SLAC NATIONAL ACCELERATOR LABORATORY

BERNHARD MISTLBERGER

LHC CROSS SECTIONS AT N3LO IN QCD

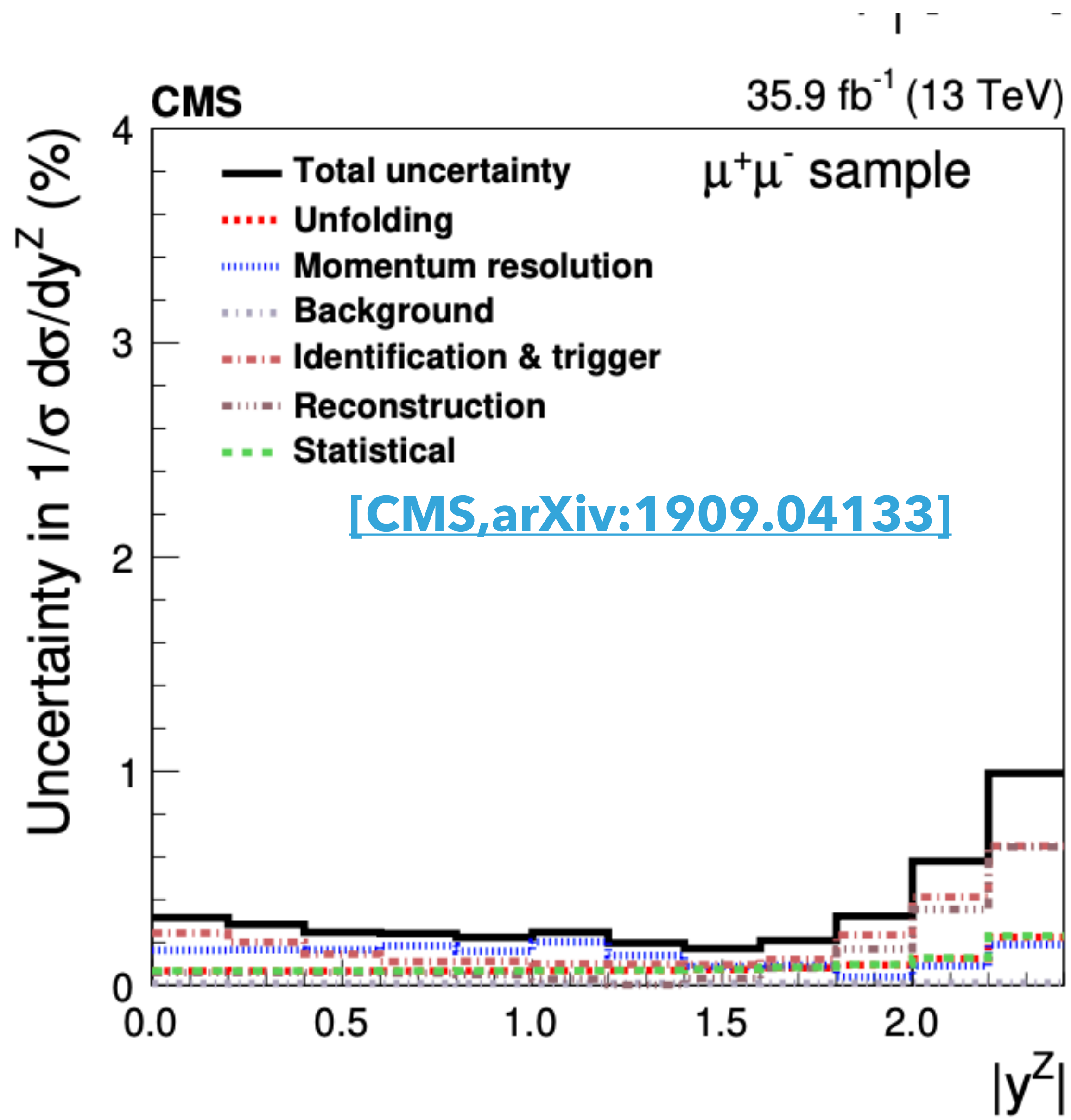
With **Claude Duhr, Falko Dulat, Robert Szafron and Julien Baglio**

arXiv:1904.09990 arXiv:2001.07717 arXiv:2004.04752 arXiv:2007.13313

+ to appear

PRECISION OBSERVABLES

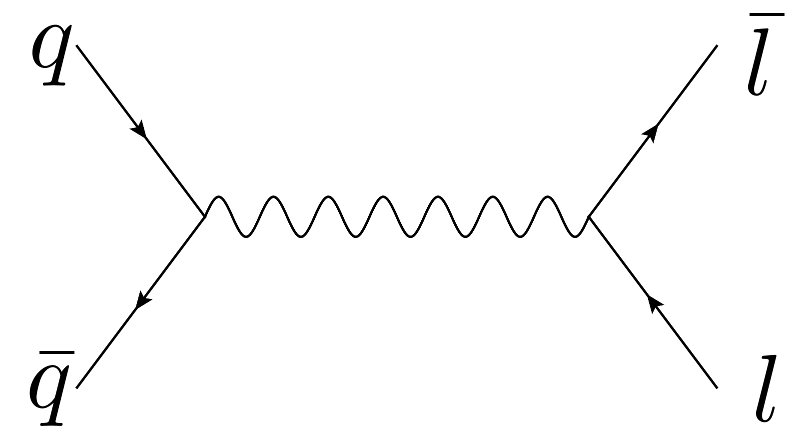
- Precision Observables at the LHC:
  - Clean Final States
  - Large Invariant Mass
  - Sizable Cross Section w.r.t. 3000 fb^-1 of data



Uncertainty on the Shape of the DY Rapidity Distribution

EXAMPLES OF PRECISION OBSERVABLES – DY

- DY: Production of lepton pair at the LHC



Clean and Abundant

- Measurement of the W boson mass:

[ATLAS, arXiv:1701.07240]

$$m_W = 80370 \pm 7\,(\text{stat.}) \pm 11\,(\text{exp. syst.}) \pm 14\,(\text{mod. syst.})\,\text{MeV}$$

$$m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}.$$

Theory!

- LHC will deliver the most precise single measurement of $\sin\theta_W$

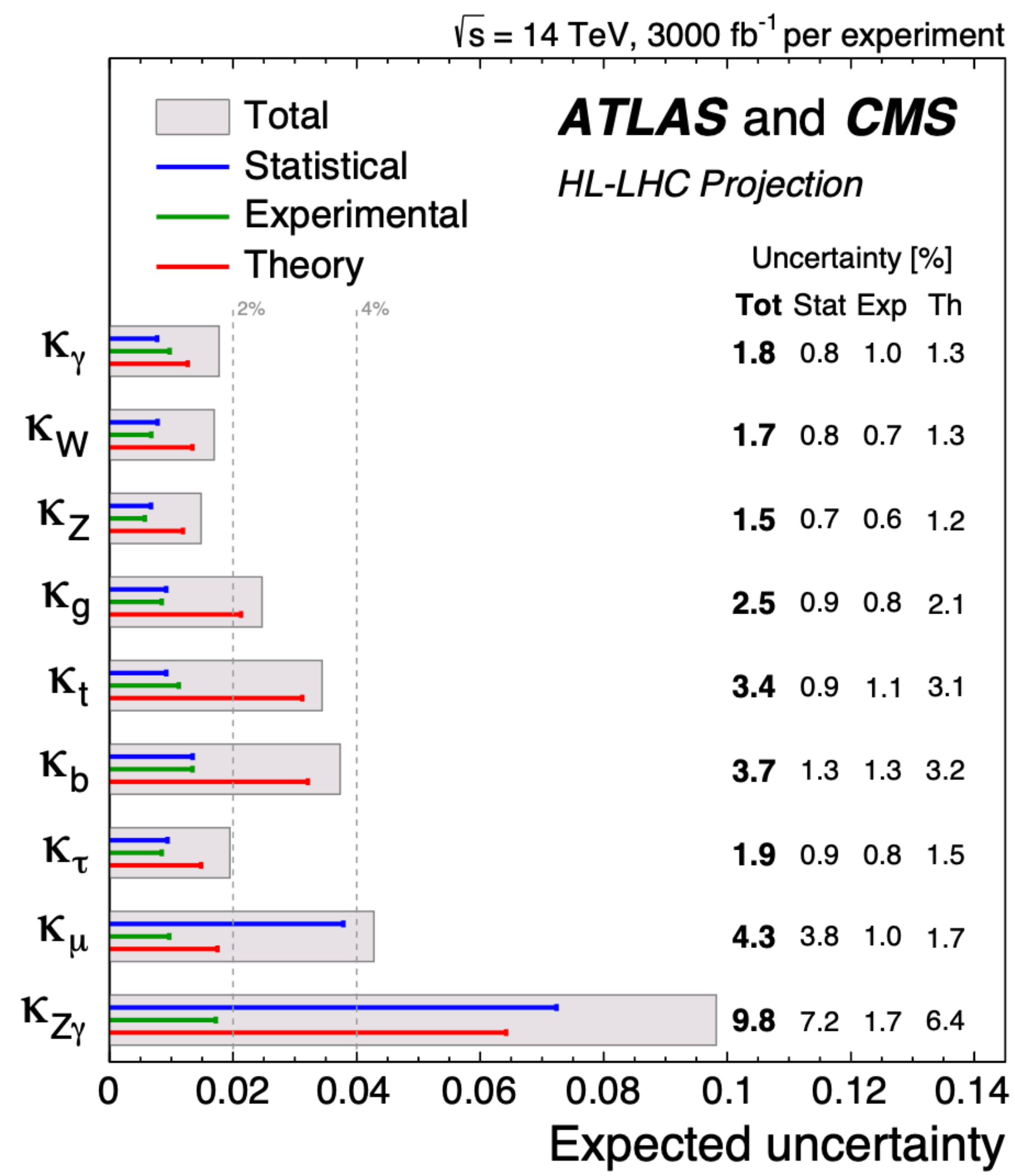
(ATLAS)

- Play a role in EFT Operators, VBS, unitarization, PDF determination, nature of the Z and W boson, strong coupling constant measurement, …

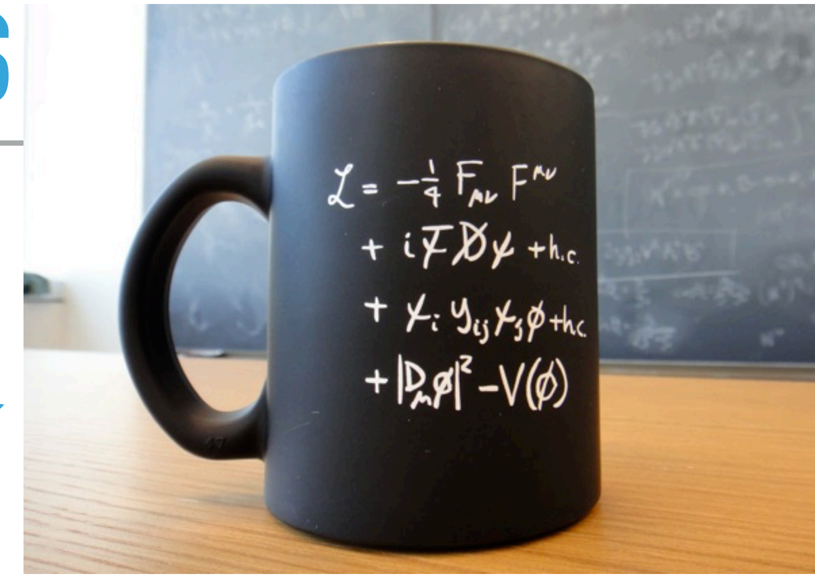
EXAMPLES OF PRECISION OBSERVABLES – HIGGS

Cross Section to Produce a Higgs

- LHC will determine the ratio $\frac{\sigma_{\mathrm{LHC}}}{\sigma_{SM}}$ to < 2.4 % with 3000 fb^-1.
- Extract coupling strengths at a couple of %.
- Transverse Momentum Distribution
- Check the nature of the boson! (EFT, POs, STXS, …)



THE WAY TO PRECISION LHC PREDICTIONS



$$\sigma \sim \int dx dy f(x) f(y) \hat{\sigma} + \mathcal{O}\left(\frac{\Lambda}{Q}\right)$$

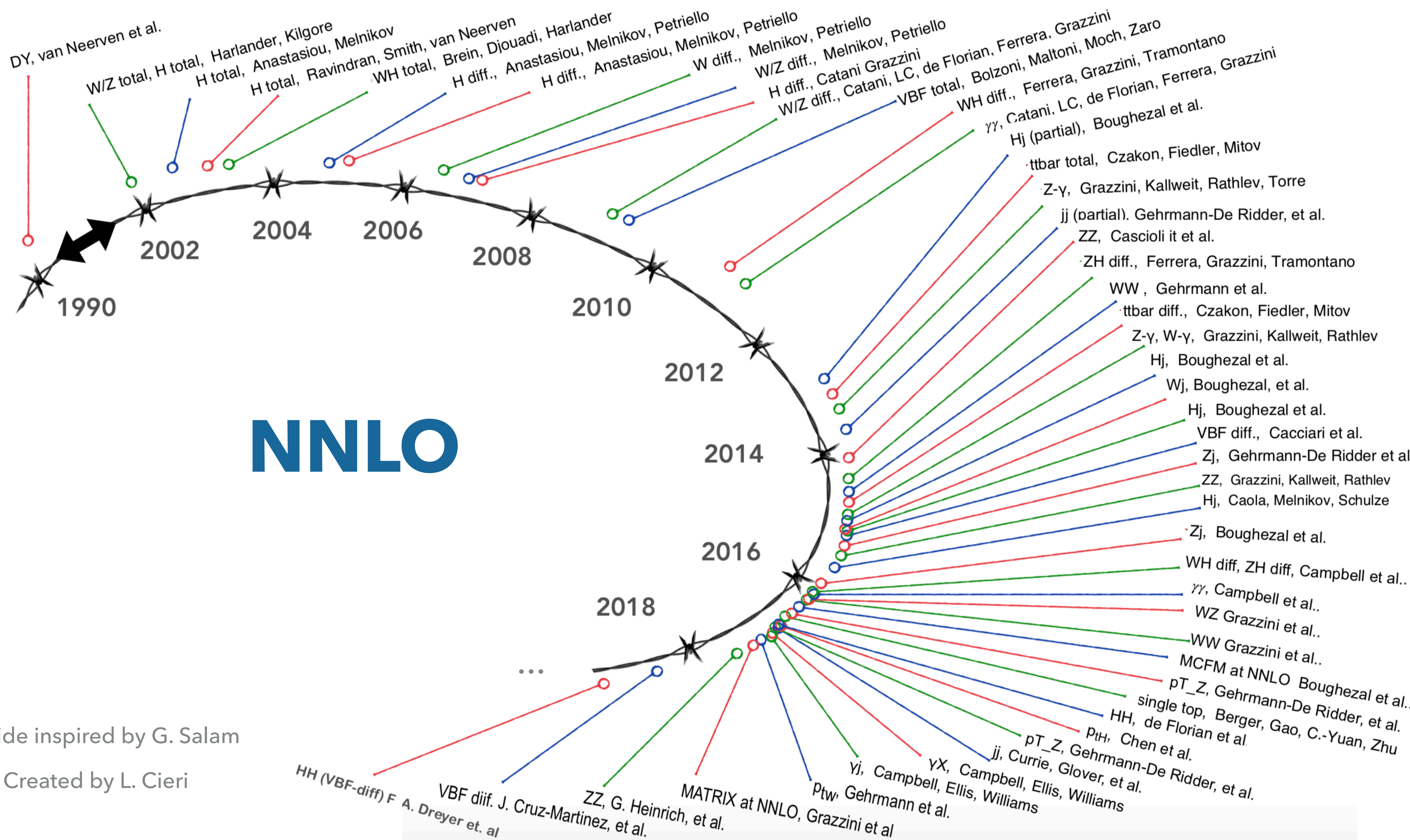
- Perturbative partonic cross sections
- QCD perturbation theory is dominant $\alpha_S = 0.118$
- Naively:

LO	NLO	NNLO	N3LO

$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S^1 \hat{\sigma}^{(1)} + \alpha_S^2 \hat{\sigma}^{(2)} + \alpha_S^3 \hat{\sigma}^{(3)} + \ldots$$

10% 1% 0.1%

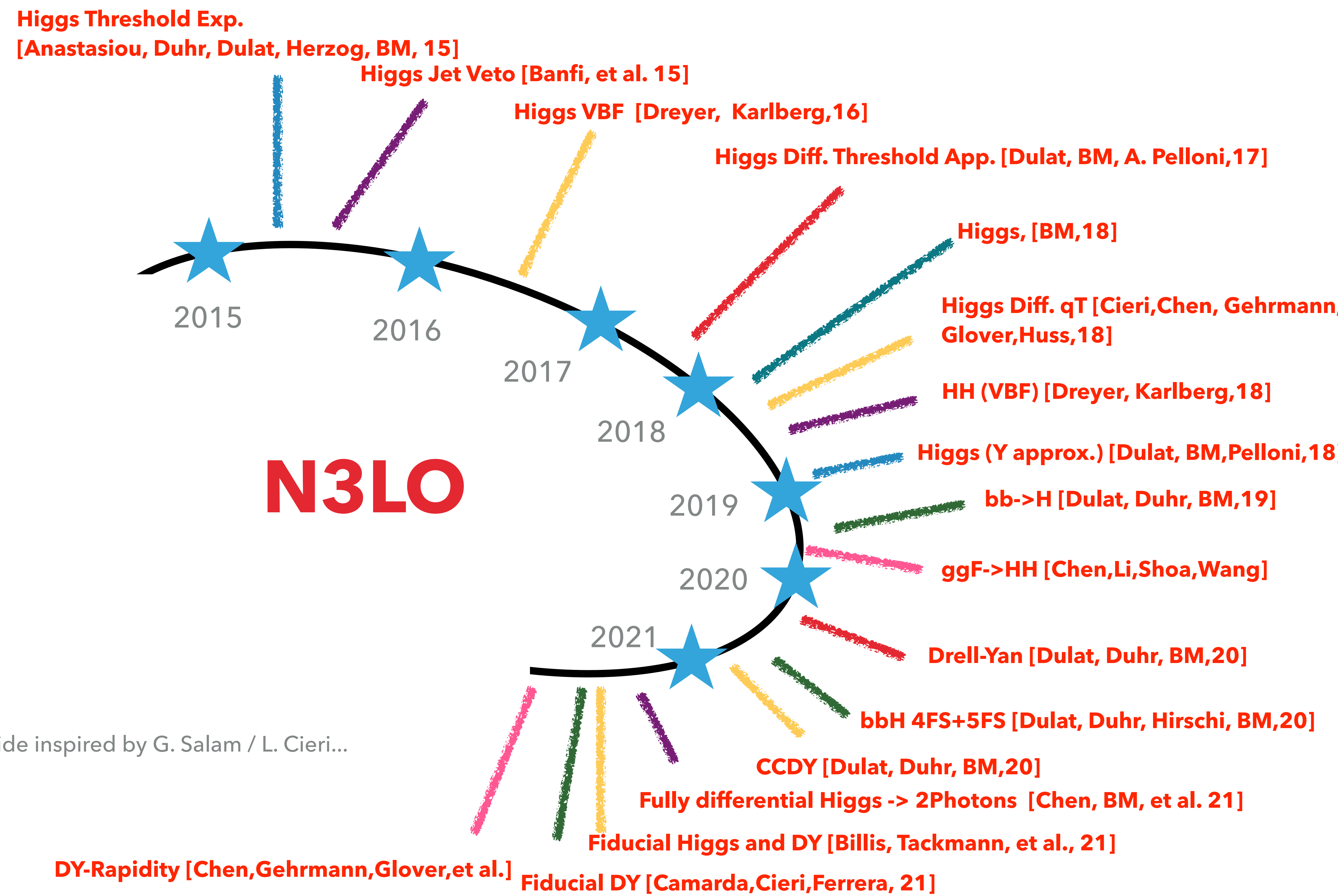
NNLO AT THE LHC OVER TIME



Slide inspired by G. Salam

Created by L. Cieri

N3LO AT THE LHC OVER TIME

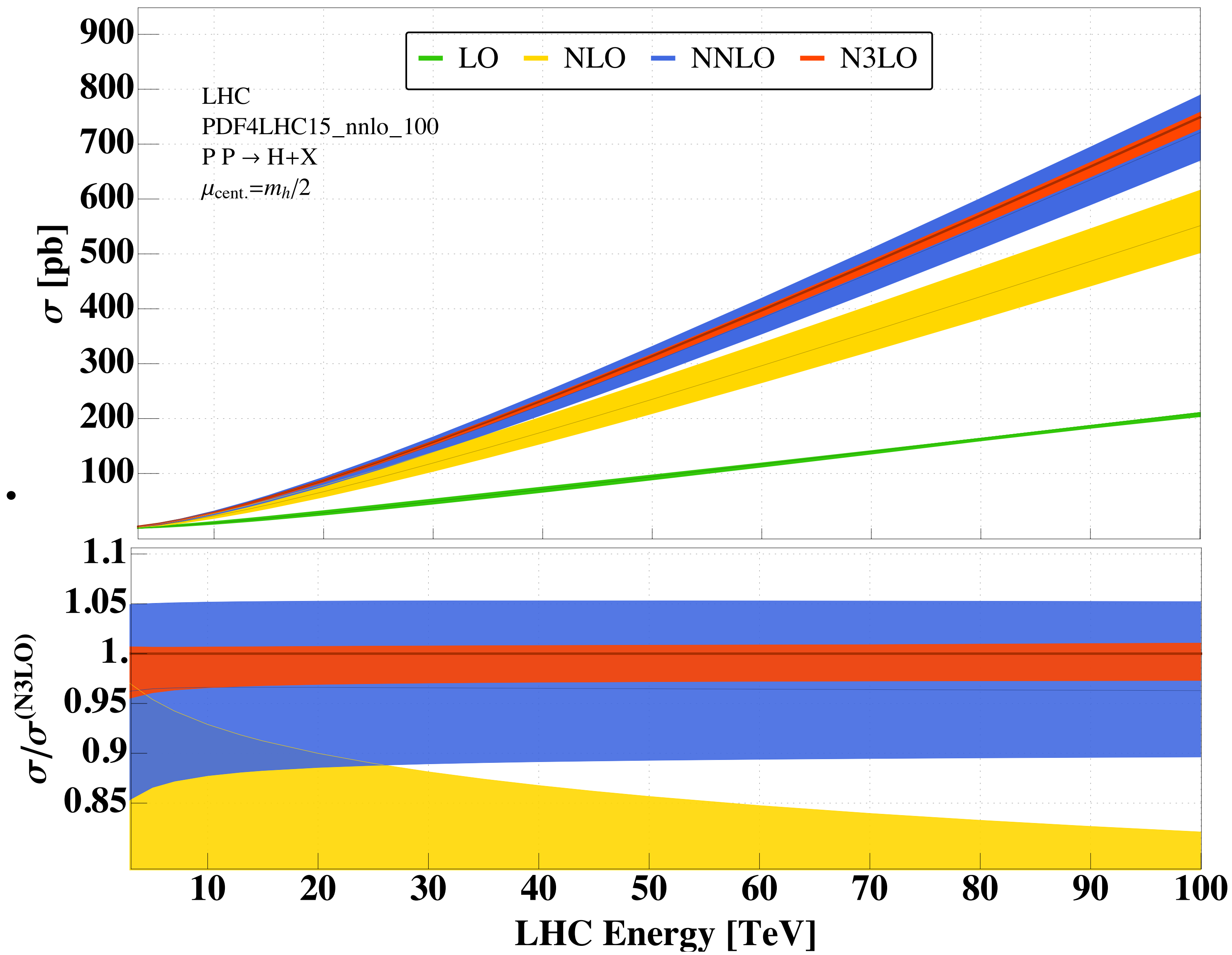


Slide inspired by G. Salam / L. Cieri...

HIGGS PRODUCTION IN GLUON FUSION

- First example of LHC cross section at N3LO.
- N3LO stabilizes the perturbative expansion.
- Reduction of perturbative uncertainty:

$$9.5\% \to 2.2\%$$



HIGGS PRODUCTION

How many Higgs bosons are produced at the LHC?

$$\mu = \frac{\sigma_{\mathrm{obs.}}}{\sigma_{\mathrm{SM}}}$$

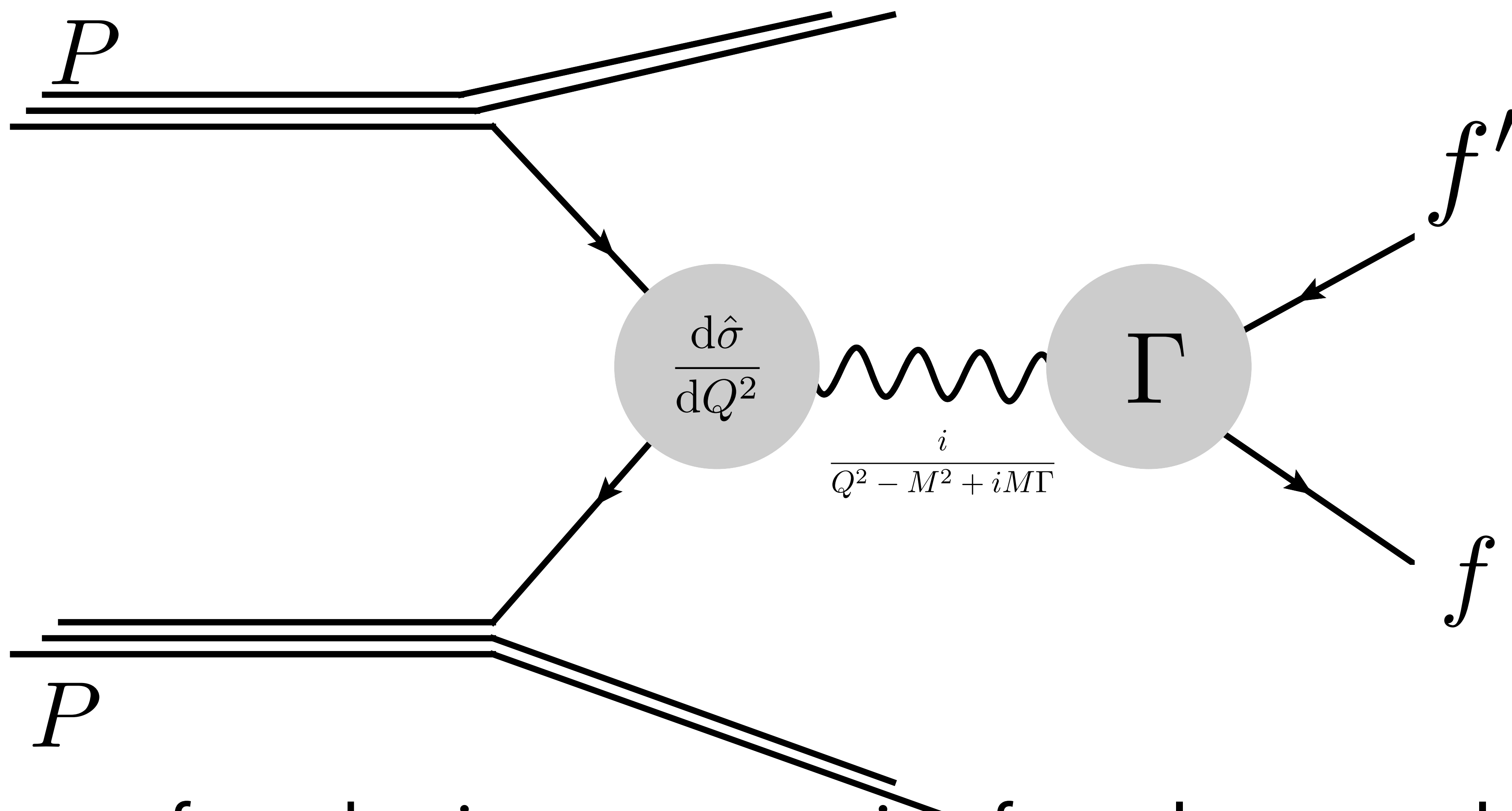
$$\mu = 1.06 \pm 0.07 = 1.06 \pm 0.04(\mathrm{stat}) \pm 0.03(\mathrm{exp.})^{+0.05}_{-0.04}(\mathrm{sig.\ th.}) \pm 0.02(\mathrm{bkg.\ th.})$$ ATLAS

$$\mu = 1.02^{+0.07}_{-0.06} = 1.02 \pm 0.04(\mathrm{stat}) \pm 0.04(\mathrm{exp.}) \pm 0.04(\mathrm{sig.+bkg.\ th.})$$ CMS

- Agreement of EXP and TH at ~ 7%!
- TH Uncertainty ~ Exp Uncertainty

INCLUSIVE CROSS SECTIONS

General structure for the production of a colorless, off-shell particle:

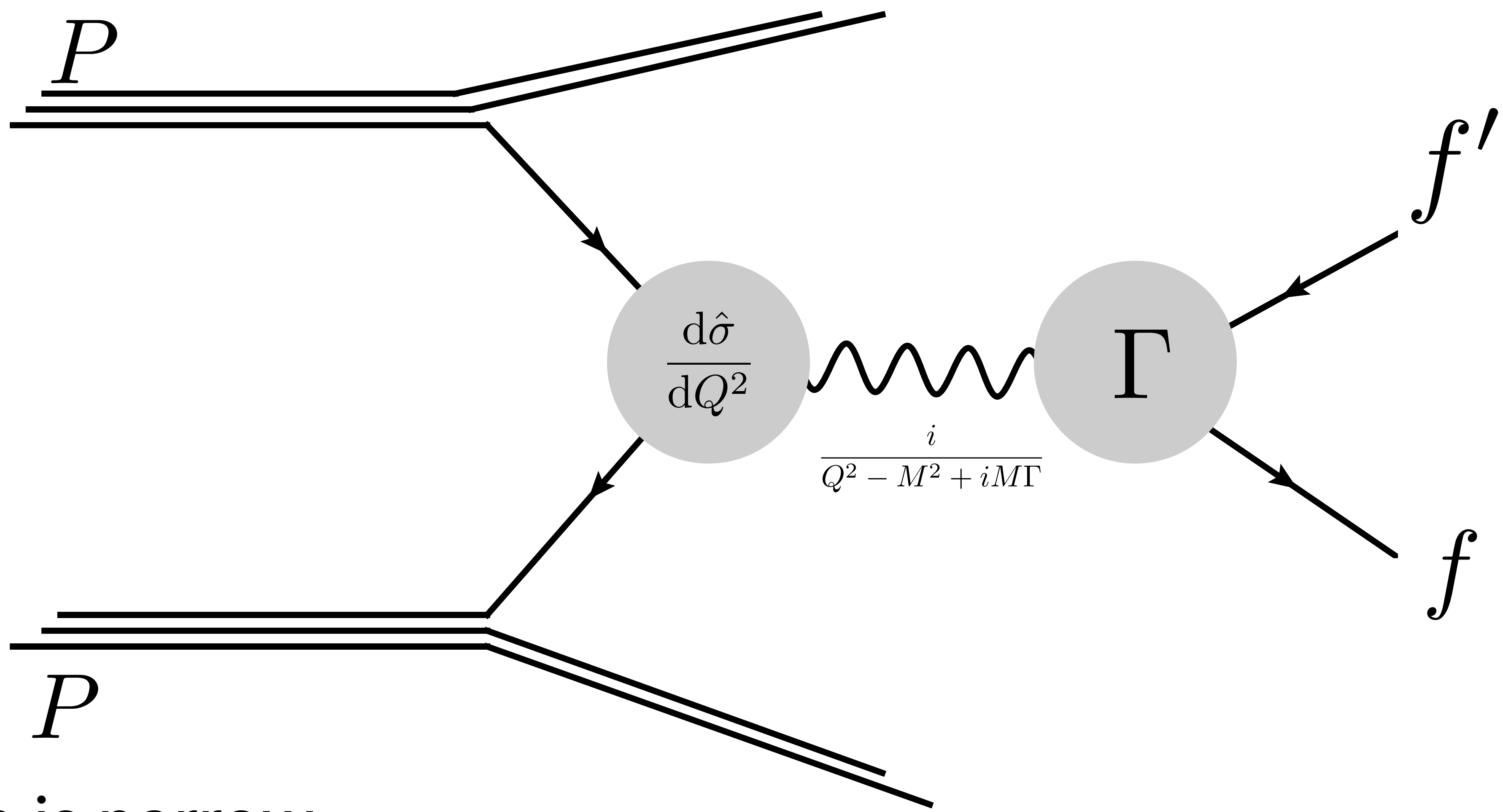


- Structure of production cross section for color neutral final states similar for a large range of processes:

$$\gamma^* \; W \; Z \; H \;\; bbH \;\; H^* \; G \quad WH \;\; ZH$$

- Components of their calculations are re-usebale: IBP relations, Master integrals, large pieces of computer code.

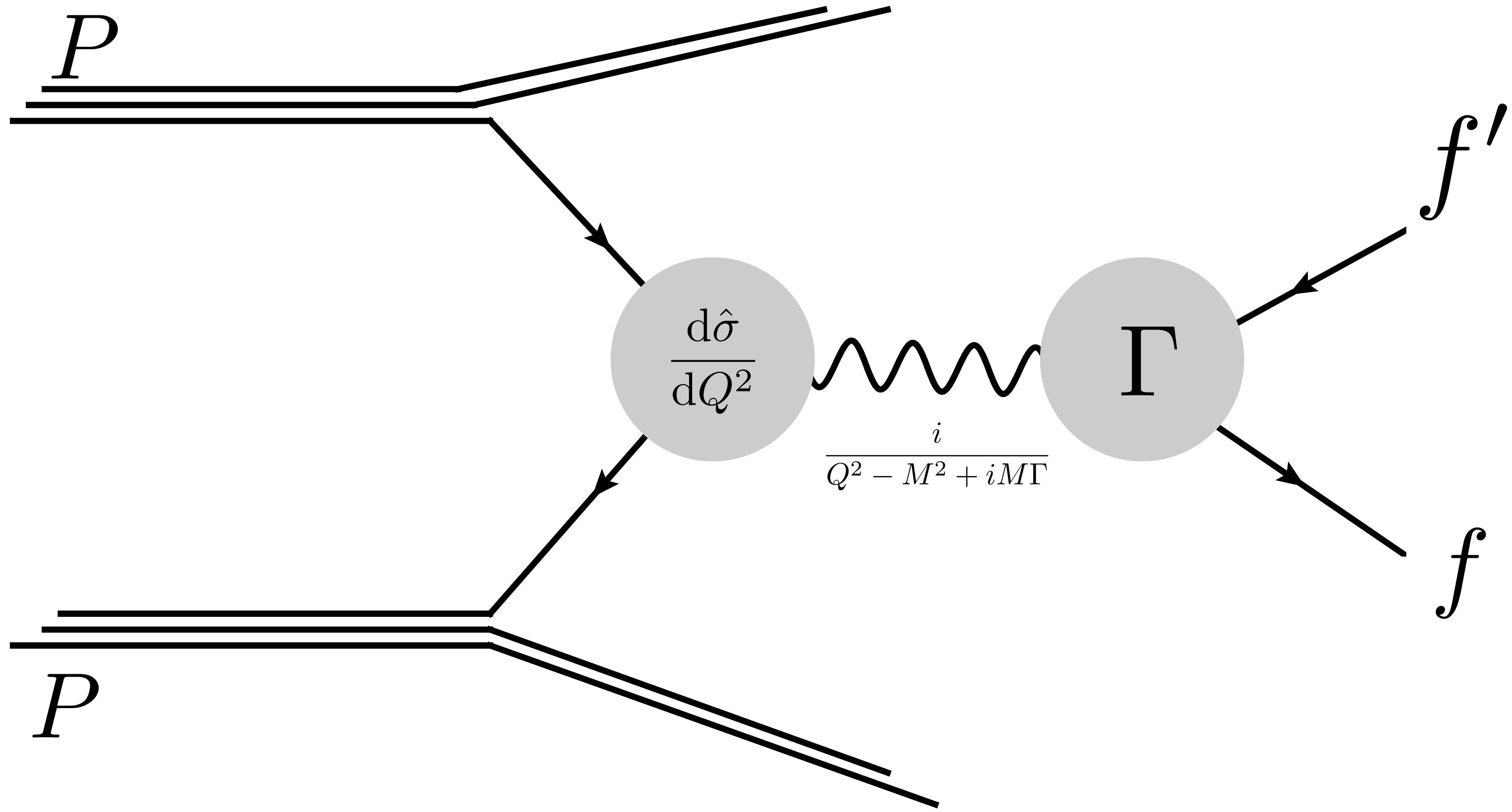
General structure for the production of a colorless, off-shell particle:



- Width is narrow
- Factorization of production and decay.
- Breit-Wigner distribution to approximate propagation of off-shell gauge boson.

INCLUSIVE CROSS SECTIONS

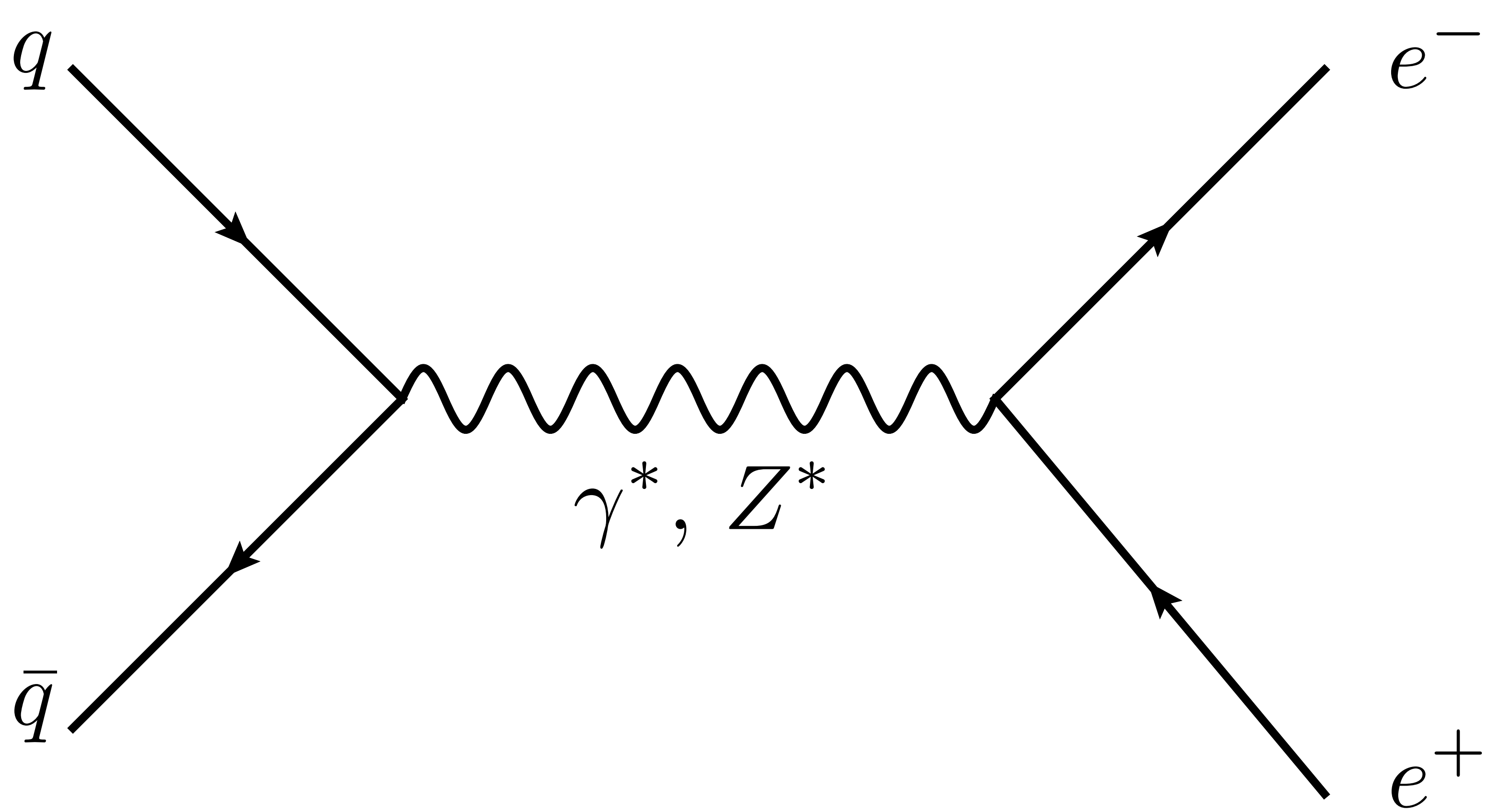
General structure for the production of a colorless, off-shell particle:



$$Q^2 \frac{d\sigma_{P\,P \to V/V' + X \to f f' + X}}{dQ^2} = Q^2 \frac{d\sigma_{P\,P \to V/V' + X}}{dQ^2} \times \Gamma_{V/V' \to f f'}(Q^2, m_f^2, m_{f'}^2) \times \mathrm{BW}(Q^2, m_V, m'_V)$$

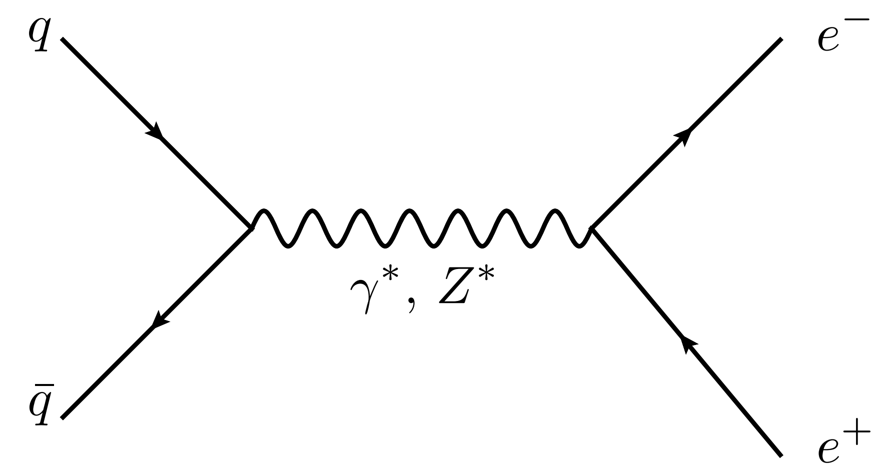
DRELL-YAN PRODUCTION

The probability to produce a $e^+ e^-$ pair

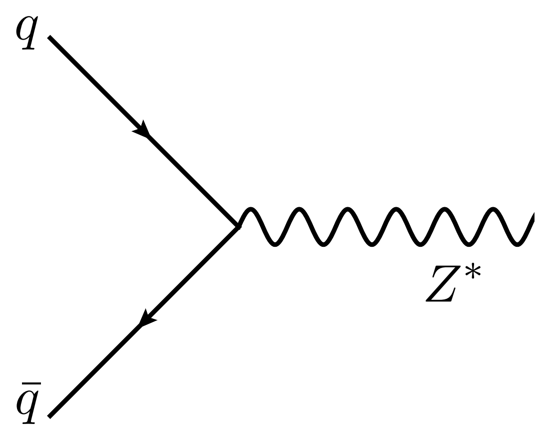


- The standard candle process at the LHC
- Parton distribution functions

DY PRODUCTION



- Computed photon cross section in arXiv:2001.07717
- Z boson vertex contains **Axial-Vector** coupling:



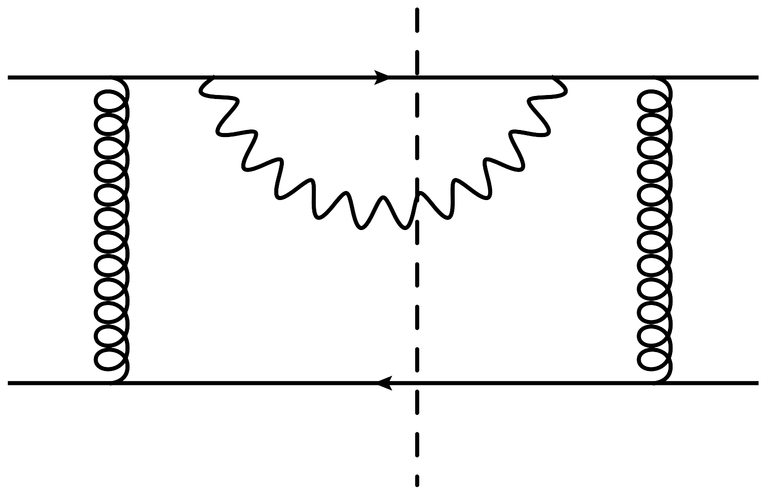
$$\sim g_W (V\gamma^\mu - A\gamma_5\gamma^\mu)$$

- How to treat γ_5 ?
  - 't Hooft -Veltman $\gamma_5 = \frac{-i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$
  - Larin: $\gamma_5\gamma^\mu \rightarrow \frac{1}{2}[\gamma_5, \gamma^\mu] = \frac{-i}{3!}\epsilon_{\mu\nu\rho\sigma}\gamma^\nu\gamma^\rho\gamma^\sigma$
  - ...

DY PRODUCTION

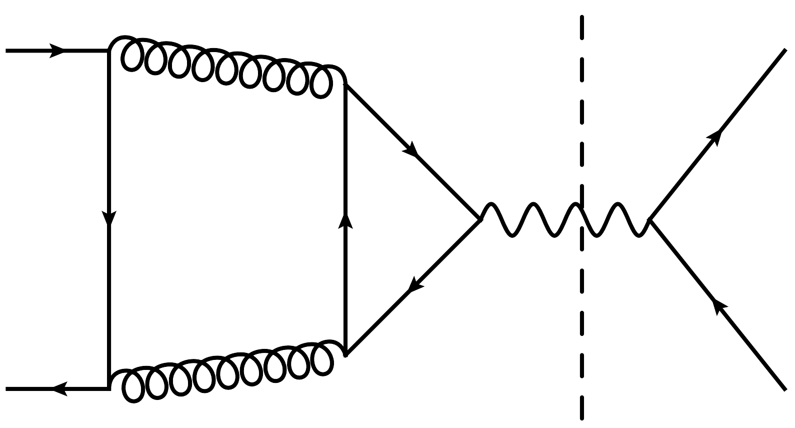
- Where does the treatment of γ_5 matter?

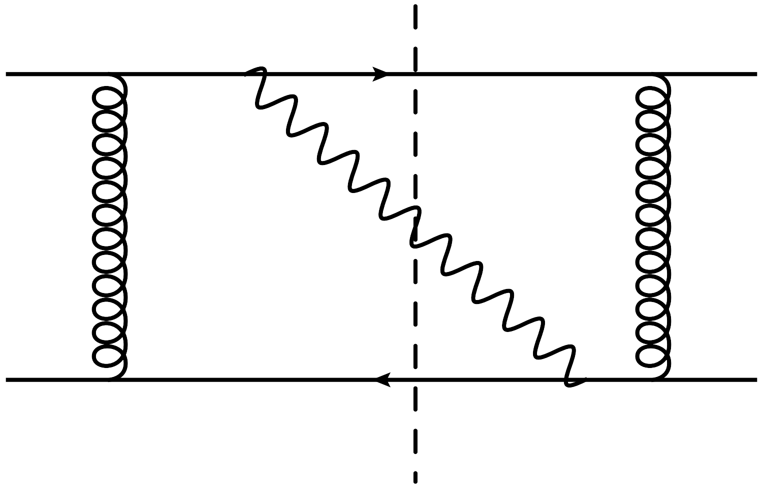
  - AV current couples on same fermion line twice:



γ_5 appears twice in the same trace. If treated in D=4, drops out identically! Such contributions can be extracted directly from calculation of vector current DY.

  - AV current couples to fermion line only once:





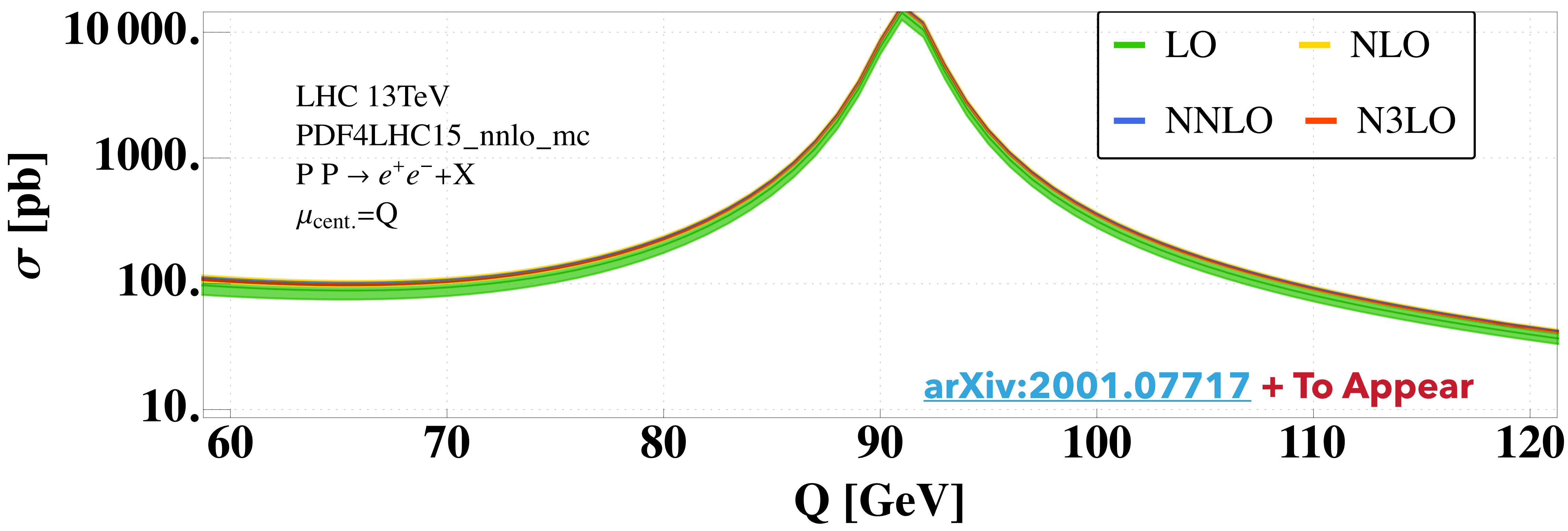
Axial anomaly diagrams. Genuinely different contributions from vector current first at NNLO.

Finite renormalisation in Latin Scheme. Top Quark contributes UV divergences in 5 flavor QCD.

DY PRODUCTION

- Inclusive Cross Section

$$P\,P \rightarrow e^+e^- + X$$

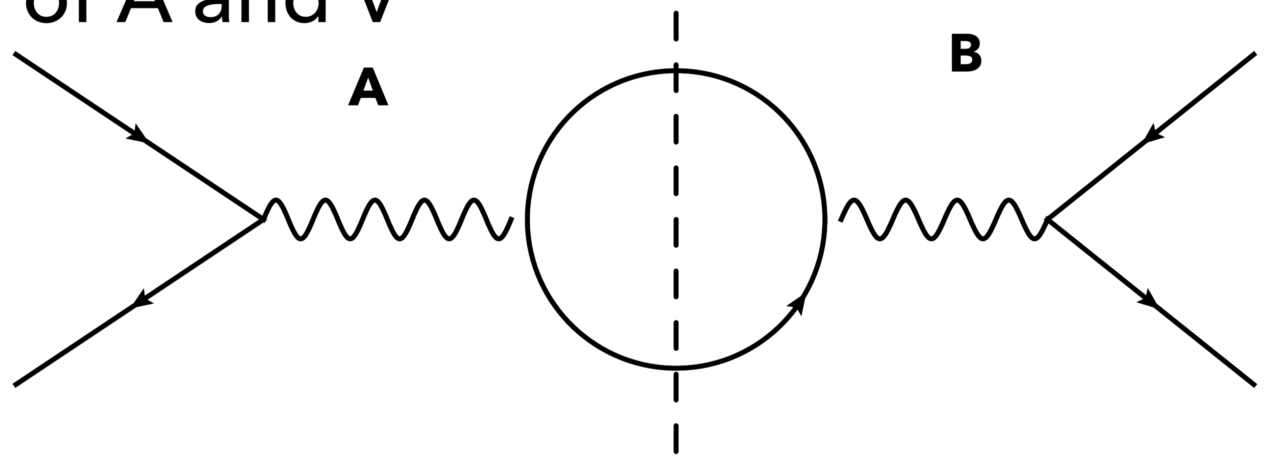


- Bulk of the cross section centered around Z-pole

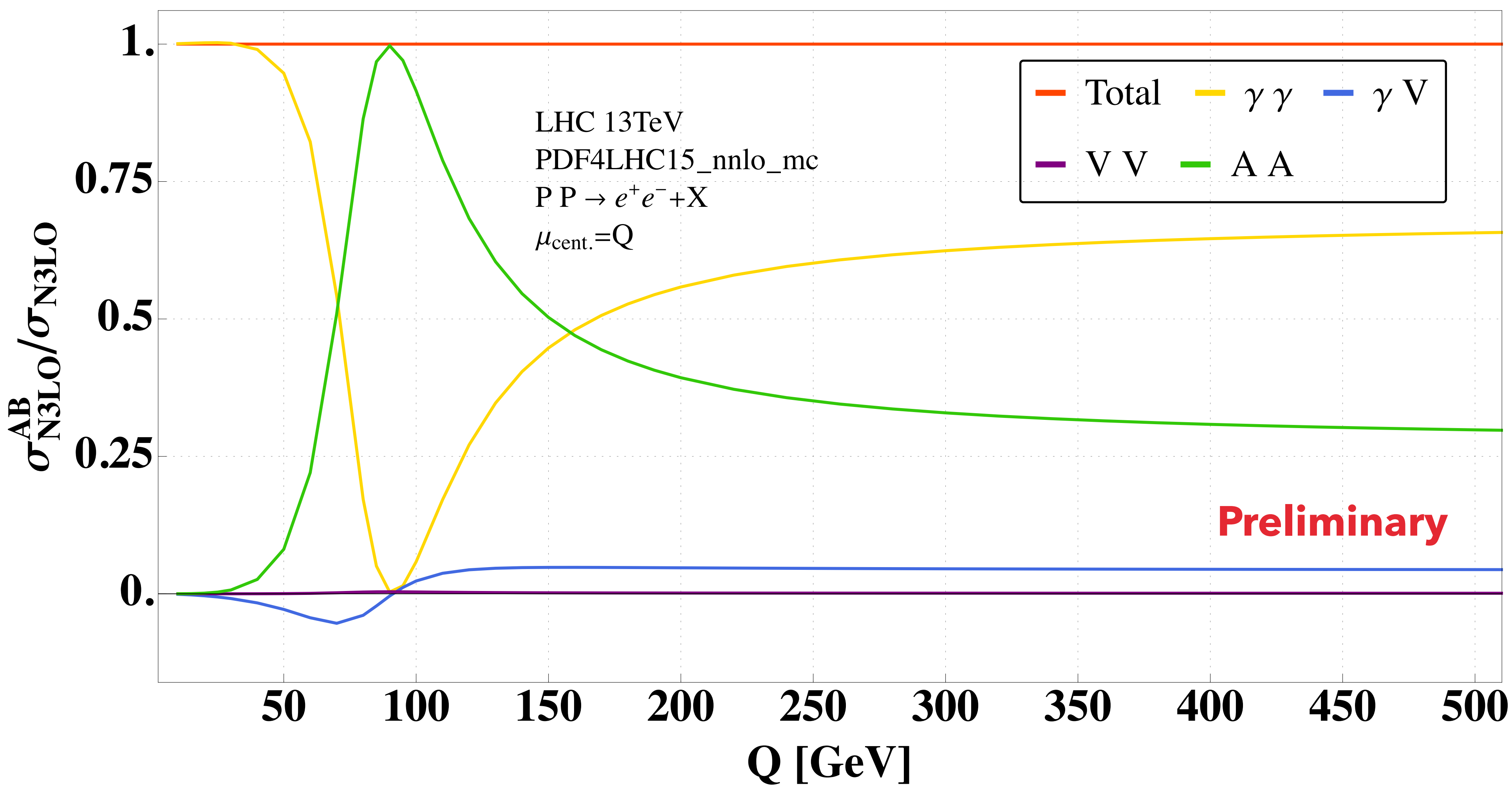
DY PRODUCTION

$$P\,P \rightarrow e^{+}e^{-} + X$$

- Interference of A and V



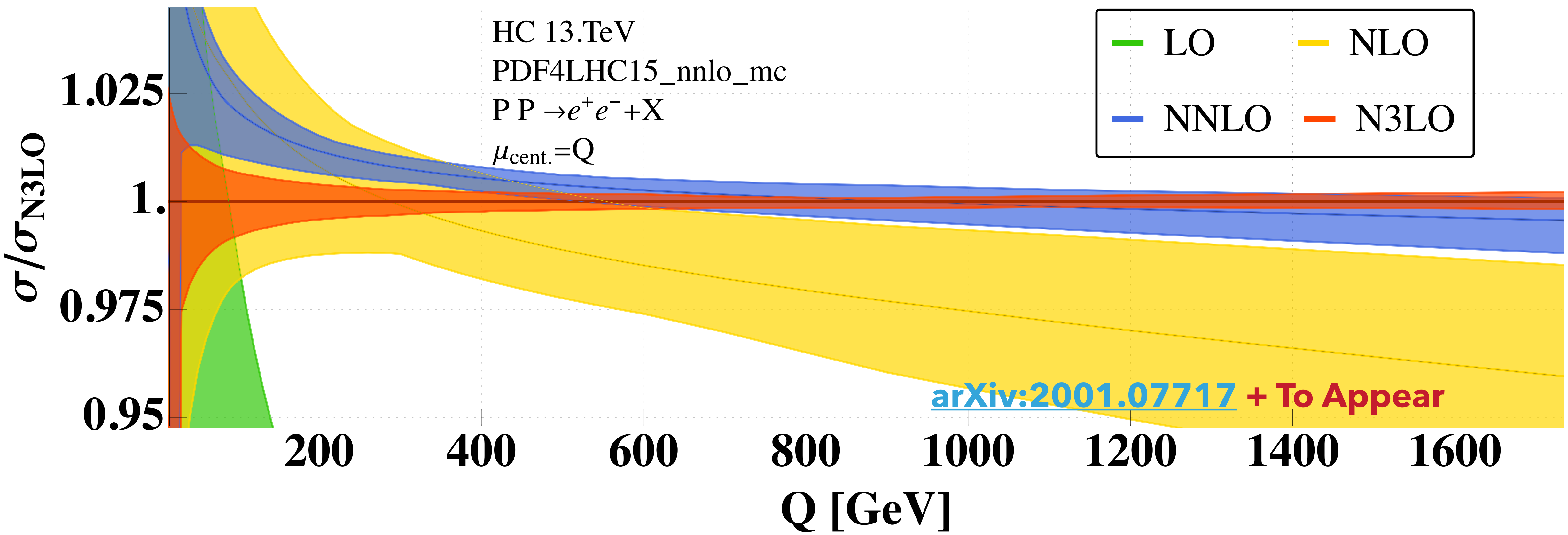
N3LO:



arXiv:2001.07717 + To Appear

- Inclusive K Factor for

$$P\,P \to e^+e^- + X$$



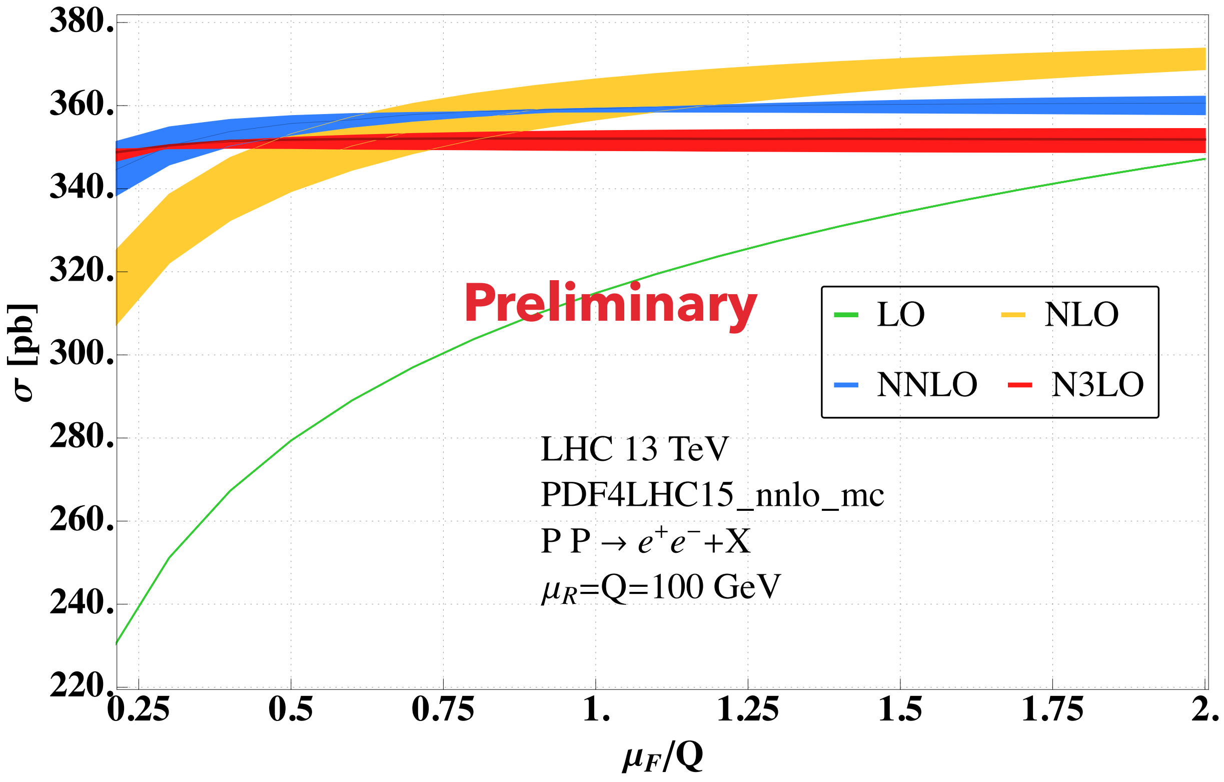
- Relatively Large corrections for Q ~ 100 GeV.
- Nice, perturbative behavior for large Q.
- Overall, reduced scale dependence.
- Extremely similar to K-Factor of DY via Photon Exchange!

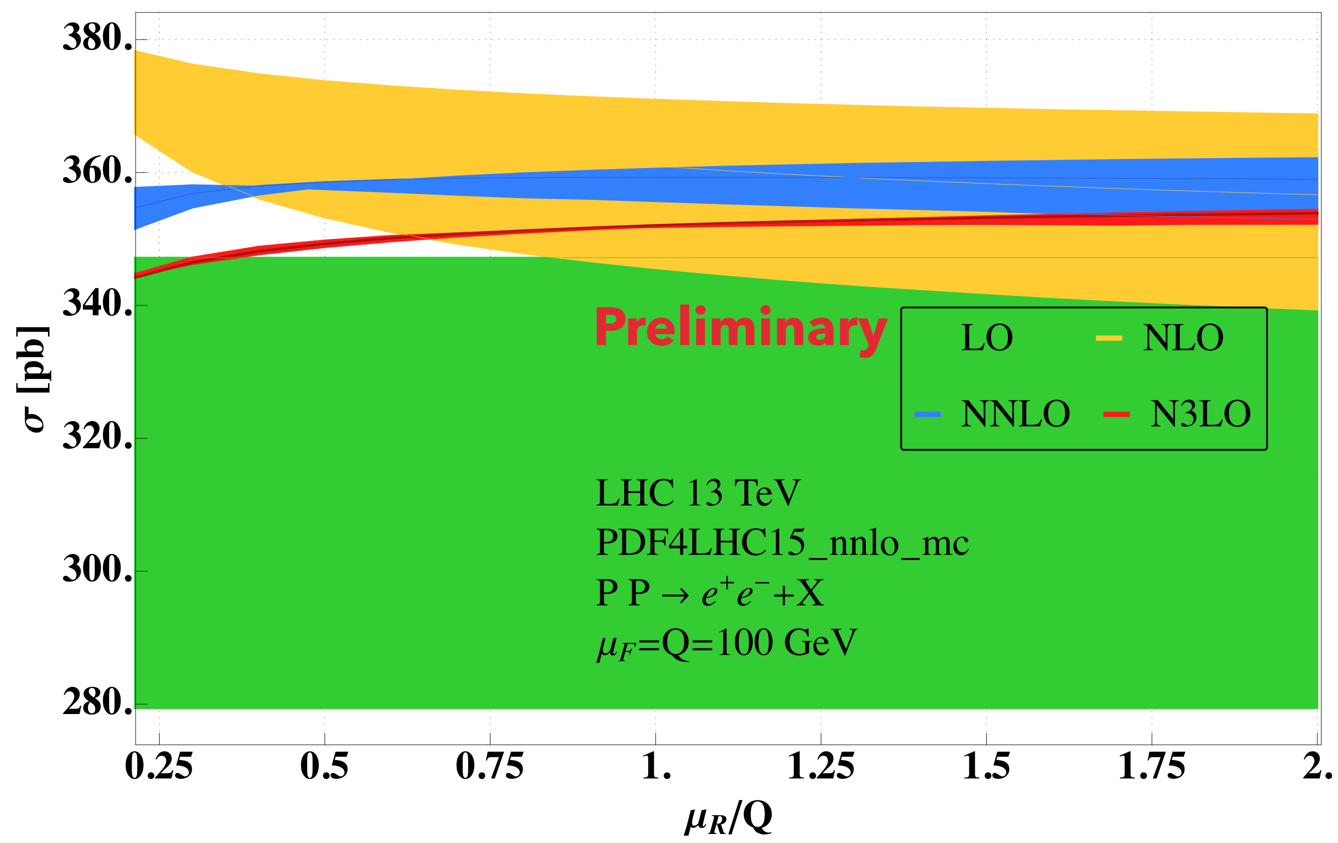
arXiv:2001.07717

DY PRODUCTION

$$P\,P \to e^{+}e^{-} + X$$

- Scale Dependence:





- Relatively flat.
- Systematic offset.
- Going to lower scales does not change the picture much.

arXiv:2001.07717 + To Appear

DY PRODUCTION

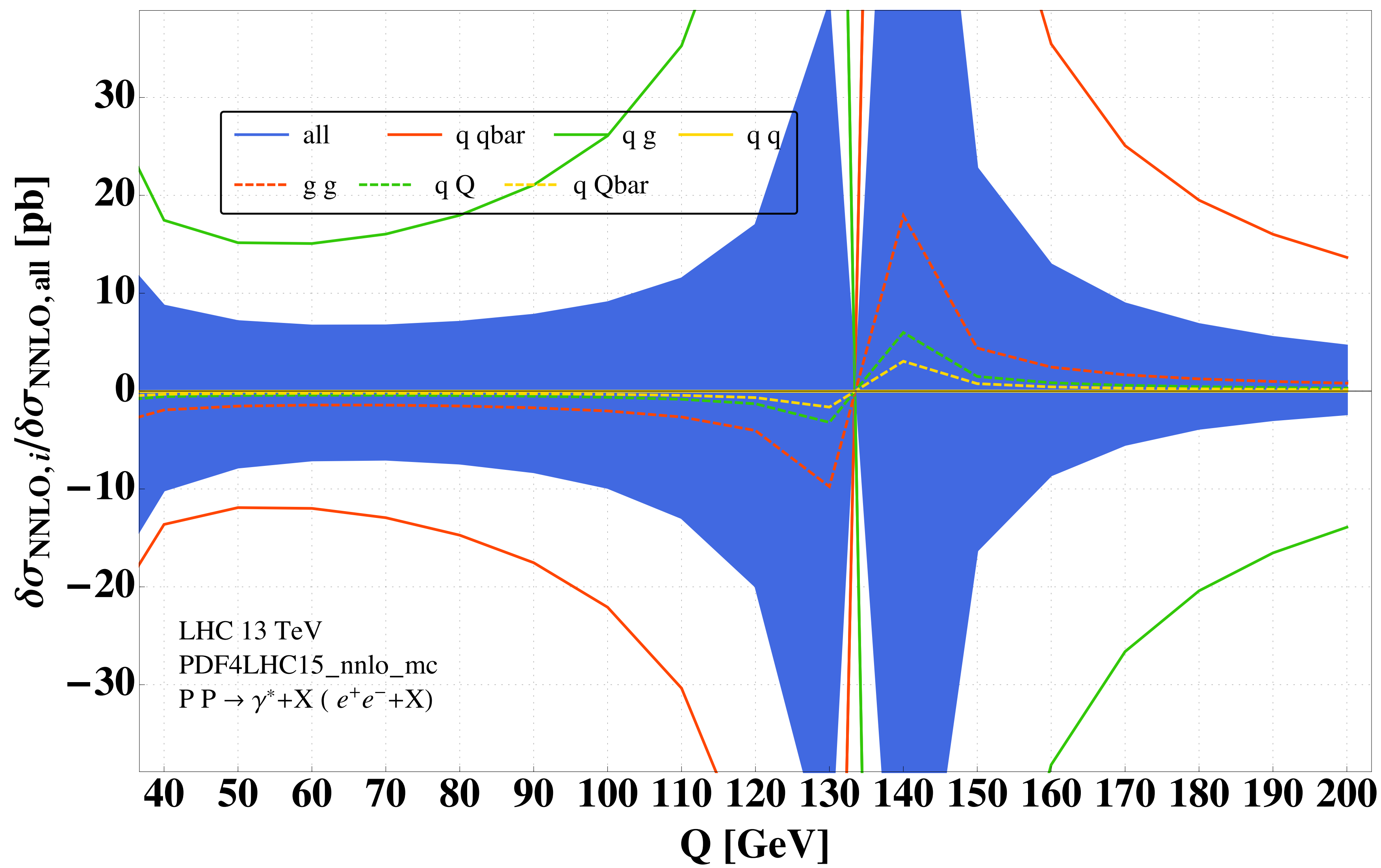
$$P\,P \rightarrow \gamma^* + X \rightarrow l^+ l^- + X$$

NNLO

- How is the cross section decomposed into initial state configurations?

Size of individual partonic channel compared to correction at NNLO

- Large! cancellation between quark and gluon induced channels.
- Small Miss-alignment of PDFs can lead to relatively large change in cross section predictions.



arXiv:2001.07717

DY PRODUCTION

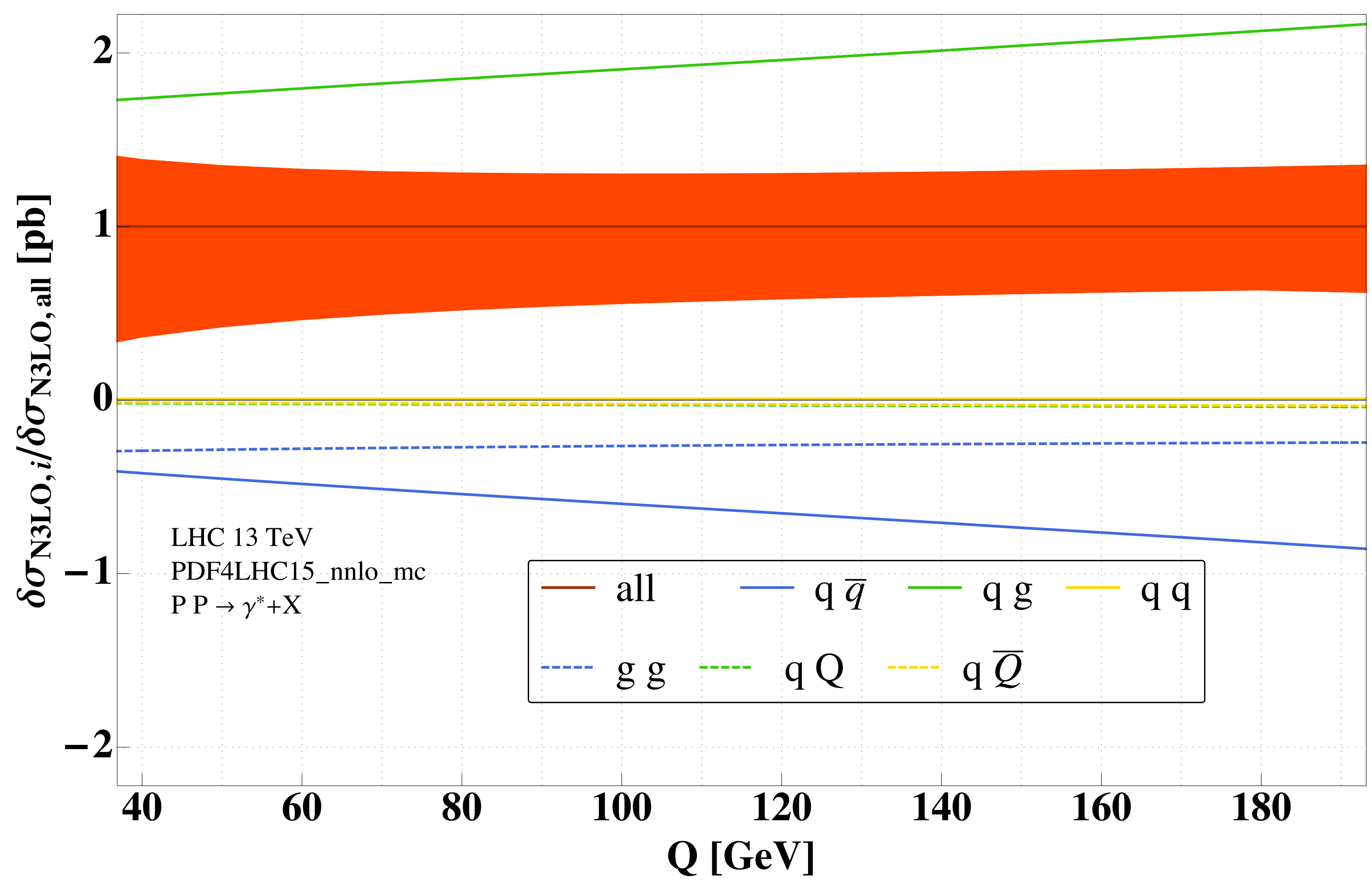
$$P\,P \to \gamma^* + X \to l^+ l^- + X$$

N3LO

- How is the cross section decomposed into initial state configurations?

Size of individual partonic channel compared to correction at N3LO

- Same feature at N3LO.
- Is this effect due to imprecise knowledge of PDFs?
- Is this because we don't have N3LO PDFs?
- Scale Variation?

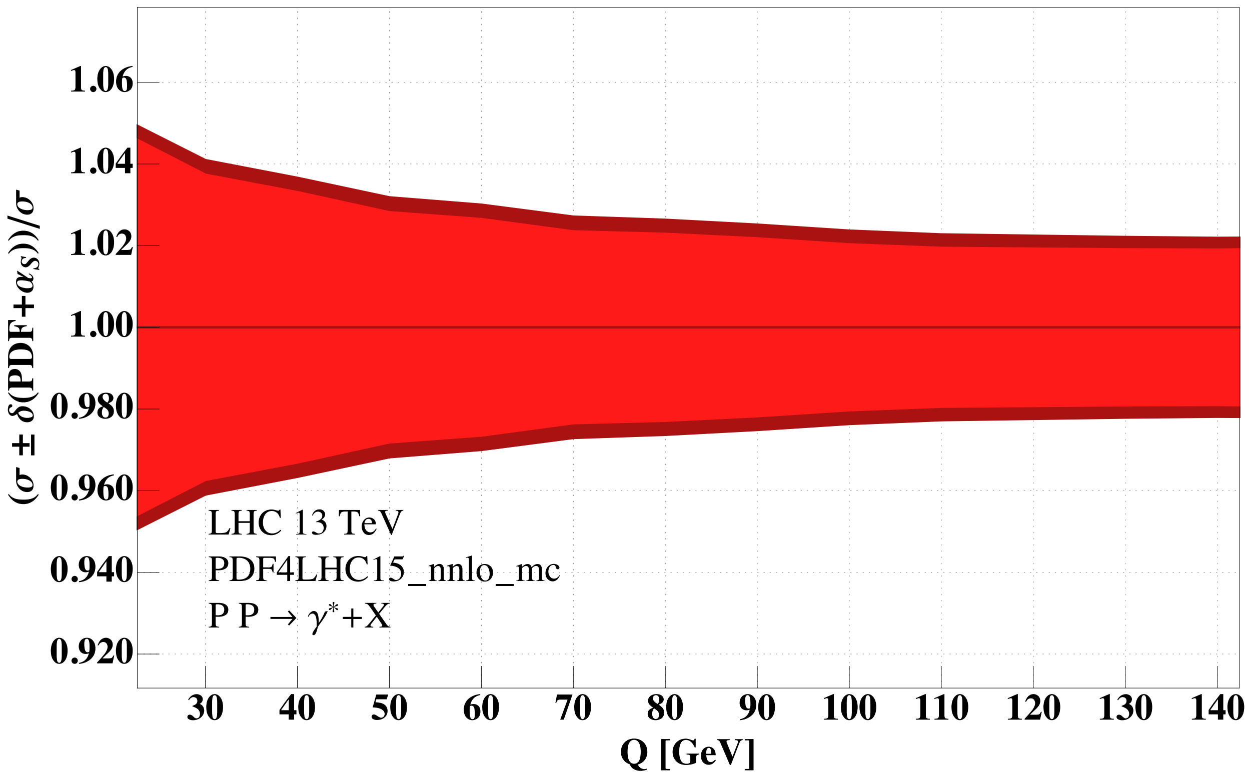


arXiv:2001.07717

DY PRODUCTION

$$P\,P \rightarrow \gamma^* + X \rightarrow l^+ l^- + X$$

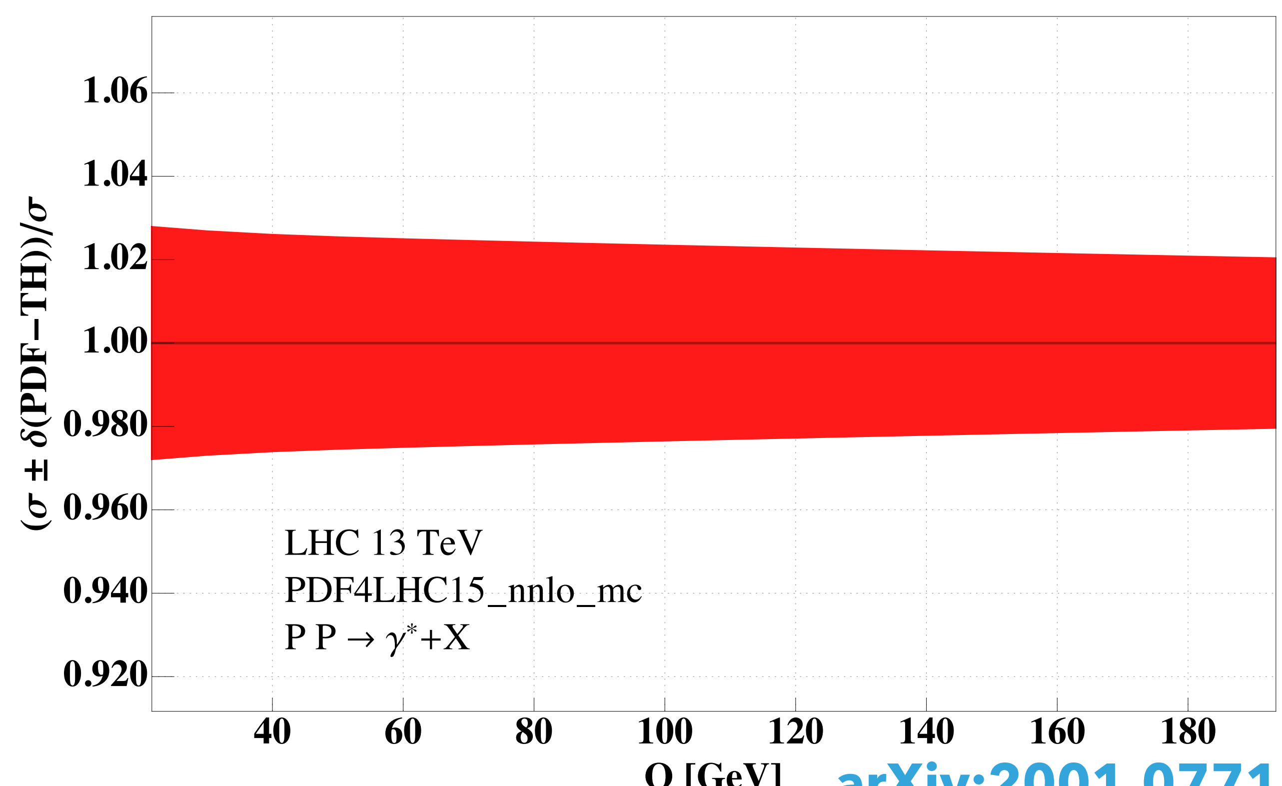
- PDF Uncertainties: Statistical and Systematic from Methods and Measurement



- Large compared to N3LO QCD uncertainties.

- Missing N3LO PDFs:

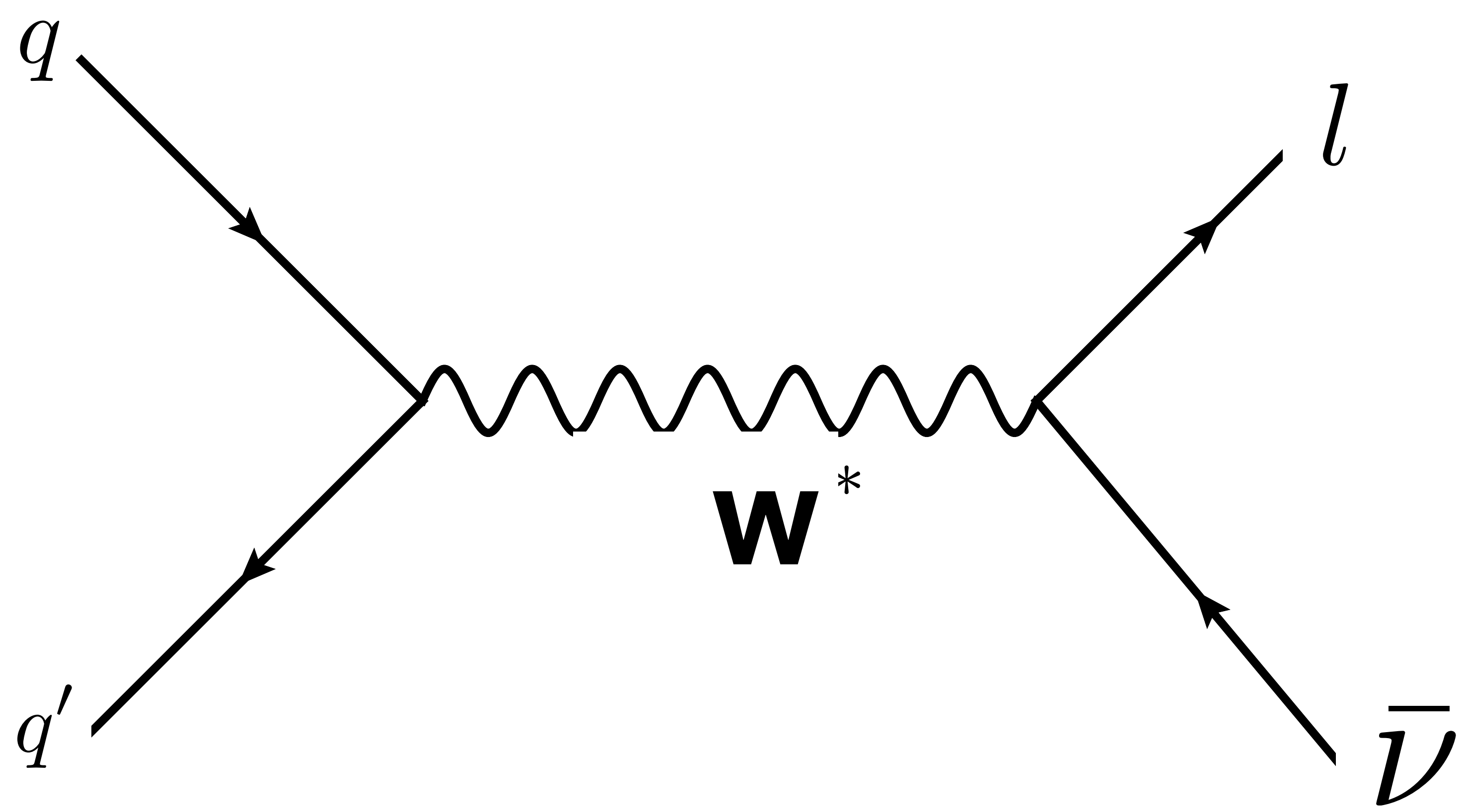
Half the effect of computing the NNLO correction with NNLO or NLO PDFs



arXiv:2001.07717

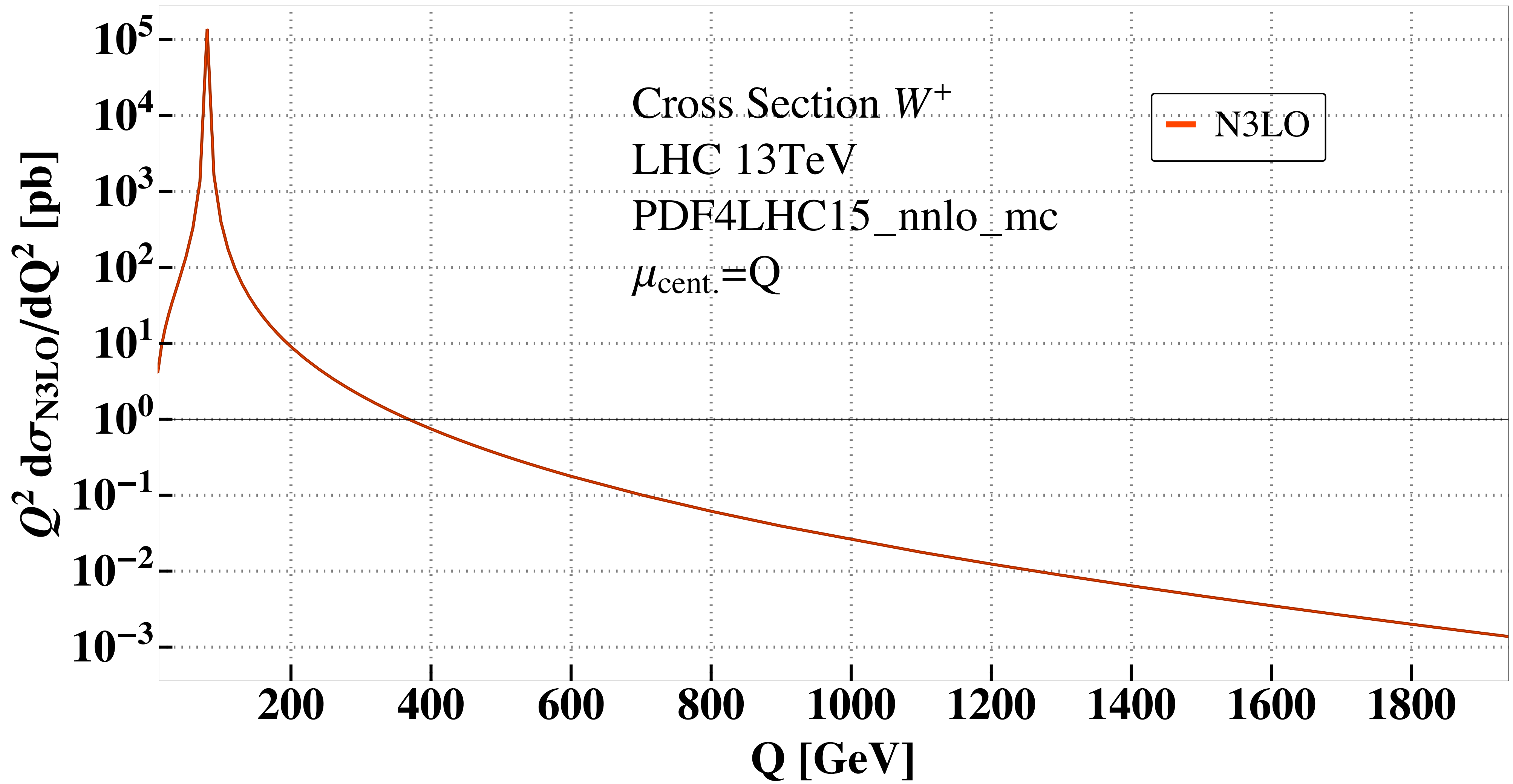
CHARGED CURRENT DRELL-YAN PRODUCTION

The probability to produce a W and its decay



- Very similar partonic cross section to DY - partial overlap.
- Different arrangement of PDFs / charges.

CHARGED CURRENT DRELL-YAN PRODUCTION



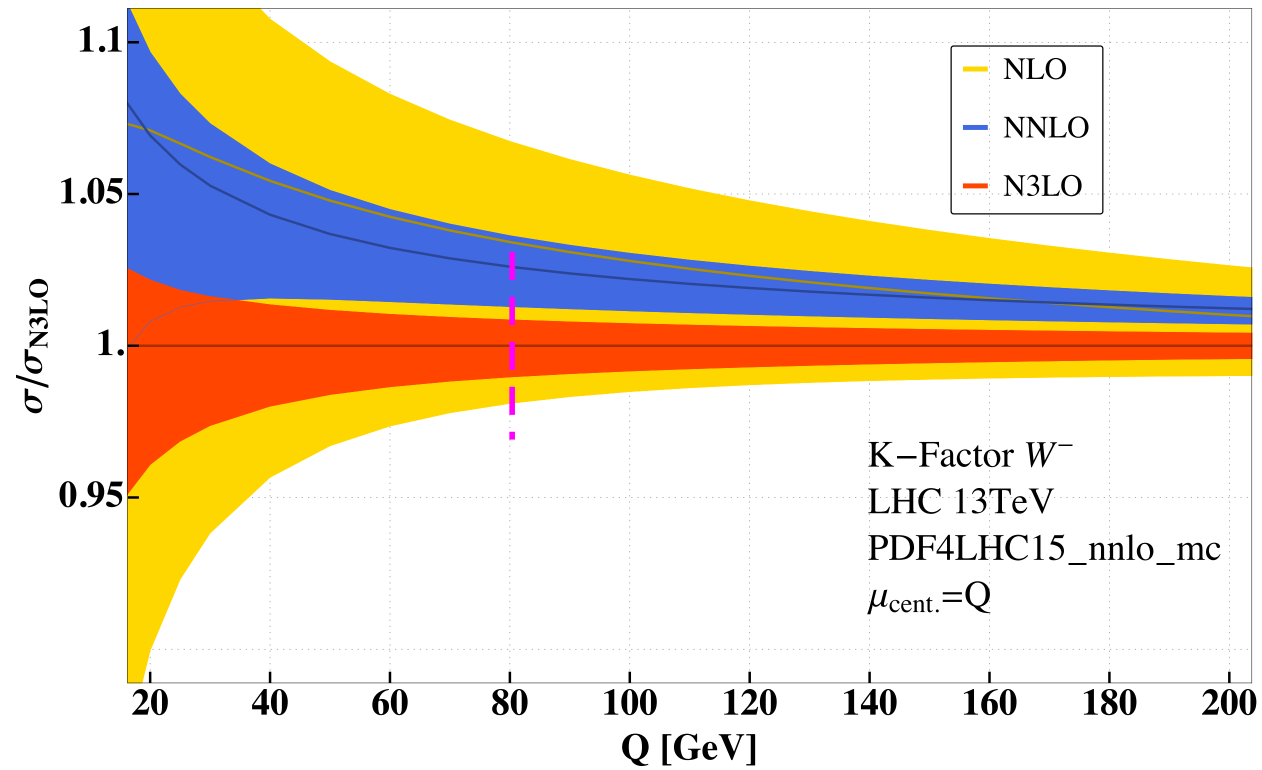
- Bulk of the cross section centered around W boson mass.
- Off-shell cross section modeled via Breit-Wigner

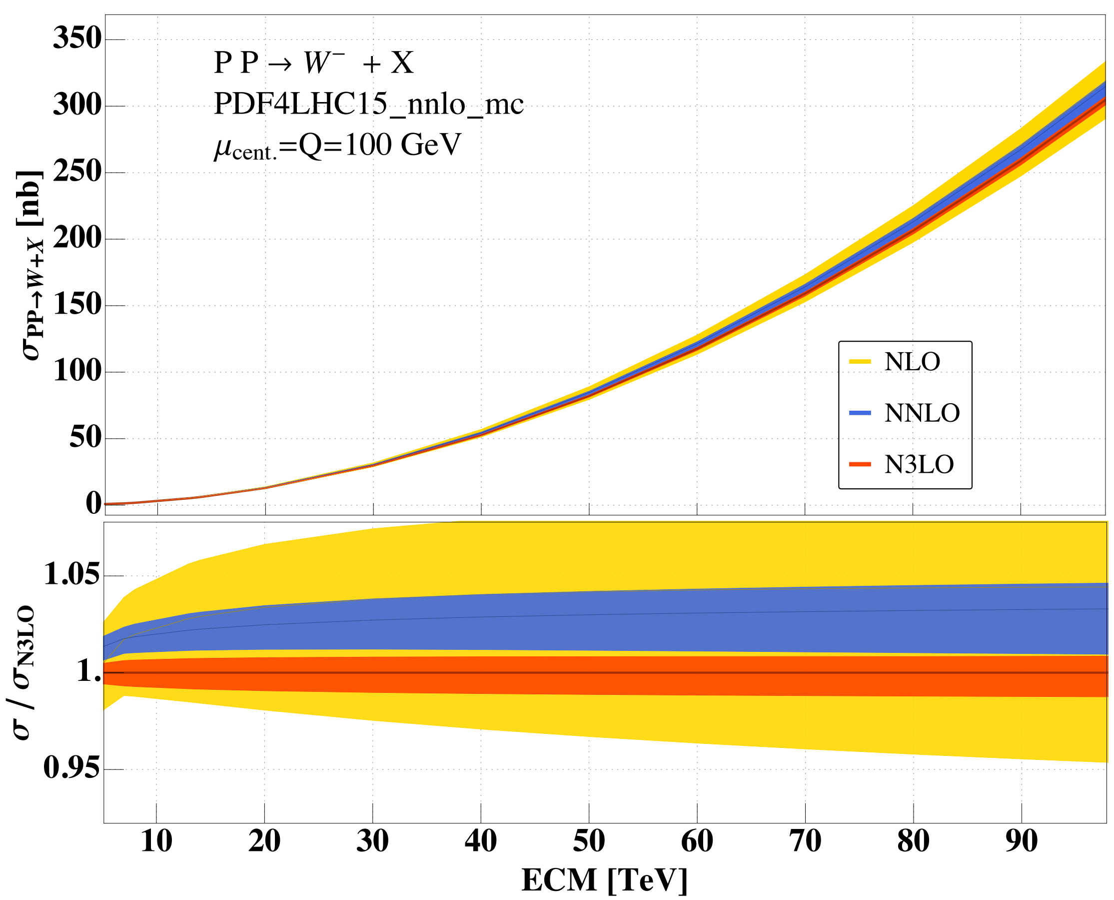
arXiv:2007.13313

CHARGED CURRENT DY PRODUCTION

$$P\,P \to W + X \to l\nu + X$$

- Scale bands do not overlap around Q ~ 100 GeV.



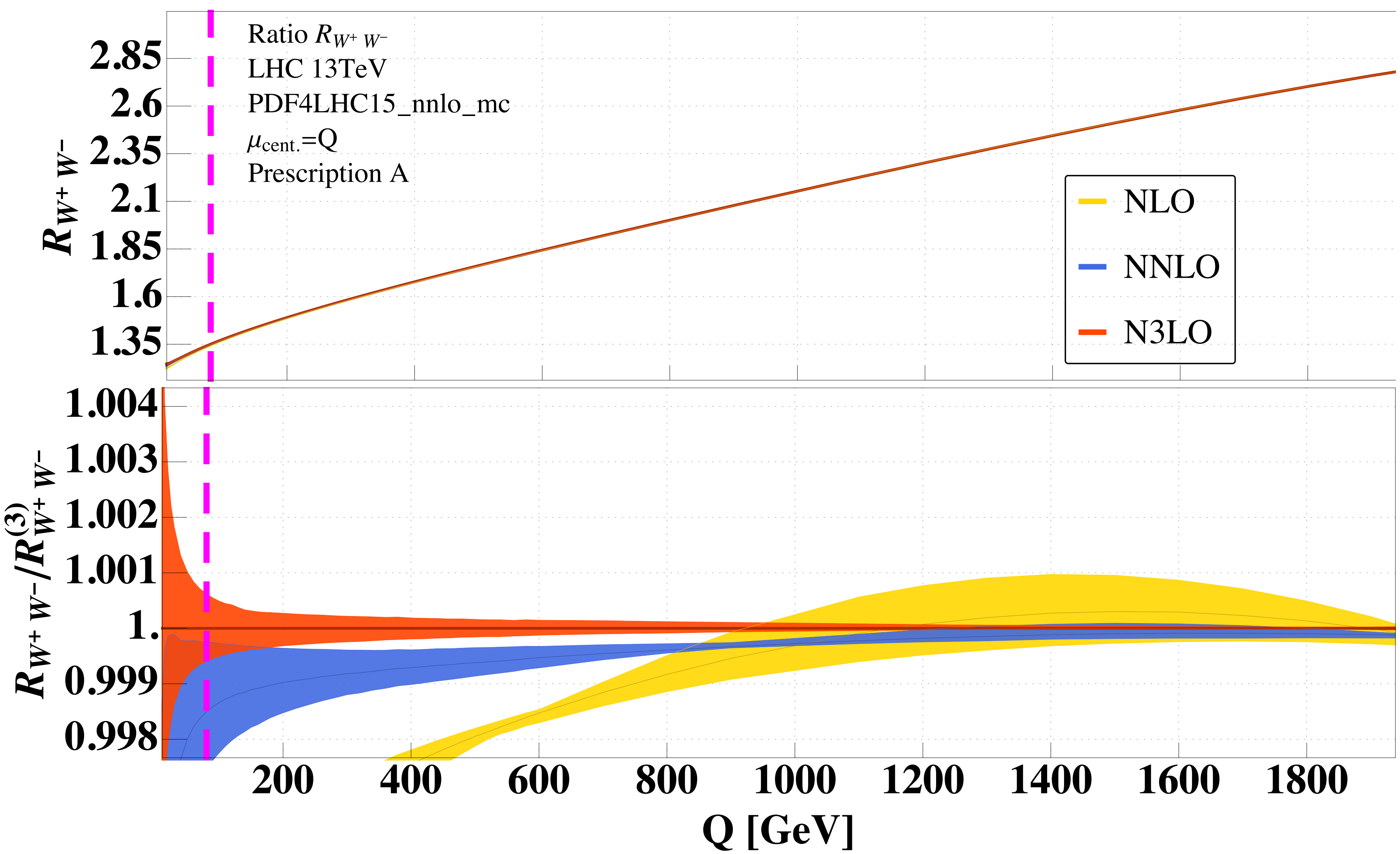


- Little change under variation of LHC energy.
- Overall: N3LO reduces scale dependence.

arXiv:2007.13313

CHARGED CURRENT DY PRODUCTION

- Ratios of Cross Sections: $$R_{AB} = \frac{\sigma_A}{\sigma_B}$$
  - Luminosity / PDF uncertainties drop out (to a degree).



- How to set uncertainties?

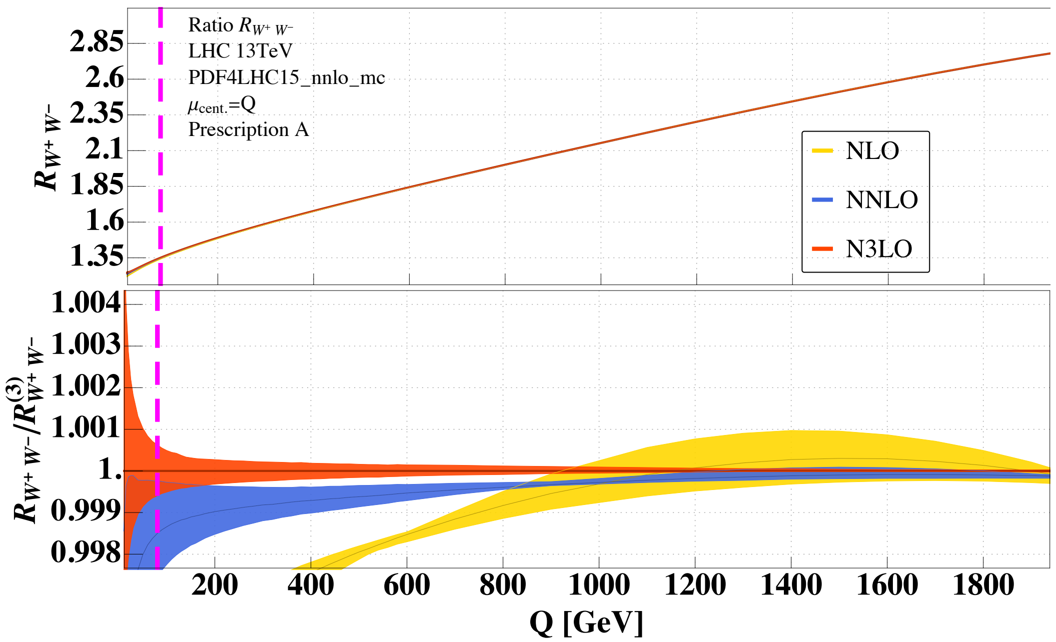
arXiv:2007.13313

CHARGED CURRENT DY PRODUCTION

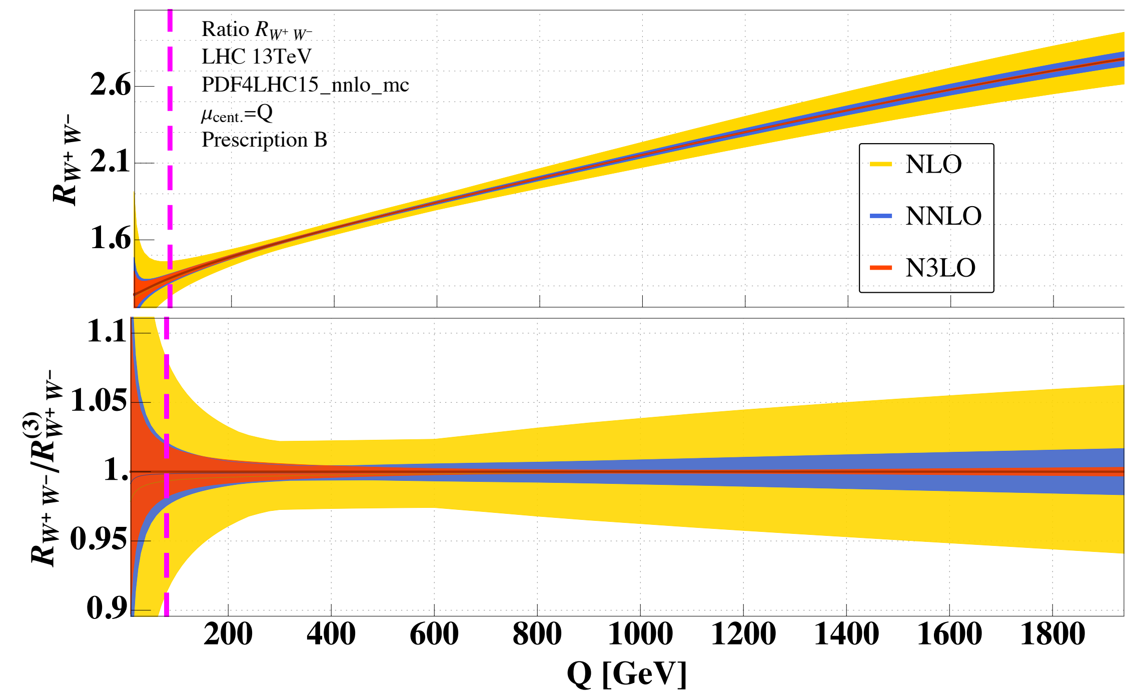
- Ratios of Cross Sections:
- How to set uncertainties?

$$R_{AB} = \frac{\sigma_A}{\sigma_B}$$

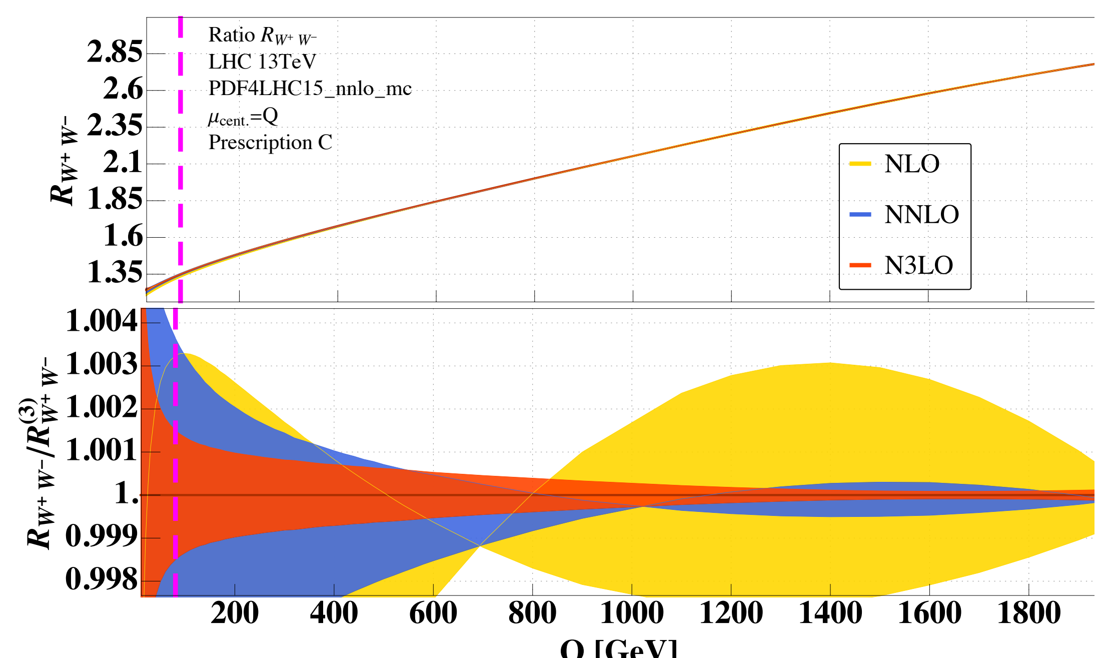
Correlated scales.



Un-Correlated scales.



Progression of the series:
Size of the last correction



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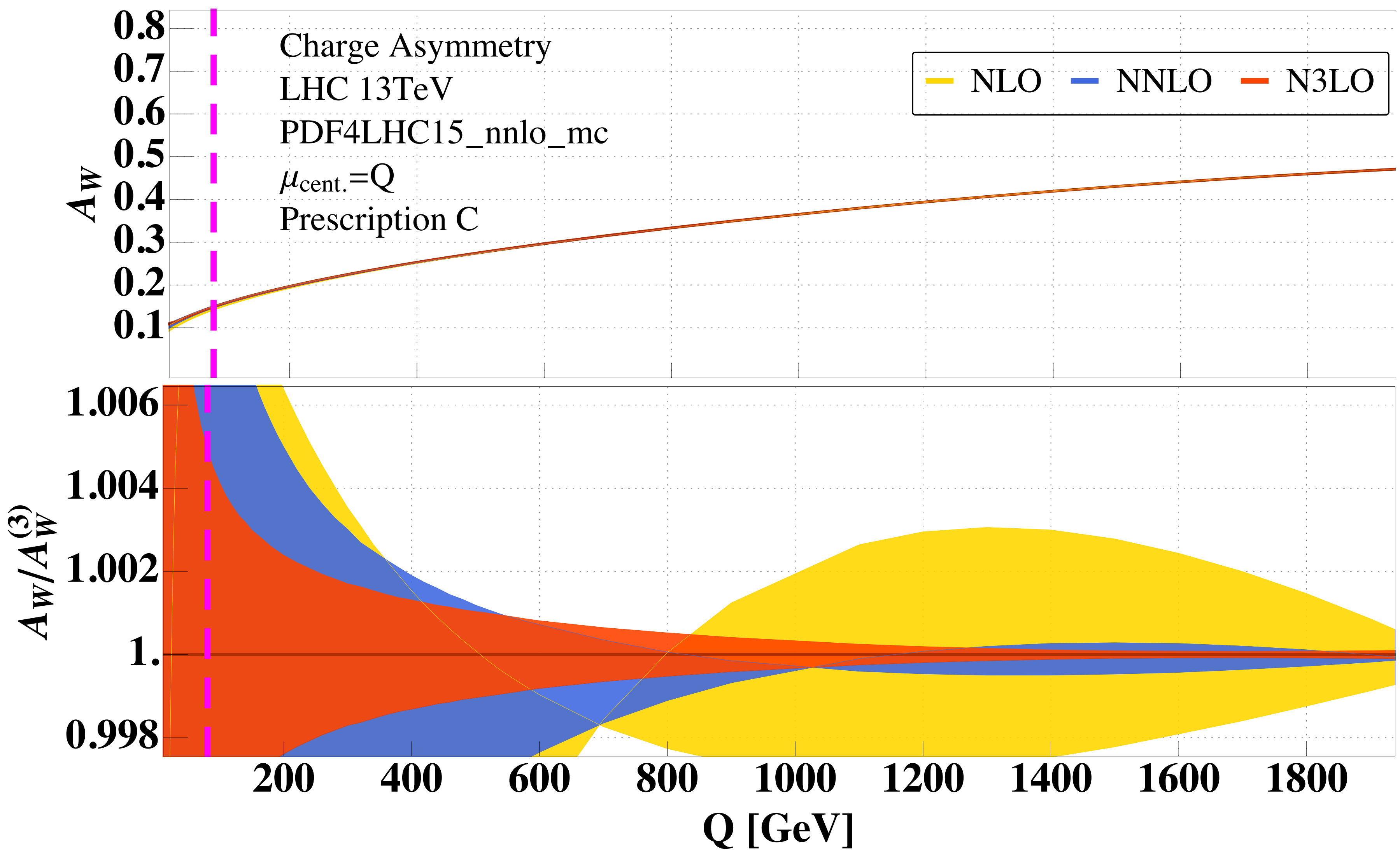
CHARGED CURRENT DY PRODUCTION

Charge Asymmetry:



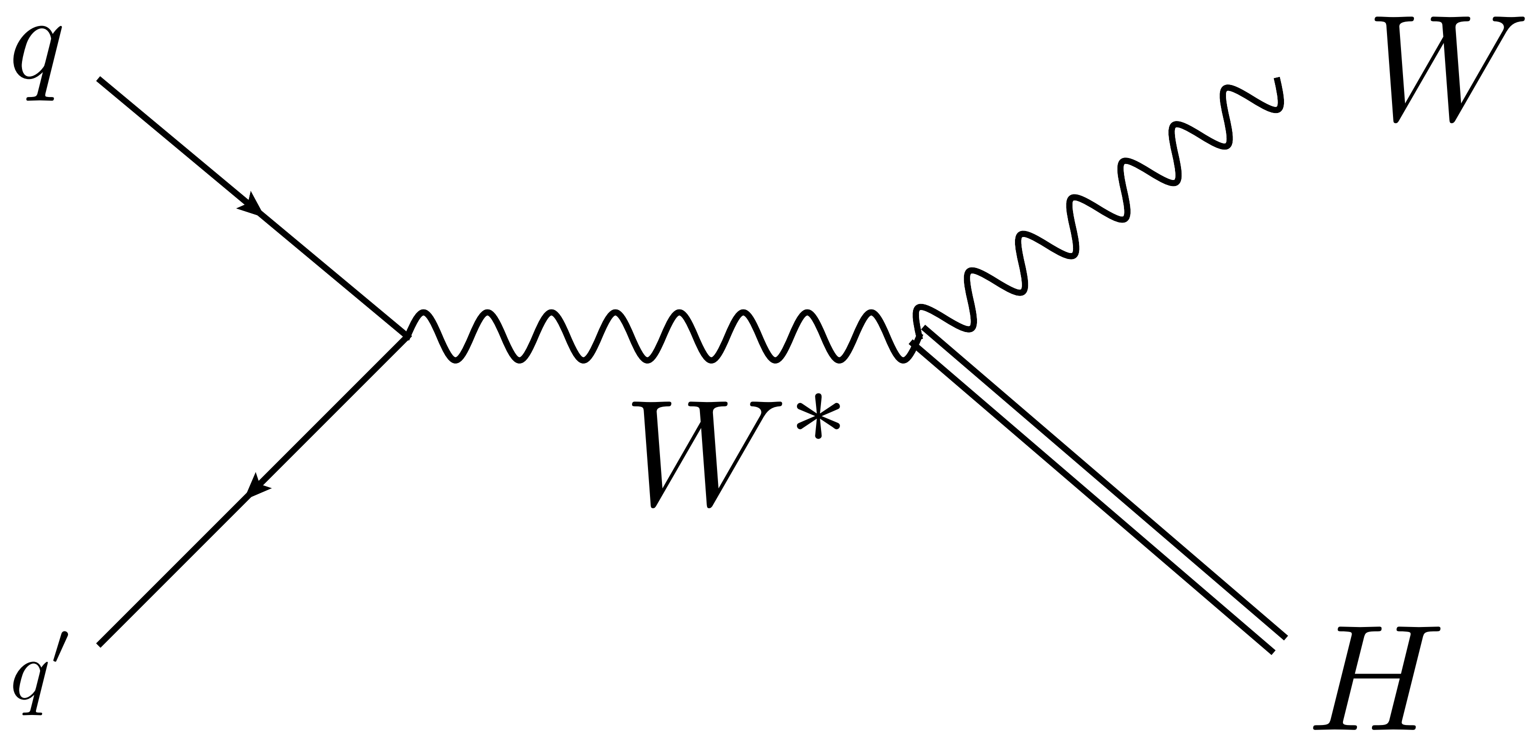
$$A_W(Q) = \frac{\sigma_{W^+}(Q) - \sigma_{W^-}(Q)}{\sigma_{W^+}(Q) + \sigma_{W^-}(Q)}$$

$$A_W^{(2)}(m_W) = 0.148^{+0.41\%}_{-0.34\%},$$
$$A_W^{(3)}(m_W) = 0.149^{+0.15\%}_{-0.19\%}.$$

arXiv:2007.13313

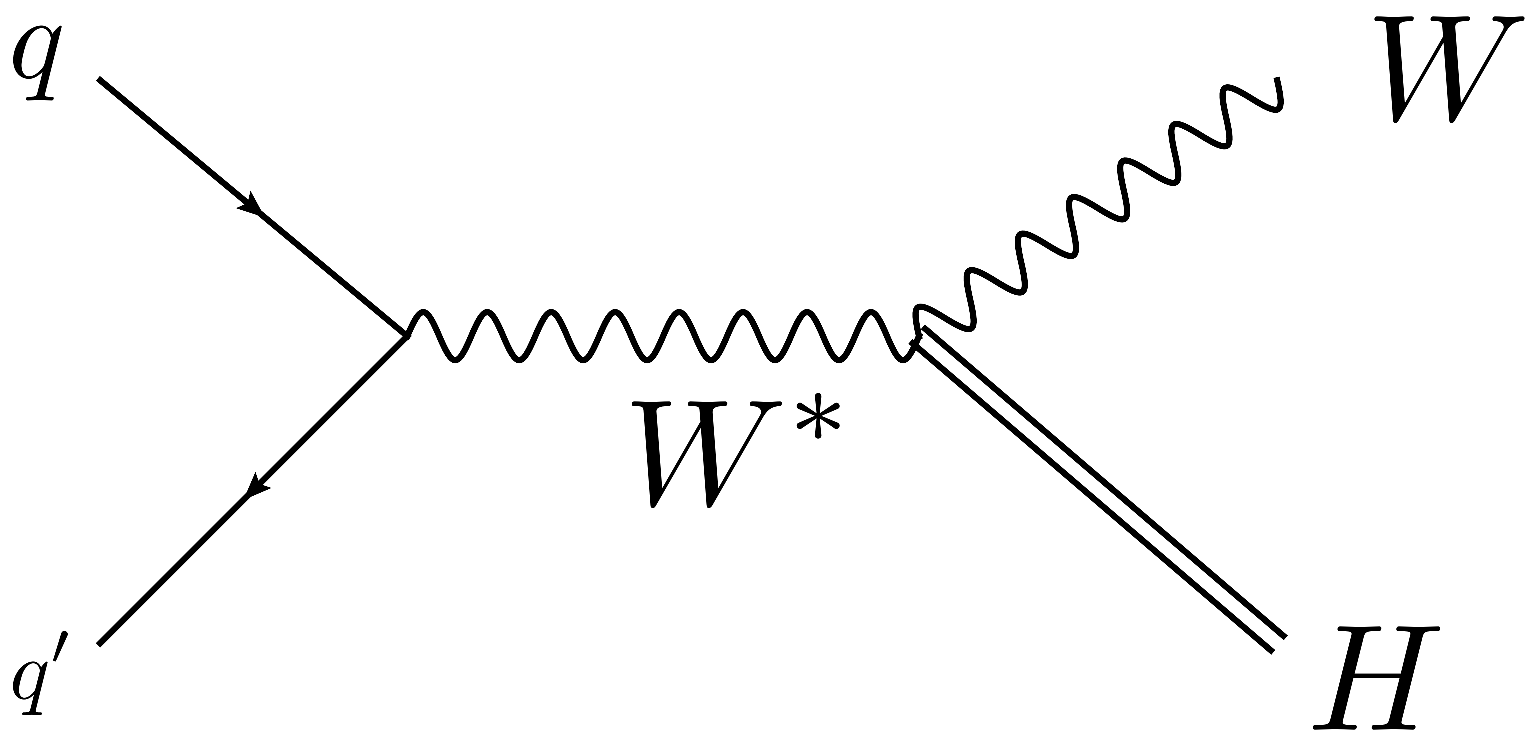
WH ASSOCIATED PRODUCTION

The probability to produce a W and a Higgs boson



- Contribution via virtual W boson exchange

The probability to produce a W and a Higgs boson



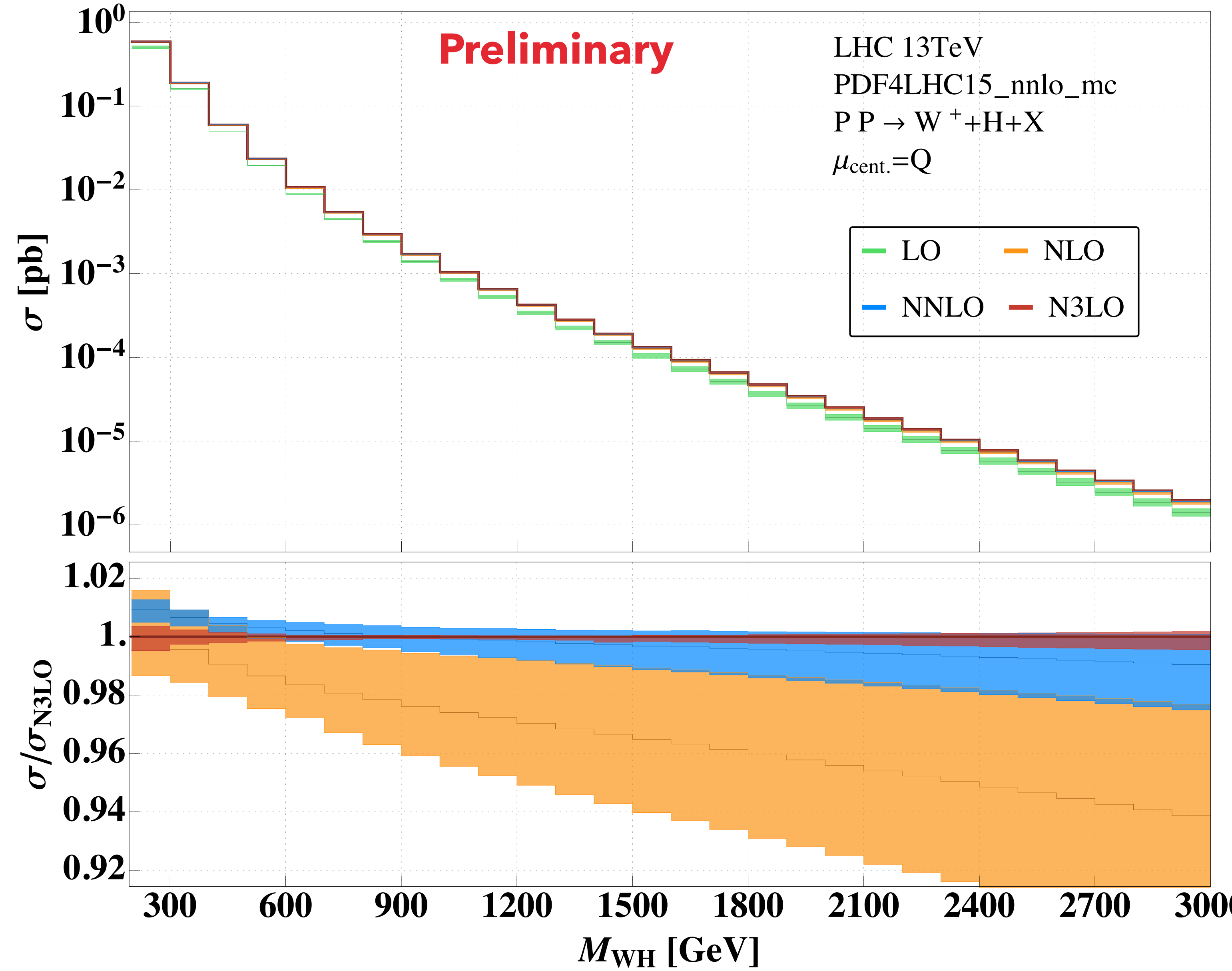
- Non-trivial decay matrix element:

$$\Gamma_{W^*\to WH}(Q^2, m_W^2, m_H^2) = \frac{m_W^2}{48\pi v^2 Q^5}\sqrt{\lambda(Q^2, m_W^2, m_H^2)} \times \left(\lambda(Q^2, m_W^2, m_H^2) + 12 m_W^2 Q^2\right) .$$

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc.$$

WH ASSOCIATED PRODUCTION

- Qualitatively similar to CCDY.
- No overlap of scale variation bands in first bin.
- Inclusive Cross Section:

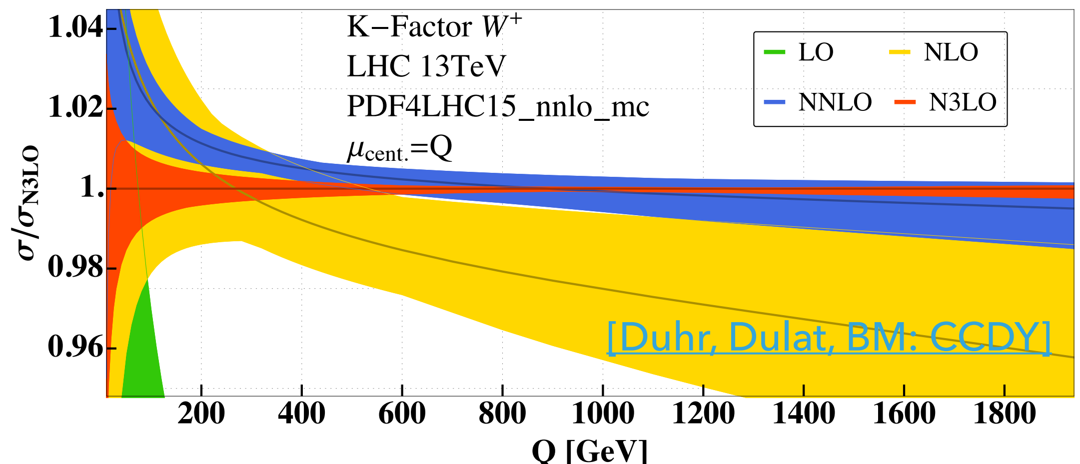


Preliminary

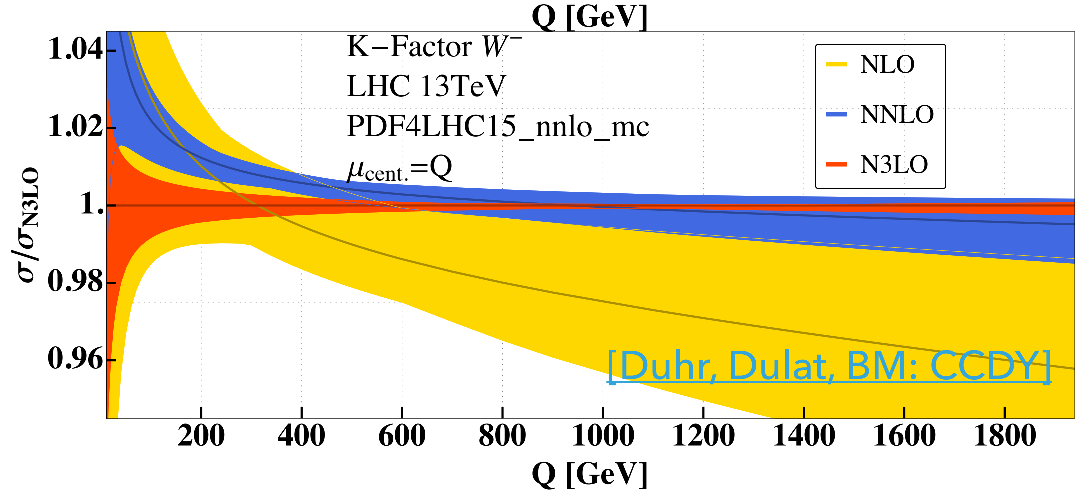
	LO	NLO	NNLO	N^{3}LO
W^+H	$0.760^{2.4\%}_{-3.5\%}$	$0.885^{1.4\%}_{-1.5\%}$	$0.891^{0.32\%}_{-0.46\%}$	$0.877^{0.58\%}_{-0.30\%}$
W^-H	$0.484^{2.5\%}_{-3.4\%}$	$0.561^{1.3\%}_{-1.4\%}$	$0.564^{0.29\%}_{-0.46\%}$	$0.559^{0.33\%}_{-0.43\%}$

CHARGED CURRENT DY PRODUCTION

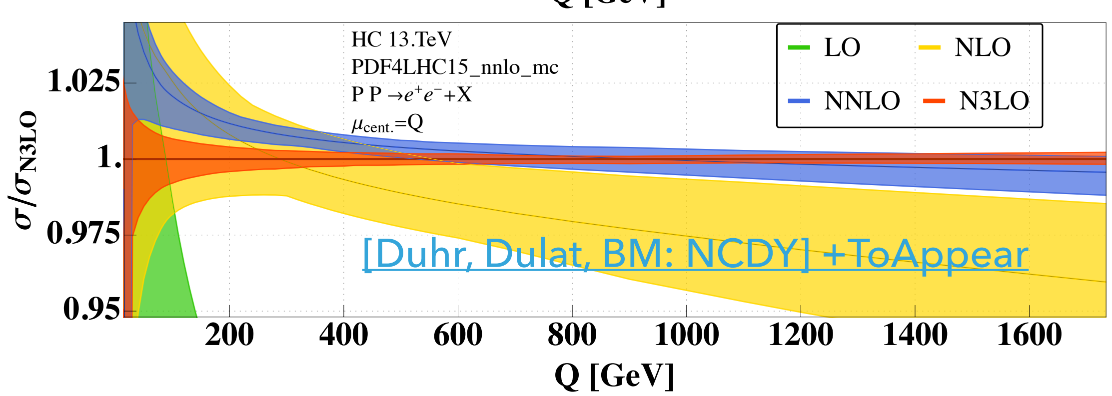
K-Factor



W+



W-

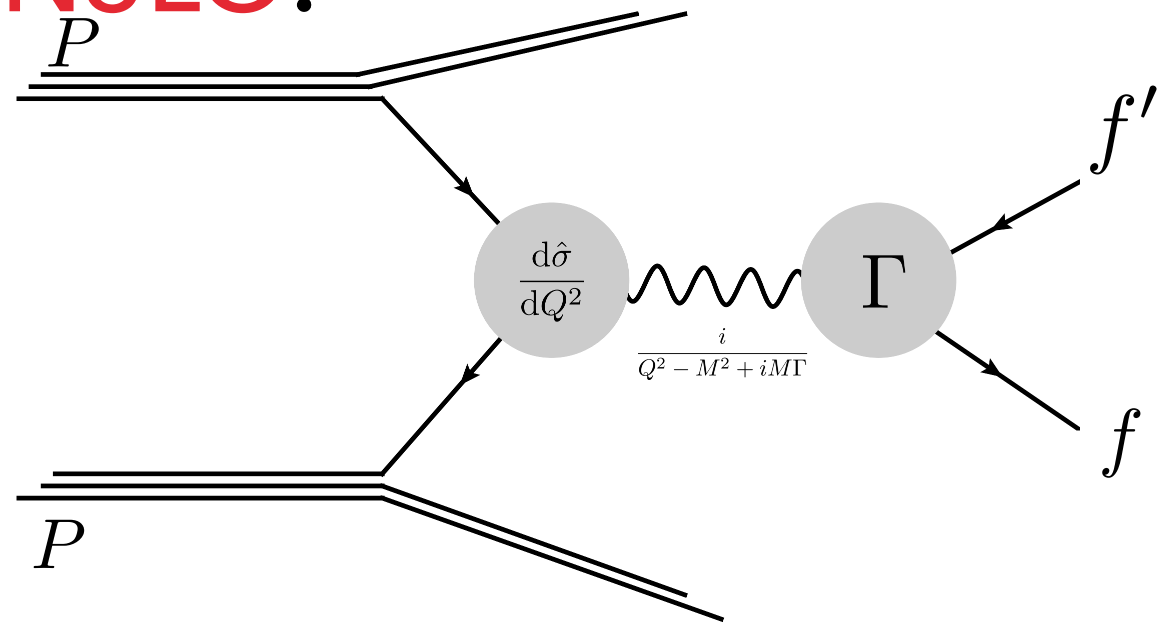


Neutral Current DY

- Very similar corrections for CC and NC DY cross sections.
- Gap at ~ 100 GeV of NNLO and N3LO bands.

CONCLUSIONS – THE EMERGING N3LO PICTURE

Inclusive Cross Sections at N3LO:



- Framework to compute inclusive cross sections at N3LO in QCD perturbation theory.
- Applied to several processes:

	$\frac{\delta\sigma_{N3LO}}{\sigma_{NNLO}}$
★ ggF Higgs	3.5%
★ Bottom quark fusion Higgs	-1.4%
★ Neutral Current DY	-2.1%
★ Charged Current DY	-2.1%
★ Associated Higgs boson production	-1.6%

CONCLUSIONS – THE EMERGING N3LO PICTURE

We know several processes at **N3LO**:

- Corrections are at the order of a few percent
- Perturbative Uncertainty only one source of uncertainties: PDFs, EWK, Masses, Coupling Constants, …
- Same as precision target of LHC phenomenology.
- We are at the **beginning** of the age of wide-spread N3LO phenomenology.

Work in progress:

- ***Public Tool*** to calculate inclusive cross sections.
- NCDY: Z boson production.

Thank you!

PREDICTIONS FOR THE LHC

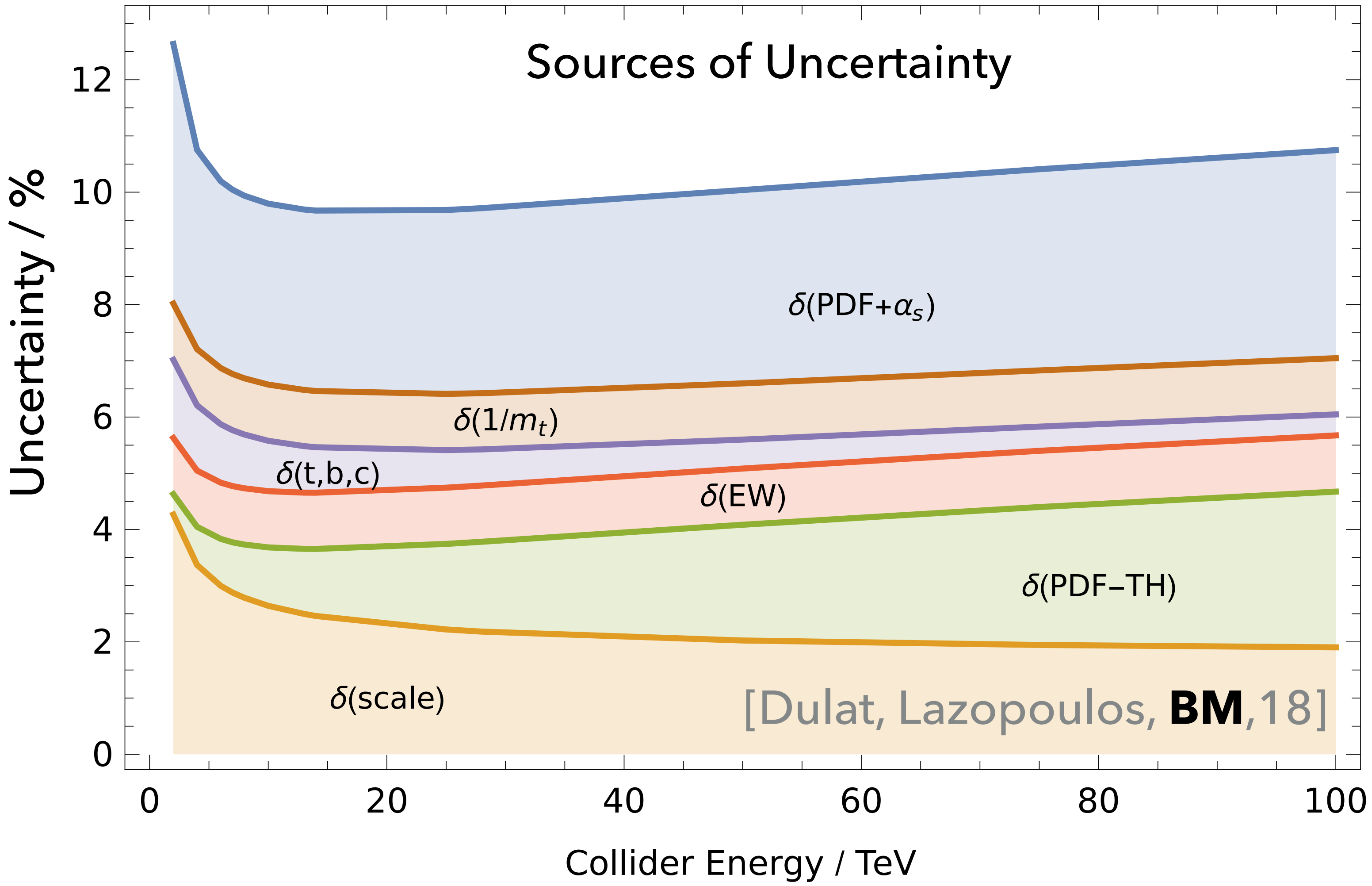
Much more than QCD corrections

- Electro-weak corrections.
- Neglected quark mass effects.
- Coupling to bottom, charm quarks.
- Estimate uncertainties.

  Truncation of perturbative series

  PDF, α_S

- …



PREDICTIONS FOR THE LHC

Much more than QCD corrections

- Electro-weak corrections.
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  Truncation of perturbative series

  PDF, α_S

- …

HL-LHC

Pessimistic scenario

Optimistic scenario