seems saturated in IIB modulispace!!!...

The upper bound is easily satisfied by Veneziano-Shapiro ($2\zeta_3 \approx 2{,}4 < 3{,}000$)

⇒ Is it saturated in some theory?



[Bern, Kosmopoulos, Zhiboedov '21]

Are we missing some constraints?

Comment: Causality vs EFT power counting

a contact interaction can be probed by dispersive sum rules if the maximal spin in its partial waves is large enough: $J > J_* \leq 1$

Ex: complex scalar $\mathcal{L} \sim g_1 (H^\dagger \partial_\mu H)^2$, $\mathcal{M} \sim g_1 t$ → dim. 6 → spin 1

Ex: graviton R^4 $\mathcal{M}_{++--} = g\,[12]^4 \langle 34 \rangle^4 \sim s^4$ → spin 4

↳ The following SM contact interactions have spin ≤ 1:

$J_{\max}$	dimension	interactions
0	dim. 3	$\cancel{\phi^{(a}\phi^b\phi^{c)}}$
	dim. 4	$\phi^{(a}\phi^b\phi^c\phi^{d)}$
$\frac{1}{2}$	dim. 4	$\psi^{(i}\psi^{j)}\phi$
	dim. 5	$\psi^{(i}\psi^{j)}\phi^{(a}\phi^{b)}$
1	dim. 5	$\cancel{F\psi^{[i}\psi^{j]}}$
	dim. 6	$F^{[a}F^bF^{c]}$, $\phi^{[a}D\phi^{b]}\phi^{[c}D\phi^{d]}$, $\psi^i\bar\psi_j\phi^{[a}D\phi^{b]}$, $\psi^{(i}\psi^{j)}\psi^{(k}\psi^{l)}$, $\psi^{(i}\psi^{j)}\bar\psi_{(k}\bar\psi_{l)}$, $F\psi^{[i}\psi^{j]}\phi$, $F^{(a}F^{b)}\phi^{(c}\phi^{d)}$
	dim. 7	$\cancel{F\phi^{[a}D\phi^bD\phi^{c]}}$, $\cancel{F^{[a}F^bF^{c]}\phi}$, $D\psi^{[i}\psi^j\psi^{k]}\bar\psi$
higher-points	dim. 6	ϕ^6, $\psi^2\phi^3$
	...	

unprobed by disp. rels.

potentially so? disp. rels. may converge?

Table 1. Interactions which have spin ≤ 1 in all channels and are thus not probed by dispersion relations; ϕ are scalars, ψ Weyl fermions, and F field strengths. Adding any further derivative or graviton coupling pushes these above the $J_{\max} = 1$ threshold. Struck-out interactions $\cancel{\phi\phi\phi}$ are incompatible with SM gauge invariance.

[Unpublished; see: Durieux, Kitahara, Machado, Shadmi + Weiss '20]

Right now, even assuming spin-1 convergence, we don't get linear bounds on dim. 6 SM operators: spin-1 sum rules are typically not positive ("$s-u$")

$\Rightarrow$ nonlinear bounds $g_1^2 \leq \# g_2$? (stronger than $g_1^2 \lesssim \frac{4\pi^2}{M^2}$)

Cool fact with gravity:

i) All contacts involving a graviton grow with spin ≥ 2!

[Chowdhury, Gadde, Gopalka, Halder, Janagal, Minwalla '20]

ii) only 3 modifications to GR (in generic D) have spin 2

[Camanho, Edelstein, Maldacena, Zhiboedov '14]

$J_{\max}$	dimension	interactions
$\frac{3}{2}$	dim. 7	$\psi\psi\phi\phi D^2$, $F\psi\bar{\psi}\phi D$, $FF\psi\psi$, $FF\bar{\psi}\bar{\psi}$, $\cancel{RF\psi\psi}$
	dim. 8	$\psi\bar{\psi}\phi\phi D^3$, $F\psi\psi\phi D^2$, $F\bar{\psi}\bar{\psi}\phi D^2$, $FF\psi\bar{\psi}D$
2	dim. 8	$\phi\phi\phi\phi D^4$, $\psi\psi\psi\psi D^2$, $F\bar{F}\psi\bar{\psi}D$, $FFFF$, $FF\bar{F}\bar{F}$
	dim. 9	$\psi\psi\phi\phi D^4$, $\psi\psi\psi\bar{\psi}D^3$, $F\phi\phi\phi D^2$, $F\psi\bar{\psi}\phi D^3$, $F\bar{F}\psi\psi D^2$, $FF\psi\psi D^2$, $FFF\phi D^2$, $FF\bar{F}\phi D^2$
	dim. 10	$\phi\phi\phi\phi D^6$, $\psi\psi\psi\psi D^4$, $\psi\psi\bar{\psi}\bar{\psi}D^4$, $FF\phi\phi D^4$, $F\bar{F}\phi\phi D^4$, $FFF\bar{F}D^2$, F^4D^2
2 w/ gravity	dim. ≤ 6	S_{GB}, S_{R^3}, $S'^{(D\geq 7)}_{R^3}$, RFF, $RR\phi\phi$,
	dim. 7	$RFF\phi$, $RRF\phi$, $RRR\phi$,
	dim. 8	$RF\phi\phi D^2$, $R\psi\psi\phi D^2$, $RFFF$, $RRFF$
	dim. 9	$\cancel{R\phi^3D^2}$, $\cancel{RFF\phi D^2}$, $\cancel{RRFFD^2}$

Conclusion

$$\boxed{\text{Causality} \implies \text{two-sided bounds on generic EFT coefficients}}$$

"causal EFT" is a pleonasm! (= "causal whatever")

↳ Limits on causal modifications of GR? (ongoing)

↳ Lots to explore: $0 \leq I_n \leq 2$? Kinks? dim 6? loops?

↳ Is AdS more constraining than flat space?

Causality certainly holds more surprises...