INSTITUT POLYTECHNIQUE DE PARIS

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# ASYMPTOTIC SYMMETRIES, DOUBLE COPY & CONFORMAL DRESSINGS IN

# ◇ CELESTIAL DIAMONDS ◇

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AMPLITUDES, 19 AUGUST 2021

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# $\mathcal{S}$-matrix *and holography*

$\mapsto$ basic observable of quantum gravity in asymptotically flat spacetimes.

*Holographic* flavor: use on-shell data to learn about bulk physics.

$$ {}_{\text{boost}}\langle out \,|\, \mathcal{S} \,|\, in \rangle_{\text{boost}} = \langle \mathcal{O}^{\pm}_{\Delta_1, J_1}(w_1, \bar{w}_1) \ldots \mathcal{O}^{\pm}_{\Delta_n, J_n}(w_n, \bar{w}_n) \rangle_{\text{celestial CFT}} $$

***Celestial Holography programme:***

*attempts to apply holographic principle to asymptotically flat spacetimes.*

Observation: soft theorems in QFT = Ward ID of asymptotic symmetries.

Symmetry generators look like currents living on the celestial sphere.

# $\mathcal{S}$-matrix *as celestial amplitude*

$\mapsto$ basic observable of quantum gravity in asymptotically flat spacetimes.

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$$_{\text{boost}}\langle out \,|\, \mathcal{S} \,|\, in \rangle_{\text{boost}} = \langle \mathcal{O}^{\pm}_{\Delta_1, J_1}(w_1, \bar{w}_1) \ldots \mathcal{O}^{\pm}_{\Delta_n, J_n}(w_n, \bar{w}_n) \rangle_{\text{celestial CFT}}$$

## *CFT-inspired approach to amplitudes:*

Celestial CFT bootstrap machinery to compute and constrain scattering?

Collinear limits $\Rightarrow$ OPE data for celestial CFT.

Conformally soft limits: constraints from symmetries & conformal dressings.

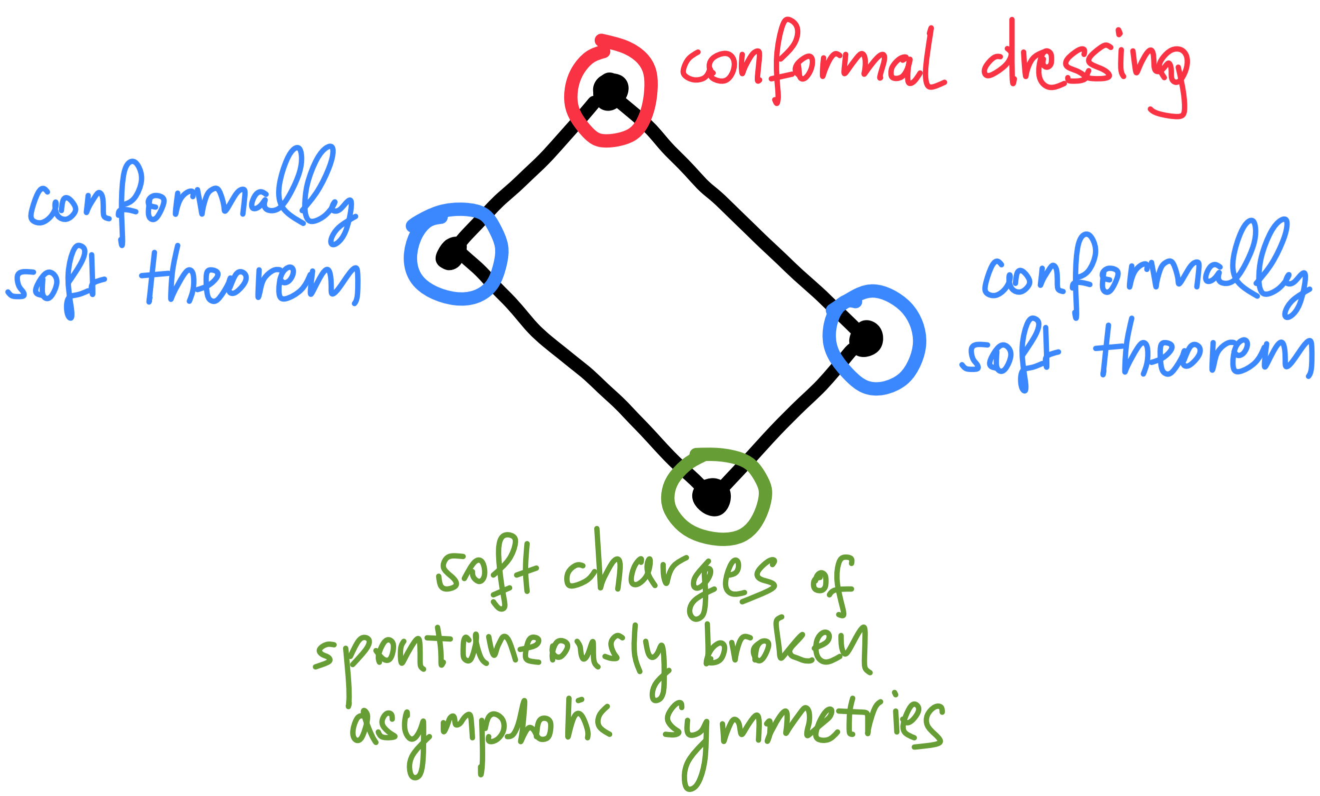
Anti-Wilsonian paradigm: celestial amplitudes probe all energy scales.

# Plan of the talk

BASED ON
2012.15694 - SABRINA PASTERSKI
2105.03516 & 2105.09792 - SABRINA PASTERSKI & EMILIO TREVISANI
ALSO
2007.15027 - EDUARDO CASALI
2108.xxxxx- YORGO PANO & SABRINA PASTERSKI

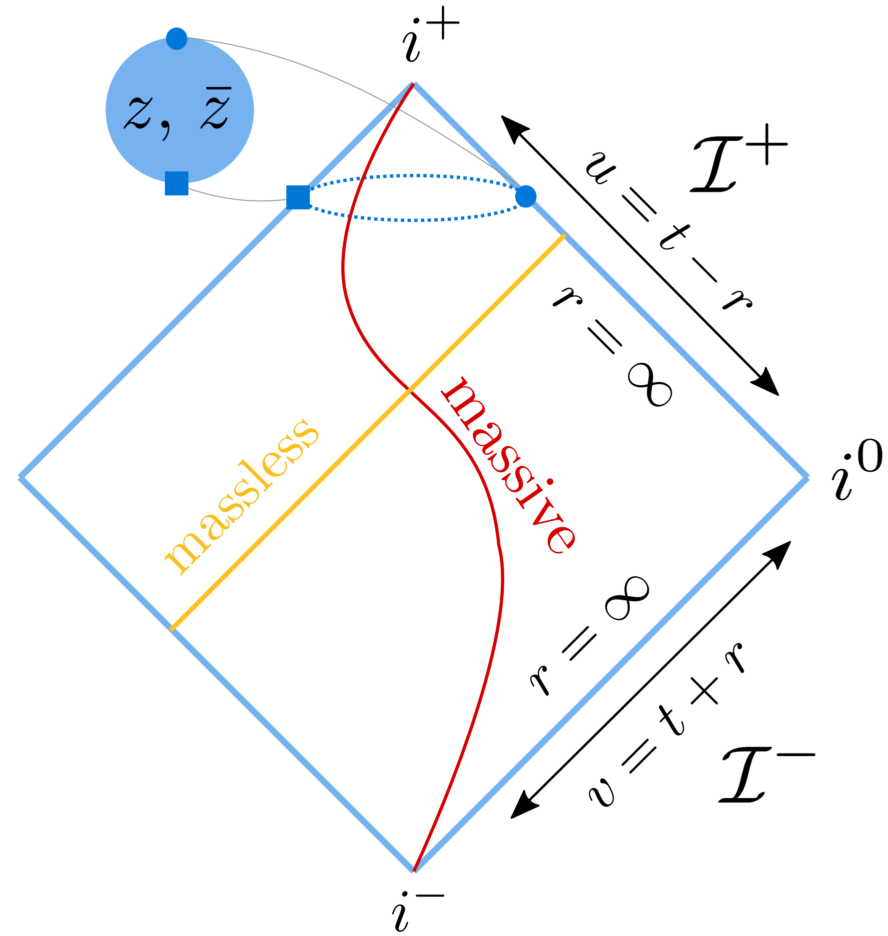
## CELESTIAL DIAMONDS:

*Natural structure in celestial CFT that unifies the discussion of* ***soft sector****:*




Appearance of *celestial double copy* and *shockwaves* in the diamond.

# Symmetries




$$\mathscr{L}\eta_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu = 0$$

Poincare generators in Bondi coordinates $(u, r, z, \bar z)$:

$$\xi = (1+\frac{u}{2r})Y^z\partial_z - \frac{u}{2r}D^{\bar z}D_z Y^z \partial_{\bar z} - \frac{1}{2}(u+r)D_z Y^z \partial_r + \frac{u}{2}D_z Y^z \partial_u + c.c.$$
$$+f\partial_u - \frac{1}{r}(D^z f \partial_z + D^{\bar z} f \partial_{\bar z}) + D^z D_z f \partial_r$$

Translations: $f_0 = 1, f_1 = \frac{z+\bar z}{1+z\bar z}, f_2 = \frac{i(\bar z - z)}{1+z\bar z}, f_3 = \frac{1-z\bar z}{1+z\bar z}$

Lorentz transformations:

$$Y^z_{12} = iz,\ Y^z_{13} = -\frac{1}{2}(1+z^2),\ Y^z_{23} = -\frac{i}{2}(1-z^2)$$
$$Y^z_{03} = z,\ Y^z_{02} = -\frac{i}{2}(1+z^2),\ Y^z_{01} = -\frac{1}{2}(1-z^2)$$

Asymptotic symmetry group of Einstein gravity:

$$\eta_{flat} \Rightarrow \eta_{flat} + \ldots$$ ← e.g. allow radiation

[Bondi,van der Burg,Metzner,Sachs'62]
[Barnich,Troessaert'11] ...

BMS supertranslations and superrotations: $f(z,\bar z)$ & $Y^z(z,\bar z)$

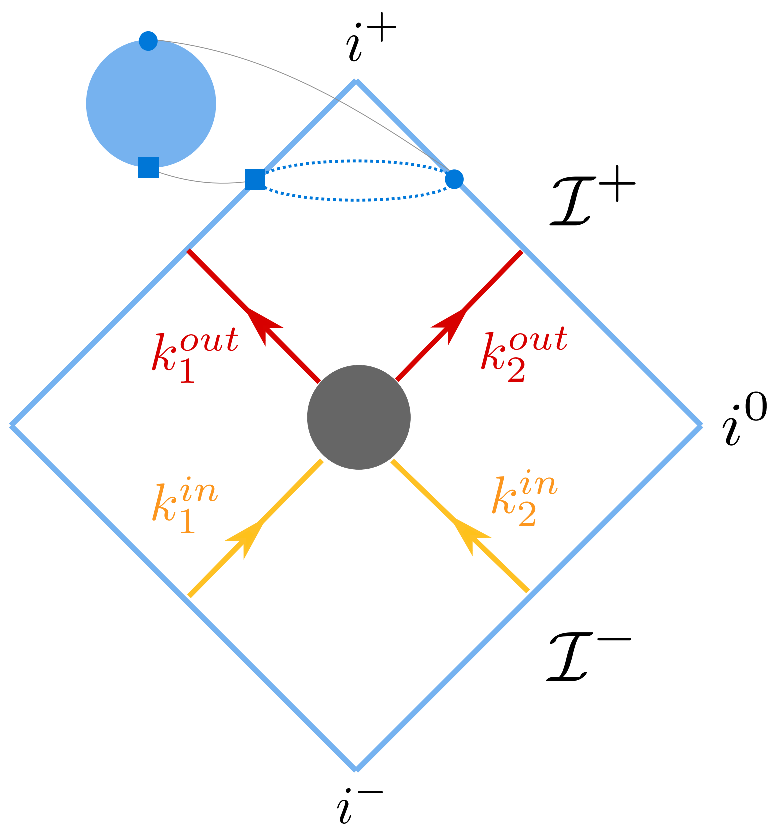
Asymptotic symmetries manifest in scattering amplitudes as soft theorems.

[Strominger'13] ...

# 4D $\mathcal{S}$-matrix ⇒ 2D correlator

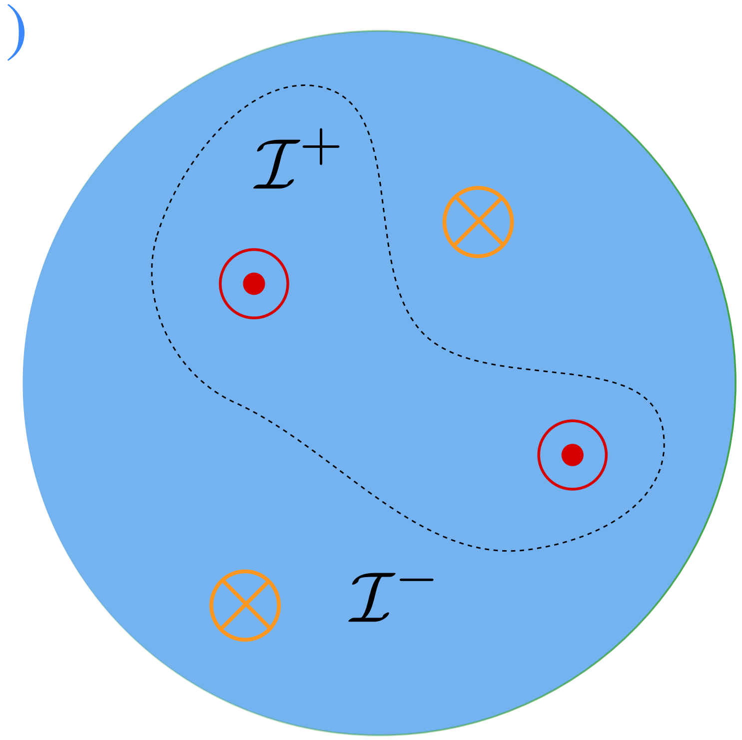
[de Boer,Solodhukin'03] [Cheung,de la Fuente,Sundrum'16][Pasterski,Shao,Strominger'16]

For massless scattering the map is a Mellin transform:




$$\mathscr{M}(\,.\,) = \int_0^\infty \frac{d\omega}{\omega}\omega^\Delta(\,.\,)$$

$$=$$




energy, helicity, point on $S^2$:

$$\mathscr{A}_n(\{k_i^\mu = \omega_i q^\mu(w_i,\bar{w}_i);\ell_i\})$$

conformal dimension, spin, point on $S^2$:

$$\langle \prod_{i=1}^n \mathscr{O}_{\Delta_i,J_i}(w_i,\bar{w}_i)\rangle$$

Lorentz symmetry

conformal symmetry

# Conformal Primary Operators

Given a 4D operator $O^s(X^\mu)$ of spin-$s$ in the Heisenberg picture, we can define a 2D operator $\mathcal{O}^{s,\pm}_{\Delta,J}(w,\bar{w})$ in the celestial CFT via a suitble inner product with *conformal primay wavefunctions* $\Phi^s_{\Delta,J}(X^\mu_\mp; w, \bar{w})$.

2D celestial CFT operator

4D wavefunction

$$\mathcal{O}^{s,\pm}_{\Delta,J}(w,\bar{w}) \equiv i\left(O^s(X^\mu), \Phi^s_{\Delta^*,-J}(X^\mu_\mp; w, \bar{w})\right)_\Sigma$$

[Donnay,Pasterski,AP'20]
[Pasterski,AP,Trevisani'21]

4D bulk operator
expanded into modes

inner product = integral over Cauchy slice $\Sigma$ in the 4D bulk

The $\pm$ label indicates *in* vs *out* states selected by prescription for analytically continuing the wavefunctions as $X^\mu_\pm = X^\mu \pm i\varepsilon\{-1,0,0,0\}$.

# Conformal Primary Wavefunctions

$$\Phi^{s=|J|}_{\Delta,J}(X^\mu; w, \bar{w}) \simeq \int_0^\infty \frac{d\omega}{\omega} \omega^\Delta \epsilon_{\mu_1 \ldots \mu_s} e^{\pm i \omega q \cdot X}$$


4D spin-$s$ field under Lorentz transformations

2D conformal primary with conformal dimension $\Delta$ and spin $J$

Lorentz transformation

bulk point $X^\mu \mapsto \Lambda^\mu{}_\nu X^\nu$

conformal transformation

boundary point $w \mapsto \frac{aw+b}{cw+d}$ $\bar{w} \mapsto \frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}$

$ad - bc = 1 = \bar{a}\bar{d} - \bar{b}\bar{c}$

$$\Phi^s_{\Delta,J}\left(\Lambda^\mu{}_\nu X^\nu; \frac{aw+b}{cw+d}, \frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}\right) = (cw+d)^{\Delta+J}(\bar{c}\bar{w}+\bar{d})^{\Delta-J} D_s(\Lambda)\Phi^s_{\Delta,J}(X^\mu; w, \bar{w})$$

3+1D spin-$s$ representation of the Lorentz algebra

[Pasterski,Shao'17]

***Radiative***: $J = \pm s$ & solve the linearized eom for massless spin-$s$ particles

***Generalized***: $|J| \leq s$ & allow sources and distributions [Pasterski,AP'20]

# Celestial amplitudes

[Pasterski,Shao,Strominger'16+'17]

$\mathcal{S}$-matrix elements constructed as

$$\widetilde{\mathcal{A}}(\Delta_i, z_i, \bar{z}_i) = \prod_{i=1}^{n} \int_0^\infty \frac{d\omega_i}{\omega_i} \omega_i^{\Delta_i} \mathcal{A}(\omega_i, z_i, \bar{z}_i)$$

transform by construction as correlators of (quasi)-primaries

$J_j = j$-th helicity

$$\widetilde{\mathcal{A}}(\Delta_i, \frac{az_i + b}{cz_i + d}, \frac{\bar{a}\bar{z}_i + \bar{b}}{\bar{c}\bar{z}_i + \bar{d}}) = \prod_{j=1}^{n} (cz_j + d)^{\Delta_j + J_j} (\bar{c}\bar{z}_j + \bar{d})^{\Delta_j - J_j} \widetilde{\mathcal{A}}(\Delta_i, z_i, \bar{z}_i) \,.$$

since the external particles are in boost eigenstates.

# Spectrum vs Symmetries

The transformation can be inverted easily when $\Delta$ is on the principal continuous series of the Lorentz group $\Delta = 1 + i\mathbb{R}$ which captures finite energy radiation:

$$\mathscr{A}(\omega_i, z_i, \bar{z}_i) = \prod_{i=1}^{n} \int_{1-i\infty}^{1+i\infty} \frac{d\Delta_i}{2\pi i} \omega_i^{-\Delta_i} \widetilde{\mathscr{A}}(\Delta_i, z_i, \bar{z}_i)\,.$$

Analytic continuation to $\Delta \in \mathbb{C}$ of great interest to celestial CFT.

More on this later!

Translations shift the conformal dimension:

$$\mathscr{P}^\mu = q^\mu e^{\partial_\Delta} \Leftrightarrow \Delta \mapsto \Delta + 1$$

[Donnay,AP,Strominger'18]
[Stieberger,Taylor'18]

*What happens to properties of amplitudes that exploit manifest translation symmetry?*

# Gravity = Gauge Theory$^2$

[Bern,Carrasco,Johansson'10]

*manifest translation symmetry*

gluon amplitude

$e^{ik\cdot X}$ ↓

color, kinematic, propagator

$$\mathscr{A}_n^{YM} = \delta^{(4)}\Big(\sum_{i=1}^{n} k_i^\mu\Big) \sum_{\gamma\in\Gamma} \frac{c_\gamma n_\gamma}{\Pi_\gamma}$$

double copy

$$c \mapsto n$$

$$\mathscr{A}_n^{G} = \delta^{(4)}\Big(\sum_{i=1}^{n} k_i^\mu\Big) \sum_{\gamma\in\Gamma} \frac{n_\gamma^2}{\Pi_\gamma}$$

graviton amplitude

# Celestial Gravity = Gauge Theory$^2$

[Bern,Carrasco,Johansson'10] $\mathcal{M}(\,.\,) = \int_0^\infty \frac{d\omega}{\omega}\omega^{\Delta}(\,.\,)$ [Casali,AP'20]

*manifest translation symmetry* $\mapsto$ *manifest conformal symmetry*

gluon amplitude

$e^{ik\cdot X}$ ↓ color kinematic

$$\mathcal{A}_n^{YM} = \delta^{(4)}\Big(\sum_{i=1}^{n} k_i^\mu\Big) \sum_{\gamma\in\Gamma} \frac{c_\gamma n_\gamma}{\Pi_\gamma}$$

propagator

double copy $c \mapsto n$

$$\mathcal{A}_n^{G} = \delta^{(4)}\Big(\sum_{i=1}^{n} k_i^\mu\Big) \sum_{\gamma\in\Gamma} \frac{n_\gamma^2}{\Pi_\gamma}$$

graviton amplitude

celestial gluon amplitude

$\Phi_{\Delta,J}$ ↓

$$\widetilde{\mathcal{A}}_n^{YM} = \mathcal{M}(\mathcal{A}_n^{YM})$$

?

$$\widetilde{\mathcal{A}}_n^{G} = \mathcal{M}(\mathcal{A}_n^{G})$$

celestial graviton amplitude

# Celestial Gravity = Gauge Theory$^2$

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$$\mathcal{A}_n^{YM} = \delta^{(4)}\Big(\sum_{i=1}^n k_i^\mu\Big) \sum_{\gamma\in\Gamma} \frac{c_\gamma n_\gamma}{\Pi_\gamma}$$

color, kinematic, propagator

double copy

$c \mapsto n$

$$\mathcal{A}_n^{G} = \delta^{(4)}\Big(\sum_{i=1}^n k_i^\mu\Big) \sum_{\gamma\in\Gamma} \frac{n_\gamma^2}{\Pi_\gamma}$$

graviton amplitude

| kinematic functions | $\mapsto$ | kinematic operators |
|---|---|---|
| $n(\omega)$ | | $\mathcal{N}(e^{\partial_\Delta})$ |

details: QCD meets gravity 2020

celestial gluon amplitude

$\Phi_{\Delta,J}$ ↓

$$\widetilde{\mathcal{A}}_n^{YM} = \sum_{\gamma\in\Gamma} c_\gamma \mathcal{N}_\gamma \widetilde{\mathcal{A}}_n^{scalar}$$

$c \mapsto \mathcal{N}$

celestial double copy

$$\widetilde{\mathcal{A}}_n^{G} = \sum_{\gamma\in\Gamma} \mathcal{N}_\gamma^2 \widetilde{\mathcal{A}}_n^{scalar}$$

celestial graviton amplitude

# Celestial Gravity = Gauge Theory$^2$

[Bern,Carrasco,Johansson'10] [Casali,AP'20]

$\mathscr{M}(\,.\,) = \int_0^\infty \frac{d\omega}{\omega}\omega^\Delta(\,.\,)$

*manifest translation symmetry* $\mapsto$ *manifest conformal symmetry*

**gluon amplitude**

$e^{ik\cdot X}$ ↓

$$\mathscr{A}_n^{YM} = \delta^{(4)}\Big(\sum_{i=1}^n k_i^\mu\Big)\sum_{\gamma\in\Gamma}\frac{c_\gamma n_\gamma}{\Pi_\gamma}$$

color, kinematic, propagator

double copy ↓ $c \mapsto n$

$$\mathscr{A}_n^{G} = \delta^{(4)}\Big(\sum_{i=1}^n k_i^\mu\Big)\sum_{\gamma\in\Gamma}\frac{n_\gamma^2}{\Pi_\gamma}$$

**graviton amplitude**

**celestial gluon amplitude**

$\Phi_{\Delta,J}$ ↓

$$\widetilde{\mathscr{A}}_n^{YM} = \sum_{\gamma\in\Gamma} c_\gamma \mathscr{N}_\gamma \widetilde{\mathscr{A}}_n^{scalar}$$

celestial double copy ↓ $c \mapsto \mathscr{N}$

$$\widetilde{\mathscr{A}}_n^{G} = \sum_{\gamma\in\Gamma} \mathscr{N}_\gamma^2 \widetilde{\mathscr{A}}_n^{scalar}$$

**celestial graviton amplitude**

| kinematic functions | $\mapsto$ | kinematic operators |
|---|---|---|
| $n(\omega)$ | | $\mathscr{N}(e^{\partial_\Delta})$ |

details: QCD meets gravity 2020

Double copy is fundamental property of gauge and gravity amplitudes.

Operator-valued celestial double $\Rightarrow$ *curved* spacetimes?

# Building blocks for celestial CFT

[Pasterski,Shao'17]
[Pasterski,AP'20]

Embedding $S^2$ into $\mathbb{R}^{1,3}$ lightcone: $q^\mu = (1 + w\bar{w}, w + \bar{w}, -i(w - \bar{w}), 1 - w\bar{w})$

$$\sqrt{2}\epsilon_+^\mu = \partial_w q^\mu, \sqrt{2}\epsilon_-^\mu = \partial_{\bar{w}} q^\mu$$

Mellin transform of plane waves :

$$\mathscr{M}(e^{\pm ik\cdot X}) = \int_0^\infty \frac{d\omega}{\omega}\omega^\Delta e^{\pm i\omega q\cdot X_\pm} = \frac{(\mp i)^\Delta \Gamma(\Delta)}{(-q\cdot X_\pm)^\Delta}$$

&

null tetrad

$$\{\underbrace{l, n}_{J=0}, \underbrace{m, \bar{m}}_{J=\pm 1}\}$$

spin frame

$$\{\underbrace{o, \bar{o}, \iota, \bar{\iota}}_{J=\pm\frac{1}{2}}\}$$

Spin $s = 0$ :

$$\varphi^\Delta = \frac{1}{(-q\cdot X)^\Delta}$$

Spin $s = \frac{1}{2}, 1, \frac{3}{2}, 2$:

$$\psi_{\Delta, J=+\frac{1}{2}} = o\varphi^\Delta$$

$$A_{\Delta, J=+1} = m\varphi^\Delta$$

$$\chi_{\Delta, J=+\frac{3}{2}} = om\varphi^\Delta$$

$$h_{\Delta, J=+2} = mm\varphi^\Delta$$

$$o_a = \sqrt{q\cdot X}\,|q]_a$$

$$o_a \mapsto (cw+d)^{\frac{1}{2}}(\bar{c}\bar{w}+\bar{d})^{-\frac{1}{2}}(Mo)_a$$

$$m^\mu = \epsilon_+^\mu + (\epsilon_+\cdot X)l^\mu$$

spacetime dependent polarization vector

$$m^\mu \mapsto \frac{cw+d}{\bar{c}\bar{w}+\bar{d}}\Lambda^\mu{}_\nu m^\nu$$

# Spin-$s$ via double copy and supersymmetry

**Supersymmetry:** relates primaries by $\frac{1}{2}$ integer spin steps

$$\mathcal{Q}_a = \partial_\theta |q]_a e^{\partial_\Delta/2} \qquad \bar{\mathcal{Q}}_{\dot{a}} = \theta \langle q|_{\dot{a}} e^{\partial_\Delta/2} \qquad \{\mathcal{Q}_a, \bar{\mathcal{Q}}_{\dot{a}}\} = -\sigma^\mu_{a\dot{a}} \mathcal{P}_\mu \qquad \mathcal{P}_\mu = q_\mu e^{\partial_\Delta}$$

[Fotopoulos,Stieberger,Taylor,Zhu'20]

**Kerr-Schild double copy:**

$$g_{\mu\nu} = \eta_{\mu\nu} + \underbrace{m_\mu m_\nu \varphi^\Delta}_{h_{\Delta,J=\pm 2;\mu\nu}}$$

[Pasterski,AP'20]

Kerr-Schild vector:

$$g^{\mu\nu} m_\mu m_\nu = 0$$
$$\eta^{\mu\nu} m_\mu m_\nu = 0$$

null

$$m^\mu \nabla_\mu m_\nu \propto m_\nu$$
$$m^\mu \partial_\mu m_\nu \propto m_\nu$$

geodesic

[Monteiro,O'Connell,White'14]

$h_{\Delta,J=\pm 2;\mu\nu}$ is *exact* sol to Einstein: Petrov type N

$$R^{\Delta,J}_{\mu\nu} = 0$$

$$\Psi_4 = -\frac{1}{2}\Delta(\Delta-1)\varphi^\Delta$$

$\Delta = 0,1$ : asymptotic symmetries!

# Finite conformal primary backgrounds

[Pasterski,AP'20]

- Aichelburg-Sexl shockwave or ultraboosted Schwarzschild [Aichelburg,Sexl'71]

$$g_{\mu\nu} = \eta_{\mu\nu} - \underbrace{4G_N\alpha q_\mu q_\nu \log(X^2)\delta(q\cdot X)}_{h^{gen}_{\Delta=-1,J=0;\mu\nu}} \qquad E = \alpha q^0$$

generalized conformal primary metric

- Dray-'t Hooft planar shell [Dray,t'Hooft'86]

$$g_{\mu\nu} = \eta_{\mu\nu} - 4G_N\alpha q_\mu q_\nu \frac{(X^2)}{D-2}\delta(q\cdot X) \qquad \rho = \alpha q^0$$

- Beam-like gravitational wave [Ferrari,Pendenza,Veneziano'88]

$$g_{\mu\nu} = \eta_{\mu\nu} - 4G_N\alpha q_\mu q_\nu f(X^2)\delta(q\cdot X)$$

$$E = \alpha q^0 \sqrt{X^2} f'(X^2)$$

- Kerr gyraton or ultraboosted Kerr see [Cristofoli'20] [Arkani-Hamed,Huang,O'Connell'20]...

$$g_{\mu\nu} = \eta_{\mu\nu} - 4G_N\alpha q_\mu q_\nu \log(|X^2 - a^2|)\delta(q\cdot X)$$

$$a^\mu = s^\mu/m$$

$\mapsto$ *Amplitudes methods to explore non-perturbative bulk physics in celestial CFT.*

# Soft limits as celestial currents

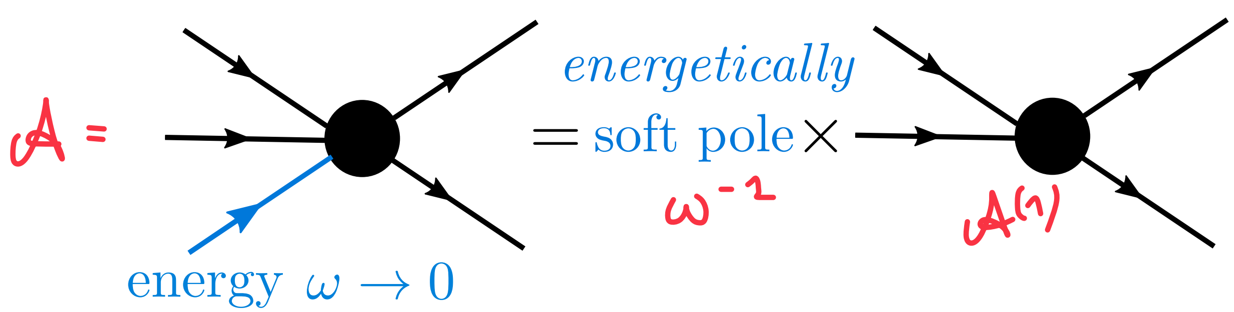
Ward ID of asymptotic symmetries = soft theorems in QFT. [Strominger'13] ...

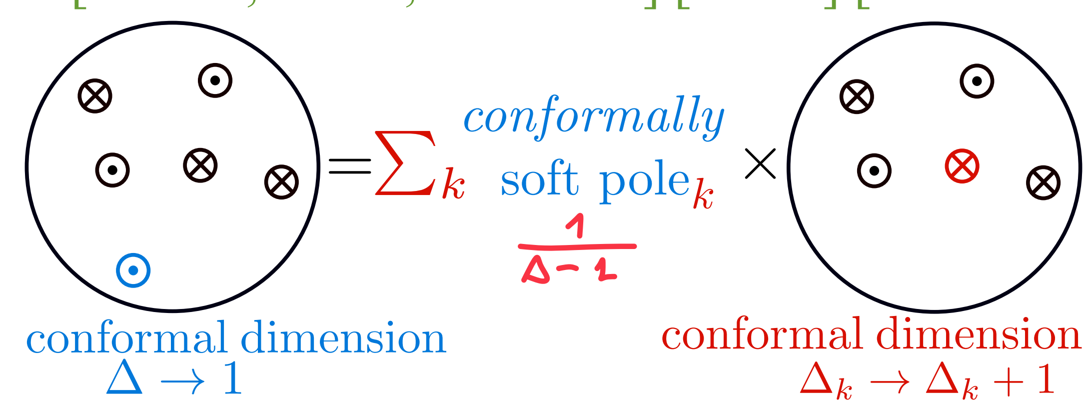
$$\lim_{\omega\to 0} \mathscr{A} = \omega^{-1}\mathscr{A}^{(1)} + \omega^{0}\mathscr{A}^{(0)} + \dots$$

Soft limits at different orders in $\omega \to 0 \Leftrightarrow$ "conformally soft" $\Delta$ poles.

e.g. leading soft graviton: [Weinberg'65]

[Adamo,Mason,Sharma'19] [AP'19] [Guevara'19]




$$\mathscr{P}_w \mathscr{O}_\omega(z,\bar{z}) \sim \frac{\omega}{w-z}\mathscr{O}_\omega(z,\bar{z})$$

$$\mathscr{P}_w \mathscr{O}_\Delta(z,\bar{z}) \sim \frac{1}{w-z}\mathscr{O}_{\Delta+1}(z,\bar{z})$$

[Donnay,AP,Strominger'18]

*supertranslation current*

# Conformally soft primaries

symmetry generator

$$\mathcal{O}_{\Delta,J} \equiv i(\hat{O}, \Phi^{G}_{\Delta^*,-J})_\Sigma$$

pure gauge

[Donnay,AP,Strominger'18]
[Donnay,Pasterski,AP'20]
[Pasterski,AP'20]
[Pano,Pasterski,AP-to appear]

| | $A_{\Delta,J=\pm 1}$ | $\chi_{\Delta,J=+\frac{3}{2}}$ | $h_{\Delta,J=\pm 2}$ | |
|---|---|---|---|---|
| $\Delta$ | 1 | $\frac{1}{2}$ | 1 | 0 |
| symmetry | large U(1) $\mathcal{J}_w$ | large SUSY | supertranslation $\mathcal{P}_w$ | shadow superrotation |

Goldstones of spontaneously broken asymptotic symmetries for $1 \leq s \leq 2$

Diff($S^2$)

Kerr-Schild double copy: BMS = (large U(1))$^2$

[Huang,Kol,O'Connell'19]

| | $\psi_{\Delta,J=+\frac{1}{2}}$ | $A_{\Delta,J=\pm 1}$ | $\chi_{\Delta,J=+\frac{3}{2}}$ | $h_{\Delta,J=\pm 2}$ |
|---|---|---|---|---|
| $\Delta$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | -1 |

not pure gauge

[Fotopoulos,Taylor, Stieberger,Zhu'20]
[Adamo,Mason,Sharma'19]
[Guevara,19]

Soft theorems without conformal Goldstones for $1 \leq s \leq 2$

# Conformally soft shadow primaries

$$\widetilde{\Phi}_{\Delta,J} \equiv \widetilde{\Phi_{2-\Delta,-J}}$$

pure gauge

| | $\widetilde{A}_{\Delta,J=\pm1}$ | $\widetilde{\chi}_{\Delta,J=-\frac{3}{2}}$ | $\widetilde{h}_{\Delta,J=\pm2}$ | |
|---|---|---|---|---|
| $\Delta$ | 1 | $\frac{3}{2}$ | 1 | 2 $\mathscr{T}_{ww}$ |
| symmetry | large U(1) | large SUSY | supertranslation | superrotation |

Goldstones of spontaneously broken asymptotic symmetries for $1 \leq s \leq 2$

Virasoro

[Donnay,AP,Strominger'18]
[Donnay,Pasterski,AP'20]
[Pasterski,AP'20]
[Pano,Pasterski,AP-to appear]

SUSY current

[Fotopoulos,Taylor, Stieberger,Zhu'20]

stress tensor for celestial CFT

[Kapec,Mitra,Raclariu,Strominger'16]

| | $\widetilde{\psi}_{\Delta,J=-\frac{1}{2}}$ | $\widetilde{A}_{\Delta,J=\pm1}$ | $\widetilde{\chi}_{\Delta,J=-\frac{3}{2}}$ | $\widetilde{h}_{\Delta,J=\pm2}$ |
|---|---|---|---|---|
| $\Delta$ | $\frac{3}{2}$ | 2 | $\frac{5}{2}$ | 3 |

not pure gauge

Soft theorems without conformal Goldstones for $1 \leq s \leq 2$

# A unified treatment of soft modes

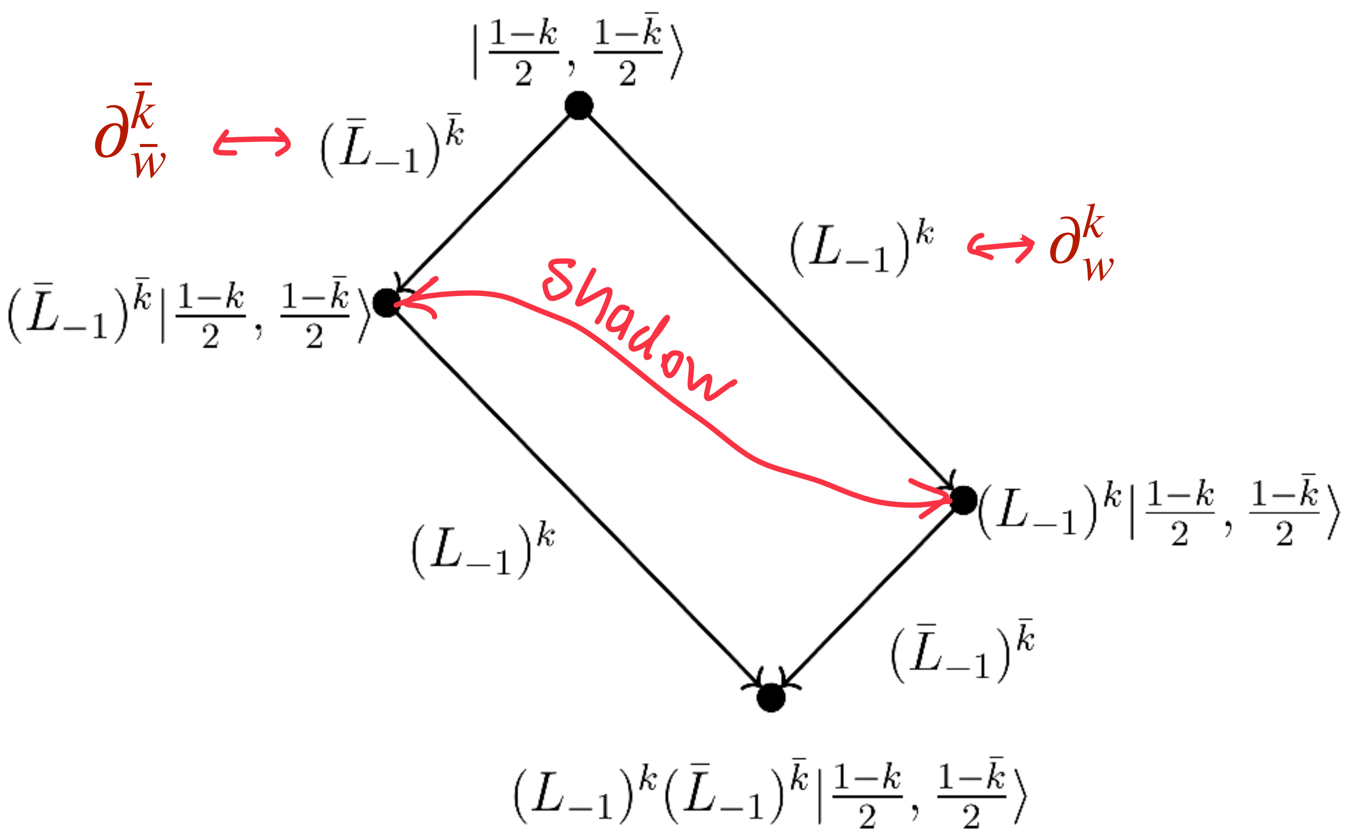
Asymptotic symmetry generators, their associated soft charges as well as conformal dressings for celestial amplitudes are all captured by the structure of the conformal multiplets of global $SL(2,\mathbb{C})$.

Primary $|h,\bar{h}\rangle$ annihilated by action of $L_1, \bar{L}_1$.

Primary descendants from action of $L_{-1}, \bar{L}_{-1}$ w/

$(L_{-1})^k|h,\bar{h}\rangle$ when $h = \frac{1-k}{2}$ with $k \in \mathbb{Z}_>$

$(\bar{L}_{-1})^{\bar{k}}|h,\bar{h}\rangle$ when $\bar{h} = \frac{1-\bar{k}}{2}$ with $k \in \mathbb{Z}_>$




"celestial diamonds"

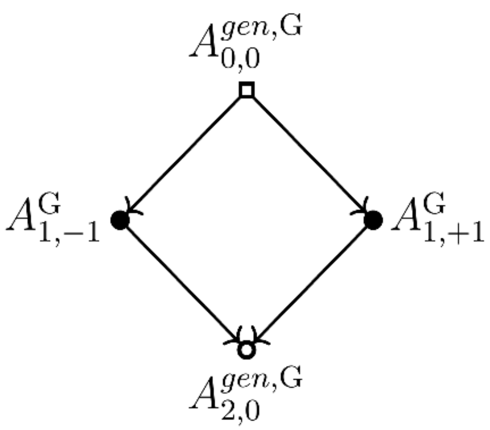
[Pasterski,AP,Trevisani'21]

# Celestial diamonds

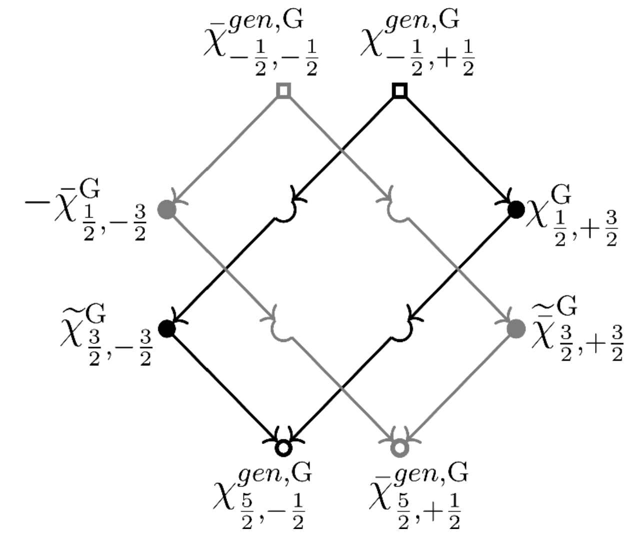
$\Phi^{gen,s} \mapsto \Phi^{s=|J|}$ or $\Phi^{s=|J|} \mapsto \Phi^{gen,s}$

[Pasterski,AP,Trevisani'21]

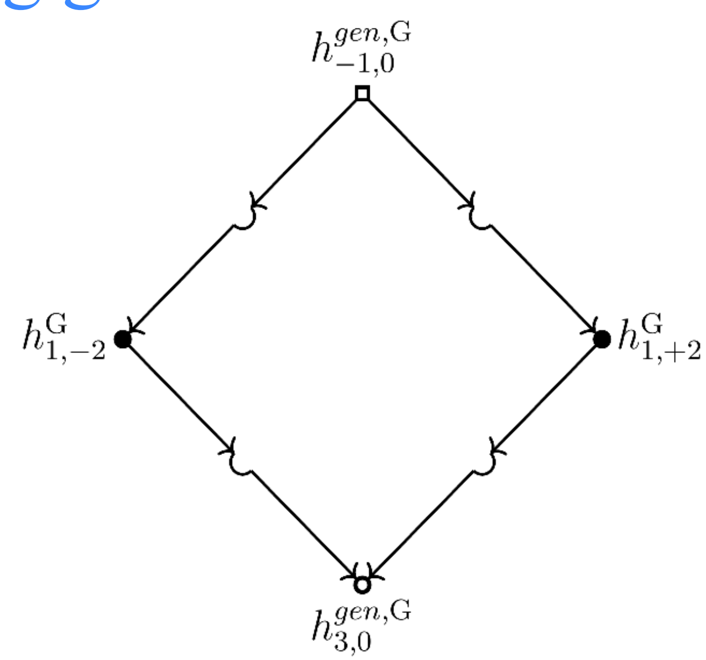
Leading photon:



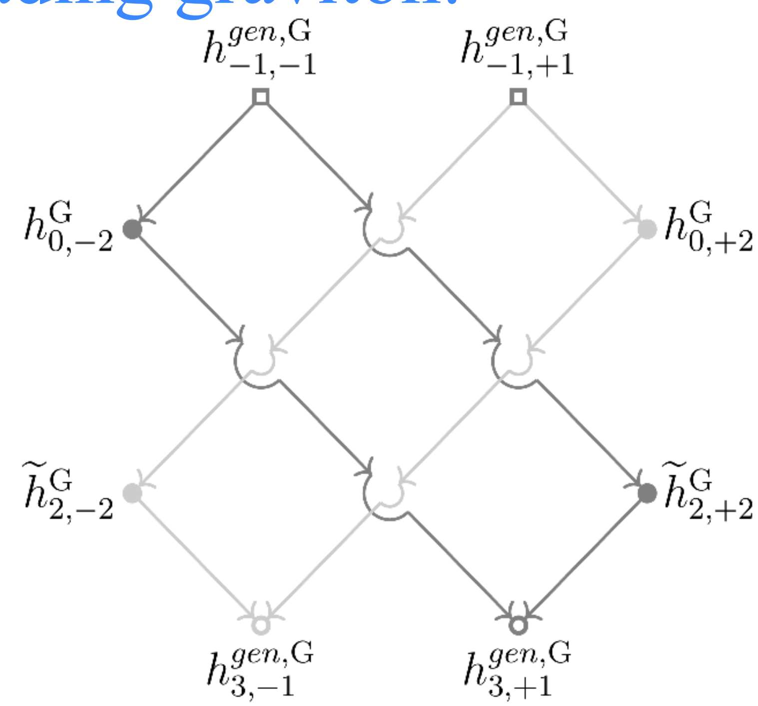

Leading gravitino:




Leading graviton:




Subleading graviton:




$\Rightarrow$ Capture most leading (conformally) soft theorems.

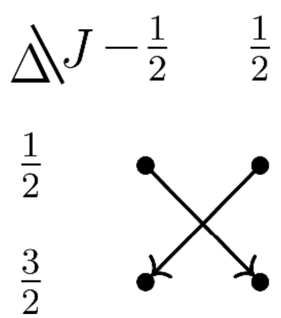
*Shockwaves are at the top corner of the associated memory diamonds.*

# Degenerate diamonds

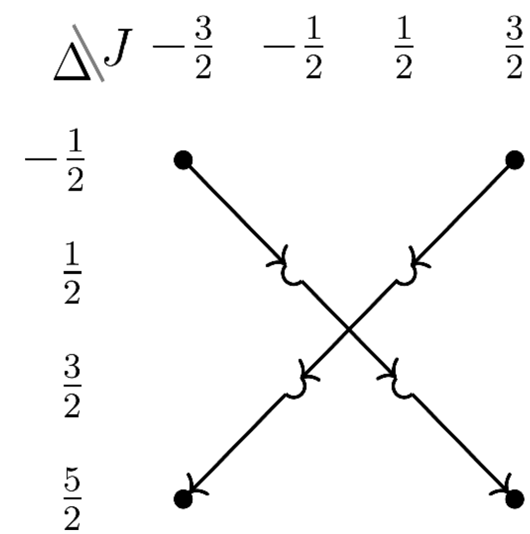
$\Phi^{s=|J|} \mapsto \widetilde{\Phi}^{s=|J|}$

[Pasterski,AP,Trevisani'21]

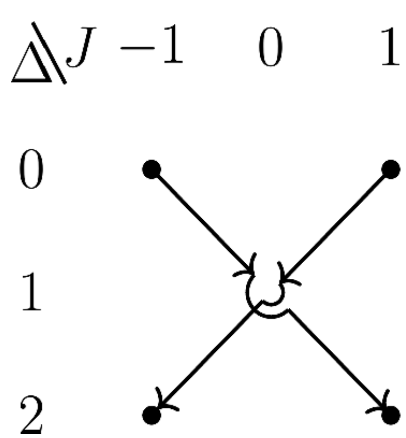
Leading photino:



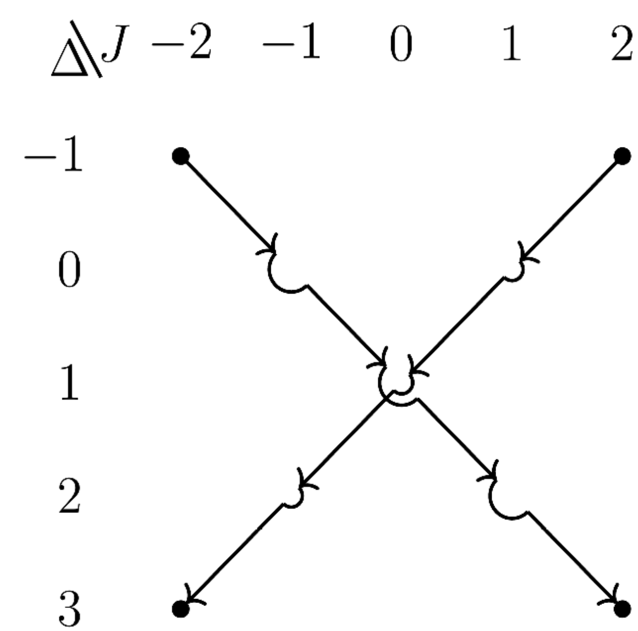

Subleading gravitino:




Subleading photon:




Subsubleading graviton:




$\Rightarrow$ Capture most subleading (conformally) soft theorems.

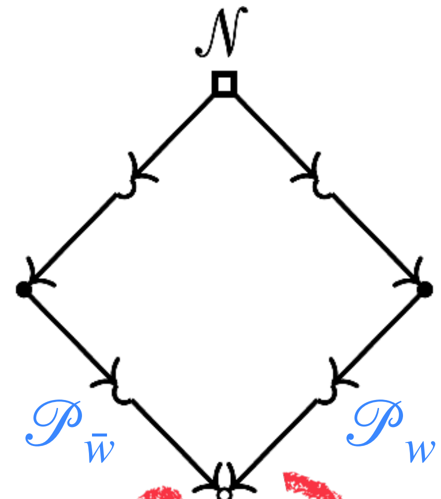
*Celestial diamonds unify pure gauge & non-pure gauge soft modes.*

# Soft charges

$$[\mathcal{O}^{\rm G}_{\Delta,\pm 2}, \hat{h}_{\mu\nu}] = i(\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu) = i h^{\rm G}_{\Delta,\pm 2;\mu\nu}$$

shift of bulk field

## Supertanslations:




$(\Delta, J) = (1, +2)$

$h^{\rm G}_{1,+2;\mu\nu}$

$$\mathcal{N}_{\bar{w}\bar{w}} = \frac{1}{2!}\partial_{\bar{w}}^2 \mathcal{N} \qquad \mathcal{N}_{ww} = \frac{1}{2!}\partial_w^2 \mathcal{N} = \int d^2 z f_{ww} \mathcal{O}^{soft}_{3,0} = Q^{soft}_f$$

$$\beta \mathcal{O}^{soft}_{3,0} = \frac{\beta}{4\pi\kappa} \lim_{\Delta\to 1} (\Delta - 1)(1 + z\bar{z})\left[\partial_{\bar{z}}^2 (a_{\Delta,+2} + a^\dagger_{\Delta,+2}) + \partial_z^2 (a_{\Delta,-2} + a^\dagger_{\Delta,-2})\right]$$

## Superrotations:

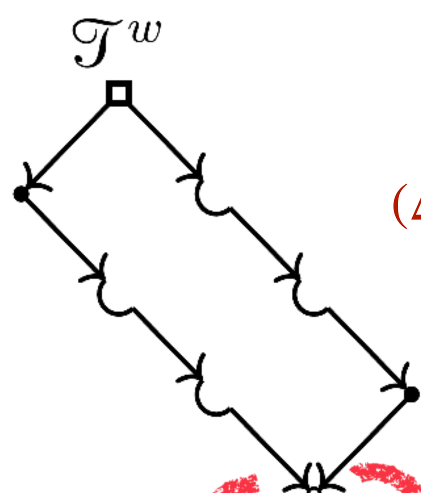
[Banerjee'18]...

[Pasterski,AP,Trevisani'21]

$h^{\rm G}_{0,-2;\mu\nu}$ $\qquad (\Delta, J) = (0, -2)$

$$Q^{soft}_Y = \int d^2 z Y^z_{\bar{w}\bar{w}} \mathcal{O}^{soft}_{3,+1} = \widetilde{\mathcal{T}}_{\bar{w}\bar{w}} = \partial_{\bar{w}} \mathcal{T}^w$$

Diff($S^2$)




$(\Delta, J) = (2, +2)$

$\widetilde{h}^{\rm G}_{2,+2;\mu\nu}$

$$\mathcal{T}_{ww} = -\frac{1}{3!}\partial_w^3 \mathcal{T}^w = \int d^2 z Y^z_{ww} \mathcal{O}^{soft}_{3,+1} = Q^{soft}_Y$$

Virasoro

$$\frac{i\beta}{4\pi\kappa} \lim_{\Delta\to 0} \Delta \left[\partial_{\bar{z}}^3 (a_{\Delta,-2} - a^\dagger_{\Delta,-2})\right] = \beta \mathcal{O}^{soft}_{3,+1}$$

The symmetry parameters $\{f_{ww}, Y^z_{ww}, Y^z_{\bar{w}\bar{w}}\}$ are Green's fcts $\{\partial_{\bar{z}}^{-2}, \partial_{\bar{z}}^{-1}, \partial_z^{-3}\}$.

# Conformal dressings

$$W_{G,j}[\phi_G]\,|p_j\rangle$$

$j$-th hard momentum

Fadeev-Kulish dressings render IR divergent amplitudes finite.

Conformally soft dressing (subleading order in soft expansion, leading in coupling $\kappa$): [Akhoury,Choi'19]

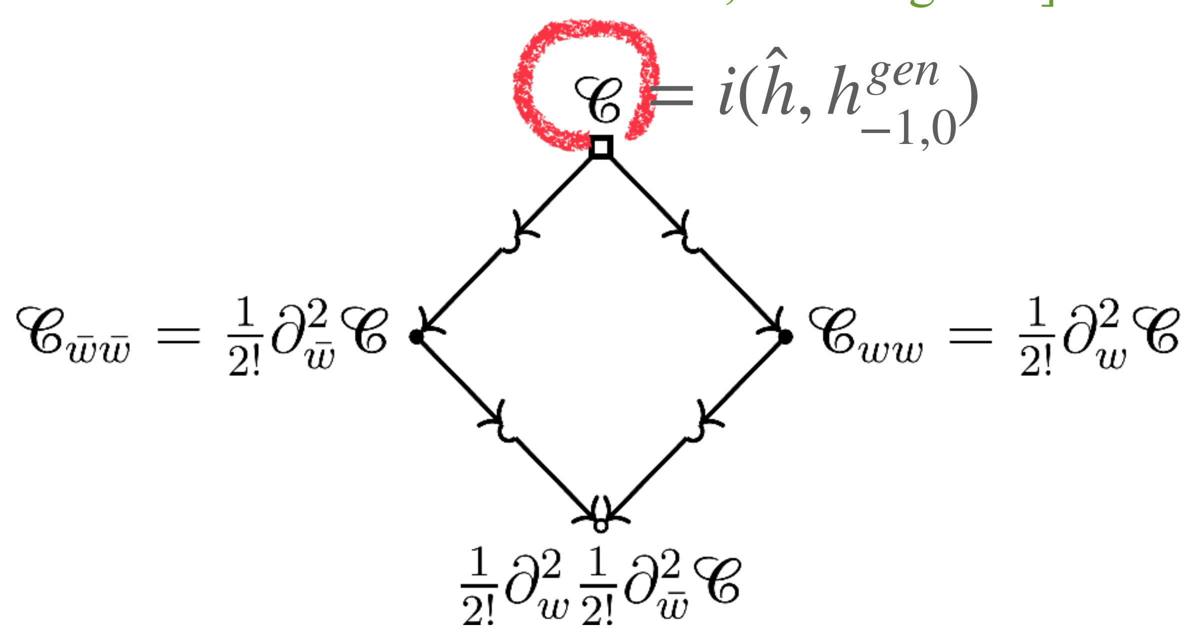
$$W_{G,j} = exp\left\{\frac{\kappa}{2}\int \frac{d^3k\, \phi_G(\overrightarrow{k})}{(2\pi)^3 2k^0}\frac{p_j^\mu}{p_j\cdot k}\left[(p_j^\nu - ik_\rho J_j^{\rho\nu})a^\dagger_{\mu\nu} - (p_j^\nu + ik_\rho J_j^{\rho\nu})a_{\mu\nu}\right]\right\}$$

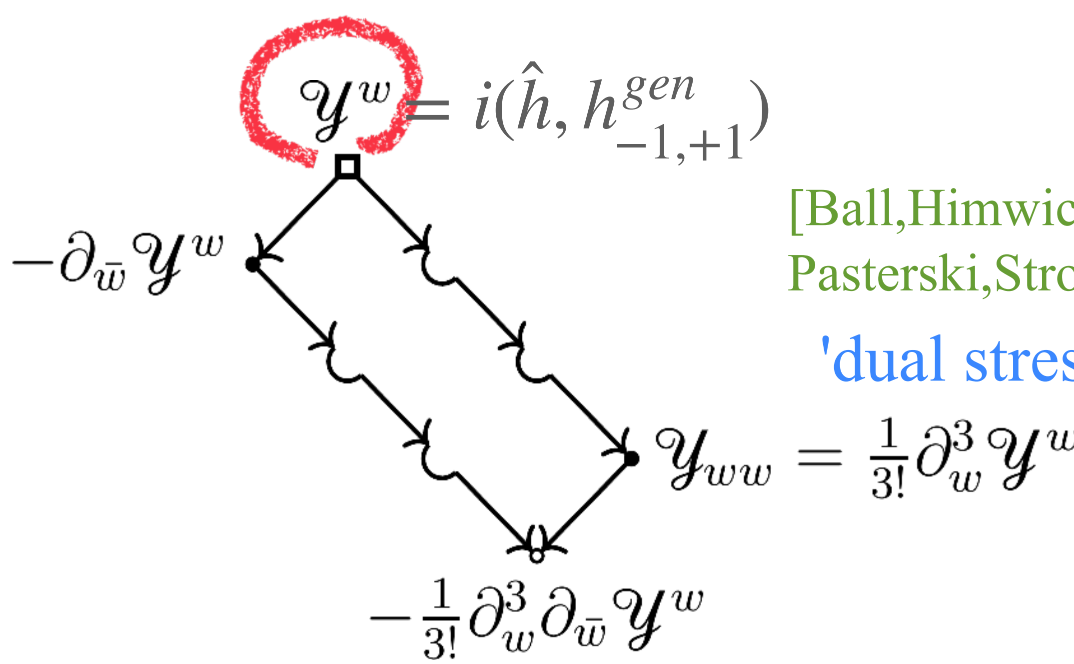
$$= exp\left\{ -i\kappa\left[\omega_j\mathscr{C} + \bar{h}_j\partial_{\bar{w}_j}\mathscr{Y}^{\bar{w}_j} + \mathscr{Y}^{\bar{w}_j}\partial_{\bar{w}_j} + h_j\partial_{w_j}\mathscr{Y}^{w_j} + \mathscr{Y}^{w_j}\partial_{w_j}\right]\right\}$$

$\phi_G(\overrightarrow{k}) = 1$ — conformal dressing

$(\Delta, J) = (-1,0)$ [Arkani-Hamed,Pate, Raclariu,Strominger'20]

$(\Delta, J) = (-1, \pm 1)$ [Pasterski,AP,Trevisani'21]

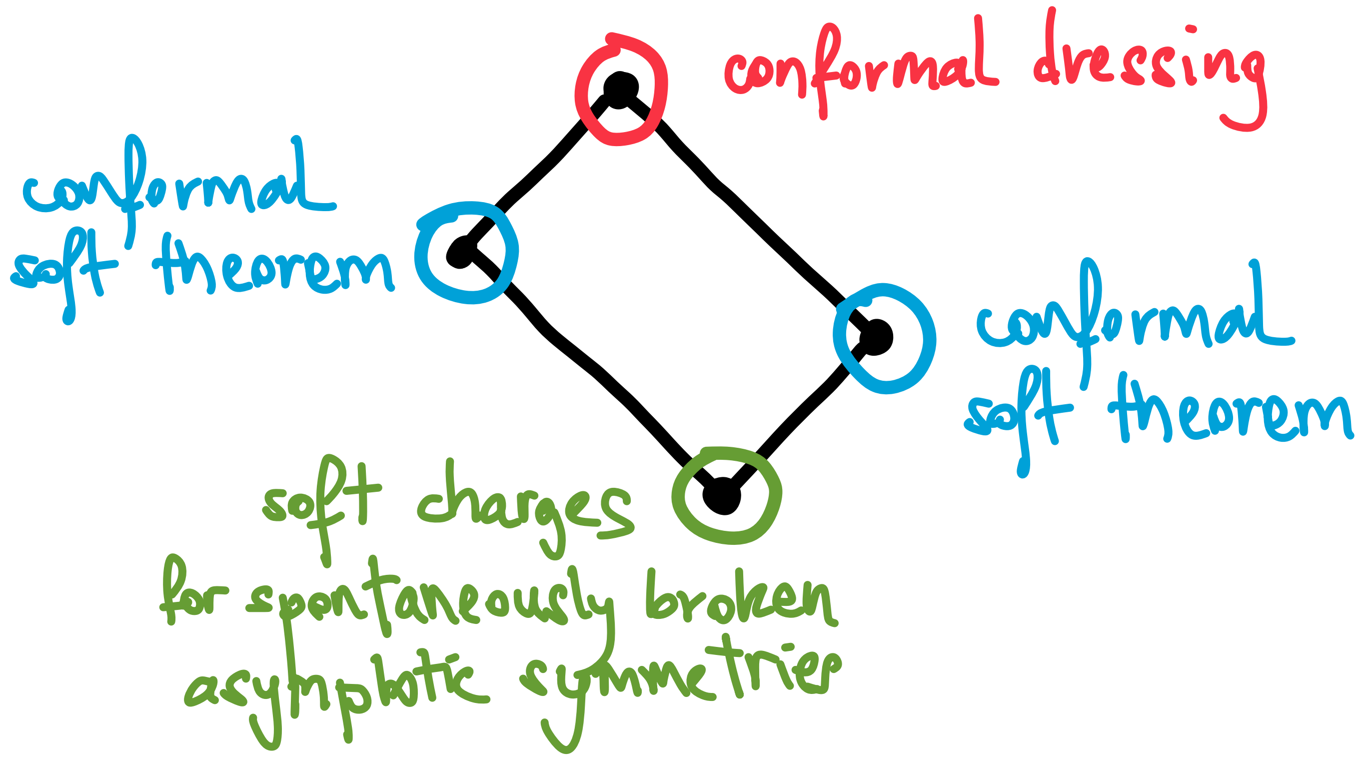



[Ball,Himwich,Narayanan, Pasterski,Strominger'19]

'dual stress tensor'

# Summary




**Celestial diamonds** reveal power of symmetry to organize conformally soft physics. Stack via SUSY into celestial pyramids.

2D operators at the top of the diamond:

- non-trivial levels and central extensions
- effective 2D descriptions of soft sector

Conformal primary **wavefunctions** satisfy classical double copy in the variety of forms: Kerr-Schild, Weyl and operator-valued.

Celestial **amplitudes** satisfy operator-valued double copy.

Generalized conformal primaries capture interesting bulk metrics as **exact sol to Einstein's equations**: ultraboosted black holes, shockwaves,...

$\mapsto$ ***Combining tools from amplitudes, classical GR & CFT:***

***insight into bulk physics from Celestial Holography programme.***

***Exploration of celestial territory has only begun!***

# Thank you!