

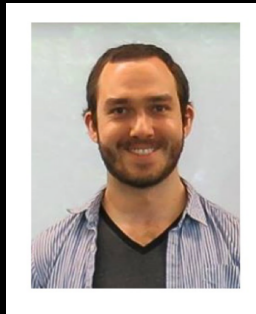
Études in Effective  
Double Copy

AMPLITUDES 2021

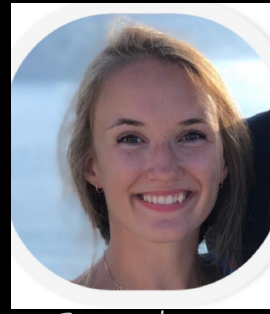
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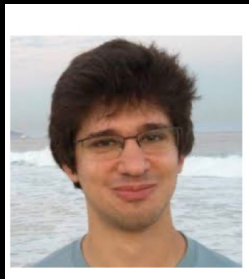
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## Functional Graph Based Double Copy

color-kinematics  $\hat{=}$  functional representation

- Each topology  $\overline{H}$  gets its own function.
- Small # of basis graphs  $\rightarrow$  encode the full amplitude
- at tree level for gluons



color-kinematics makes locality manifest  
 $\hat{=}$  it preserves gauge-invariance

If you want local GI expressions

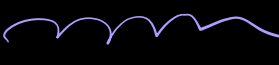
- just need a small # of building blocks

What's going on with Double Copy?

- (Gravity = "YM<sup>2</sup>") + SUSY + related EFT's

- Open string (tree-level at least)

is a field theory double copy between

Yang-Mills 

• a magical scalar  $\xrightarrow{\alpha'}$  Z-theory  
 • all orders in  $\alpha'$

\* Turn string theory KLT into

a field theory KLT w/ HD-YM

$$\underline{OS} = \sum_{\Gamma_3} \frac{\overset{f^{abc} d^{abc} \delta_{ij}}{\check{Z}_g \tilde{\eta}_g^{ij}}}{d_g} \leftarrow k_i, \epsilon_i$$

$Z_g$  obey Jacobi & anti-symmetry

just like normal  $c_g \leftarrow f^{abc}$

$$OS_I = \underset{\nearrow \text{Chen-Peter ordering}}{Z_I} \otimes A^{YM}$$

$$\underline{OS} = \sum_{I \in S_{n-1}} \text{Tr}(T^{I_1} T^{I_2} \dots T^{I_n}) OS_I$$

$$\underline{CS} = OS_I \otimes_{I_5}^{\alpha'} OS_S$$

$$= \underline{A_{YM}} \otimes \left( \underline{Z_I \otimes_{I_5}^{\alpha'} Z_S \otimes A_{YM}} \right)$$

field theory relations

$(n-2)!$  KK

$(n-3)!$  BCJ

$A^{YM+HD}$

$$= \sum_{g \in \Gamma_3} \frac{n_g^{vec} n_g^{vec+HD}}{dg}$$

- BLG consistent w/ color-kinematics

$\hookrightarrow 3D$

$$A^{BLG} = \sum_{\Gamma_4} \frac{c_g^{BLG} n_g^{BLG}}{dg} \quad \begin{array}{c} \text{Y} \\ \text{X} \\ \text{X} \end{array} - \begin{array}{c} \text{X} \\ \text{X} \\ \text{X} \end{array} - \begin{array}{c} \text{X} \\ \text{X} \\ \text{X} \end{array} - \begin{array}{c} \text{X} \\ \text{X} \\ \text{X} \end{array} = 0$$

$$A^{SG} = \sum_{\Gamma_4} \frac{n_g^{BLG} n_g^{BLG}}{dg} = \sum_{\Gamma_3} \frac{n_g^{sym} n_g^{sym}}{dg}$$

$$\text{Diagram 1} = \textcircled{1} \frac{(\text{Diagram 1a})(\text{Diagram 1b})}{(\text{Diagram 1c})} + \text{perms}$$

$$= \textcircled{2} \frac{(\text{Diagram 2a})(\text{Diagram 2b})}{(\text{Diagram 2c})} + \frac{(\text{Diagram 2d})(\text{Diagram 2e})}{(\text{Diagram 2f})}$$

$$+ \text{perms}$$

$$\text{Diagram 3} = \text{Diagram 4} - \text{Diagram 5}$$

Two strategies of the same physical predictions into building blocks of different & distinct algebraic properties

Z-theory makes it clear that  
EFT is a great playground to  
start exploring some of these ideas

- Important

•  $N=4$  SG has a div at 4 loops in 4D  
( $N=8$  SG??)

Finite # of CT?  $\infty$

- Results:

① finite # of build'g blocks span  
HD adjoint-type predictions at each  
multiplicity

② their amplitudes admit multiple stratifications  
along different algebras

③ composition rather than ansatz but  
lets us climb the ladder of HD corrections

$$\begin{array}{c} \text{HD}^{(m-1)} \\ \text{kinematics} \end{array} \xrightarrow{\text{composition}} \text{HD}^m$$





# Study Stratification at 4pts

$$A^{ym} = \frac{C_s \Pi_s}{s} + \frac{C_t \Pi_t}{t} + \frac{C_u \Pi_u}{u}$$

$C_s \equiv \Pi_s$  both are adjoint-type

$$C_s = C_t + C_u$$

$$\Pi_s = \Pi_t + \Pi_u$$

$$C_t = C_s - C_u$$

$$\Rightarrow A^{ym} = \underbrace{\left( \frac{\Pi_s}{s} + \frac{\Pi_t}{t} \right)}_{A(s,t)} C_s + \underbrace{\left( \frac{\Pi_u}{u} - \frac{\Pi_t}{t} \right)}_{A(u,t)} C_u$$

must be gauge inv.

$$\text{If } \Pi_s = \Pi_t + \Pi_u \Rightarrow$$

$$\frac{A(s,t)}{u} = \frac{A(u,t)}{s} = \frac{A(s,u)}{t}$$

All perm inv  $\downarrow$   $s \in A^{ym}(s,t) = \text{B.I.}$   $s \in A^{BAS}(s,t)$   
 $\downarrow = \text{NLSM}$

$$A^{dim} = - \frac{(t \Pi_s + s \Pi_t)(t C_s + s C_t)}{s t u}$$

$$s \in A^{NLSM}(s,t) = S_{\text{Petit}}^{\text{Galileo}}$$

$$= - \frac{\left( \overset{\text{BI}}{\times} \right) \left( \overset{\text{NCSM}}{\times} \right)}{\left( \overset{\text{Spec Gal}}{\times} \right)} \leftarrow \text{perm int var of wty KLT}$$

$$= \frac{(H-1)(H-1)}{(H-1)} + \text{perms}$$



### 4 pt Summary

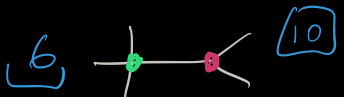
two structures


$\rangle \text{---} \langle$   
 a disjoint type  
 $f^{abc}$   
 $C_S = f^{abe} f^{ecd}$

$\times$   
 perm int  
 eg  $d^{abcd}$

$A^{HD, \text{vec}} \sim \underbrace{C_S \quad \overset{HD}{n_S}^{vec}}_{d_S} \leftarrow \begin{matrix} \swarrow 3 \text{ Built blocks} \\ 8 \text{ diff vector build blocks} \end{matrix}$

5 pt summary  
 UNIQUE closure post composition  
 6.  10  $\leftarrow$  antisym on every vertex  
 Facchi on every edge  $f^{abc} f^{cde} f^{efg}$   
 5.  8  
 relaxed  $\rightarrow f^2 d^3 f^3$   
 supposes

 Hybrid  $d^4 f^3$

 Perm

Scherzo What operators are we spanning?

Double Copy Effective Actions:

A QCD Approach to Space-Time

JJM, LR, SZ

How?

We cheat in the best possible way

- we promote amplitudes directly to operators

$n$ -field ops  $\rightarrow n$  field quanta

$\mathcal{L}(\underline{\text{Amplitude}})$

- Hard code locality w/ a few tricks

- GI manifest multiplicity by multiplicity

$\sqrt{s} R \sim \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \dots$

- Simple b/c  $\mathcal{L}$  is a machine for  
generating predictions via explicit penalties.

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Refs: 1910.12850 2104.08370

[ See also talks by H. Elvang &  
S. Paranjape for related &  
complementary KLT  
approaches... ]