Radiative Contributions to Gravitational Scattering

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Mao Zeng's talk has beautifully presented the recent progress in the dynamics of gravitationally interacting binary systems using modern scattering amplitudes techniques, including: generalized unitarity, the double copy, eikonal resummation and advanced multiloop integration methods (IBP, DE, reverse unitarity,…). (see upcoming Heissenberg's and Solon's talks)

Main approximation methods used for the 2-body pb:

Post-Newtonian (PN) approximation: expansion in 1/c, i.e. v/c

Post-Minkowskian (PM) approximation: expansion in G, i.e. GM/(c^2 b)

bound systems: GM/(c^2 b) ~ (v/c)^2

at low orders radiation-reaction effects involve an odd power of 1/c:

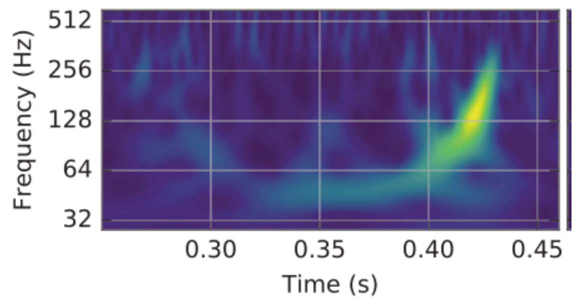
$$\text{rad reac} \sim \frac{1}{c^5} + \frac{1}{c^7} + \frac{1}{c^8} + \frac{1}{c^9} + \cdots$$

Tail effects due to ultrasoft gravitons causing long-range interactions (both conservative and dissipative): 4PN (1/c^8) and 4PM (G^4=3-loop)

LIGO-Virgo data analysis

$$\left[\frac{\delta L}{L}\right]_{\text{noise}} \sim 10^{-18} \gg h_{\text{GW}} \lesssim 10^{-21}$$

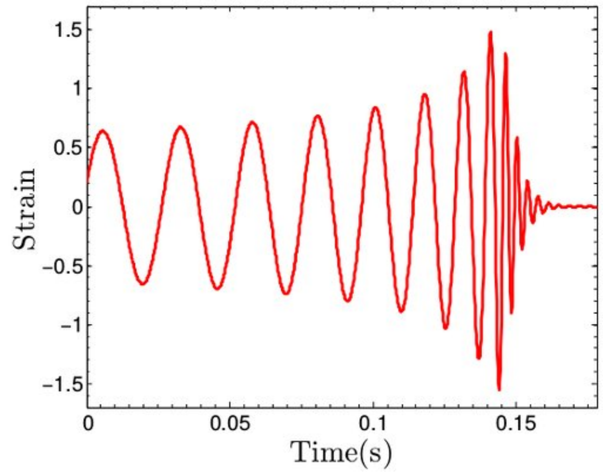
Various levels of search and analysis of such weak signals:



Online trigger searches:

CoherentWaveBurst Time-frequency
(Wilson, Meyer, Daubechies-Jaffard-Journe, Klimenko et al.)
Omicron-LALInference sine-Gaussians
Gabor-type wavelet analysis **(Gabor,…,Lynch et al.)**

Matched-filter:
PyCBC (f-domain), gstLAL (t-domain)



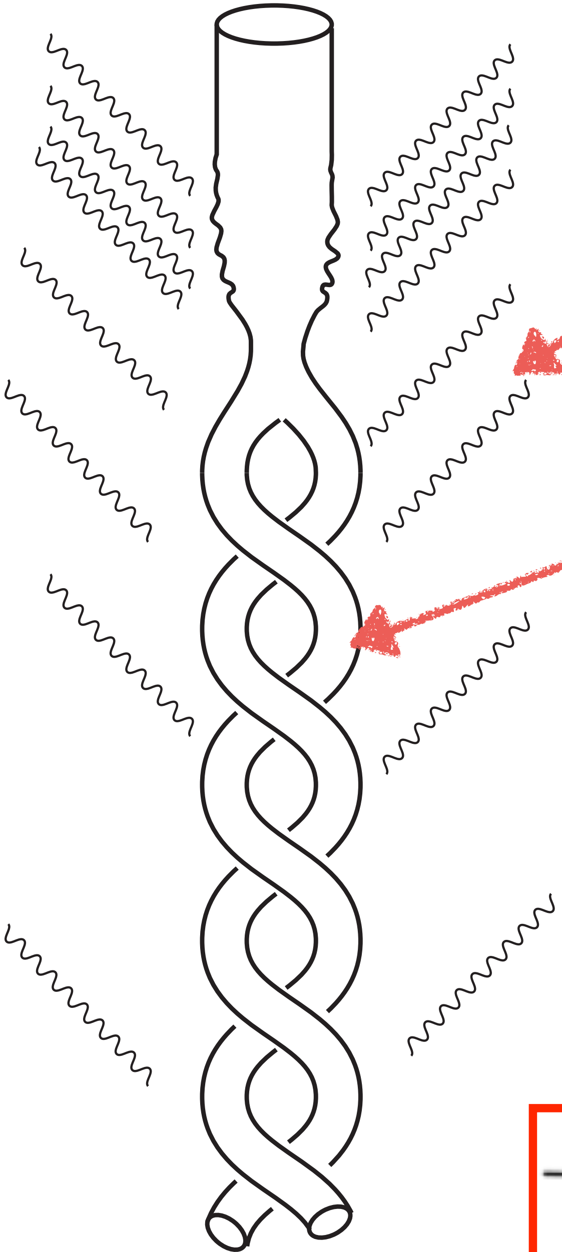
Offline data analysis:

Generic transient searches
Binary coalescence searches

Here: focus on matched-filter definition
(crucial for high SNR, significance assessment, and parameter estimation)

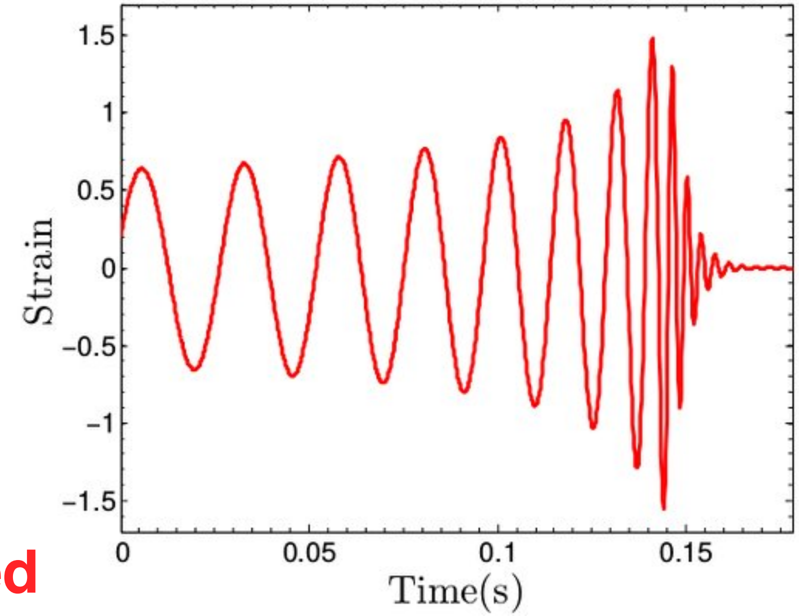
Matched Filtering

$$\langle output | h_{\text{template}} \rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\text{template}}(f)$$



GW emission

from **radiation-reacted** binary dynamics



$$ds^2 = g_{\mu\nu}(x^\lambda)\, dx^\mu dx^\nu$$

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

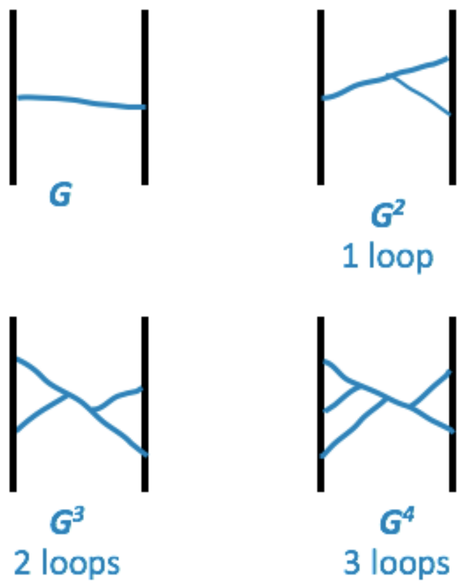
$$R_{\mu\nu} = 0$$

$$-g^{\mu\nu} g_{\alpha\beta,\mu\nu} + g^{\mu\nu} g^{\rho\sigma} \big(g_{\alpha\mu,\rho} g_{\beta\nu,\sigma} - g_{\alpha\mu,\rho} g_{\beta\sigma,\nu} + g_{\alpha\mu,\rho} g_{\nu\sigma,\beta} + g_{\beta\mu,\rho} g_{\nu\sigma,\alpha} - \frac{1}{2} g_{\mu\rho,\alpha} g_{\nu\sigma,\beta} \big) = 0$$

Effective One-Body (EOB) approach: H + Rad-Reac Force

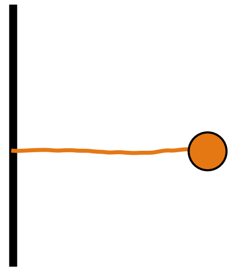
Historically rooted in QM: Brezin-Itzykson-ZinnJustin'70
eikonal scattering amplitude in QED

Real 2-body system
(in the c.o.m. frame)



An effective particle of mass mu in some effective metric

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$



$$g^{\mu\nu}_{\text{eff}}(X)$$

mass-shell constraint

$$0 = g^{\mu\nu}_{\text{eff}}(X) P_\mu P_\nu + \mu^2 + Q(X, P)$$

Level correspondence
in the semi-classical limit:
Bohr-Sommerfeld ->
identification of
quantized action variables

$$J = \ell\hbar = \frac{1}{2\pi}\oint p_\varphi d\varphi$$
$$N = n\hbar = I_r + J$$
$$I_r = \frac{1}{2\pi}\oint p_r dr$$

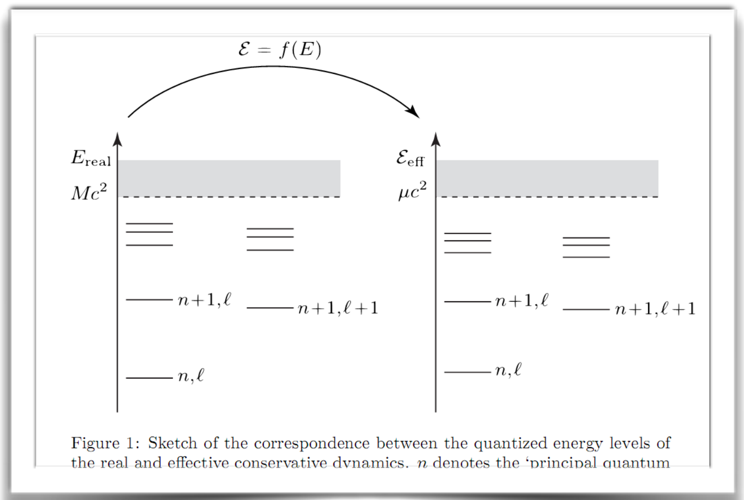


Figure 1: Sketch of the correspondence between the quantized energy levels of the real and effective conservative dynamics. n denotes the 'principal quantum

Crucial energy map

$$\mathcal{E}_{\text{eff}} = \frac{(\mathcal{E}_{\text{real}})^2 - m_1^2 - m_2^2}{2(m_1 + m_2)}.$$

EOB

$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}},$$

$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r},$$

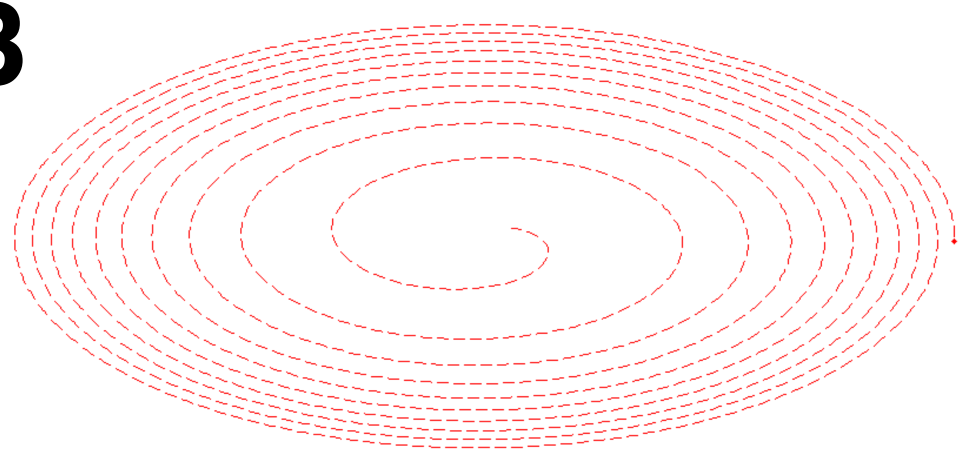
$$\Omega \equiv \frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi},$$

$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi .$$

Hamiltonian

Rad Reac Force

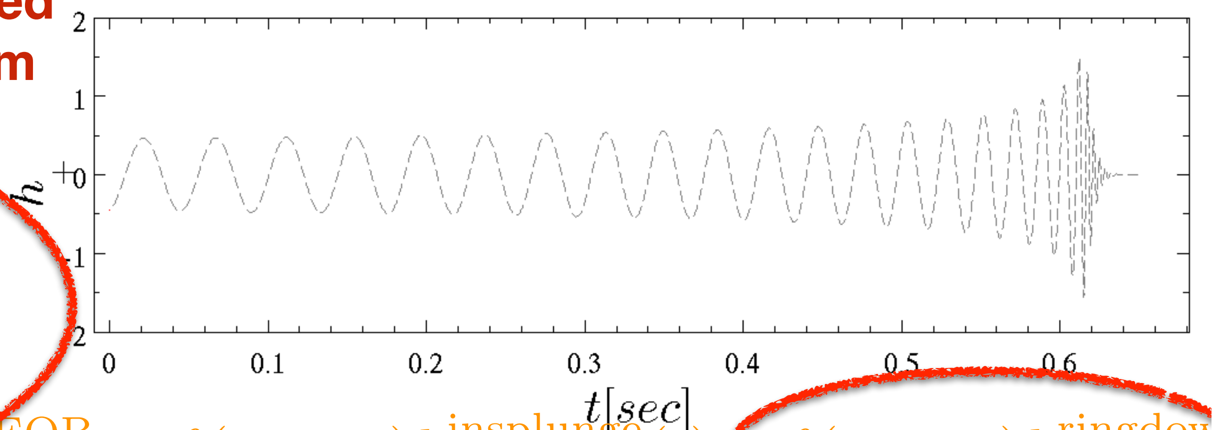
Resummed waveform



$$h_{\ell m} \equiv h_{\ell m}^{(N,\epsilon)} \hat{h}_{\ell m}^{(\epsilon)} \hat{h}_{\ell m}^{\rm NQC}$$

$$\hat{h}_{\ell m}^{(\epsilon)} = \hat{S}_{\rm eff}^{(\epsilon)} T_{\ell m} e^{i\delta_{\ell m}} \rho_{\ell m}^{\ell}$$

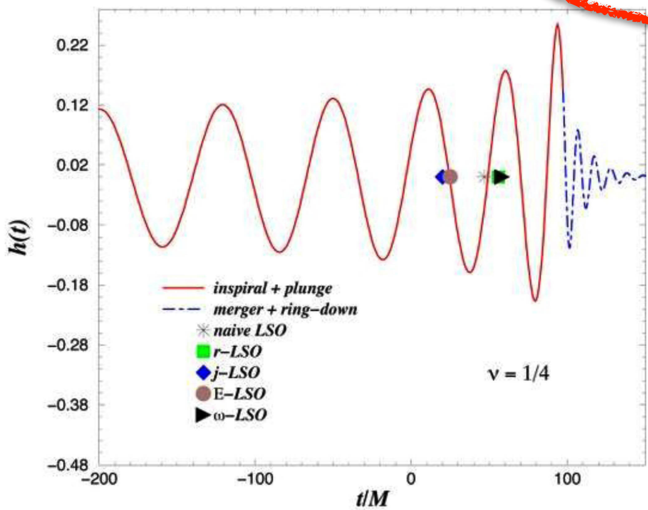
$$\mathcal{F}_\varphi \equiv -\frac{1}{8\pi\Omega} \sum_{\ell=2}^{\ell_{\max}} \sum_{m=1}^{\ell} (m\Omega)^2 |R h_{\ell m}^{(\epsilon)}|^2$$



$$h_{\ell m}^{\rm EOB} = \theta(t_m - t) h_{\ell m}^{\rm insplunge}(t) + \theta(t - t_m) h_{\ell m}^{\rm ringdown}$$

$$h_{\ell m}^{\rm ringdown}(t) = \sum_N C_N^+ e^{-\sigma_N^+ (t - t_m)}$$

$$T_{\ell m} = \frac{\Gamma(\ell + 1 - 2i\hat{\hat{k}})}{\Gamma(\ell + 1)} e^{\pi \hat{\hat{k}}} e^{2i\hat{\hat{k}} \log(2kr_0)},$$



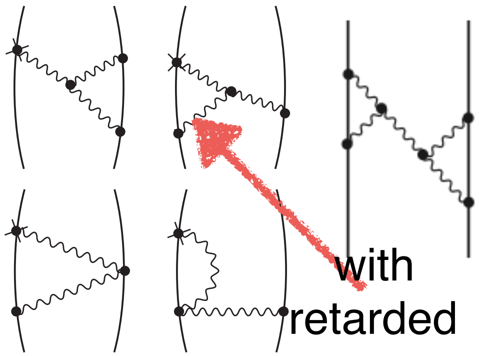
First complete waveforms for BBH coalescences: analytical EOB

(Buonanno-Damour'00, Buonanno-Chen-Damour'05)

State of the art for PN dynamics

- 1PN (including v^2/c^2) [Lorentz-Droste '17], Einstein-Infeld-Hoffmann '38
- 2PN (inc. v^4/c^4) Ohta-Okamura-Kimura-Hiida '74, Damour-Deruelle '81
  Damour '82, Schäfer '85, Kopeikin '85
- 2.5 PN (inc. v^5/c^5) Damour-Deruelle '81, Damour '82, Schäfer '85,
  Kopeikin '85
- 3 PN (inc. v^6/c^6) Jaranowski-Schäfer '98, Blanchet-Faye '00,
  Damour-Jaranowski-Schäfer '01, Itoh-Futamase '03,
  Blanchet-Damour-Esposito-Farèse' 04, Foffa-Sturani '11
- 3.5 PN (inc. v^7/c^7) Iyer-Will '93, Jaranowski-Schäfer '97, Pati-Will '02,
  Königsdörffer-Faye-Schäfer '03, Nissanke-Blanchet '05, Itoh '09
- **4PN** (inc. v^8/c^8) Jaranowski-Schäfer '13, Foffa-Sturani '13,'16
  Bini-Damour '13, Damour-Jaranowski-Schäfer '14, Marchand+'18, Foffa+'19

First complete 2PN and 2.5PN dynamics obtained by using 2PM (G^2) EOM of Bel et al.'81

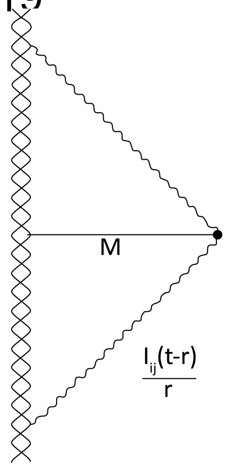


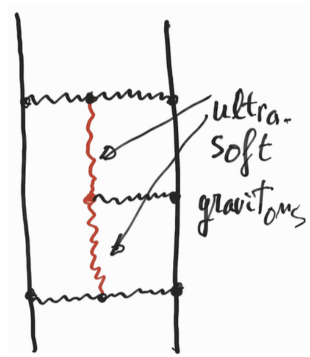
with retarded propagator

New feature at G^4/c^8 (4PN and 4PM) : **non-locality in time** (linked to IR divergences of formal PN-expansion) (Blanchet,TD '88)

- 5PN (inc. v^{10}/c^{10} and **G^6**) Bini-Damour-Geralico'19: complete **modulo two**
- **numerical** parameters; potential-graviton contrib.: Bluemlein et al'21

- 6PN (inc. v^{12}/c^{12} and **G^7**) Bini-Damour-Geralico'20: complete **modulo four**
- additional parameters





Inclusion of **spin-dependent effects**: Barker-O' Connell'75, Faye-Blanchet-Buonanno'06, Damour-Jaranowski-Schaefer'08, Porto-Rothstein '06, Levi '10, Steinhoff-Hergt-Schaefer '10, Steinhoff'11, Levi-Steinhoff'15-18, Bini-TD, Vines , Guevara-Ochirov-Vines,….

State of the art for PN GW flux from (bound) binary system

- lowest order : Einstein 1918 Peters-Mathews 63
- 1 + (v^2/c^2) : Wagoner-Will 76
- … + (v^3/c^3) : Blanchet-Damour 92, Wiseman 93
- … + (v^4/c^4) : Blanchet-Damour-Iyer Will-Wiseman 95
- … + (v^5/c^5) : Blanchet 96
- … + (v^6/c^6) : Blanchet-Damour-Esposito-Farèse-Iyer 2004
- … + (v^7/c^7) : Blanchet

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

Current bottleneck

4PN quadrupole (Marchand+'20)

$$x = \left(\frac{v}{c}\right)^2 = \left(\frac{G(m_1+m_2)\Omega}{c^3}\right)^{\frac{2}{3}} = \left(\frac{\pi G(m_1+m_2)f}{c^3}\right)^{\frac{2}{3}}$$

$$\begin{aligned}\mathcal{F} = \frac{32c^5}{5G}\nu^2 x^5 \Big\{ 1 &+ \left(-\frac{1247}{336} - \frac{35}{12}\nu\right) x + 4\pi x^{3/2} \\
&+ \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right) x^2 + \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2} \\
&+ \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_\mathrm{E} - \frac{856}{105}\ln(16\,x) \right. \\
&\qquad \left. + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right] x^3 \\
&+ \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Big\}.\end{aligned}$$

3.5PN

State of the art for PM dynamics and GW losses

PM dynamics

- 1PM O(G) Bertotti'56, Portilla'79

- 2PM O(G^2) (1-loop) Westpfahl-Goller '79 Bel-Damour-Deruelle-Ibanez-Martin'81,Westpfahl'85

- **3 PM O(G^3) (2-loop)** massless: Amati-Ciafaloni-Veneziano'90, massive;conservative:
- Bern-Cheung-Roiban-ShenSolonZeng'19, Kalin-Liu-Porto'20, Bjerrum-Bohr+'21
- massive-radiation-reacted: DiVecchia-Heissenberg-Russo-Veneziano'20, TD'20,
- DiVecchia+'21, Hermann-Parra-Martinez-Ruf-Zeng'21,Bjerrum-Bohr+'21

- **4 PM** O(G^4) (3-loop) conservative+ potential-graviton-only: Bern +'21; Dlapa+21

GW losses

LO 4-momentum loss O(G^3): Kovacs-Thorne'77; **Hermann-Parra-Martinez-Ruf-Zeng'21, Mougiakakos'21,**

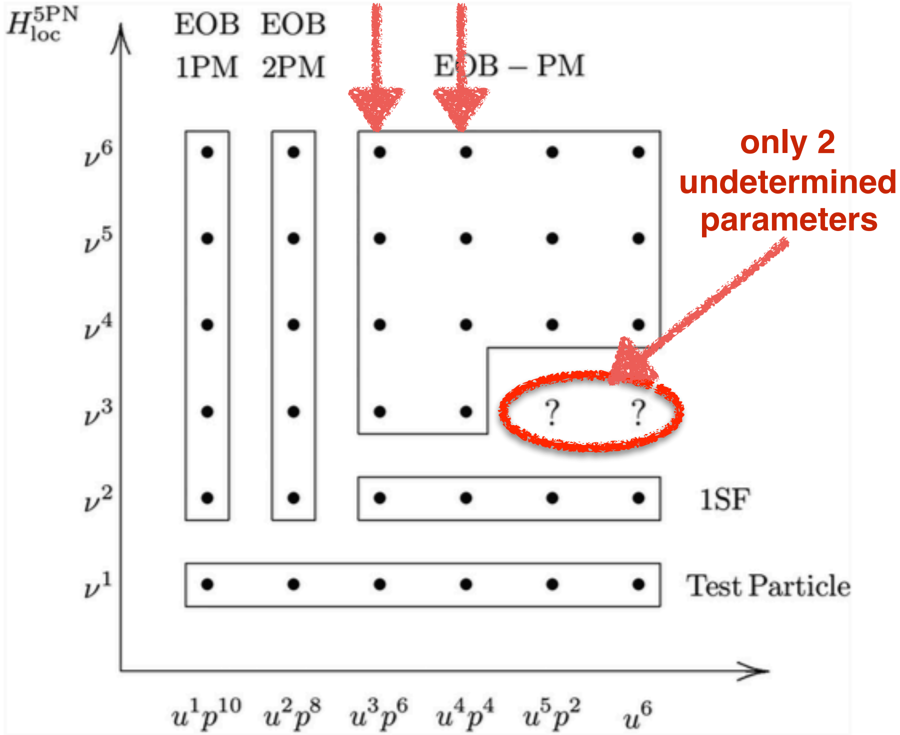
LO angular-momentum loss O(G^2): TD'20

Novel Approach to Binary Dynamics: Application to the Fifth Post-Newtonian Level

Donato Bini[1,2] Thibault Damour,[3] and Andrea Geralico[1]

Tutti Frutti: combine several efficient, complementary tools:

PN
PM
MPM
EOB
SF
Delaunay averaging



Step 1: Use MPM + EFT to separate off the nonlocal part

$$S^{\leq n\mathrm{PN}}_{\mathrm{tot}}[x_1(s_1), x_2(s_2)] = S^{\leq n\mathrm{PN}}_{\mathrm{loc}}[x_1(s_1), x_2(s_2)] + S^{\leq n\mathrm{PN}}_{\mathrm{nonloc}}[x_1(s_1), x_2(s_2)]$$

Step 2: Compute z_1^SF to e^6

Step 3: Use 1st law to transform z_1^SF into a pr^6 EOB Hamilt.

$$\delta z_1^{e^6} = \nu\left[\frac{1}{4}u_p^3 + \left(-\frac{53}{12} - \frac{41}{128}\pi^2\right)u_p^4 + C_5 u_p^5 + C_6 u_p^6 + o(u_p^6)\right] + O(\nu^2),$$

Step 4: Determine H^loc_1SF by subtracting the averaged H^nonloc

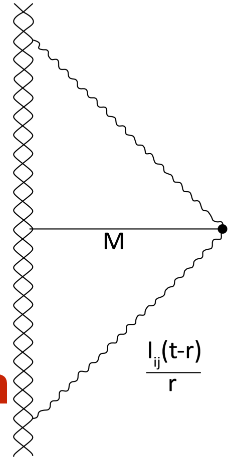
Step 5: Use EOB-PM theory to determine most of the nonlinear in nu dependence

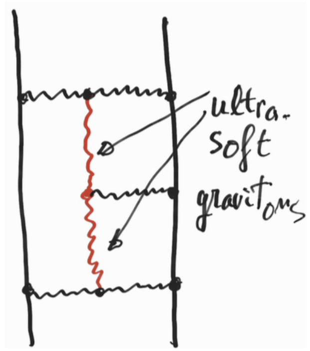
$$Q = q_4(u;\nu)p_r^4 + q_6(u;\nu)p_r^6 + q_8(u;\nu)p_r^8 + \cdots$$

$$q_6(u;\nu) = \nu q_{62}^{\nu^1} u^2 + \nu q_{63}^{\nu^1} u^3 + O(u^{7/2}) + O(\nu^2)$$

Inclusion of conservative nonlocal radiation-graviton effects

Use **Delaunay-averaging expansions in e or p_r**
(TD-Jaranowski-Schaefer'15,Bini-TD-Geralico…)





Starting at G^4/c^8, dynamics contains a nonlocal action

$$S^{4+5\mathrm{PN}}_{\mathrm{nonloc}}[x_1(s_1),x_2(s_2)] = \frac{G^2\mathcal{M}}{c^3}\int dt\,\mathrm{PF}_{2r^h_{12}(t)/c} \times \int \frac{dt'}{|t-t'|}\mathcal{F}^{\mathrm{split}}_{1\mathrm{PN}}(t,t').$$

$$\mathcal{F}^{\mathrm{split}}_{1\mathrm{PN}}(t,t') = \frac{G}{c^5}\left(\frac{1}{5}I^{(3)}_{ab}(t)I^{(3)}_{ab}(t') + \frac{1}{189c^2}I^{(4)}_{abc}(t)I^{(4)}_{abc}(t') + \frac{16}{45c^2}J^{(3)}_{ab}(t)J^{(3)}_{ab}(t')\right). \quad (\ldots$$

For elliptic motions, the 4PN nonlocal EOB Hamiltonian reads

$$A(u) = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41\pi^2}{32}\right)\nu u^4 + \left(\left(\frac{2275\pi^2}{512} - \frac{4237}{60} + \frac{128}{5}\gamma_E + \frac{256}{5}\ln 2\right)\nu + \left(\frac{41\pi^2}{32} - \frac{221}{6}\right)\nu^2 + \frac{64}{5}\nu\ln u\right)u^5, \quad (8.1a)$$

$$\bar{D}(u) = 1 + 6\nu u^2 + (52\nu - 6\nu^2)u^3 + \left(\left(-\frac{533}{45} - \frac{23761\pi^2}{1536} + \frac{1184}{15}\gamma_E - \frac{6496}{15}\ln 2 + \frac{2916}{5}\ln 3\right)\nu + \left(\frac{123\pi^2}{16} - 260\right)\nu^2 + \frac{592}{15}\nu\ln u\right)u^4, \quad (8.1b)$$

$$\hat{Q}(\mathbf{r}',\mathbf{p}') = \left(2(4-3\nu)\nu u^2 + \left(\left(-\frac{5308}{15} + \frac{496256}{45}\ln 2 - \frac{33048}{5}\ln 3\right)\nu - 83\nu^2 + 10\nu^3\right)u^3\right)(\mathbf{n}'\cdot\mathbf{p}')^4 + \left(\left(-\frac{827}{3} - \frac{2358912}{25}\ln 2 + \frac{1399437}{50}\ln 3 + \frac{390625}{18}\ln 5\right)\nu - \frac{27}{5}\nu^2 + 6\nu^3\right)u^2(\mathbf{n}'\cdot\mathbf{p}')^6 + \mathcal{O}[\nu u(\mathbf{n}'\cdot\mathbf{p}')^8].$$

SIXTH POST-NEWTONIAN LOCAL-IN-TIME DYNAMIC

(Bini-TD-Geralico'20)

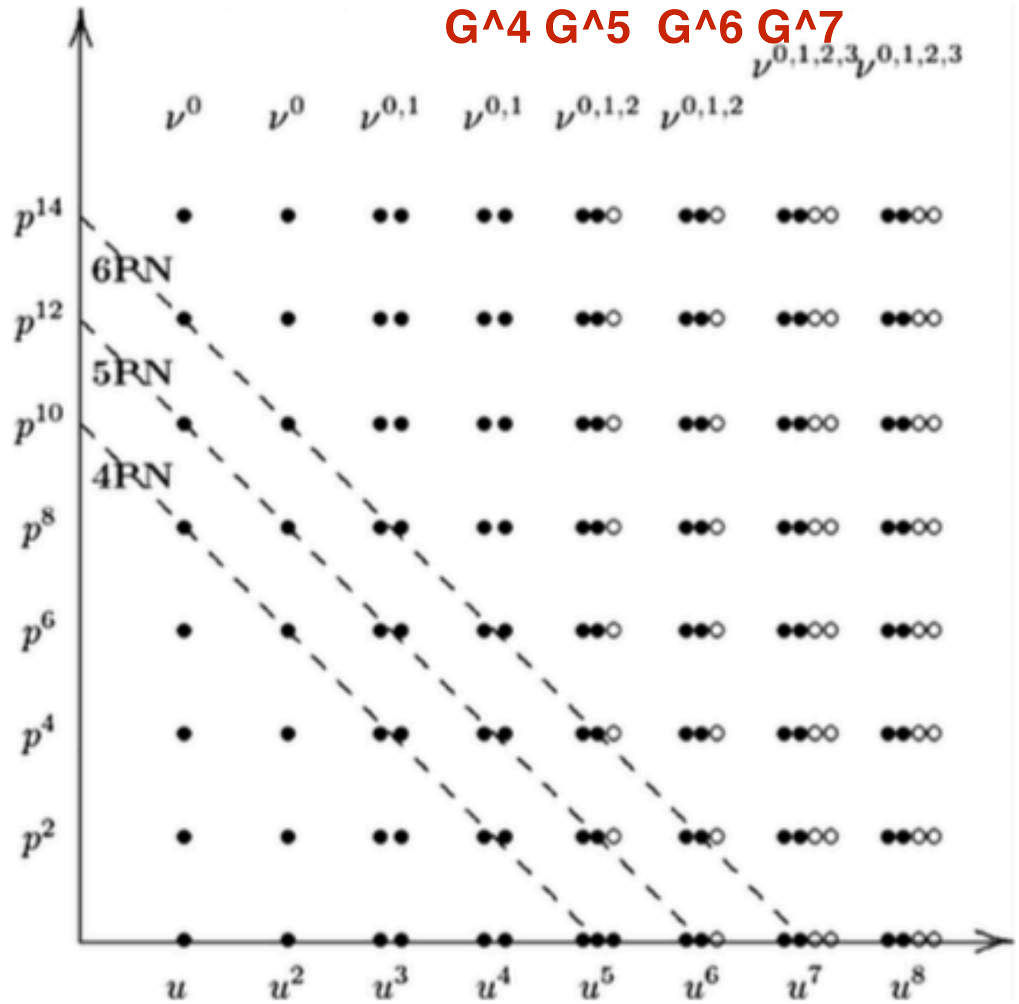


FIG. 1. Schematic representation of the irreducible information contained, at each post-Minkowskian level (keyed by a power of $u = GM/r$), in the local dynamics. Each vertical column of dots describes the post-Newtonian expansion (keyed by powers of p^2) of an energy-dependent function parametrizing the scattering angle. The various columns at a given post-Minkowskian level correspond to increasing powers of the symmetric mass-ratio ν. See text for details.

6PN dynamics complete at 3PM and 4PM

Conservative Radiative Contributions to the Classical Scattering at 5PN and G^4+ G^5

Recent amplitude computation of **potential-graviton** contribution to **conservative** 4PM (G^4) dynamics (Bern et al ’21)

Need to add several types of radiation-related contributions:
radiation-graviton conservative nonlocal contribution
radiation-reaction contributions

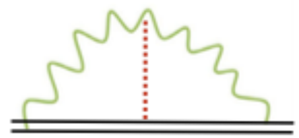
(relevant works: Foffa-Sturani’19, Bluemlein et al ’21, Hermann-Parra-Martinez-Ruf-Zeng’21,Bini-TD-Geralico’21)

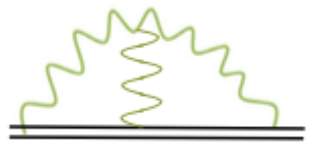
only missing 5PN parameters with rational terms R6,R5:

$$a_6^{\nu^2} = r_{a_6} + \frac{25911}{256}\pi^2 ,$$
$$\bar{d}_5^{\nu^2} = r_{\bar{d}_5} + \frac{306545}{512}\pi^2$$

$$R_{a_6} = -\frac{654389}{525} + 700 C_{QQL} - \frac{884}{3} C_{QQQ_1} - \frac{680}{3} C_{QQQ_2} ,$$
$$R_{\bar{d}_5} = -\frac{1773479}{315} - 2720 C_{QQL} + \frac{9568}{9} C_{QQQ_1} + \frac{1792}{3} C_{QQQ_2} .$$

Discrepancy with Foffa-Sturani’19





$$S_{\rm rad} = \eta^8 S_{I_2} + \eta^{10} S_{I_3} + \eta^{10} S_{J_2} + \eta^{10}[S_{QQL} + S_{QQQ_1} + S_{QQQ_2}]$$

$$S_{QQL} = C_{QQL} G^2 \int dt I^{(4)}_{is} I^{(3)}_{js} \varepsilon_{ijk} L_k$$
$$S_{QQQ_1} = C_{QQQ_1} G^2 \int dt I^{(4)}_{is} I^{(4)}_{js} I_{ij} ,$$
$$S_{QQQ_2} = C_{QQQ_2} G^2 \int dt I^{(3)}_{is} I^{(3)}_{js} I^{(2)}_{ij} .$$

A tale of many Green's functions

$$G_{\rm ret}(x) = \frac{\delta(t - r/c)}{r} \qquad G_{\rm ret} = \mathrm{P}\frac{1}{k^2} + i\pi \mathrm{sign}(k^0)\delta(k^2)$$

$$G_{\rm sym}(x) = \frac{\delta(t - r/c) + \delta(t + r/c)}{2r} \qquad G_{\rm sym} = \mathrm{P}\frac{1}{k^2}$$

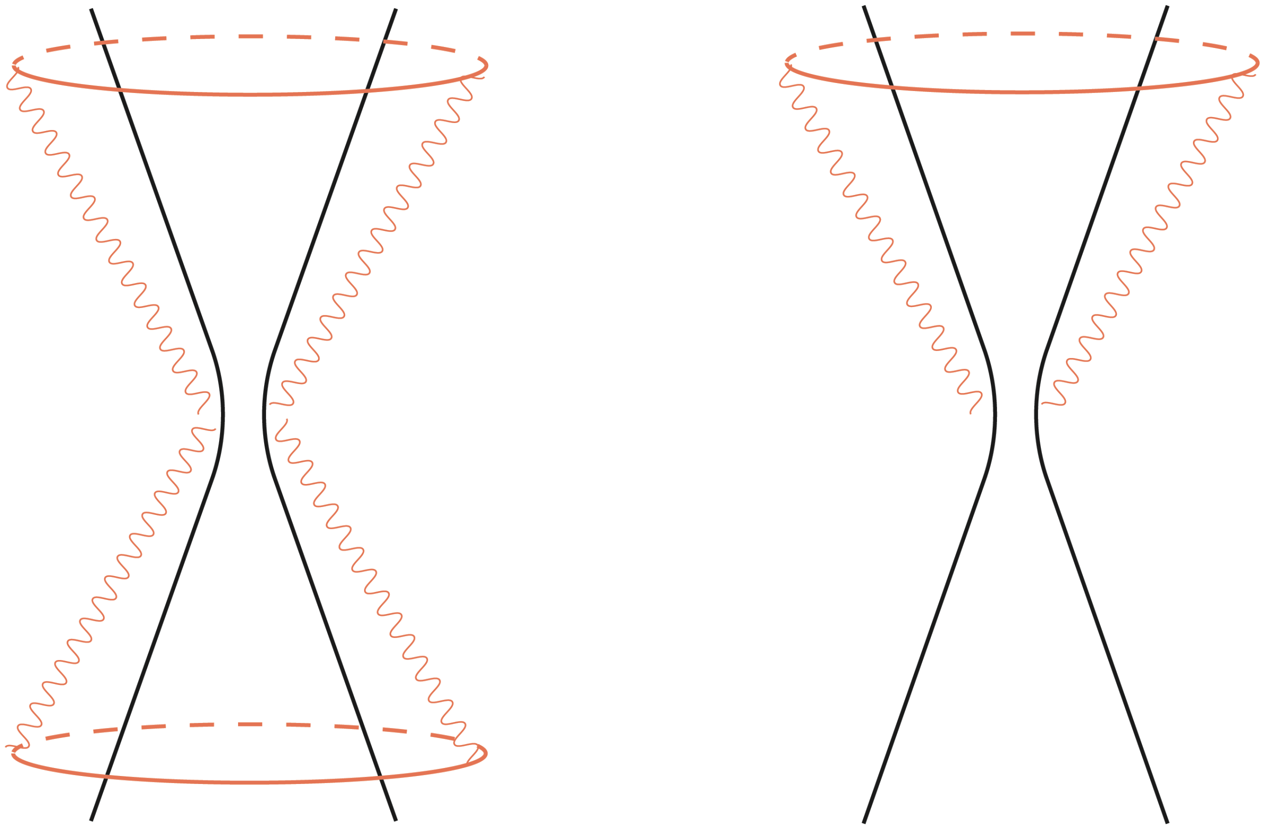
$$G^{\rm PN}_{\rm sym}(x) = \frac{\delta(t)}{r} + \frac{r}{2c^2}\ddot{\delta}(t) + \cdots \qquad G^{\rm PN}_{\rm sym} = \frac{1}{\mathbf{k^2}} + \frac{\omega^2}{c^2\mathbf{k^4}} + \cdots$$

$$G_{\rm F}(x) = \frac{i}{\pi(t^2 - r^2 + i0)} \qquad G_{\rm F} = \mathrm{P}\frac{1}{k^2} + i\pi\delta(k^2)$$

+ issues of: <in,out>; <in,in>, FWF, Schwinger-Keldysh,…

+ issue of scattering effects quadratic in radiation-reaction

Conservative vs Radiation-reacted
Classical Gravitational Scattering



Radiation-reaction effects enter scattering at G^3/c^5 (Bini-TD'12)

$$\frac{1}{2}\chi^{\rm rad} = +\frac{8G^3}{5c^5}\frac{m_1^3 m_2^3}{J^3}\nu v^2 + \cdots$$

Radiation-reaction effects in scattering play a crucial role at **high-energy**

(DiVecchia-Heissenberg-Russo-Veneziano'21, TD'21, Hermann-Parra-Martinez-Ruf-Zeng'21,….)

Simple Map: ConservativeScattering angle <-> EOB dynamics

$$\frac{1}{2}\chi = \Phi(E_{\rm real}, J; m_1, m_2, G)$$

TD'16-18
Bini-TD-Geralico'20

$$\frac{1}{2}\chi_{\rm class}(E,J) = \frac{1}{j}\chi_1(\hat{E}_{\rm eff},\nu) + \frac{1}{j^2}\chi_2(\hat{E}_{\rm eff},\nu) + O(G^3)$$

$$0 = g_{\rm eff}^{\mu\nu} P_\mu P_\nu + \mu^2 + Q$$

$$\chi_1(\hat{\mathcal{E}}_{\rm eff},\nu) = \frac{2\hat{\mathcal{E}}_{\rm eff}^2 - 1}{\sqrt{\hat{\mathcal{E}}_{\rm eff}^2 - 1}},$$

$$\chi_2(\hat{\mathcal{E}}_{\rm eff},\nu) = \frac{3\pi}{8}\frac{5\hat{\mathcal{E}}_{\rm eff}^2 - 1}{\sqrt{1+2\nu(\hat{\mathcal{E}}_{\rm eff}-1)}}.$$

$$g_{\rm eff}^{\mu\nu}$$

Schwarzschild metric M=m1+m2

$$Q = \left(\frac{GM}{R}\right)^2 q_2(E) + \left(\frac{GM}{R}\right)^3 q_3(E) + O(G^4)$$

$$q_2(\hat{\mathcal{E}}_{\rm eff},\nu) = -\frac{4}{\pi}[\chi_2(\hat{\mathcal{E}}_{\rm eff},\nu) - \chi_2^{\rm Schw}(\hat{\mathcal{E}}_{\rm eff})].$$

$$q_3(\hat{\mathcal{E}}_{\rm eff},\nu) = \frac{4}{\pi}\frac{2\hat{\mathcal{E}}_{\rm eff}^2 - 1}{\hat{\mathcal{E}}_{\rm eff}^2 - 1}(\chi_2(\hat{\mathcal{E}}_{\rm eff},\nu) - \chi_2^{\rm Schw}(\hat{\mathcal{E}}_{\rm eff})) - \frac{\chi_3(\hat{\mathcal{E}}_{\rm eff},\nu) - \chi_3^{\rm Schw}(\hat{\mathcal{E}}_{\rm eff})}{\sqrt{\hat{\mathcal{E}}_{\rm eff}^2 - 1}}.$$

$$\mathcal{E}_{\rm eff} = \frac{(\mathcal{E}_{\rm real})^2 - m_1^2 - m_2^2}{2(m_1+m_2)}.$$

$$\frac{\mathcal{E}_{\rm eff}}{\mu} = \gamma = -\frac{p_1\cdot p_2}{m_1 m_2}$$

$$q_2(\gamma,\nu) = \frac{3}{2}(5\gamma^2-1)\left(1 - \frac{1}{h(\gamma,\nu)}\right)$$

$$h(\gamma,\nu) = \sqrt{1+2\nu(\gamma-1)}$$

Linear combinations of the scattering coefficients!

Application to the ACV eikonal scattering phase (massless or ultra-relativistic scattering)

Amati-Ciafaloni-Veneziano'90+ Ciafaloni-Colferai'14+ Bern et al'20+ DiVecchia et al'20

$$\delta^{\text{eikonal}} = \frac{1}{\hbar}(\delta^{\text{R}} + i\delta^{\text{I}}) + \text{quantum corr.}$$

$$\frac{1}{2}\chi^{\text{eikonal}} = 2\frac{\gamma}{j} + \frac{16}{3}\frac{\gamma^3}{j^3} + \cdots$$

valid in the HE limit gamma-> infty

Using the chi—> Q dictionary this corresponds to the HE limits:

$$q_2^{\text{HE}} = \frac{15}{2}\gamma^2$$

$$q_3^{\text{HE}} = \gamma^2$$

i.e. an HE limit for the EOB mass-shell condition (TD'18)

$$0 = g_{\text{eff}}^{\mu\nu}(X)P_\mu P_\nu + \mu^2 + Q(X,P)$$

$$0 = g_{\text{Schw}}^{\mu\nu}P_\mu P_\nu + \left(\frac{15}{2}\left(\frac{GM}{R}\right)^2 + \left(\frac{GM}{R}\right)^3\right)P_0^2$$

3PM computation (Bern-Cheung-Roiban-Shen-Solon-Zeng'19)

using a combination of techniques: generalized unitarity; BCJ double-copy; 2-loop amplitude of quasi-classical diagrams; **EFT transcription** (Cheung-Rothstein-Solon'18); **resummation of PN-expanded integrals for potential-gravitons**

$$\chi_3^{\rm cons} = \chi_3^{\rm Schw} - \frac{2\nu\sqrt{\gamma^2-1}}{h^2(\gamma,\nu)}\bar{C}^{\rm cons}(\gamma)$$

$$q_3^{\rm cons} = \frac{3}{2}\frac{(2\gamma^2-1)(5\gamma^2-1)}{\gamma^2-1}\left(\frac{1}{h(\gamma,\nu)}-1\right) + \frac{2\nu}{h^2(\gamma,\nu)}\bar{C}^{\rm cons}(\gamma)$$

$$\overline{C}^{\rm cons}(\gamma) = \frac{2}{3}\gamma(14\gamma^2+25) + 2(4\gamma^4-12\gamma^2-3)\frac{\mathcal{A}(v)}{\sqrt{\gamma^2-1}}$$

$$h(\gamma,\nu) \equiv \frac{\sqrt{s}}{M} = \sqrt{1+2\nu(\gamma-1)}$$

$$\mathcal{A}(v) \equiv \mathrm{arctanh}(v) = \frac{1}{2}\ln\frac{1+v}{1-v} = 2\,\mathrm{arcsinh}\sqrt{\frac{\gamma-1}{2}}$$

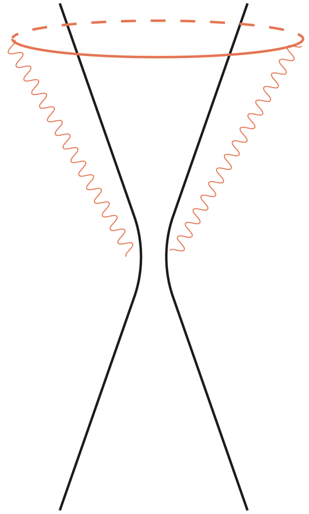
puzzling HE limits when compared to ACV and Akcay et al'12

$$\frac{1}{2}\chi^{\rm cons} = 2\frac{\gamma}{j} + (12-8\ln(2\gamma))\frac{\gamma^3}{j^3} + O(G^4)$$

$q_3^{\rm cons} \approx +8\ln(2\gamma)\gamma^2$ instead of $q_3^{\rm ACV} \approx +1\gamma^2$

confirmations: 5PN (Bini-TD-Geralico'19); 6PN (Blümlein-Maier-Marquard-Schäfer'20, Bini-TD-Geralico'20); 3PM (Cheung-Solon'20, Kälin-Porto'20)

Radiation-Reaction Contribution to the Transverse Classical Scattering Angle at G^3 (TD 2010.01641)



$$\chi^{\rm tot} = \chi^{\rm cons} + \chi^{\rm rad}$$

where, to first order in Rad-Reac, one has (Bini-TD'12)

$$\chi^{\rm rad}(E,J) = -\frac{1}{2}\frac{\partial \chi^{\rm cons}}{\partial E} E^{\rm rad} - \frac{1}{2}\frac{\partial \chi^{\rm cons}}{\partial J} J^{\rm rad}$$

O(G^2)
[TD-Deruelle'81]

chi^cons=O(G^1)

O(G^3)

O(G^4)

O(G^3)

$$h_{ij}^{\rm TT} = \frac{f_{ij}(t-r,\theta,\phi)}{r} + O\left(\frac{1}{r^2}\right)$$

$$J_k^{\rm rad} = \frac{\epsilon_{kij}}{16\pi G}\int du\, d\Omega \left[f_{ia}\partial_u f_{ja} - \frac{1}{2} x^i \partial_j f_{ab} \partial_u f_{ab} \right]$$

DeWitt'71, Thorne'80
Kovacs-Thorne'77, Bel et al'81,
Westpfahl'85

$$\mathcal{I}(v) = -\frac{16}{3} + \frac{2}{v^2} + \frac{2(3v^2-1)}{v^3}\mathcal{A}(v) \qquad \mathcal{A}(v) \equiv {\rm arctanh}(v) = \frac{1}{2}\ln\frac{1+v}{1-v}$$

$$\frac{1}{2}\chi^{\rm rad}(\gamma, j, \nu) = +\frac{\nu}{h^2(\gamma,\nu) j^3}(2\gamma^2-1)^2 \mathcal{I}(v) + O(G^4)$$

$$\frac{1}{2}(\chi^{\rm cons} + \chi^{\rm rad}) = 2\frac{\gamma}{j} + \frac{16}{3}\frac{\gamma^3}{j^3} = \chi^{\rm ACV}$$

Radiation-Reaction Contributions to the Classical Scattering Angle at G^3 (Bini-TD-Geralico 2107.08896)

In the incoming c.m frame: impulses Delta p_1, Delta p_2 **to O(rad-reac^1)**

$$\Delta p_1^0 = \sqrt{m_1^2 + (\mathbf{p}^+)^2} - \sqrt{m_1^2 + (\mathbf{p}^-)^2} + \frac{\mathbf{p}^+ \cdot \mathbf{P}^+}{(E_1 + E_2)^+},$$

$$\Delta \mathbf{p}_1 = \mathbf{p}^+ - \mathbf{p}^- + \frac{E_1^+}{(E_1 + E_2)^+}\mathbf{P}^+,$$

$$\Delta p_2^0 = \sqrt{m_2^2 + (\mathbf{p}^+)^2} - \sqrt{m_2^2 + (\mathbf{p}^-)^2} - \frac{\mathbf{p}^+ \cdot \mathbf{P}^+}{(E_1 + E_2)^+},$$

$$\Delta \mathbf{p}_2 = -(\mathbf{p}^+ - \mathbf{p}^-) + \frac{E_2^+}{(E_1 + E_2)^+}\mathbf{P}^+, \qquad (3.32)$$

effect of recoil (ie momentum loss)

effect of E and J losses in the incoming c.m. frame

At order **G^3** reproduces the KMOC-derived result of Hermann +'21

$$\Delta p_1^{\rm rr,3PM} = \frac{G^3 m_1^2 m_2^2}{b^3}\left[-\frac{2(2\gamma^2-1)^2}{(\gamma^2-1)}\mathcal{I}(v)\hat{b} + \pi\frac{\widehat{\mathcal{E}}(\gamma)}{\gamma+1}(u_2^- - \gamma u_1^-)\right].$$

transverse impulse (TD'20)

longitudinal impulse (Hermann+'21)

At order **G^4 and G^5** we predict the rad-reac contributions to scattering (with PN-accuracy

$$\text{rad reac} \sim \frac{1}{c^5} + \frac{1}{c^7} + \frac{1}{c^8} + \frac{1}{c^9} +$$

6PN evaluation of G^6 and G^7 classical radiative scattering integrals

(Bini-TD-Geralico-Laporta-Mastrolia’ 21)

Scattering coefficients:

$$A_{mnk} = \int_{-1}^{+1}\int_{-1}^{+1} \frac{dT\,dT'}{|T-T'|}\, a_{mnk}(T,T')$$

$$a_{mnk}(T,T') = \sum_{p,q\geq 0} R^{mnk}_{pq}(T,T')\,Ath(T,T')^p\,At(T,T')^q$$

$$At(T,T') \equiv \arctan(T) - \arctan(T'),$$

$$Ath(T,T') \equiv \mathrm{arctanh}(T) - \mathrm{arctanh}(T'),$$

$$J(x) \equiv i\int_0^1 dT(1-x^2) \times \frac{-2\ln^3\left(\frac{1-T}{1+T}\right) - 3\mathrm{Li}_3\left[\left(\frac{(1-T)(1-x)}{(1+T)(1+x)}\right)^2\right]}{2x(T+x)(T+1/x)}.$$

$$\begin{aligned}
iJ(x) = {} & \frac{23}{240}\pi^4 - 21\ln(2)\zeta(3) + \pi^2\ln^2(2) - \ln^4(2) - 24a_4 + \frac{21}{2}H_{-1}(x)\zeta(3) - \frac{3}{2}H_0(x)\zeta(3) + \frac{21}{2}H_1(x)\zeta(3) \\
& + \frac{1}{2}\pi^2 H_{0,-1}(x) + \frac{1}{2}\pi^2 H_{0,1}(x) - \frac{3}{2}\pi^2 H_{-1,-1}(x) - \frac{3}{2}\pi^2 H_{-1,1}(x) - \frac{3}{2}\pi^2 H_{1,-1}(x) - \frac{3}{2}\pi^2 H_{1,1}(x) \\
& + 12H_{0,1,-1}(x)\ln(2) + 12H_{0,1,1}(x)\ln(2) - 12H_{0,-1,-1,-1}(x) + 6H_{0,-1,-1,0}(x) - 12H_{0,-1,1,-1}(x) \\
& + 6H_{0,-1,1,0}(x) - 12H_{0,1,-1,-1}(x) + 6H_{0,1,-1,0}(x) - 12H_{0,1,1,-1}(x) + 6H_{0,1,1,0}(x) - 6H_{-1,-1,-1,0}(x) \\
& - 6H_{-1,-1,1,0}(x) - 6H_{-1,1,-1,0}(x) - 6H_{-1,1,1,0}(x) - 6H_{1,-1,-1,0}(x) - 6H_{1,-1,1,0}(x) - 6H_{1,1,-1,0}(x) \\
& - 6H_{1,1,1,0}(x) + 12H_{0,-1,-1}(x)\ln(2) + 12H_{0,-1,1}(x)\ln(2).
\end{aligned}$$

TABLE I. Analytical results for the $O(G^6)$ scattering coefficients A_{2nk}.

Coefficient	Value
$\pi^{-1}A_{200}$	$-\frac{99}{4} - \frac{2079}{8}\zeta(3)$
$\pi^{-1}A_{220}$	$-\frac{41297}{112} - \frac{9216}{7}\ln(2) + \frac{49941}{64}\zeta(3)$
$\pi^{-1}A_{221}$	$\frac{1937}{8} + \frac{3303}{4}\zeta(3)$
$\pi^{-1}A_{240}$	$\frac{1033549}{4536} + \frac{10704}{7}\ln(2) - \frac{40711}{128}\zeta(3)$
$\pi^{-1}A_{241}$	$\frac{8008171}{8064} + \frac{75520}{21}\ln(2) - \frac{660675}{256}\zeta(3)$
$\pi^{-1}A_{242}$	$-\frac{583751}{864} - \frac{100935}{64}\zeta(3)$

Challenges in translating quantum scattering amplitudes into classical dynamical information

$$\mathcal{M}(s,t) = \mathcal{M}^{(\frac{G}{\hbar})}(s,t) + \mathcal{M}^{(\frac{G^2}{\hbar^2})}(s,t) + \cdots,$$

$$\mathcal{M}^{(\frac{G}{\hbar})}(s,t) = 16\pi \frac{G}{\hbar} \frac{2(p_1 \cdot p_2)^2 - p_1^2 p_2^2}{-t}.$$

$$\alpha_g \equiv \frac{G E_1 E_2}{\hbar}$$

Problem: The domain of validity of the Born-Feynman expansion is GE_1 E_2/(hbar v) << 1, while the domain of validity of the classical scattering is GE_1 E_2/(hbar v) >> 1! (Bohr 1948)

$$\mathcal{M} \sim \frac{Gs}{\hbar} + \left(\frac{Gs}{\hbar}\right)^2 + \left(\frac{Gs}{\hbar}\right)^3 + \ldots$$
$$\sim \alpha_g + \alpha_g^2 + \alpha_g^3 + \ldots$$

$$\hbar \to \infty \text{ vs } \hbar \to 0$$

Ways of recovering the classical information from M(s,t) ?

Focus on non-analytic terms in q =sqrt(-t) in the q-> limit ?
(Donoghue'94,….,Neill-Rothstein'13,…,Cachazo-Guevara'17,Damour'17, Cheung et al'18,Bern et al.'19,…)

Control and resum the exponentiated terms in an eikonal-like approximation?
('tHooft'86, ACV'86-90,…,Akhoury et al'13,Bjerrum-Bohr et al'18,Koemans-Collado et al'19)

Compute the quasi-classical impulse Delta p from amplitude ?
(Kosower-Maybee-O'Connell'19)

Not very efficient way of including radiation-reaction effects?

The G^4 scattering includes several subtleties that will put to the test the hope that modern amplitude techniques can efficiently provide LIGO-useful results

Conclusions

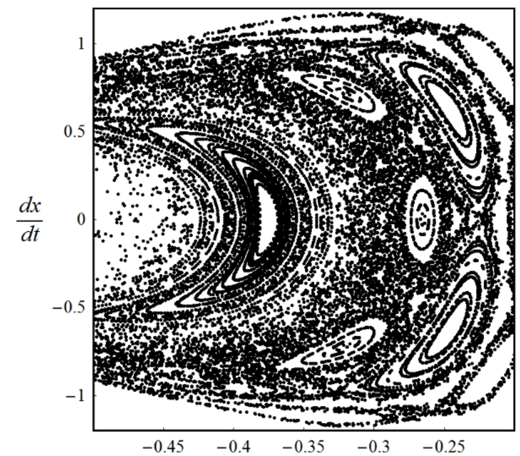
- Analytical approaches to GW signals have played a crucial role (in conjunction with Numerical Relativity simulations) for the detection, interpretation and parameter estimation of coalescing binary systems (BBH and BNS). It is important to further improve our analytical knowledge for future GW detectors: second generation ground-based detectors, space detectors, second generation ground-based detectors.

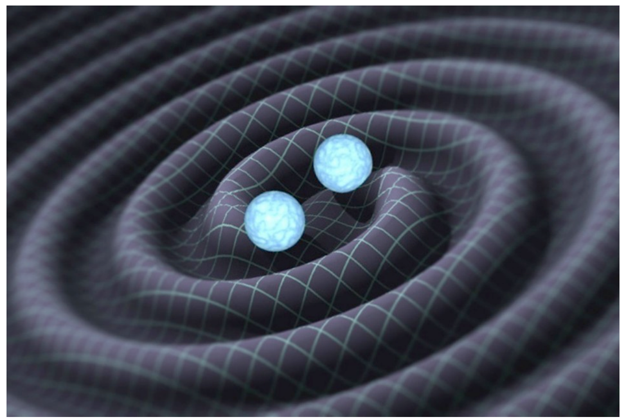
- The development of new analytical approaches (quantum (and classical) scattering, EFT, Tutti-Frutti,…) has started to bring new results of interest for GW detection and must be pursued vigorously in parallel. Discrepancies must be resolved to complete the determination of the **5PN** dynamics (of direct utility for LIGO-Virgo)

Henri Poincaré

«Il n’y a pas de problèmes résolus, il y a seulement des problèmes plus ou moins résolus »

«There are no (definitely) solved problems, there are only more or less solved problems »





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