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Integrability and chaos in SYM theories from anomalous-dimension spectra

arXiv:2011.04633 & ongoing work
with Tristan McLoughlin & Raul Pereira

Amplitudes 2021

Anne Spiering

Anomalous dimensions from the dilatation operator

- Goal: understand universal statistical properties of YM theories
  here: one-loop anomalous dimensions in conformal SYM theories

$$[\mathcal{D}, \mathcal{O}(x)] = (\Delta^0_{\mathcal{O}} + \gamma_{\mathcal{O}} + x^\mu \partial_\mu)\mathcal{O}(x)$$

- $\mathfrak{su}(2)$ sector of $SU(N)$ $\mathcal{N} = 4$ SYM theory

$$D\ \psi\ \bar{\psi}\ \phi \longrightarrow X = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), Z = \frac{1}{\sqrt{2}}(\phi_5 + i\phi_6) \longrightarrow \mathrm{tr}(Z^\ell) \quad \mathrm{tr}(XZ^{\ell_1}XZ^{\ell_2}\ldots) \quad \mathrm{tr}(XZ^{\ell_1})\mathrm{tr}(XZ^{\ell_2}XX)$$

$$\mathcal{D} = :\mathrm{Tr}(Z\check{Z} + X\check{X}): - \frac{2\lambda}{N} :\mathrm{Tr}([X,Z][\check{X},\check{Z}]): + \mathcal{O}(\lambda^2)$$

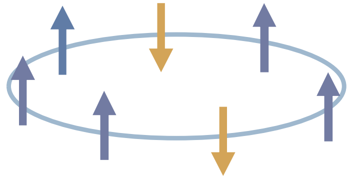
Planar integrability and finite N

$$\mathcal{D}_2 = -\frac{2\lambda}{N} : \mathrm{Tr}([X, Z][\check{X}, \check{Z}]) :$$

planar limit

$\mathcal{D}_2$ maps to integrable XXX Hamiltonian $\rightarrow$ diagonalisation via Bethe ansatz techniques [Minahan, Zarembo '02].

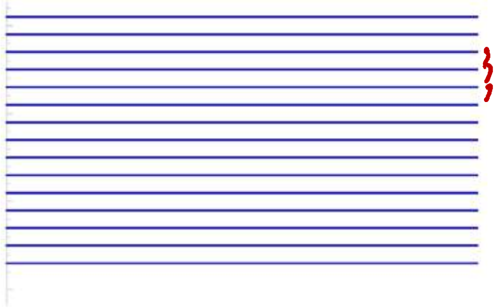
$\mathrm{tr}(ZZZXZZX)$



finite N

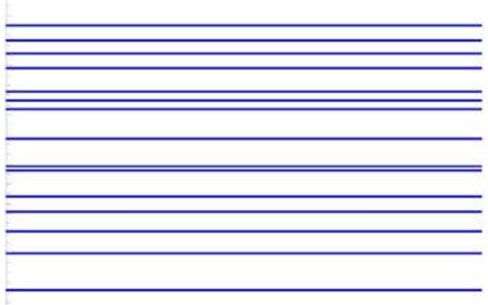
Do not know how to analytically diagonalise in general case. Here: analyse *numerical* spectrum obtained from direct diagonalisation.

Integrability & chaos from the spectrum



uniform spectrum

e.g. harmonic oscillator



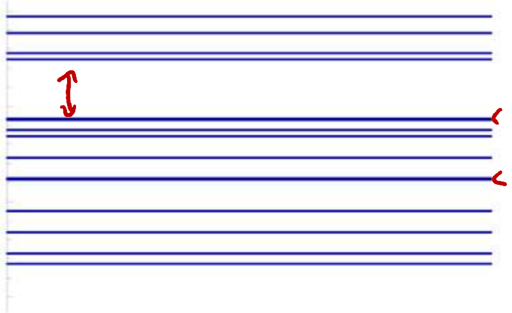
RMT (GOE)

e.g. heavy nuclei

[Wigner '51]

[Bohigas, Giannoni, Schmidt '84]

quantum chaos



uncorrelated spectrum

e.g. XXX spin chain

[Poilblanc, Ziman, Bellissard, Mila, Montabaux '93]

[Berry, Tabor '77]

integrability

Correlations in spectra give insight into the nature of the underlying model, in particular the existence/absence of integrability and chaos.

β-deformed $\mathcal{N} = 4$ SYM theory

For better statistics and an interesting generalisation also study less symmetric β-deformed $\mathcal{N} = 4$ SYM theory:

- special case of an $\mathcal{N} = 1$ four-dimensional CFT [Leigh, Strassler '95]
- planar integrability for $\beta \in \mathbb{R}$ and planar dilatation operator known [Roiban '04] [Berenstein, Cherkis '04] [Beisert, Roiban '05] [Fokken, Sieg, Wilhelm '15]
- gravity dual [Lunin, Maldacena '05]
- planar amplitudes inherited from undeformed amplitudes up to phase [Khoze '06]
  non-planar amplitudes already at tree-level contain multi-trace contributions [Jin, Roiban '12]

- Introduce deformed structure constants: $f_\beta^{abc} = |\tilde{g}_\beta| \mathrm{Tr}([T^a, T^b]_\beta T^c)$

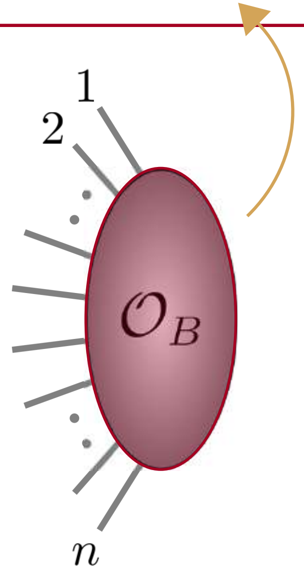
$$\mathcal{M}_\beta^{(0)}(X^a, Z^b, \bar{X}^c, \bar{Z}^d) = -2g_{\mathrm{YM}}^2 \left(f_\beta^{abe} f_\beta^{cde} + \frac{\langle 12 \rangle \langle 34 \rangle}{\langle 24 \rangle \langle 31 \rangle} f^{ace} f^{bde} \right)$$

Anomalous dimensions from on-shell methods

Non-planar dilatation operator from on-shell methods:

$$\mathcal{D}^{(1)}_{AB} \langle 1, ..., n|\mathcal{O}_B|0\rangle^{(0)} = -\frac{1}{\pi} \langle 1, ..., n|\mathcal{M} \otimes \mathcal{O}_A|0\rangle^{(0)} + \gamma^{(1)}_{IR} \langle 1, ..., n|\mathcal{O}_A|0\rangle^{(0)}$$

[Caron-Huot, Wilhelm '16]



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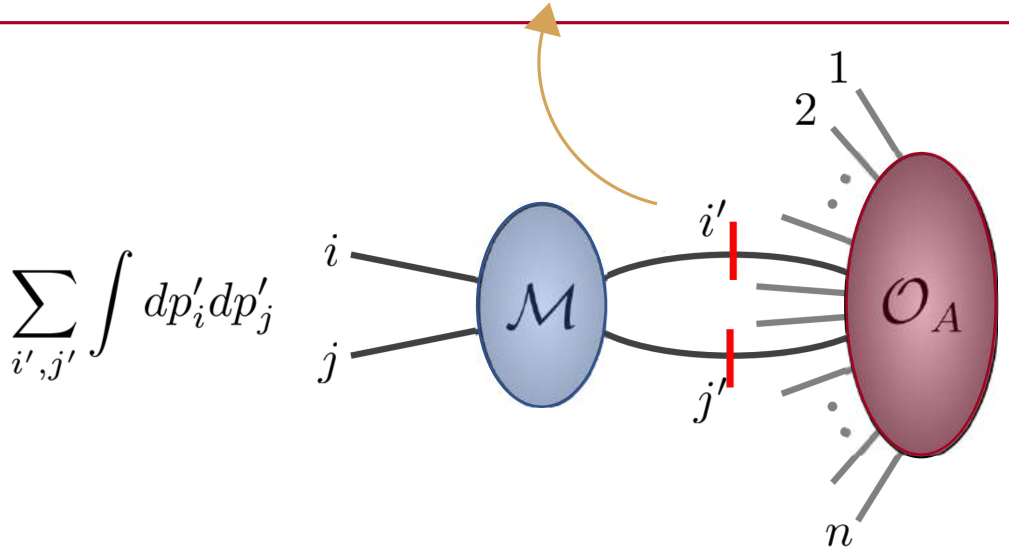

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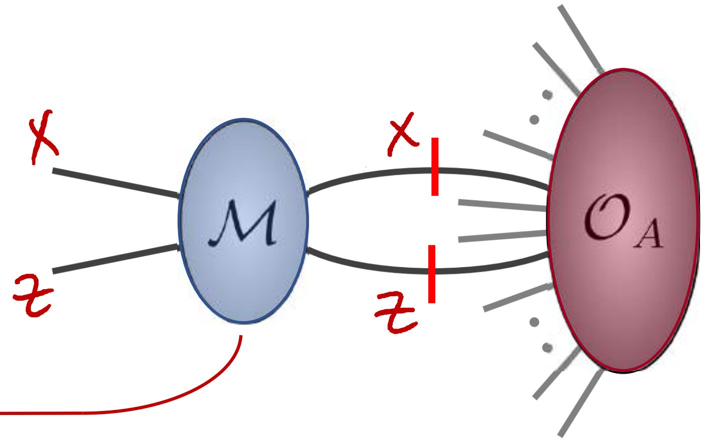
[Caron-Huot, Wilhelm '16]

$$\sum_{i',j'} \int dp'_i dp'_j$$



$$\mathcal{M}^{(0)}(X^a, Z^b, \bar{X}^c, \bar{Z}^d) = -2g^2_{\mathrm{YM}}\left(f^{abe}f^{cde} + \frac{\langle 12\rangle\,\langle 34\rangle}{\langle 24\rangle\,\langle 31\rangle} f^{ace}f^{bde}\right)$$

$$\mathcal{D}_2 = -\frac{2\lambda}{N} : \mathrm{Tr}([X,Z][\check{X},\check{Z}]) :$$

Anomalous dimensions from on-shell methods

Non-planar dilatation operator from on-shell methods:

$$\mathcal{D}^{(1)}_{AB}\langle 1,\dots,n|\mathcal{O}_B|0\rangle^{(0)} = -\frac{1}{\pi}\langle 1,\dots,n|\mathcal{M}\otimes\mathcal{O}_A|0\rangle^{(0)} + \gamma^{(1)}_{IR}\langle 1,\dots,n|\mathcal{O}_A|0\rangle^{(0)}$$

[Caron-Huot, Wilhelm ‘16]

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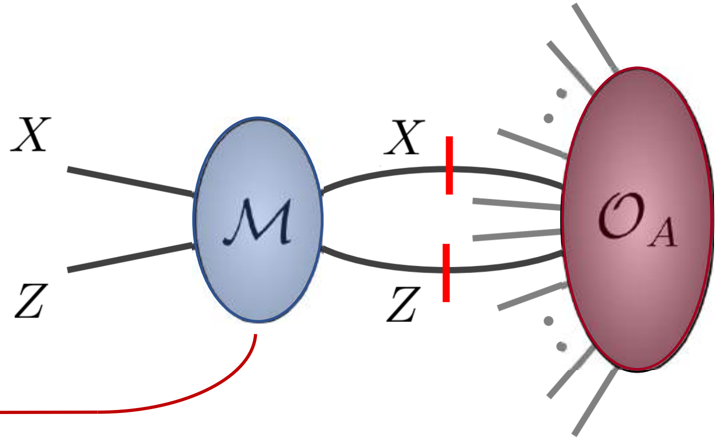
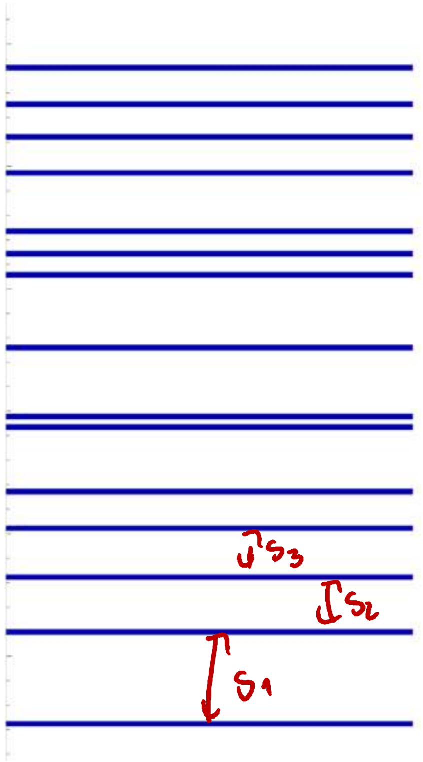


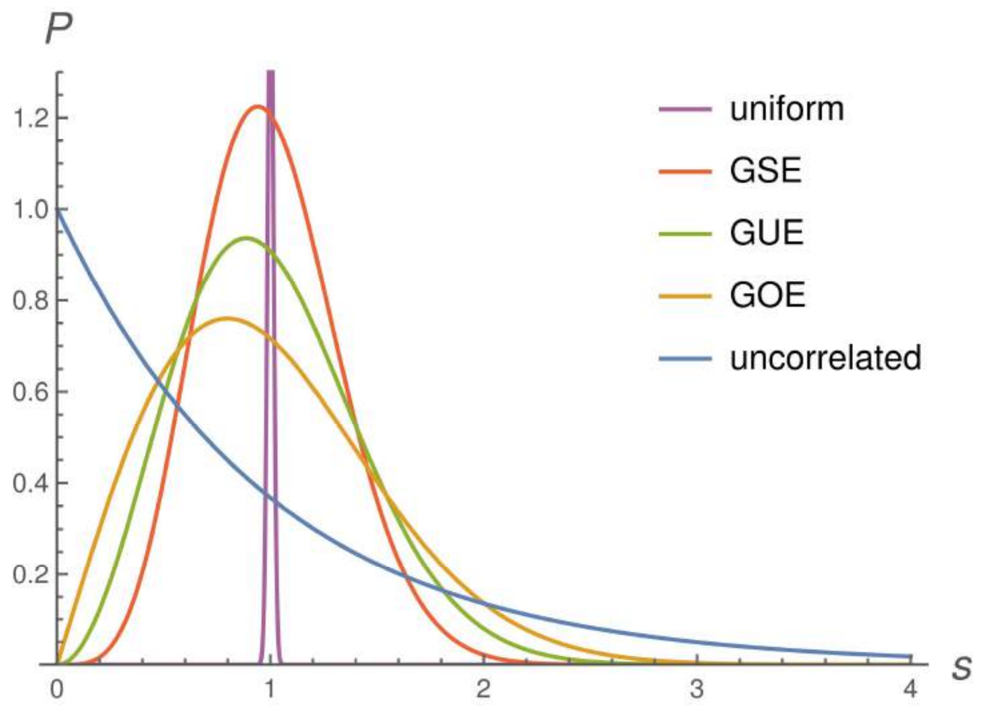
$$\mathcal{M}^{(0)}_\beta(X^a, Z^b, \bar{X}^c, \bar{Z}^d) = -2g^2_{\mathrm{YM}}\left(f^{abe}_\beta f^{cde}_\beta + \frac{\langle 12\rangle\langle 34\rangle}{\langle 24\rangle\langle 31\rangle} f^{ace} f^{bde}\right)$$

$$\mathcal{D}_2 = -\frac{2\lambda_\beta}{N}\left(:\mathrm{Tr}([X,Z]_\beta[\check{X},\check{Z}]_\beta): -\frac{1}{N}:\mathrm{Tr}[X,Z]_\beta\mathrm{Tr}[\check{X},\check{Z}]_\beta:\right)$$

Short-range correlations from level spacings

- Measure for level correlations: nearest-neighbour spacing distribution

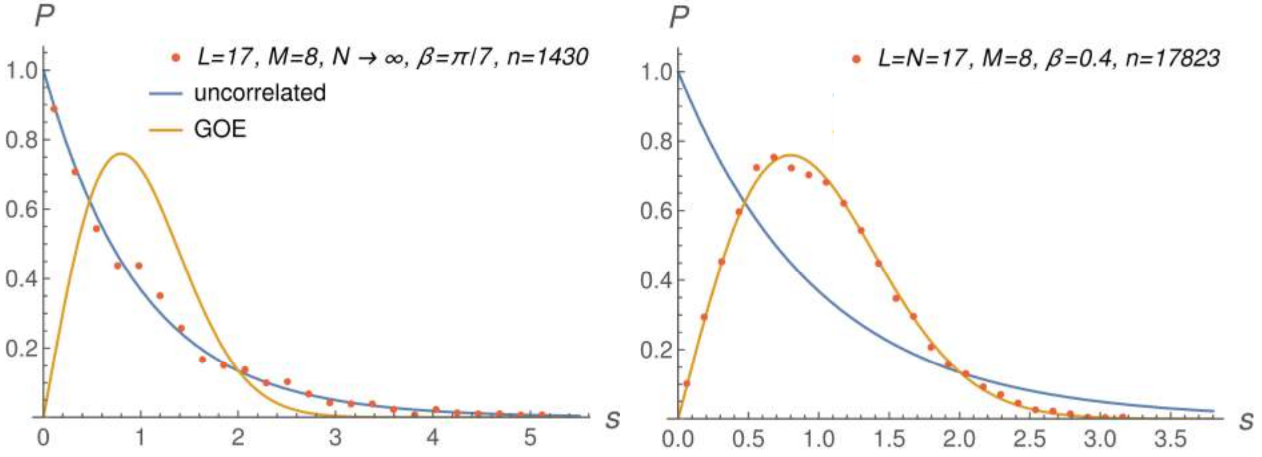




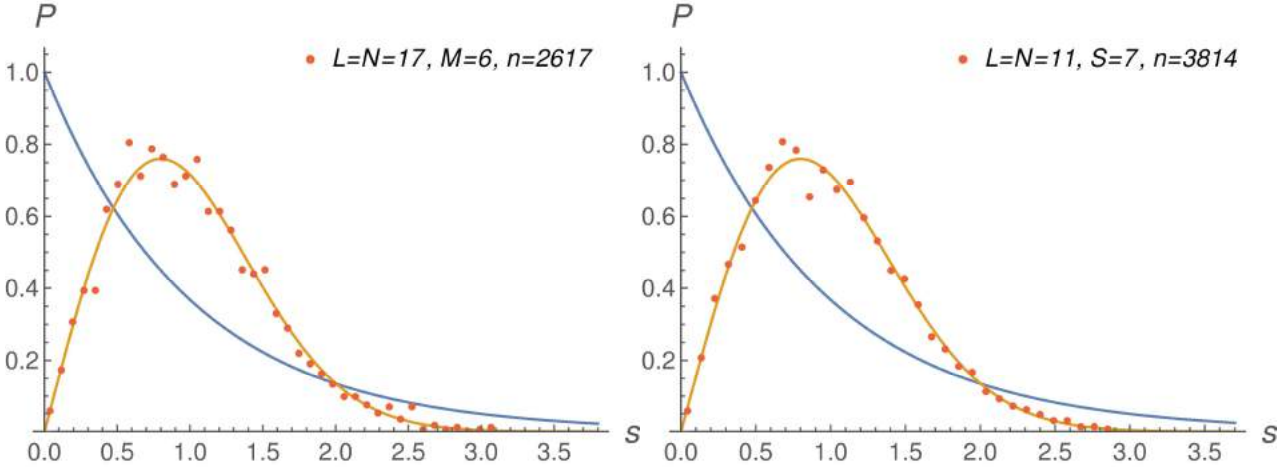
Gaussian
{ symplectic
unitary
orthogonal }
ensemble

Integrability and chaos in SYM theories

- Results for β-deformed $\mathcal{N} = 4$ SYM theory



- Results for undeformed $\mathcal{N} = 4$ SYM theory



planar spectrum
~ Poisson distribution
↓
integrability

non-planar spectrum
~ RMT (GOE)
↓
quantum chaos

Conclusions & Outlook

- Short-range correlations of planar one-loop anomalous dimension spectra compatible with planar integrability
- At finite N spectrum well described by RMT
  also from long-range correlations and statistics of eigenvectors

> Conjecture: $\mathcal{N} = 4$ SYM theory and its β-deformed version are chaotic at finite N and described by GOE RMT

- Connect with properties of the dual gravity theory?
  e.g. late-time fluctuations of large AdS black holes in SYK model described by RMT
  [Cotler et al ‘16]
- Chaos directly from the scattering amplitudes?
  [Rosenhaus ‘20] [Gross, Rosenhaus ‘21]