Massive Spin-2's at High Energies

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How do you construct a UV complete theory which has an isolated massive high-spin particle in the IR?

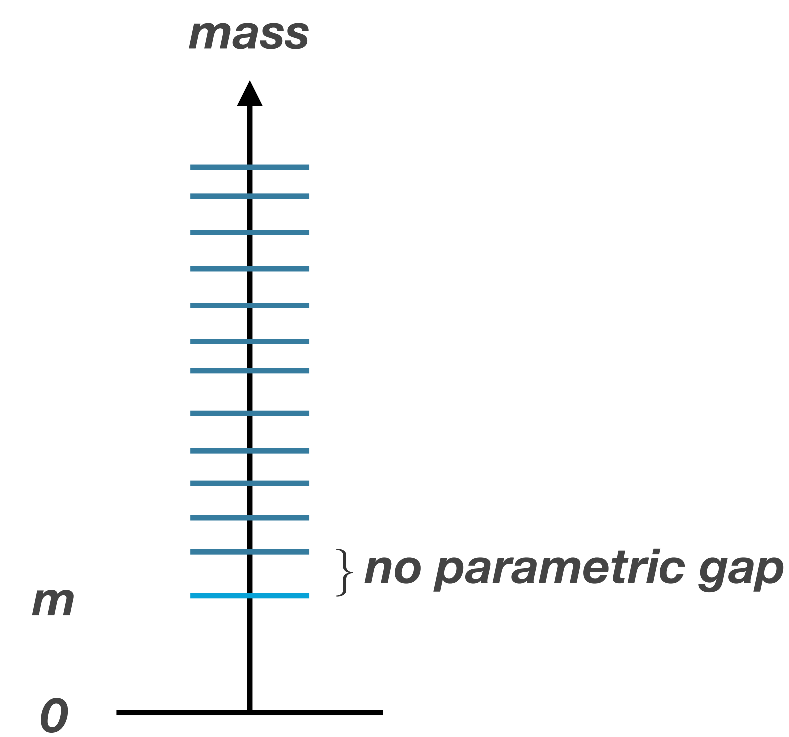
How do you construct a UV complete theory which has an isolated massive high-spin particle in the IR?

More modestly: can you construct a low-energy EFT of a massive high-spin particle with a parametrically large separation between the mass and the cutoff?

Massive Particles

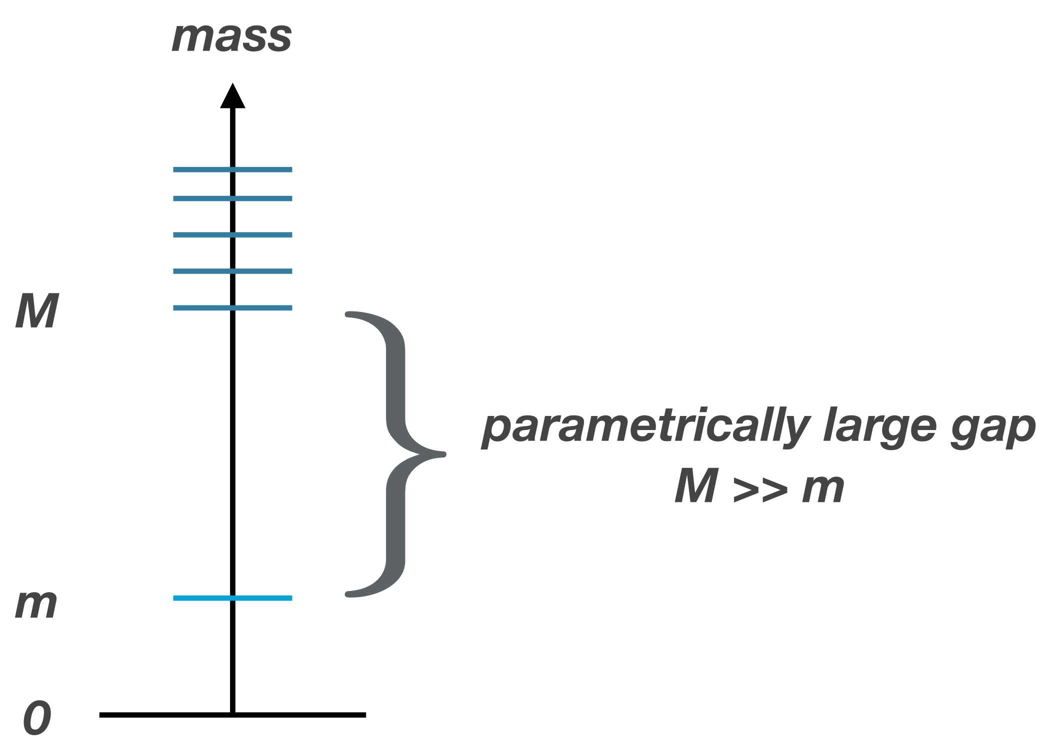
Interacting, massive high-spin particles *do* exist

- **QCD:** $m^2 \sim \Lambda_{QCD}^2$
- **Kaluza-Klein Theory:** $m^2 \sim \frac{1}{R^2}$
- **String Theory:** $m^2 \sim \frac{1}{\alpha'}$



Massive Particles

Can the spectrum look like this?



spin-0: yes!

spin-1: yes!

spin≥2: ???

Need to construct a low energy EFT with cut-off >> mass

Massive Spin-2's:

* First "high-spin" particle
* Possible relevance to gravity

Massive Spin-2's at High Energies

Outline

- Intro to massive spin-2s
- Causality constraints
- Raising the cutoff

Work with:

James Bonifacio, Kurt Hinterbichler, Austin Joyce

1708.05716, 1712.10020, 1903.09643

1. Intro to Massive Spin-2s

Massive Spin-2

The Free Theory

$$\mathcal{S}_2 = \int d^4x \, \left(-\tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \tfrac{1}{2}\partial_\lambda h \partial^\lambda h \right.$$
$$\left. + m_1^2\, h_{\mu\nu}h^{\mu\nu} + m_2^2 h^2\right)$$

add a mass!

Massive Spin-2

The Free Theory

$$\mathcal{S}_2 = \int d^4x \, \left(-\tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \tfrac{1}{2}\partial_\lambda h \partial^\lambda h \right.$$
$$\left. -\tfrac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)\right)$$

← FIERZ-PAULI MASS TERM

- 4 Bianchi constraints: $m_1^2\partial^\mu h_{\mu\nu} + m_2^2\partial_\nu h = 0$
- additional constraint: $m^2 h = 0$

$10 - 4 - 1 = 5$ DOF ✓

Massive Spin-2

The Interacting Theory

$$\mathcal{S} \;=\int d^4x\;\Big(-\tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu}+\partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda}-\partial_\mu h^{\mu\nu}\partial_\nu h+\tfrac{1}{2}\partial_\lambda h\partial^\lambda h$$
$$-\tfrac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu}-h^2)+\mathcal{O}(h^3)+\mathcal{O}(h^4)+\dots\Big)$$

add interaction terms

- Is there a *non*-linear theory of a massive spin-2 particle that maintains a constraint at the fully non-linear level and thus avoids an extra, pathological DOF?

Boulware, Deser (1972)

Massive Spin-2

The Interacting Theory

$$\mathcal{S} = \frac{M_{Pl}^2}{2}\left\{\int d^4x \, \det e \, R[e] - m^2 \int \sum_{n=0}^{4} \beta_n S_n[e]\right\}$$

$$S_0[e] = \epsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge e^d$$

$$S_1[e] = \epsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge \mathbf{1}^d$$

$$S_2[e] = \epsilon_{abcd}\, e^a \wedge e^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d$$

$$S_3[e] = \epsilon_{abcd}\, e^a \wedge \mathbf{1}^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d$$

$$S_4[e] = \epsilon_{abcd}\, \mathbf{1}^a \wedge \mathbf{1}^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d$$

Free parameters: $M_{Pl}, \ \Lambda, \ m, \ \beta_2, \ \beta_3$

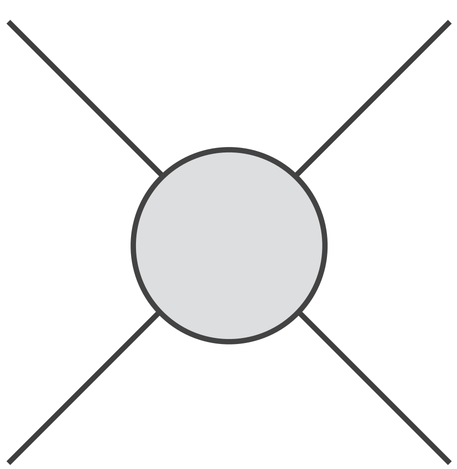
C. de Rham, G. Gabadadze, A. J. Tolley (2010) S. F. Hassan, RAR (2011) K. Hinterbichler, RAR (2012)

What about the cut-off?

Generic massive gravity:

$$\mathcal{S} = \int d^4x \, \Big(-\tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \tfrac{1}{2}\partial_\lambda h \partial^\lambda h \\ -\tfrac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \mathcal{O}(h^3) + \mathcal{O}(h^4) + \dots \Big)$$

2-2 scattering of the helicity-0 mode:



$$\mathcal{M} \sim \left(\frac{E}{\Lambda_5}\right)^{10}$$ where $$\Lambda_5 = (m^4 M_P)^{1/5}$$

What about the cut-off?

Ghost-free massive gravity:

$$\mathcal{S} = \frac{M_{Pl}^2}{2} \int d^4x \left\{ \det e \, R[e] - m^2 \sum_{n=0}^{4} \beta_n S_n[e] \right\}$$

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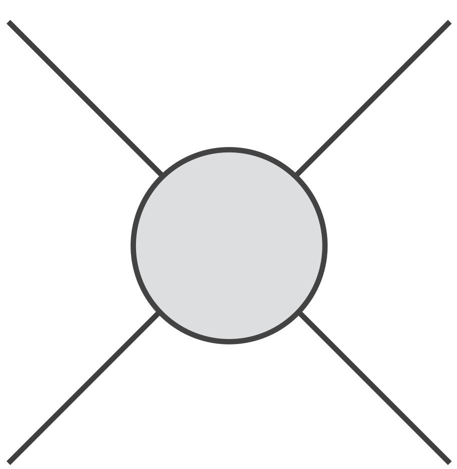


$$\mathcal{M} \sim \left(\frac{E}{\Lambda_3} \right)^6$$ where $$\Lambda_3 = (m^2 M_P)^{1/3}$$

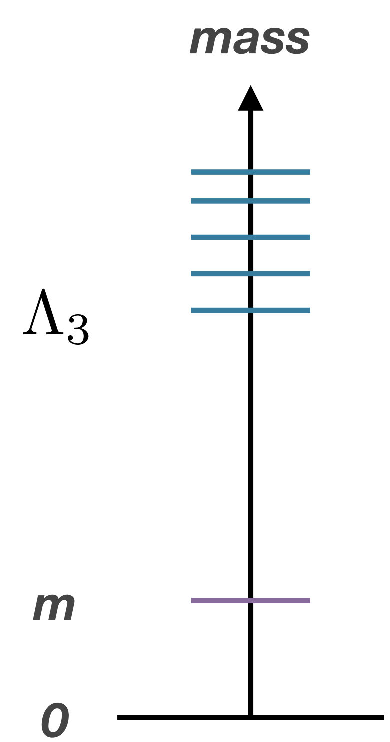
What about the cut-off?

$$\mathcal{S} = \frac{M_{Pl}^2}{2} \int d^4x \left\{ \det e\, R[e] - m^2 \sum_{n=0}^{4} \beta_n S_n[e] \right\}$$

EFT cut-off:

$$\Lambda_3 = (M_{Pl} m^2)^{1/3}$$

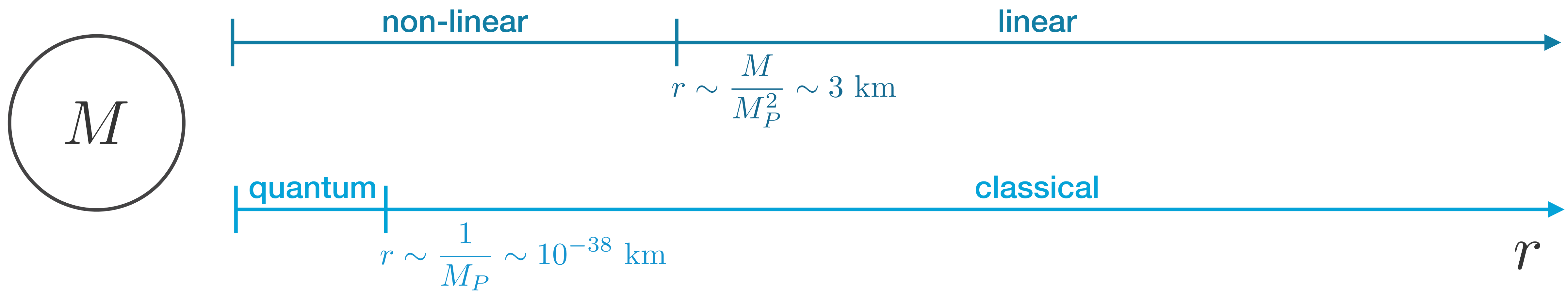
$$\Lambda_3 \gg m$$



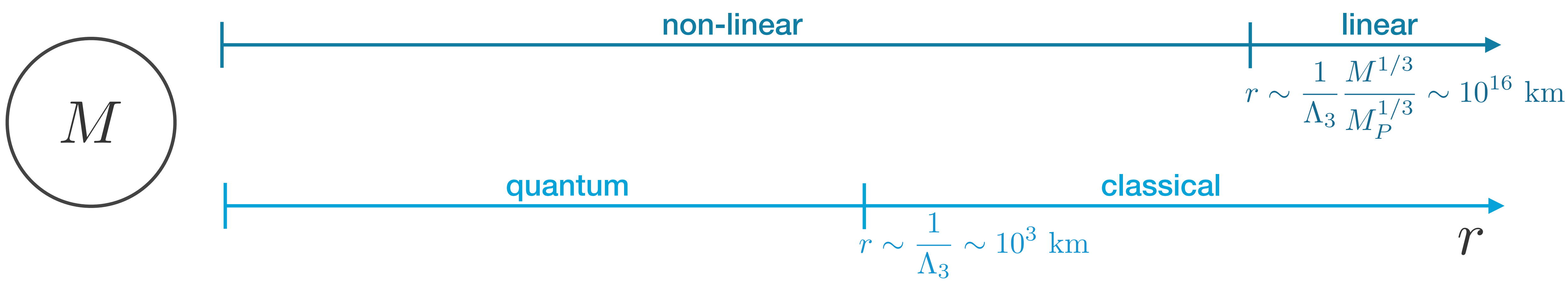
hierarchy is stable under quantum corrections

What about the cut-off?

General Relativity:



Massive Gravity:



(NOT TO SCALE)

2.

Causality Constraints

Causality Contraints

In the S-Matrix, causality manifests as momentum space analyticity

$$\psi_f(t) = \int dt' \int d\omega \, S(\omega) \mathrm{e}^{-i\omega(t-t')} \psi_i(t')$$

if $\psi_i(t) = 0 \text{ for } t < 0 \;\; \to S(\omega)$ *analytic in upper half plane*

Causality Contraints

Analyticity Constraints

$$\mathcal{L} = (\partial\phi)^2 + \frac{c}{\Lambda^4}(\partial\phi)^4 + \ldots$$

forward limit $t \to 0$

- analyticity gives: $$I = \frac{1}{2\pi i}\oint ds\, \frac{\mathcal{A}(s,0)}{(s-\mu^2)^3} > 0$$

$$\Rightarrow c > 0$$

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi (2006)

Causality Contraints

Analyticity Constraints

$$\mathcal{L} = (\partial\phi)^2 + \frac{c}{\Lambda^4}(\partial\phi)^4 + \dots$$

forward limit $t \to 0$

- analyticity gives: $I = \frac{1}{2\pi i}\oint ds\, \frac{\mathcal{A}(s,0)}{(s-\mu^2)^3} > 0$

$$\Rightarrow c > 0$$

- low energy EFT: c must be positive to avoid superluminality around non-trivial backgrounds

constraints agree!

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi (2006)

Causality Contraints

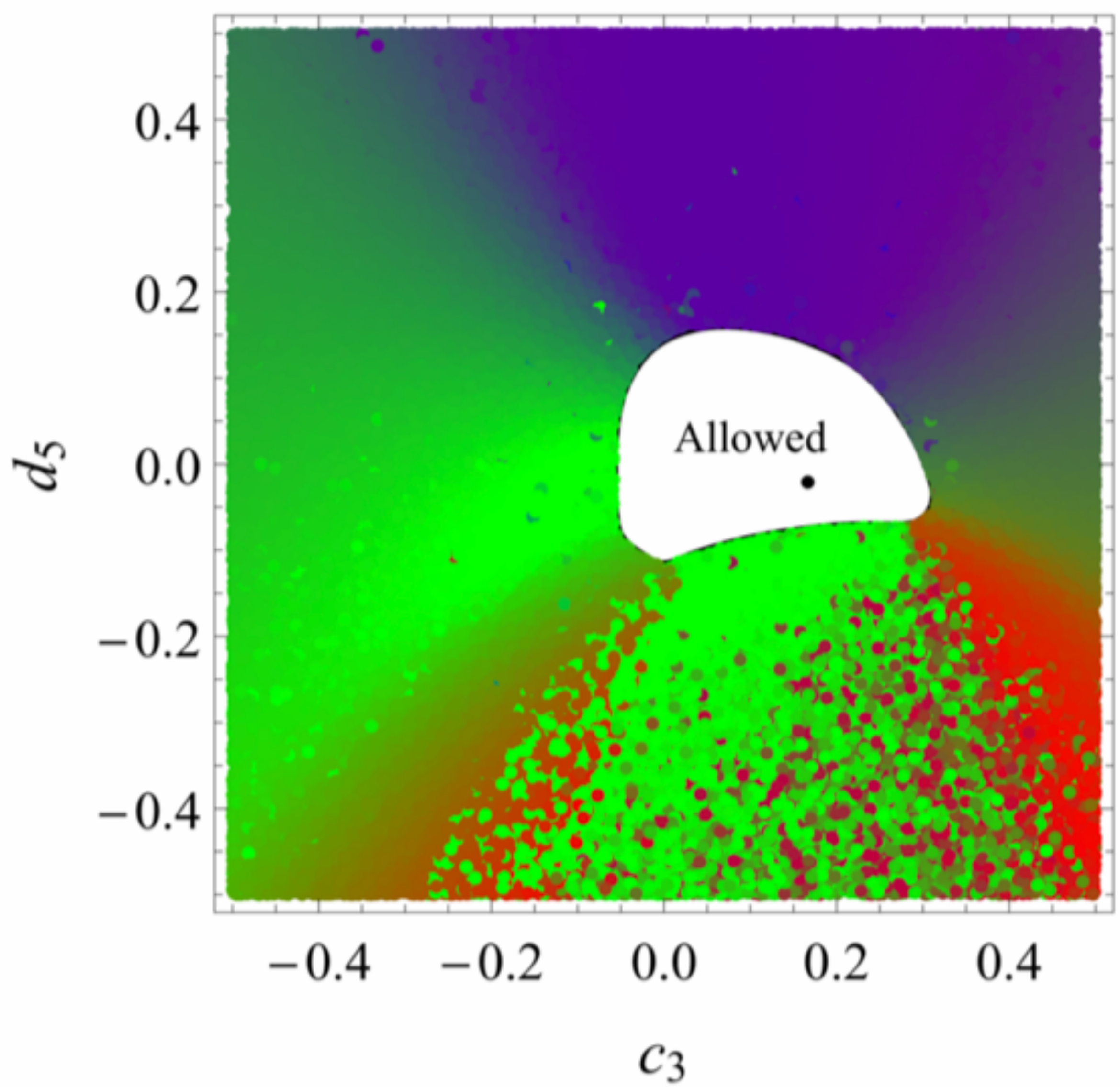
Analyticity Constraints

Is this connection always manifest?

Let's try a different example…

Massive Spin-2 Particles

Analyticity Constraints



$(\beta_2, \beta_3) \to (c_3, d_5)$

Consistency of scattering amplitudes in the forward limit constrains the two free parameters of a massive spin-2 particle

Cheung, Remmen (2016)

Massive Spin-2 Particles

What about superluminality?

- *Can the backgrounds in question can be reached dynamically within the regime of validity of the effective theory?*

- *Is the superluminality itself is visible within the effective theory?*

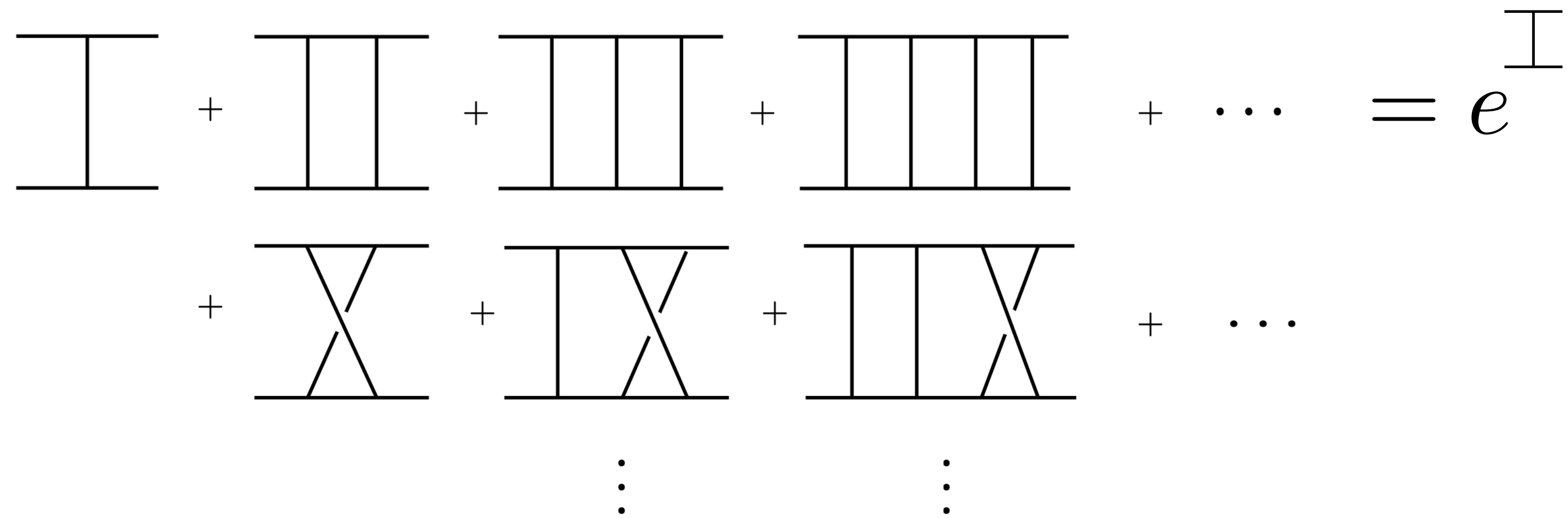
Asymptotic **superluminality: S-matrix observable, doesn't depend on existence of non-trivial background solution**

Causality Contraints

Eikonal Scattering

2-2 scattering at large CoM energy,
large impact parameter $t/s \to 0$:

$$\mathcal{M}_{\rm eik}(s,t) =$$



$$i\mathcal{M}_{\rm eik}(s,t) = 2s\int d^2\vec{b}\, \mathrm{e}^{i\vec{q}\cdot\vec{b}}\left(\mathrm{e}^{i\delta(s,\vec{b})} - 1\right)$$

time delay: $\Delta x^- = \frac{1}{|p^-|}\delta(s,b)$ $\delta(s,b) > 0$ ← eikonal phase

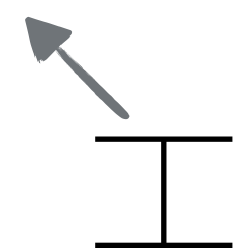
t' Hooft (1987), Kabat & Ortiz (1992)

Causality Contraints

Eikonal Scattering

eikonal phase depends only on *on-shell* cubic vertices:

$$\delta(s,b) = \frac{1}{2s}\int \frac{d^2\vec{q}}{(2\pi)^2} \mathrm{e}^{-i\vec{b}\cdot\vec{q}} \mathcal{M}_{\mathrm{tree}}(\vec{q})$$



factorizes!

Powerful: *Can take CoM energy well above EFT cutoff and only the on-shell cubic vertices are relevant*

Camanho, Edelstein, Maldacena, Zhiboedov (2014)

Massive Spin-2 Particles

Only 5 on-shell cubic vertices

$\mathcal{A}_1$	$z_1 \cdot z_2 \;\; z_2 \cdot z_3 \;\; z_3 \cdot z_1$	$h^3_{\mu\nu}$
$\mathcal{A}_2$	$(p_1 \cdot z_3 \;\; z_1 \cdot z_2 + p_3 \cdot z_2 \;\; z_1 \cdot z_3 + p_2 \cdot z_1 \;\; z_2 \cdot z_3)^2$	$\sqrt{-g}R\big\vert_{(3)}$
$\mathcal{A}_3$	$(p_1 \cdot z_3)^2 (z_1 \cdot z_2)^2 + (p_3 \cdot z_2)^2 (z_1 \cdot z_3)^2 + (p_2 \cdot z_1)^2 (z_2 \cdot z_3)^2$	$\delta^{[\mu_1}_{\nu_1} \delta^{\mu_2}_{\nu_2} \delta^{\mu_3}_{\nu_3} \delta^{\mu_4]}_{\nu_4} \partial_{\mu_1} \partial^{\nu_1} h_{\mu_2}^{\ \nu_2} h_{\mu_3}^{\ \nu_3} h_{\mu_4}^{\ \nu_4}$
$\mathcal{A}_4$	$p_1 \cdot z_3 \; p_2 \cdot z_1 \; p_3 \cdot z_2 \;\; (p_1 \cdot z_3 \;\; z_1 \cdot z_2 + p_3 \cdot z_2 \;\; z_1 \cdot z_3 + p_2 \cdot z_1 \;\; z_2 \cdot z_3)$	$\sqrt{-g}\left(R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2\right)\Big\vert_{(3)}$
$\mathcal{A}_5$	$(p_1 \cdot z_3)^2 (p_2 \cdot z_1)^2 (p_3 \cdot z_2)^2$	$\sqrt{-g}\, R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta} R^{\alpha\beta}{}_{\mu\nu}\Big\vert_{(3)}$

Free coefficients: a_1, a_2, a_3, a_4, a_5

Joyce, Hinterbichler, RAR (2017)

Massive Spin-2 Particles

Order-by-order constraints $mb \ll 1$

Highest order: $\delta(s,b) = \pm \frac{2520 s}{\pi M_{\rm Pl}^2} \frac{a_5^2}{(mb)^8}$

$a_5 = 0$
kills R^3

Next order:
$$\delta(s,b) = \begin{cases} \pm \frac{(2a_2-a_3)^2 s}{\sqrt{2}\pi M_{\rm Pl}^2} \frac{1}{(mb)^4}\,, \\ \pm \frac{3(2a_2-a_3)^2 s}{16\pi M_{\rm Pl}^2} \frac{1}{(mb)^4}\,, \\ \pm \frac{(2a_2-a_3)^2 s}{2\pi M_{\rm Pl}^2} \frac{1}{(mb)^4}\,, \\ \pm \frac{\sqrt{\frac{3}{2}}(2a_2-a_3)^2 s}{4\pi M_{\rm Pl}^2} \frac{1}{(mb)^4}\,, \end{cases}$$

$a_3 = 2a_2$
kills pseudo-linear
(for D>4 also have a_4 =0)

Next order:
$$\delta(s,b) = \begin{cases} \pm \frac{a_1^2 s}{24\sqrt{2}\pi M_{\rm Pl}^2} \frac{1}{(mb)^2}\,, \\ \pm \frac{a_1^2 s}{48\pi M_{\rm Pl}^2} \frac{1}{(mb)^2}\,, \end{cases}$$

$a_1 = 0$
fixes h^3 coefficient

Finally: $\delta(s,b) = \frac{a_2^2 s}{16\pi M_{\rm Pl}^2} K_0(mb)$

positive!

Joyce, Hinterbichler, RAR (2017)

Massive Spin-2 Particles

Order-by-order constraints $mb \ll 1$

Highest order: $\delta(s,b) = \pm\frac{2520s}{\pi M_{\rm Pl}^2}\frac{a_5^2}{(mb)^8}$ $a_5 = 0$

Next order: $\delta(s,b) = \begin{cases} \pm\frac{(2a_2-a_3)^2 s}{\sqrt{2}\pi M_{\rm Pl}^2}\frac{1}{(mb)^4}\,, \\ \pm\frac{3(2a_2-a_3)^2 s}{16\pi M_{\rm Pl}^2}\frac{1}{(mb)^4}\,, \\ \pm\frac{(2a_2-a_3)^2 s}{2\pi M_{\rm Pl}^2}\frac{1}{(mb)^4}\,, \\ \pm\frac{\sqrt{\frac{3}{2}}(2a_2-a_3)^2 s}{4\pi M_{\rm Pl}^2}\frac{1}{(mb)^4}\,, \end{cases}$ $a_3 = 2a_2$

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Finally: $\delta(s,b) = \frac{a_2^2 s}{16\pi M_{\rm Pl}^2}K_0(mb)$ *positive!*

**from EFT point of view, coefficients do not need to be strictly zero but extremely small*

Joyce, Hinterbichler, RAR (2017)

Massive Spin-2 Particles

Eikonal Scattering

constrains cubic interactions
of the massive spin-2:

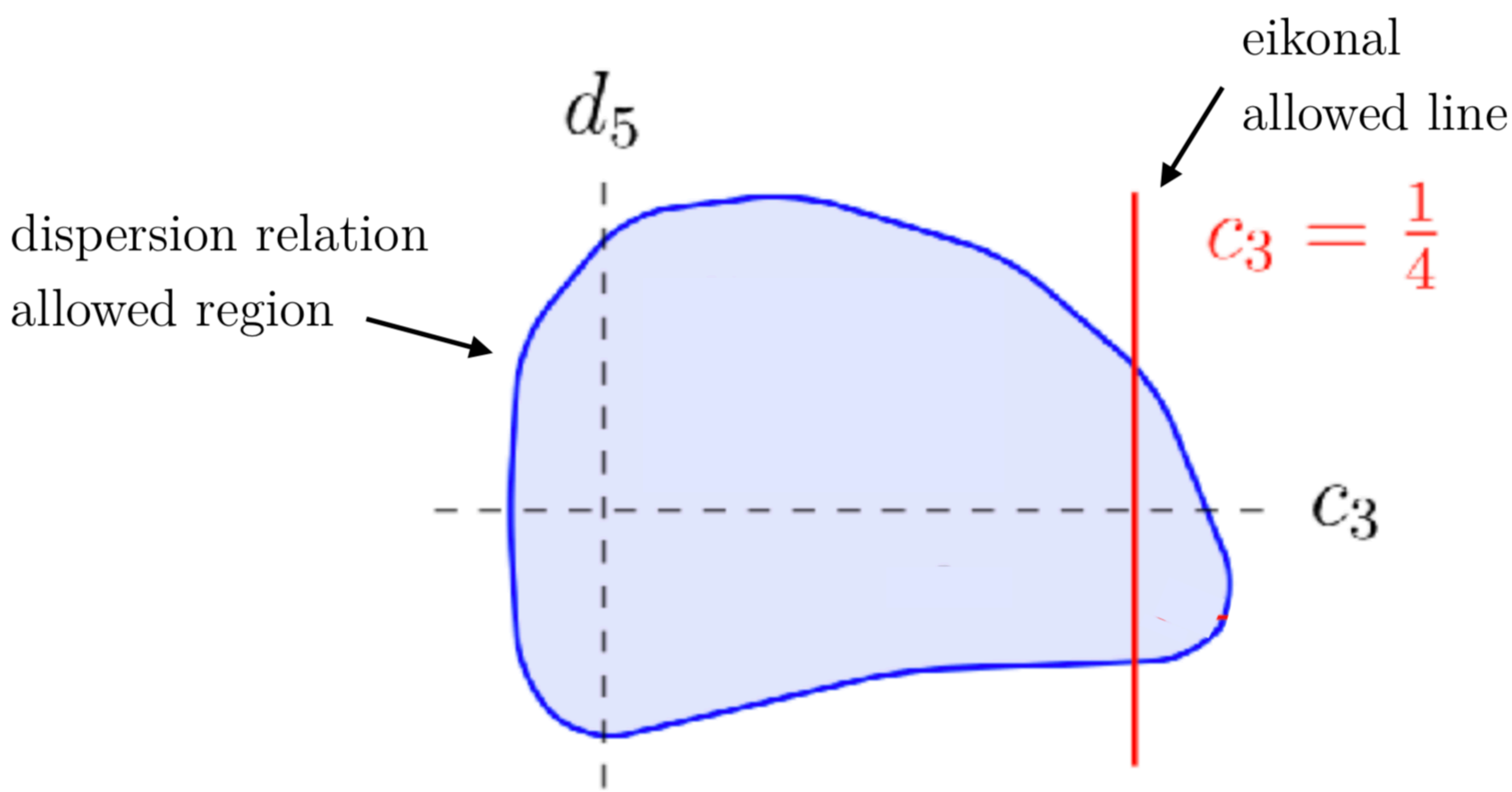
$$\mathcal{L}_3 = \frac{1}{2M_p} R^{(3)}_{\rm EH} + \frac{m^2}{2M_p} h^3_{\mu\nu}$$

If the cubic Lagrangian is not of this form then new physics must arise at the scale m.

Joyce, Hinterbichler, RAR (2017)

Massive Spin-2 Particles

Eikonal Scattering



Joyce, Hinterbichler, RAR (2017)

Massive Spin-2 Particles

agreement for pseudo-linear massive gravity

$$\mathcal{L} = \mathcal{L}_{FP} + \frac{1}{M_p}\lambda_1\mathcal{L}_{2,3} + \frac{m^2}{M_p}\lambda_3\mathcal{L}_{0,3} + \frac{m^2}{M_p^2}\lambda_4\mathcal{L}_{0,4}$$

$$\mathcal{L}_{2,3} = 12\,\delta^{[\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\nu_3}\delta^{\mu_4]}_{\nu_4}\,(\partial_{\mu_1}\partial^{\nu_1}h_{\mu_2}^{\ \nu_2})\,h_{\mu_3}^{\ \nu_3}\,h_{\mu_4}^{\ \nu_4}$$

$$\mathcal{L}_{0,3} = \frac{1}{6}\left([h]^3 - 3[h][h^2] + 2[h^3]\right)$$

$$\mathcal{L}_{0,4} = \frac{1}{24}\left([h]^4 - 6[h]^2[h^2] + 3[h^2]^2 + 8[h][h^3] - 6[h^4]\right)$$

Hinterbichler (2013)

NO consistency with positivity, NO consistency with luminality

Hinterbichler, RAR (2016); Joyce, Hinterbichler, RAR (2017)

Causality Contraints

Caveats about about asymptotic superluminality

- In flat spacetime, no direct derivation of the absence of asymptotic time advances in the S-matrix as a consequence of more fundamental notions such as analyticity or locality

- No proof that eikonal phase is always given by resummation of ladder graphs

- Bounds don't apply for cosmological applications

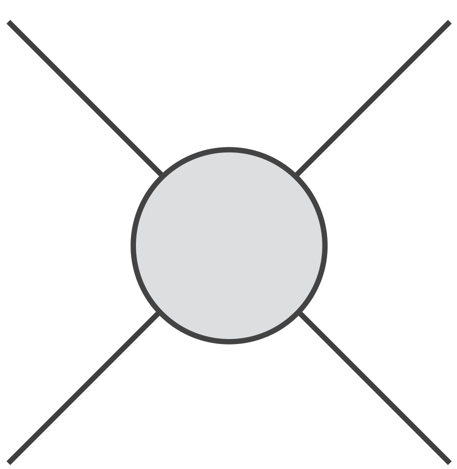
3. Raising the Cutoff

Raising the Cutoff

Ghost-free massive gravity:

$$\mathcal{S} = \frac{M_{Pl}^2}{2} \int d^4x \left\{ \det e \, R[e] - m^2 \sum_{n=0}^{4} \beta_n S_n[e] \right\}$$

2-2 scattering of the helicity-0 mode:

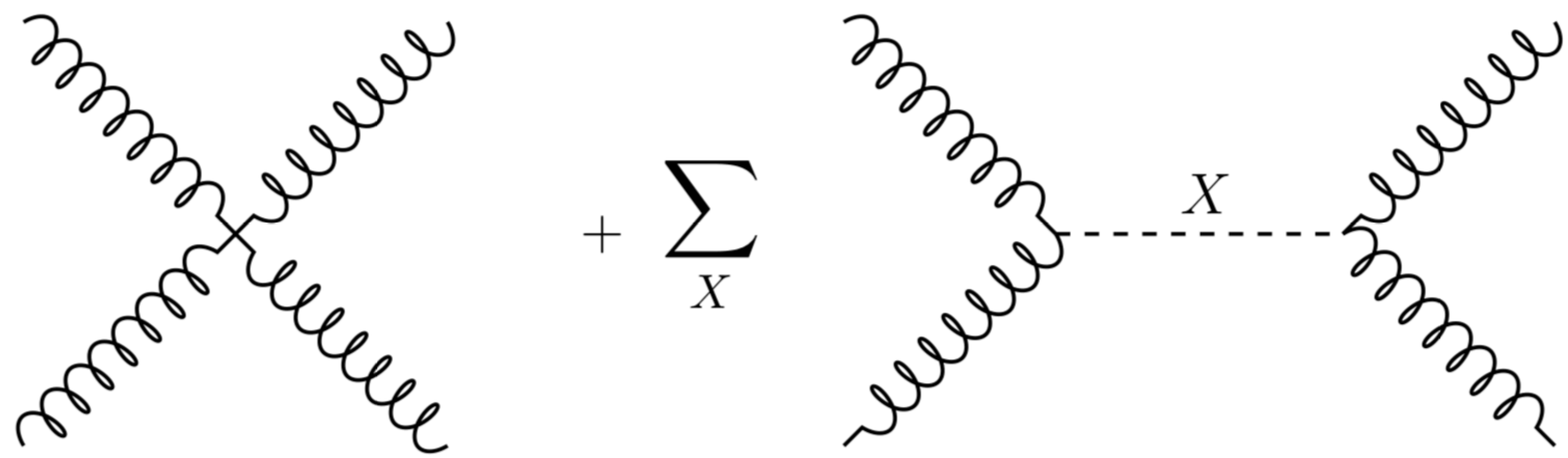


$$\mathcal{M} \sim \left(\frac{E}{\Lambda_3} \right)^6$$ where $\Lambda_3 = (m^2 M_P)^{1/3}$

Raising the Cutoff

Gravitational Higgs Mechanism

ADD VECTORS AND SCALARS



Improved behavior?

$$\mathcal{M} \sim E^{2n} \quad \Rightarrow \quad \Lambda_n = (M_{Pl} m^{n-1})^{1/n}$$

Bonifacio, Hinterbichler, RAR (2019)

Raising the Cutoff

Approach 1:

- Start with Einstein Hilbert kinetic term plus a potential
- Consider all relevant cubic vertices:

$$\begin{aligned}\mathcal{L}_{\hat{h}\hat{h}\phi_j} &= \frac{m^2}{2M_p}\sum_{l\geq 0}\Big(c_{1,l,j}\hat{h}_{\mu\nu}\hat{h}^{\mu\nu} + c_{2,l,j}m^{-2}\partial_\lambda\hat{h}_{\mu\nu}\partial^\nu\hat{h}^{\mu\lambda} + c_{3,l,j}m^{-4}\partial_\lambda\partial_\rho\hat{h}_{\mu\nu}\partial^\mu\partial^\nu\hat{h}^{\lambda\rho} \\ &+ \tilde{c}_{1,l,j}m^{-2}\varepsilon^{\mu\nu\lambda\rho}\partial_\mu\hat{h}_{\lambda\sigma}\partial_\nu\hat{h}_\rho{}^\sigma + \tilde{c}_{2,l,j}m^{-4}\varepsilon^{\mu\nu\lambda\rho}\partial_\mu\partial^\sigma\hat{h}_{\lambda\gamma}\partial_\nu\partial^\gamma\hat{h}_{\rho\sigma}\Big)m^{-2l}\Box^l\phi_j, \\ \mathcal{L}_{\hat{h}\hat{h}A_j} &= \frac{m_{A_j}m}{M_p}\sum_{l\geq 0}\Big(d_{1,l,j}m^{-1}\hat{h}_{\mu\nu}\partial^\mu\hat{h}^{\nu\lambda} + d_{2,l,j}m^{-3}\partial_\rho\hat{h}_{\mu\nu}\partial^\mu\partial^\nu\hat{h}^{\rho\lambda} \\ &+ \tilde{d}_{1,l,j}m^{-1}\varepsilon^{\mu\nu\rho\lambda}\partial_\mu\hat{h}_{\nu\sigma}\hat{h}_\rho{}^\sigma + \tilde{d}_{2,l,j}m^{-3}\varepsilon^{\mu\nu\rho\sigma}\partial_\mu\partial_\gamma\hat{h}_\rho{}^\lambda\partial_\nu\hat{h}_\sigma{}^\gamma\Big)m^{-2l}\Box^l A_{j,\lambda} \\ &+ \frac{m_{A_j}}{2M_p}\sum_{l\geq 0}\Big(d_{3,l,j}\hat{h}_{\mu\nu}\hat{h}^{\mu\nu} + d_{4,l,j}m^{-2}\partial_\lambda\hat{h}_{\mu\nu}\partial^\nu\hat{h}^{\mu\lambda} + d_{5,l,j}m^{-4}\partial_\lambda\partial_\rho\hat{h}_{\mu\nu}\partial^\mu\partial^\nu\hat{h}^{\lambda\rho} \\ &+ \tilde{d}_{3,l,j}m^{-2}\varepsilon^{\mu\nu\lambda\rho}\partial_\mu\hat{h}_{\lambda\sigma}\partial_\nu\hat{h}_\rho{}^\sigma + \tilde{d}_{4,l,j}m^{-4}\varepsilon^{\mu\nu\lambda\rho}\partial_\mu\partial^\sigma\hat{h}_{\lambda\gamma}\partial_\nu\partial^\gamma\hat{h}_{\rho\sigma}\Big)m^{-2l}\Box^l\partial^\lambda A_{j,\lambda}\end{aligned}$$

- Calculate tree-level amplitude, fix coefficients

$\Rightarrow$ recover dRGT potential, no improvement

Bonifacio, Hinterbichler, RAR (2019)

Raising the Cutoff

Approach 2: Model Independent

- Write down the most general 4-point amplitude consistent with Lorentz invariance, locality, unitarity, crossing symmetry and a bounded number of derivatives using on-shell cubic and quartic vertices

- Finite number of free parameters for the exchange terms, finite number of free polynomials for the contact terms

- Expand at high energies, fix coefficients

NO improved behavior

Bonifacio, Hinterbichler, RAR (2019)

See also: Bonifacio, Hinterbichler (2018)

Outlook

- Raising the low cutoff of a massive spin-2 requires massive particles of spin 2 or higher
- Can you raise the cutoff with a finite number of particles?
- Can you have a parametrically large gap between the mass and new physics?

Thank you!