Retrospective: Lessons from the Past

Zvi Bern

Amplitudes 2021

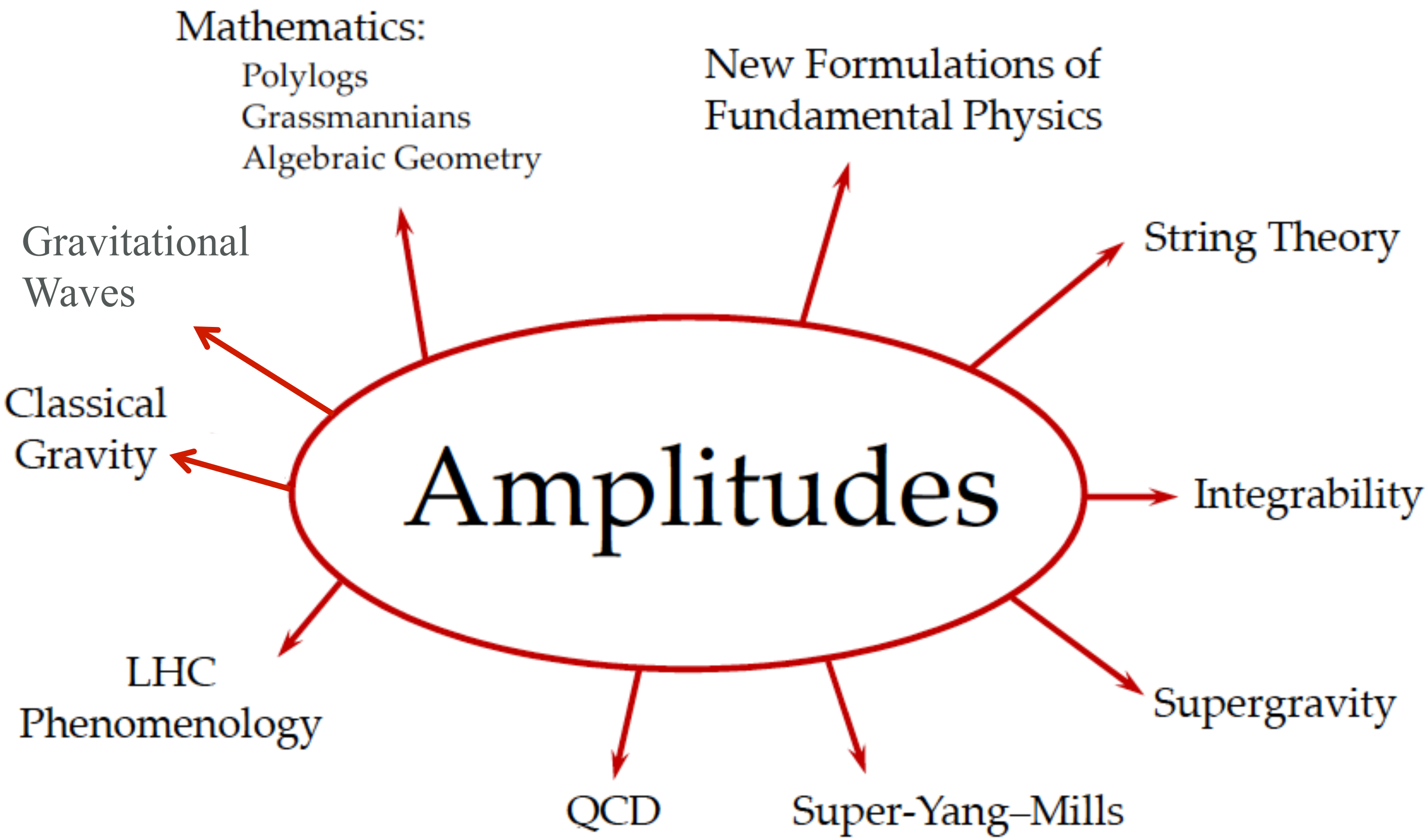
UCLA Mani L. Bhaumik Institute for Theoretical Physics

Outline

Organizers asked me look back at early history, some of reasons for our success and prospects for future.

Today Many Directions

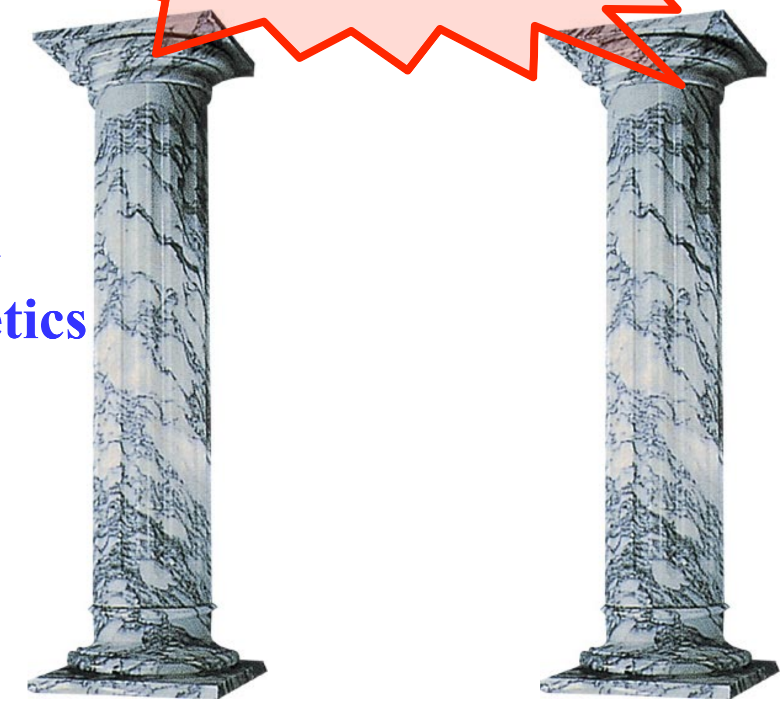
Looking at the broad range of talks, our field has blossomed in many directions.



We have seen many examples of these directions this workshop.

Two Pillars of Our Field

Amplitudes



structure
symmetry
beauty aesthetics

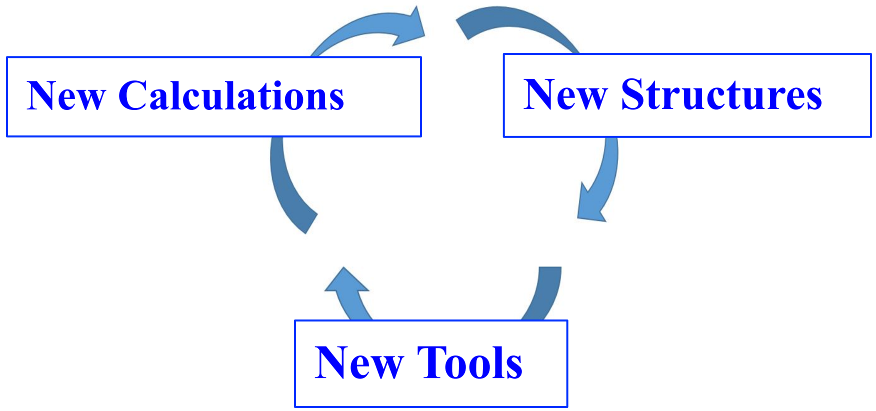
obviously important

explicit
results useful
outside our field

I want to emphasize this

To thrive any field needs the support of *both* pillars

The search for new structures.



A virtuous cycle.

- Key priority for new calculations is to uncover new and useful structures.
- Simultaneously push state of the art for physical quantities of interest.
- This philosophy was clear from the beginning.

Examples of structure:

1. Parke-Taylor
2. Curves in Twistor space and MHV rules
3. Geometric interpretations. Amplituhedron
4. $N = 4$ sYM and link to strong coupling
5. Double copy

Will show examples of how some of these were found.

Simplicity in Scattering Amplitudes

Amplitudes began for me when I was a postdoc at the Niels Bohr Institute.

David Kosower (postdoc at Columbia) was visiting for a month

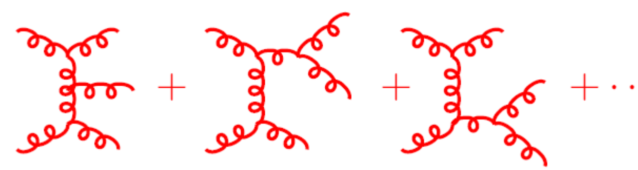
One of the great aspects of NBI is the flow of visitors.

Volume 201, number 1 PHYSICS LETTERS B 28 January 1988

MULTI-GLUON SCATTERING: A STRING-BASED CALCULATION ☆

David A. KOSOWER, Bum-Hoon LEE and V.P. NAIR
Department of Physics, Columbia University, New York, NY 10027, USA

Received 4 August 1987



- **String theory is an amplitudes-based approach. Not Feynman diagrams.**
- **Manifested key features already noted by Parke and Taylor.**
- **Spinor helicity and compact expressions.**
- **Color decompositions. U(1) decoupling identities.**

Mangano, Parke, Xu

Can we apply these types of ideas to loop level?
It took a while before we could decisively answer this.

Spinor Helicity

Nuclear Physics B291 (1987) 392–428
North-Holland, Amsterdam

HELICITY AMPLITUDES FOR MULTIPLE BREMSSTRAHLUNG IN MASSLESS NON-ABELIAN GAUGE THEORIES*

Zhan XU, Da-Hua ZHANG and Lee CHANG

Department of Physics, Tsinghua University, Beijing, The People's Republic of China

Received 22 January 1985
(Revised 10 December 1986)

Many earlier papers but this was cleanest

$$\boxed{\varepsilon_\mu^+(k,q) = \frac{\langle q_-|\gamma_\mu|k_-\rangle}{\sqrt{2}\,\langle qk\rangle} \qquad \varepsilon_\mu^-(k,q) = \frac{\langle q_+|\gamma_\mu|k_+\rangle}{\sqrt{2}\,\langle qk\rangle^*}}$$

A clean way to describe physical degrees of freedom

$$A_5^{\text{tree}}(1^\pm,2^+,3^+,4^+,5^+) = 0$$

$$A_5^{\text{tree}}(1^-,2^-,3^+,4^+,5^+) = i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

$$A_5^{\text{tree}}(1^-,2^+,3^-,4^+,5^+) = i\frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

Parke-Taylor Formula

VOLUME 56, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1986

Amplitude for *n*-Gluon Scattering

Stephen J. Parke and T. R. Taylor
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 17 March 1986)

A nontrivial squared helicity amplitude is given for the scattering of an arbitrary number of gluons to lowest order in the coupling constant and to leading order in the number of colors.

$$|\mathcal{M}_n(+++++\cdots)|^2 = c_n(g,N)[0+O(g^4)],$$

$$|\mathcal{M}_n(-++++\cdots)|^2 = c_n(g,N)[0+O(g^4)],$$

$$|\mathcal{M}_n(--+++\cdots)|^2 = c_n(q,N)[(p_1\cdot p_2)^4 \times \textstyle\sum_P[(p_1\cdot p_2)(p_2\cdot p_3)(p_3\cdot p_4)\cdots(p_n\cdot p_1)]^{-1} + O(N^{-2}) + O(g^2)],$$

Paper that started the field

Checked explicitly for *n* = 4,5,6 (6 point numerical)
This was an "educated guess".

Spinor helicity form came later.

Mangano, Parke and Xu (1988)

$$A(1^-,2^-,3^+,\ldots,n^+) = i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

Some important lessons:

- **You can guess further than you can calculate.**
- **Analytic structure very powerful. Factorization for checking.**
- **Amplitudes can have remarkably simple structures. Look for them!**

Simplicity in Scattering Amplitudes

34 years ago David Kosower mentioned the "Parke-Taylor formula".

I said, "What's that?" (Words to be forgotten!)

David Kosower's response should be immortalized:

"Everyone needs to know the Parke-Taylor formula!"

David was right.
34 years later everyone does indeed know it!

MHV amplitude in spinor notation: Mangano, Parke and Xu (1988)

$$A(1^-, 2^-, 3^+, \ldots, n^+) = i \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

Response from QCDers: "So what?"

The importance of Parke-Taylor formula was not immediately recognized. Took about 20 years before its significance became known to the wider community.

$$A(1^-,2^-,3^+,\ldots,n^+) = i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

Basic concern were that it was too special:

- **Special helicities. For collider need all helicities**
- **Massless**
- **Only tree level**
- **Not proven**
- **What is it good for? What can you do with it?**

- **Only a few of us (young) believers could see the potential past the flaws.**
- **It took a while before each problem was dealt with.**
- **"Amplitudes? Wasn't that all understood in the 1960s?"**

Berends-Giele Recursion

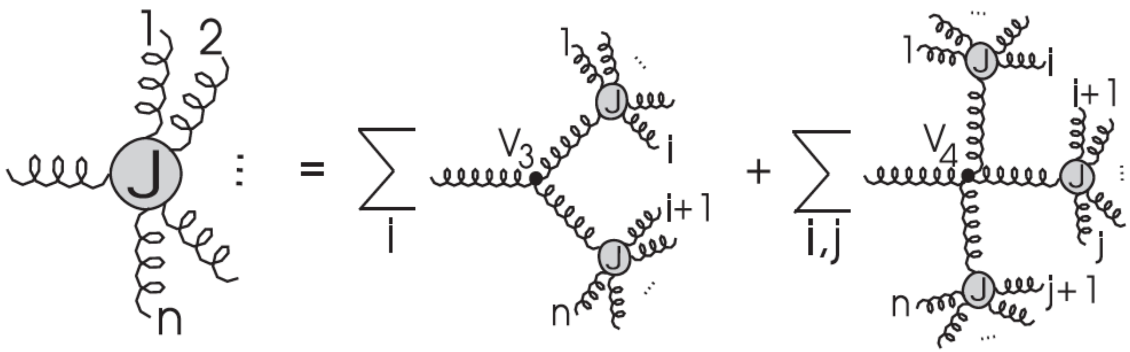
Nuclear Physics B306 (1988) 759–808
North-Holland, Amsterdam

RECURSIVE CALCULATIONS FOR PROCESSES WITH *n* GLUONS

F.A. BERENDS and W.T. GIELE*

Instituut-Lorentz, University of Leiden, P.O.B. 9506, 2300 RA Leiden, The Netherlands

Received 30 December 1987



Recursive definition of current

$$J^\mu(1^+,2^+,\ldots,n^+) = \frac{\langle q^-|\gamma^\mu\, \not{P}_{1,n}|q^+\rangle}{\sqrt{2}\,\langle q\,1\rangle\,\langle 1\,2\rangle\cdots\langle n-1,\,n\rangle\,\langle n\,q\rangle}$$

$$J^\mu(1^-,2^+,\ldots,n^+) = \frac{\langle 1^-|\gamma^\mu\, \not{P}_{2,n}|1^+\rangle}{\sqrt{2}\,\langle 1\,2\rangle\cdots\langle n\,1\rangle} \sum_{m=3}^{n} \frac{\langle 1^-|\, \not{k}_m\, \not{P}_{1,m}|1^+\rangle}{P^2_{1,m-1}P^2_{1,m}}$$

- **Succeeded in recursively *proving* Parke-Taylor MHV amplitude formula.**
- **Showed the importance of recursive approaches.**
- **Beyond MHV hard to use analytically. (Numerically fast.)**

All the ideas from 1980s collected

PHYSICS REPORTS (Review Section of Physics Letters) 200, No. 6 (1991) 301–367. North-Holland

MULTI-PARTON AMPLITUDES IN GAUGE THEORIES

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Istituto Nazionale di Fisica Nucleare, Scuola Normale Superiore and Dipartimento di Fisica, Pisa, Italy

and

Stephen J. PARKE

Fermi National Accelerator Laboratory,⁾ P.O. Box 500, Batavia, IL 60510, USA*

Editor: R. Petronzio Received July 1990

Abstract:

In this report we review recent developments in perturbation-theory methods for gauge theories. We present techniques and results that are useful in the calculation of cross sections for processes with many final-state partons which have applications in the study of multi-jet phenomena in high-energy colliders.

- Our bible for next decade.
- Laid the foundation for new advances. Can we go to loops?
- Dated, but still a great review for the basic ideas.

Can we push these types of ideas to loops?

PHYSICAL REVIEW D VOLUME 38, NUMBER 6 15 SEPTEMBER 1988

New approach to one-loop calculations in gauge theories

Zvi Bern
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(Received 31 December 1987)

We propose using the technology of four-dimensional string theories to calculate amplitudes in gauge theories. Strings make such calculations much more efficient by summing a large number of Feynman diagrams all at once. We check the idea by constructing a string model reducing to a pure non-Abelian gauge theory in the infinite-tension limit and computing its β function with these techniques.

Not particularly well received.

Hard to impress anyone by repeating a 15 year old calculation.

Towards Loops

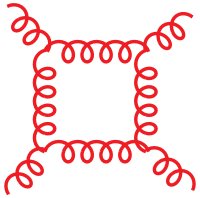
ZB and Kosower

All sorts of objections:

1) String states won't decouple
2) Infrared divergences different in string theory
3) How can you get beta function? String theory is finite
4) You cannot get a consistent string theory with SU(N)
5) Modular invariance broken
6) Can we calculate anything state of the art?

Not a simple project. It took a while to deal with these objections.

The argument that finally worked and ended debate was we reproduced Ellis and Sexton (1986) paper on NLO 2→2 scattering in QCD.



Lessons:

- **Listen to objections, but sometimes best to just move forward.**
- **To convince people (including yourself) need state of the art calculations.**

String Based Methods

Nuclear Physics B 379 (1992) 451–561
North-Holland

NUCLEAR
PHYSICS B

The computation of loop amplitudes in gauge theories

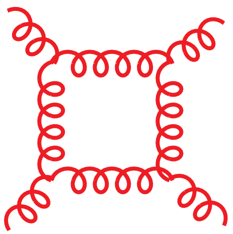
Zvi Bern

Department of Physics, University of Pittsburgh, Pittsburgh, PA 15260, USA

David A. Kosower *

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510, USA

Received 19 August 1991
Accepted for publication 21 January 1992



- **Reproduced Ellis and Sexton result — in much simpler form**
- **Back then 1 loop 4 gluons was a big deal**
- **This time some people in QCD paid attention (Al Mueller and Zoltan Kunszt)**
- **String methods were just a first step towards more modern ones, but we got to state of the art**

Some helicities astonishingly simple:

$$A(1^+,2^+,3^+,4^+) = \frac{i}{48\pi^2}\left(1+\frac{n_s}{N_c}-\frac{n_f}{N_c}\right)\frac{st}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

String Based Methods

After this success we joined forces with Lance Dixon and went on to do many more things. Great fun!

If the methods are any good one should be able to go beyond the state of the art.

VOLUME 70, NUMBER 18 PHYSICAL REVIEW LETTERS 3 MAY 1993

One-Loop Corrections to Five-Gluon Amplitudes

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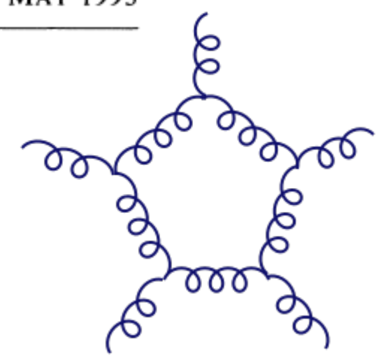
Lance Dixon[b]
Stanford Linear Accelerator Center, Stanford, California 94309

David A. Kosower[c]
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

(Received 1 March 1993)

We present the one-loop helicity amplitudes with five external gluons. The computation employs string-based methods, new techniques for performing tensor integrals, and improvements in the spinor helicity method.



New Data: Simplicity at One Loop

ZB, Dixon, Kosower 1993

Using string-based methods we obtained the one-loop QCD five-gluon.

$$A_{5;1}^{[0]} = c_\Gamma \left(V^s A_5^{\text{tree}} + i F^s \right) ,$$
$$A_{5;1}^{[1/2]} = -c_\Gamma \left((V^f + V^s) A_5^{\text{tree}} + i (F^f + F^s) \right) ,$$
$$A_{5;1}^{[1]} = c_\Gamma \left((V^g + 4V^f + V^s) A_5^{\text{tree}} + i (4F^f + F^s) \right)$$

Susy decompositions

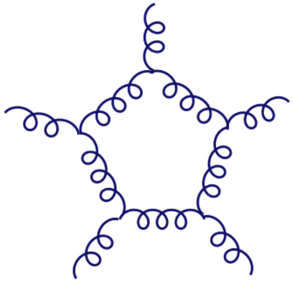
$$V^g = -\frac{1}{\epsilon^2} \sum_{j=1}^{5} \left(\frac{\mu^2}{-s_{j,j+1}} \right)^{\epsilon} + \sum_{j=1}^{5} \ln\left(\frac{-s_{j,j+1}}{-s_{j+1,j+2}} \right) \ln\left(\frac{-s_{j+2,j-2}}{-s_{j-2,j-1}} \right) + \frac{5}{6}\pi^2$$

***N* = 4 sYM**

$$V^f = -\frac{5}{2\epsilon} - \frac{1}{2}\left[\ln\left(\frac{\mu^2}{-s_{23}} \right) + \ln\left(\frac{\mu^2}{-s_{51}} \right) \right] - 2, \qquad V^s = -\frac{1}{3} V^f + \frac{2}{9}$$

$$F^f = -\frac{1}{2} \frac{\langle 1\,2\rangle^2 \left(\langle 2\,3\rangle [3\,4] \langle 4\,1\rangle + \langle 2\,4\rangle [4\,5] \langle 5\,1\rangle \right)}{\langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,5\rangle \langle 5\,1\rangle} \frac{\mathrm{L}_0\left(\frac{-s_{23}}{-s_{51}} \right)}{s_{51}}$$

$$F^s = -\frac{1}{3} \frac{[3\,4] \langle 4\,1\rangle \langle 2\,4\rangle [4\,5] \left(\langle 2\,3\rangle [3\,4] \langle 4\,1\rangle + \langle 2\,4\rangle [4\,5] \langle 5\,1\rangle \right)}{\langle 3\,4\rangle \langle 4\,5\rangle} \frac{\mathrm{L}_2\left(\frac{-s_{23}}{-s_{51}} \right)}{s_{51}^3} - \frac{1}{3} F^f$$
$$- \frac{1}{3} \frac{\langle 3\,5\rangle [3\,5]^3}{[1\,2] [2\,3] \langle 3\,4\rangle \langle 4\,5\rangle [5\,1]} + \frac{1}{3} \frac{\langle 1\,2\rangle [3\,5]^2}{[2\,3] \langle 3\,4\rangle \langle 4\,5\rangle [5\,1]} + \frac{1}{6} \frac{\langle 1\,2\rangle [3\,4] \langle 4\,1\rangle \langle 2\,4\rangle [4\,5]}{s_{23} \langle 3\,4\rangle \langle 4\,5\rangle s_{51}}$$



Amazingly it fit into PRL

- **This time beyond previous state of the art: *first* 1 loop five point.**
- **Supersymmetry exposed analytic structure.**
- **Loop-level helicity amplitudes are simple!**
- **Finally, QCD community started paying attention, though still weirdos.**
- **Bill Kilgore later used this in 3 jet production at colliders. Link to pheno.**

Be curious: Why so simple?

The supersymmetric pieces are amazingly simple, especially $N = 4$. Why?

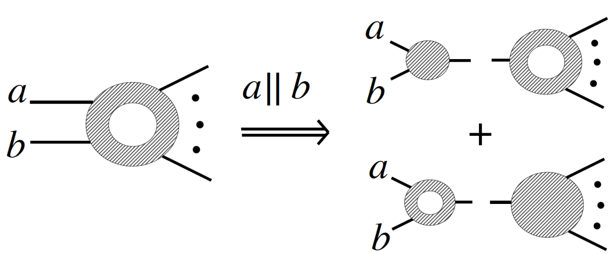
$$V^g = -\frac{1}{\epsilon^2}\sum_{j=1}^{5}\left(\frac{\mu^2}{-s_{j,j+1}}\right)^{\epsilon} + \sum_{j=1}^{5}\ln\left(\frac{-s_{j,j+1}}{-s_{j+1,j+2}}\right)\ln\left(\frac{-s_{j+2,j-2}}{-s_{j-2,j-1}}\right) + \frac{5}{6}\pi^2$$

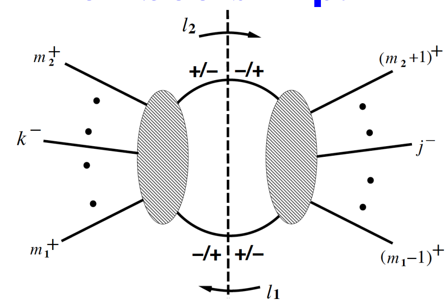
Whenever results are simpler than one might expect, a light bulb should go on.

There has to be a better way!

What is the source? Analytic behavior?

- **Unitarity. Sewing trees into loops.**
- **Factorization. Pole structure is simple. Factorization bootstrap.**





Pointed to much more powerful methods where analytic properties can be used to recycle simpler amplitudes into more complicated ones.

Lesson: If something is simpler than expected understand the origin.

Simplicity at Loops

ELSEVIER

Nuclear Physics B425 (1994) 217–260

NUCLEAR PHYSICS B

One-loop n-point gauge theory amplitudes, unitarity and collinear limits

Zvi Bern [a,1], Lance Dixon [b,2], David C. Dunbar [a,3], David A. Kosower [c,4]

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[b] *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*
[c] *Service de Physique Théorique de Saclay, Centre d'Etudes de Saclay, F-91191 Gif-sur-Yvette cedex, France*

Received 11 March 1994; accepted for publication 6 April 1994

Parke-Taylor simplicity can be imported to create loops.
Poor-man's on-shell recursion: Use analytic constraints to find new results

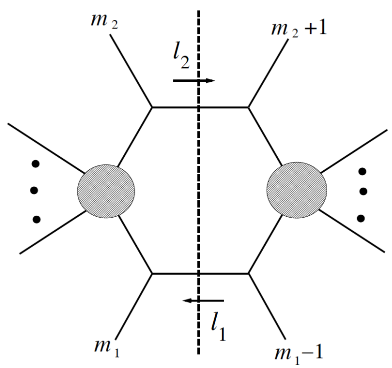
A simpler way?

Bern, Dixon, Dunbar, Kosower (1994)

Unitarity at integrand level. Integrate and combine.

$$\int d^D\mathrm{LIPS}(-\ell_1,\ell_2)\ A^{\mathrm{tree}}(-\ell_1,m_1,\ldots,m_2,\ell_2)\ A^{\mathrm{tree}}(-\ell_2,m_2+1,\ldots,m_1-1,\ell_1)$$

Key observation: simple dependence of propagators on loop momenta:



$$A^{\mathrm{tree}}(l_1^+,1^-,2^-,3^+,\ldots,m^+,l_3^+)=\frac{\langle 12\rangle^4}{\langle l_1 1\rangle\langle 12\rangle\cdots\langle m l_1\rangle\langle l_1 l_2\rangle}$$

No more than a hexagon integral needed for all *n* MHV.

***N* = 4 susy**

$$V_n^g=\sum_{i=1}^{n}-\frac{1}{\epsilon^2}\left(\frac{\mu^2}{-t_i^{[2]}}\right)^{\epsilon}-\sum_{r=2}^{\lfloor n/2\rfloor-1}\sum_{i=1}^{n}\ln\left(\frac{-t_i^{[r]}}{-t_i^{[r+1]}}\right)\ln\left(\frac{-t_{i+1}^{[r]}}{-t_i^{[r+1]}}\right)+D_n+L_n+\frac{n\pi^2}{6}$$

D_n and L_n in terms of polylogs

Arbitrary number of external legs at loop level.

Applications to QCD

ELSEVIER

Nuclear Physics B 513 (1998) 3–86

NUCLEAR PHYSICS B

One-loop amplitudes for e^+e^- to four partons

Zvi Bern [a,1], Lance Dixon [b,2], David A. Kosower [c,3]

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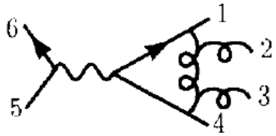
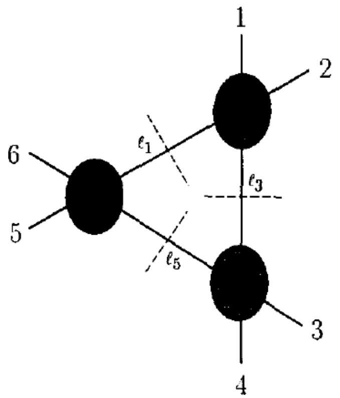
[b] Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA
[c] Service de Physique Théorique, Centre d'Etudes de Saclay, F-91191 Gif-sur-Yvette cedex, France [4]



Received 13 August 1997; accepted 20 October 1997



generalized cuts

- Full QCD Loop amplitudes needed from pheno reconstructed from their analytic properties. Gauge invariant.
- Generalized unitarity cuts introduced.
- Method hard to use. Integration generated hard to clean mess containing spurious poles.
- Campbell, Glover, Miller competed very well with traditional Feynman methods (numerical approach).

Lesson: An approach with compact analytic results can stimulate greater things to come. Integration problem solved later.

Gravity

See talks from Carrasco, Cheung, Elvang, Keeler, Paranjape, Puhm, Teng, Vazquez-Holm

Nuclear Physics B269 (1986) 1–23

A RELATION BETWEEN TREE AMPLITUDES OF CLOSED AND OPEN STRINGS*

H. KAWAI, D.C. LEWELLEN and S.-H.H. TYE

Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853, USA

Received 11 October 1985

- An astonishing idea: Gravity from gauge theory.
- Implications enormous: Gravity no more complicated than gauge theory!
- Perfectly aligned with amplitudes.
- Paper was initially pretty much ignored in string community.
- I was mystified why people were not amazed.

Sometimes the significance takes a while to be appreciated

KLT Relation Between Gravity and Gauge Theory

KLT (1985)

Kawai-Lewellen-Tye string relations in low-energy limit:

gravity

gauge-theory color ordered

$$M_4^{\rm tree}(1,2,3,4) = -is_{12}A_4^{\rm tree}(1,2,3,4)\,A_4^{\rm tree}(1,2,4,3)\,,$$

$$M_5^{\rm tree}(1,2,3,4,5) = is_{12}s_{34}A_5^{\rm tree}(1,2,3,4,5)\,A_5^{\rm tree}(2,1,4,3,5) + is_{13}s_{24}A_5^{\rm tree}(1,3,2,4,5)\,A_5^{\rm tree}(3,1,4,2,5)$$

Generalizes to explicit all-leg form. ZB, Dixon, Perelstein, Rozowsky

Tree MHV Gravity Amplitudes

Volume 211, number 1,2 PHYSICS LETTERS B 25 August 1988

ON RELATIONS BETWEEN MULTI-GLUON AND MULTI-GRAVITON SCATTERING

F.A. BERENDS, W.T. GIELE [1] and H. KUIJF
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Received 24 May 1988

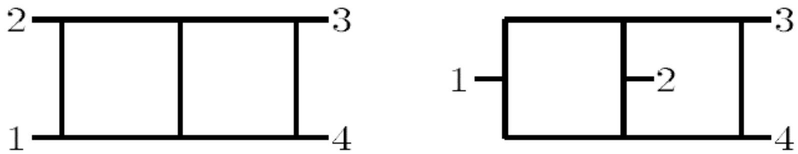
From a relation between tree amplitudes of closed and open strings it becomes possible to express a mulit-graviton amplitude into a quadratic combination of multi-gluon sub-amplitudes. Since up to six external particles the graviton and gluon amplitudes are known, we can explicitly demonstrate the correctness of the relations in these cases. For a special helicity configuration the *n*-graviton amplitude is given on the basis of the known *n*-gluon amplitudes. The soft graviton behaviour is discussed.

$$M_n^{\text{tree}}(1^-,2^-,3^+,\ldots,n^+) = -i\,\langle 1\,2\rangle^8 \times \left[\frac{[1\,2]\;[n-2\;\;n-1]}{\langle 1\;\;n-1\rangle\;N(n)} \left(\prod_{i=1}^{n-3} \prod_{j=i+2}^{n-1} \langle i\,j\rangle \right) \prod_{l=3}^{n-3} \Big(-\langle n^- | \not{K}_{l+1,n-1} | l^- \rangle \Big) \;+\; \mathcal{P}(2,3,\ldots,n-2) \right]$$

(educated guess)

- **Obviously BGK understood the significance of KLT.**
- **Parke-Taylor imported to gravity.**
- **Once again educated guessing is extremely powerful.**
- **Same unitarity ideas as for gauge theory can now be used for gravity.**

Multi Loop Amplitudes



ZB, Yan, Rozowsky (1997);
ZB, Dixon, Dunbar, Rozowsky, Perelstein (1998)

Two loop amplitudes in $N = 4$ sYM susy are simple!

$$\mathcal{A}_4^{\text{2-loop}}(1,2,3,4) = -g^6 s_{12}s_{23}\, A_4^{\text{tree}}(1,2,3,4)\Big(C^{\text{P}}_{1234}\, s_{12}\,\mathcal{I}_4^{\text{2-loop,P}}(s_{12},s_{23}) + C^{\text{P}}_{3421}\, s_{12}\,\mathcal{I}_4^{\text{2-loop,P}}(s_{12},s_{24})$$
$$+ C^{\text{NP}}_{1234}\, s_{12}\,\mathcal{I}_4^{\text{2-loop,NP}}(s_{12},s_{23}) + C^{\text{NP}}_{3421}\, s_{12}\,\mathcal{I}_4^{\text{2-loop,NP}}(s_{12},s_{24}) + \text{cyclic}\Big)\,,$$

Scalar double boxes

Integrals obtained later.

Smirnov(1999); Tausk (1999)

Simplicity remains for integrated expressions!

$N = 8$ supergravity amplitudes just as simple!

$$\mathcal{M}_4^{\text{2-loop}}(1,2,3,4) = -i\Big(\frac{\kappa}{2}\Big)^6 [s_{12}s_{23}\, A_4^{\text{tree}}(1,2,3,4)]^2 \Big(s_{12}^2\,\mathcal{I}_4^{\text{2-loop,P}}(s_{12},s_{23}) + s_{12}^2\,\mathcal{I}_4^{\text{2-loop,P}}(s_{12},s_{24})$$
$$+ s_{12}^2\,\mathcal{I}_4^{\text{2-loop,NP}}(s_{12},s_{23}) + s_{12}^2\,\mathcal{I}_4^{\text{2-loop,NP}}(s_{12},s_{24}) \quad + \text{cyclic}\Big)$$

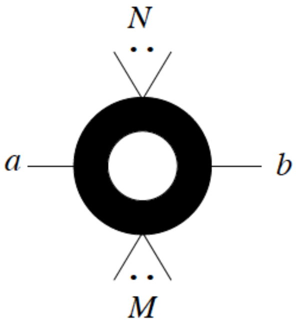
Again we can import gauge theory simplicity to gravity.

MHV One-loop Gravity Amplitudes

ZB, Dixon, Rozowsky, Perelstein (1998)

You might figure we could build all *n* gravity with gauge theory success.

But it was very hard to figure out.
Analytic properties seemed more complicated than gauge theo



A loss of faith:

1) **No one cared about scattering amplitudes.**
2) **Gravity amplitudes even worse. No experimental relevance!**

“Why am I working so hard on something no one cares about?”

An epiphany: “If you can work out a one loop all-*n* gravity amplitude, just do it.”
Impossible using ordinary methods. Almost magical.

It took about 1 year + 5 min to figure out.

JUST DO IT

Lesson: If something fantastic is within reach just do it

MHV One-Loop Gravity Amplitudes

Nuclear Physics B 546 (1999) 423–479

Multi-leg one-loop gravity amplitudes from gauge theory

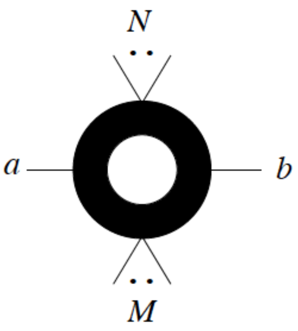
Z. Bern[a,1], L. Dixon[b,2], M. Perelstein[b,2], J.S. Rozowsky[c,3]

[a] *Department of Physics, University of California at Los Angeles, Los Angeles, CA 90095-1547, USA*

[b] *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*

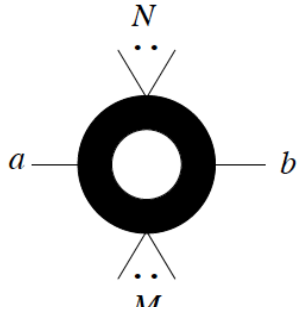
[c] *Institute for Fundamental Theory, Department of Physics, University of Florida, Gainesville, FL 32611, USA*

Received 26 November 1998; accepted 8 January 1999



MHV One-loop Gravity Amplitudes

ZB, Dixon, Rozowsky, Perelstein (1998)



Pure gravity one-loop identical helicity:

$$M_n(1^+, 2^+, \ldots, n^+) = -\frac{i\,(-1)^n}{(4\pi)^2 \cdot 960} \sum_{\substack{1 \le a < b \le n \\ M, N}} h(a, M, b) h(b, N, a) \, \mathrm{tr}^3[a\, M\, b\, N]$$

$$h(a, \{1, 2, \ldots, n\}, b) \equiv \frac{[1\,2]}{\langle 1\,2\rangle} \frac{\langle a^-| \not{K}_{1,2} |3^-\rangle \langle a^-| \not{K}_{1,3} |4^-\rangle \cdots \langle a^-| \not{K}_{1,n-1} |n^-\rangle}{\langle 2\,3\rangle\, \langle 3\,4\rangle \cdots \langle n-1,\, n\rangle\; \langle a\,1\rangle\, \langle a\,2\rangle\, \langle a\,3\rangle \cdots \langle a\,n\rangle\; \langle 1\,b\rangle\, \langle n\,b\rangle} + \mathcal{P}(2, 3, \ldots, n),$$

"half-soft function"

N = 8 supergravity MHV amplitude

$$M_n^{N=8}(1^-, 2^-, 3^+, \ldots, n^+) = \frac{(-1)^n}{8} \langle 1\,2\rangle^8 \sum_{\substack{1 \le a < b \le n \\ M, N}} h(a, M, b) h(b, N, a) \, \mathrm{tr}^2[a\, M\, b\, N]\, \mathcal{I}_4^{aMbN}$$

All *n* one-loop MHV gravity amplitudes are relatively simple (even if it was hard to derive back then).

Twistor Revolution

Commun. Math. Phys. 252, 189–258 (2004)
Digital Object Identifier (DOI) 10.1007/s00220-004-1187-3

Communications in Mathematical Physics

Perturbative Gauge Theory as a String Theory in Twistor Space

Edward Witten

Institute for Advanced Study, Princeton, NJ 08540, USA

Received: 22 April 2004 / Accepted: 3 June 2004
Published online: 7 October 2004 – © Springer-Verlag 2004

Precursor from Nair
Sigma model on CP^1

Penrose twistor transform:

$$\widetilde{A}(\lambda_i, \mu_i) = \int \prod_i \frac{d^2\widetilde{\lambda}_i}{(2\pi)^2} \exp\left(\sum_j \mu_j^{\dot{a}} \widetilde{\lambda}_{j\dot{a}} \right) A(\lambda_i, \widetilde{\lambda}_i)$$

QCD scattering amplitudes ⟷ Topological String Theory

David Kosower told me Ed was working on something related to amplitudes.

KITP collider physics workshop. Roiban, Spradlin and Volovich postdocs at KITP.
Paper landed like a meteorite, sending out shock waves to this day.

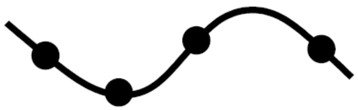
People were finally paying attention!
Finally, I could tell people I work on "Amplitudes".

"Amplitudes" became a respectable field

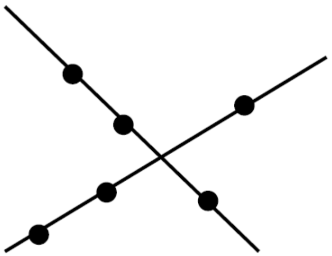
Amazing Structure

Witten conjectured that in twistor–space gauge theory amplitudes have delta-function support on curves of degree:

$$d = q - 1 + L, \quad q = \#\ \text{negative helicities}, \quad L = \#\ \text{loops},$$



Connected picture



Disconnected picture

Witten
Roiban, Spradlin and Volovich
Cachazo, Svrcek and Witten
Gukov, Motl and Neitzke
Bena. Bern and Kosower

Remarkable structures in gauge theory scattering amplitudes.

But for all its beauty what is this good for?

RSV Formula

PHYSICAL REVIEW D **70**, 026009 (2004)

Tree-level *S* matrix of Yang-Mills theory

Radu Roiban
Department of Physics, University of California, Santa Barbara, California 93106, USA

Marcus Spradlin and Anastasia Volovich
Kavli Institute for Theoretical Physics, Santa Barbara, California 93106, USA
(Received 18 May 2004; published 30 July 2004)

Turned Witten's words into a precise formula

RSV Formula

The following formula encapsulates the entire tree-level S-matrix of $\mathcal{N} = 4$ super-Yang-Mills:

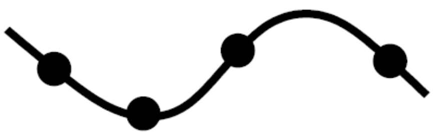
Roiban, Spradlin and Volovich

Integral over the moduli and curves

$$A_n = i(2\pi)^4 g_{\mathrm{YM}}^{n-2} \sum_{d=1}^{n-3} \int d\mathcal{M}_{n,d} \prod_{i=1}^{n} \delta^2\left(\lambda_i^\alpha - \xi_i P_i^\alpha\right) \prod_{k=0}^{d} \delta^2\left(\sum_{i=1}^{n} \xi_i \sigma_i^k \tilde{\lambda}_i^{\dot{\alpha}}\right) \delta^4\left(\sum_{i=1}^{n} \xi_i \sigma_i^k \eta_{iA}\right)$$

$$P_i^\alpha = \sum_{k=0}^{d} a_k^\alpha \sigma_i^k$$

Degree *d* polynomial in the moduli



A very strange formula from Feynman diagram viewpoint.

But it's true: impressive checks by Roiban, Spradlin and Volovich

- **An example of an amazingly beautiful formula whose practical value is still unclear. Planted seeds for CHY.**
- **Maybe in the future applications will become clear**

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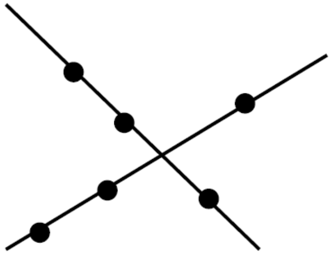
RECEIVED: *August 2, 2004*
ACCEPTED: *September 2, 2004*

MHV vertices and tree amplitudes in gauge theory



Freddy Cachazo,[a] Peter Svrcek[b] and Edward Witten[a]

[a] *School of Natural Sciences, Institute for Advanced Study, Princeton NJ 08540 U.S.A.*
[b] *Joseph Henry Laboratories, Princeton University Princeton NJ 08544 U.S.A.*
E-mail: cachazo@ias.edu, psvrcek@princeton.edu, witten@ias.edu

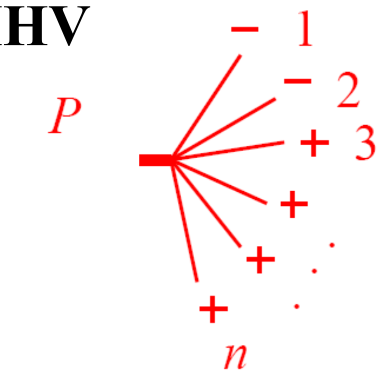
ABSTRACT: As an alternative to the usual Feynman graphs, tree amplitudes in Yang-Mills theory can be constructed from tree graphs in which the vertices are tree level MHV scattering amplitudes, continued off shell in a particular fashion. The formalism leads to new and relatively simple formulas for many amplitudes, and can be heuristically derived from twistor space.

This paper made clear to everyone that twistor insight is useful.
David Gross enthusiastic! The twistor revolution had landed.

MHV Rules

Cachazo, Svrcek and Witten

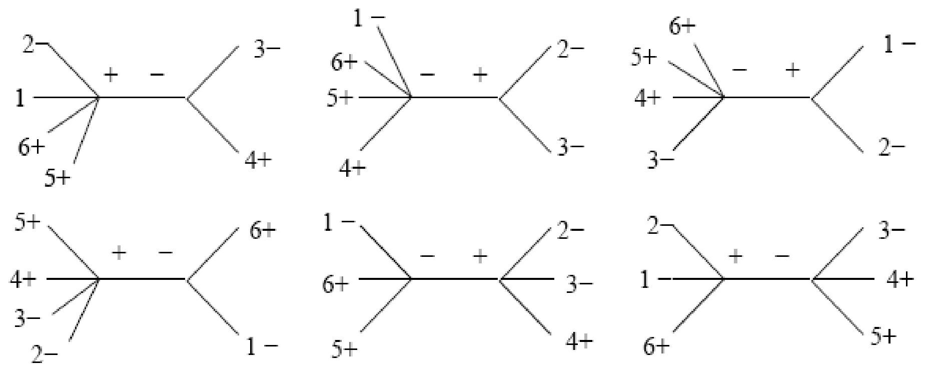
Disconnected picture suggests that in momentum space MHV amplitudes are vertices for building new amplitudes.



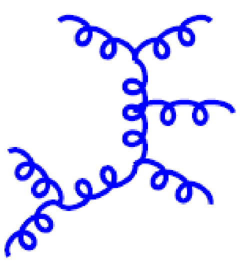
MHV amplitudes as vertices

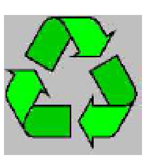
QCD gluon scattering amplitude

$- - - + + +$



$$
\begin{aligned}
A_6(1^-,2^-,3^-,4^+,5^+,6^+) = {} & \frac{\langle 1\,2\rangle^3}{\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 2|\,5+6+1\,|q\rangle\,\langle 5|\,6+1+2\,|q\rangle} \times \frac{1}{s_{34}} \times \frac{\langle 3|\,4\,|q\rangle^3}{\langle 3\,4\rangle\,\langle 4|\,3\,|q\rangle} \\
& + \frac{\langle 1|\,4+5+6\,|q\rangle^3}{\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 4|\,5+6+1\,|q\rangle} \times \frac{1}{s_{23}} \times \frac{\langle 2\,3\rangle^3}{\langle 3|\,2\,|q\rangle\,\langle 2|\,3\,|q\rangle} \\
& + \frac{\langle 3|\,4+5+6\,|q\rangle^3}{\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 6|\,3+4+5\,|q\rangle} \times \frac{1}{s_{12}} \times \frac{\langle 1\,2\rangle^3}{\langle 2|\,1\,|q\rangle\,\langle 1|\,2\,|q\rangle} \\
& + \frac{\langle 2\,3\rangle^3}{\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5|\,2+3+4\,|q\rangle\,\langle 2|\,3+4+5\,|q\rangle} \times \frac{1}{s_{61}} \times \frac{\langle 1|\,6\,|q\rangle^3}{\langle 6\,1\rangle\,\langle 6|\,1\,|q\rangle} \\
& + \frac{\langle 1|\,5+6\,|q\rangle^3}{\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 5|\,6+1\,|q\rangle} \times \frac{1}{s_{561}} \times \frac{\langle 2\,3\rangle^3}{\langle 3\,4\rangle\,\langle 4|\,2+3\,|q\rangle\,\langle 2|\,3+4\,|q\rangle} \\
& + \frac{\langle 1\,2\rangle^3}{\langle 6\,1\rangle\,\langle 2|\,6+1\,|q\rangle\,\langle 6|\,1+2\,|q\rangle} \times \frac{1}{s_{612}} \times \frac{\langle 3|\,4+5\,|q\rangle^3}{\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5|\,3+4\,|q\rangle}
\end{aligned}
$$





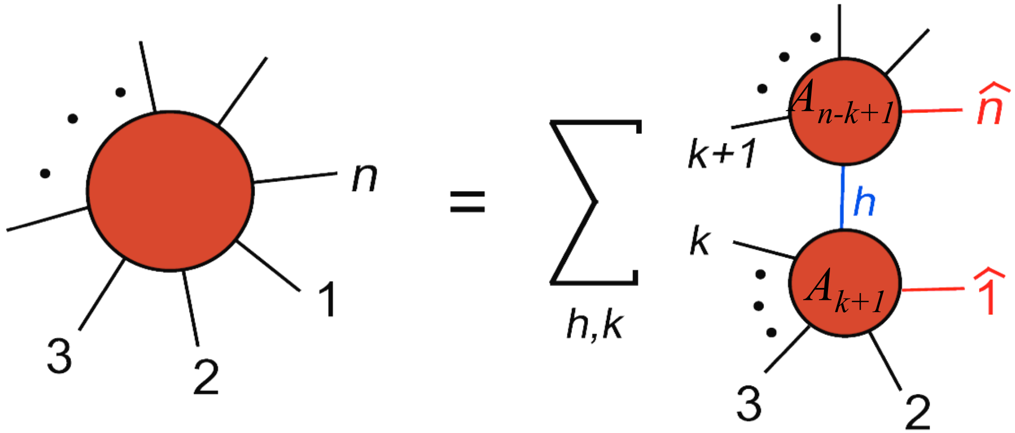
Easy to use

On-Shell Recursion

PRL **94,** 181602 (2005) PHYSICAL REVIEW LETTERS week ending 13 MAY 2005

Direct Proof of the Tree-Level Scattering Amplitude Recursion Relation in Yang-Mills Theory

Ruth Britto, Freddy Cachazo, Bo Feng, and Edward Witten
School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey 08540, USA
(Received 14 February 2005; published 10 May 2005)



Building blocks are on-shell amplitudes Inspired by form of 1 loop amplitudes.

A very general machinery for constructing tree level scattering amplitudes using on-shell recursion relations.

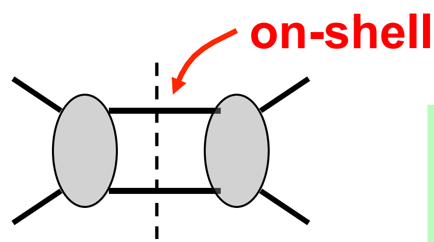
Proof relies on so little. Power comes from generality

Modern Unitarity Method

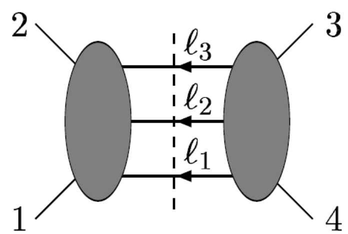
$$E^2 = p^2 + m^2$$

ZB, Dixon, Dunbar and Kosower

Two-particle cut:

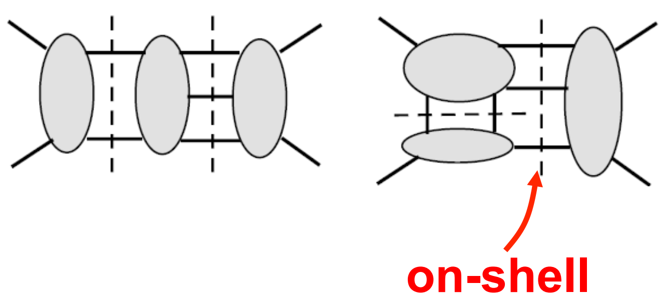


Three-particle cut:



- **Systematic assembly of complete amplitudes from other amplitudes.**
- **Works for any number of particles or loops.**

Generalized unitarity as a practical tool.



ZB, Dixon and Kosower;
ZB, Morgan;
Britto, Cachazo, Feng;
Ossala,Pittau,Papadopoulos;
Ellis, Kunszt, Melnikov;
Forde; Badger
and many others

Reproduces Feynman diagrams except intermediate steps of calculation based on physical quantities *not* unphysical ones.

Modern Unitarity Method

Also paper from
Britto, Cachazo and Feng,
Ossala, Papodopoulos and Pittau (OPP);
Badger; Anastasiou, Feng, Kunszt, Mastrolia;
etc

PHYSICAL REVIEW D **75,** 125019 (2007)

Direct extraction of one-loop integral coefficients

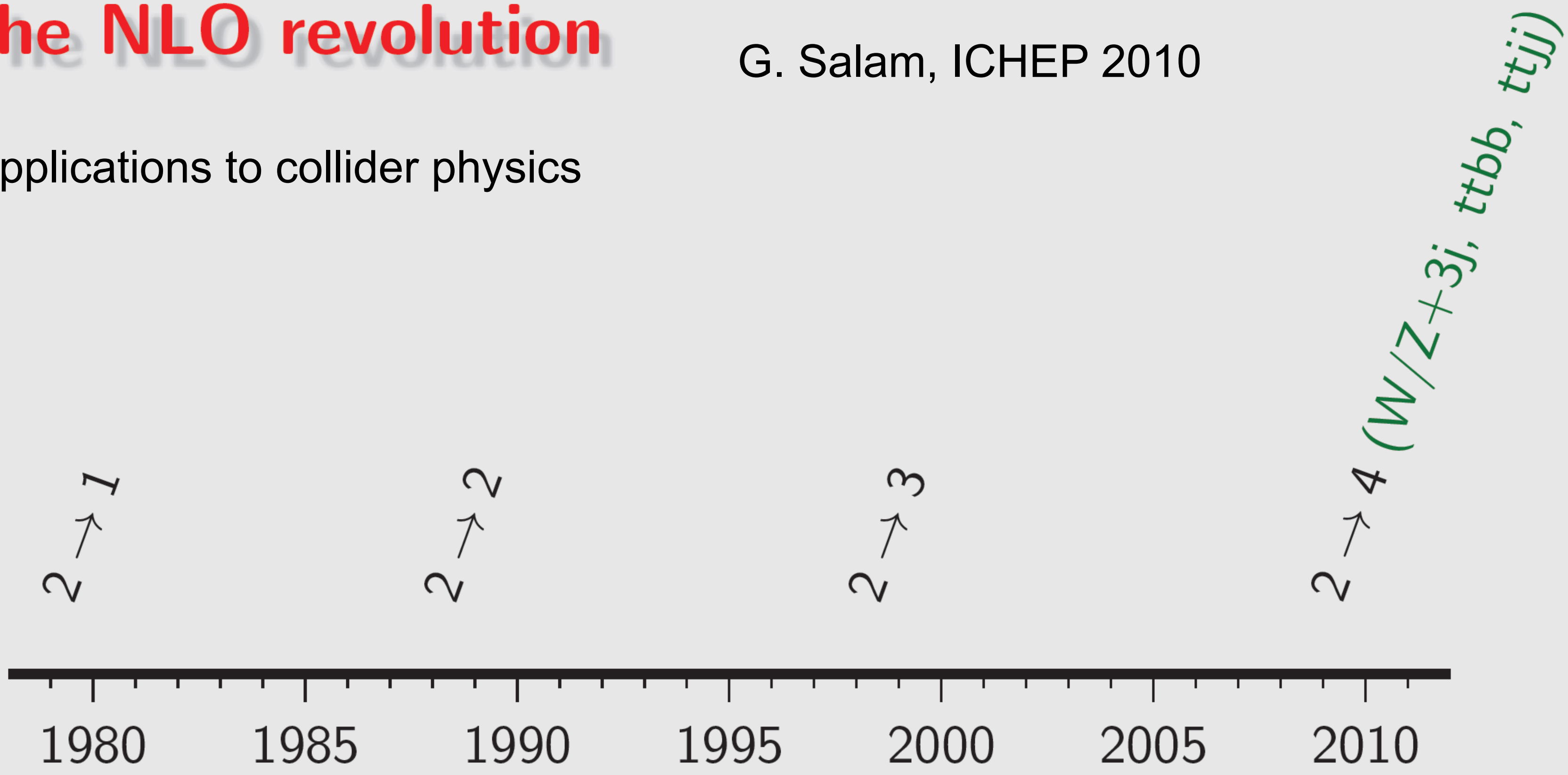
Darren Forde
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309, USA
and Department of Physics and Astronomy, UCLA, Los Angeles, California 90095-1547, USA
(Received 23 April 2007; published 19 June 2007)

We finally had a convenient way to deal with QCD integration, not only susy cases.

The NLO revolution

G. Salam, ICHEP 2010

Applications to collider physics



2009: NLO $W+3j$ [Rocket: Ellis, Melnikov & Zanderighi] [unitarity]

2009: NLO $W+3j$ [BlackHat: Berger et al] [unitarity]

2009: NLO $t\bar{t}b\bar{b}$ [Bredenstein et al] [traditional]

2009: NLO $t\bar{t}b\bar{b}$ [HELAC-NLO: Bevilacqua et al] [unitarity]

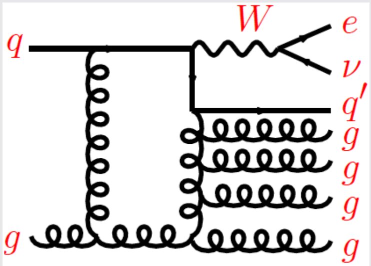
2009: NLO $q\bar{q} \to b\bar{b}b\bar{b}$ [Golem: Binoth et al] [traditional]

2010: NLO $t\bar{t}jj$ [HELAC-NLO: Bevilacqua et al] [unitarity]

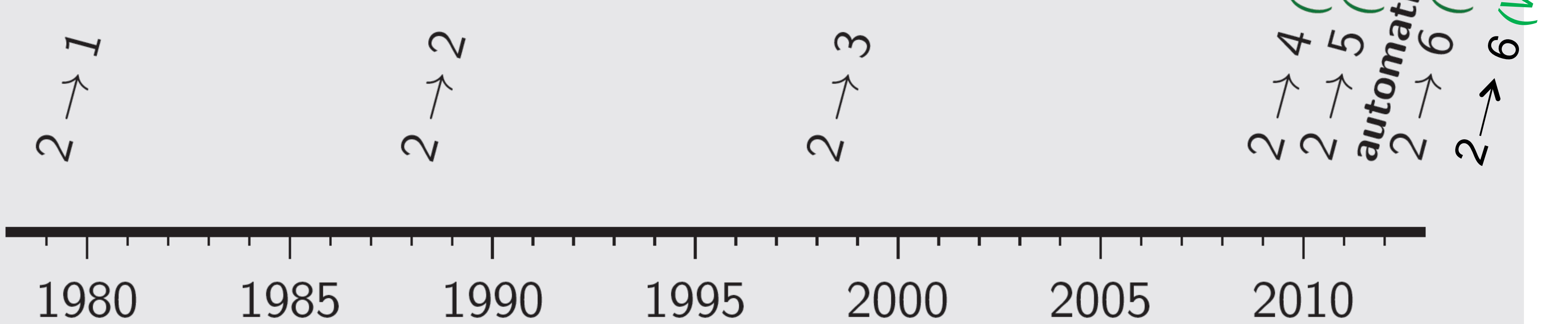
2010: NLO $Z+3j$ [BlackHat: Berger et al] [unitarity]

NLO timeline

G. Salam, La Thuile 2012



Impressive progress has continued during past decade at N^{2}LO and N^{3}LO
Talks from Badger, Duhr, Signorile-Signorile, Vita



2010: NLO W+4j [BlackHat+Sherpa: Berger et al] [unitarity]
2011: NLO $WWjj$ [Rocket: Melia et al] [unitarity]
2011: NLO Z+4j [BlackHat+Sherpa: Ita et al] [unitarity]
2011: NLO 4j [BlackHat+Sherpa: Bern et al] [unitarity]
2011: first automation [MadNLO: Hirschi et al] [unitarity + feyn.diags]
2011: first automation [Helac NLO: Bevilacqua et al] [unitarity]
2011: first automation [GoSam: Cullen et al] [feyn.diags(+unitarity)]
2011: $e^+e^- \to 7j$ [Becker et al, leading colour] [numerical loops]
2013: NLO W+5j [BlackHat+Sherpa: Bern et al] [unitarity]

Curiosities at Higher Loops

Examples of curiosities:

1. **Two loop four-point similar to (one-loop)^2 in $\mathcal{N} = 4$ sYM**

2. **Some similarities between planar and nonplanar integrands**

Curiosities at Higher Loops

VOLUME 91, NUMBER 25 PHYSICAL REVIEW LETTERS week ending 19 DECEMBER 2003

Planar Amplitudes in Maximally Supersymmetric Yang-Mills Theory

C. Anastasiou and L. Dixon

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Z. Bern

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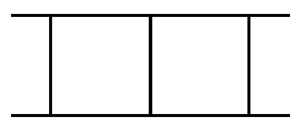
D. A. Kosower

Service de Physique, Centre d'Etudes de Saclay, F-91191 Gif-sur-Yvette cedex, France

(Received 7 September 2003; published 18 December 2003)



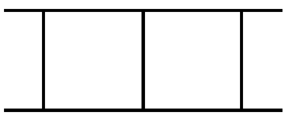
- **Worked out 2 loop 4 point amplitude in N = 4 sYM in terms of polylogs.**
- **Guessed *n*-point MHV**
- **Proposed resummation to all loop orders possible.**

Bold Guessing: Loop Iteration of the Amplitude

The planar four-point two-loop amplitude undergoes fantastic simplification.

Anastasiou, Bern, Dixon, Kosower

$$M_4^{\text{2-loop}}(s,t) = \frac{1}{2}\Big(M_4^{\text{1-loop}}(s,t)\Big)^2 + f(\epsilon) M_4^{\text{1-loop}}(s,t)\big|_{\epsilon\to 2\epsilon} - \frac{1}{2}\zeta_2^2$$



$$M_4^{\text{loop}}(s,t) = A_4^{\text{loop}}/A_4^{\text{tree}} \qquad f(\epsilon) = -\zeta_2 - \zeta_3\epsilon - \zeta_4\epsilon^2 \qquad D = 4 - 2\epsilon$$

$f(\epsilon)$ function related to IR singularities

Proposed that we might resum to all orders.

- **From 2 terms in a series we proposed to resum to all orders when no one had yet resummed even simpler anomalous dimensions.**

Early talks received a rather frosty reception.

1. **"So what?" Way too little evidence to suggest we can resum.**
2. **"*N* = 4 is a conformal field theory. *S* matrices don't make sense."**
3. **Our boldness seemed crazy. Is this an accident?**

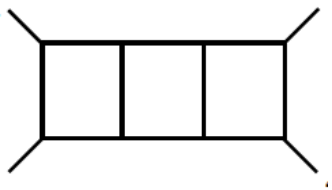
Arguing a waste of time. "Shut up and calculate"

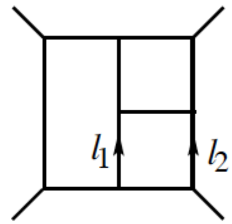
BDS Ansatz

ZB, Dixon and Smirnov (BDS)

- **Needed more evidence.**
- **Needed a precise all loop resummation formula to compare to string theory.**

“Does anyone need this integral?”





Evaluated this $N = 4$ amplitude, nontrivial

$$M_4^{\text{3-loop}}(\epsilon) = -\frac{1}{3}\Big[M_4^{\text{1-loop}}(\epsilon)\Big]^3 + M_4^{\text{1-loop}}(\epsilon)\, M_4^{\text{2-loop}}(\epsilon) + f^{\text{3-loop}}(\epsilon)\, M_4^{\text{1-loop}}(3\,\epsilon) + C^{(3)} + \mathcal{O}(\epsilon)$$

$$f^{\text{3-loop}}(\epsilon) = \frac{11}{2}\zeta_4 + \epsilon(6\zeta_5 + 5\zeta_2\zeta_3) + \epsilon^2(c_1\zeta_6 + c_2\zeta_3^2)$$

- **IR singularities complicate this. Subtract the IR singularties.**
- **With three terms can now resum in loops.**
- **We can bootstrap in legs by demanding proper collinear behavior.**

All order finite remainder

$$\mathcal{F}_n = \exp\left[\frac{1}{4}\gamma_K F_n^{(1)} + C\right]$$

Beisert-Eden-Staudacher cusp anomalous dimension

One- loop finite part

- **Alday and Maldacena confirmed this at strong coupling at four points using string theory.**
- **At 6 points and beyond this guess is wrong. Still a very active area of research.**
- **This was then understood in terms of dual conformal symmetry. “Trivial part”.**

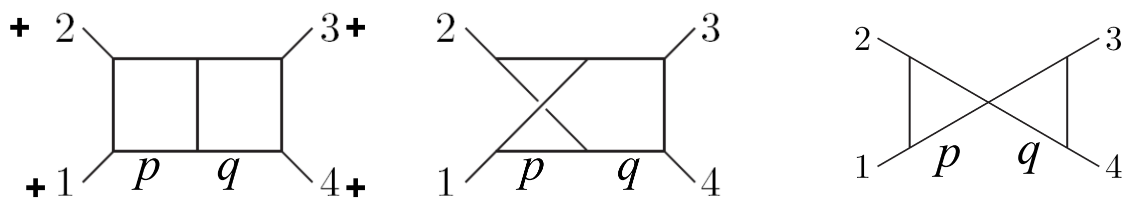
Lesson: Be bold (but not too bold)

See talk from Volovich

A Two-Loop Hint

Slide is from 2006 talk at Zurich QCD conference

Consider the four gluon all-positive helicity amplitude in QCD. This is the simplest example. If we cant find simplicity here there is no hope for any other QCD amplitudes.



$$p^2 = p_{[4]}^2 - \lambda_p^2$$
$$q^2 = q_{[4]}^2 - \lambda_q^2$$

If you expand it in polylogs it is some moderate mess.
Instead let's write it in a special basis of integrals

$D_s = 4 - 2\epsilon\delta_R$

$$A_{1234}^{\rm P} = i\,\frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}\Big\{ s_{12}\,\mathcal{I}_4^{\rm P}\Big[(D_s-2)(\lambda_p^2\,\lambda_q^2 + \lambda_p^2\,\lambda_{p+q}^2 + \lambda_q^2\,\lambda_{p+q}^2) + 16\Big((\lambda_p\cdot\lambda_q)^2 - \lambda_p^2\,\lambda_q^2\Big)\Big](s_{12}, s_{23})$$
$$\cancel{+\,4(D_s-2)\,\mathcal{I}_4^{\text{bow-tie}}[(\lambda_p^2+\lambda_q^2)(\lambda_p\cdot\lambda_q)](s_{12})}$$
$$\cancel{+\,\frac{(D_s-2)^2}{s_{12}}\,\mathcal{I}_4^{\text{bow-tie}}\Big[\lambda_p^2\,\lambda_q^2\,((p+q)^2+s_{12})\Big](s_{12},s_{23})}\Big\}\,.$$

planar

Bern, Dixon, Kosower
hep-th/0001001

non-planar

$$A_{12;34}^{\rm NP} = i\,\frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}\,s_{12}\mathcal{I}_4^{\rm NP}\Big[(D_s-2)(\lambda_p^2\,\lambda_q^2 + \lambda_p^2\,\lambda_{p+q}^2 + \lambda_q^2\,\lambda_{p+q}^2) + 16\Big((\lambda_p\cdot\lambda_q)^2 - \lambda_p^2\,\lambda_q^2\Big)\Big](s_{12}, s_{23})$$

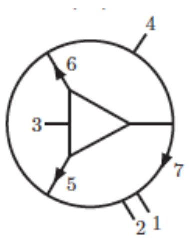
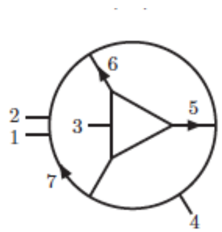
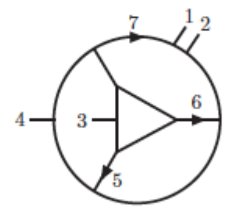
Why do the planar and non-planar double boxes look the same? I believe this is a clue.

A clue for what? Audience not impressed!

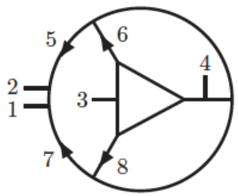
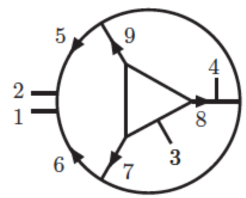
Much Better Clue: $\mathcal{N} = 4$ sYM

ZB, Carrasco, Dixon, Johansson, Roiban

$\mathcal{N} = 4$ sYM at 4 loops.

$s_{12}s_{56}s_{47}$  $s_{12}s_{56}s_{47}$  $s_{12}s_{56}s_{47}$ 

Simple relations between kinematic numerators visible

$s_{12}(s_{35}s_{38} - s_{37}s_{36})$  $s_{12}(s_{45}s_{38} - s_{12}s_{37}) - l_9^2 s_{12}s_{45} - l_6^2 l_8^2 s_{12}$ 

Some less clear

It seemed that for many kinematic numerators some relations that seemed similar to color Jacobi identities visible, though not always.

- **Key insight: If it exists at 4 loops it must exist at tree level.**
- **Once you know what you are looking for it becomes a lot simpler.**
- **Note: this could still have be curiosity without much significance.**

Color Kinematics and Double Copy

ZB, Carrasco, Johansson

See talks from Carrasco, Cheung, Elvang, Paranjape, Vazqueze-Holm, Keeler, Teng

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New relations for gauge-theory amplitudes

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The real significance is for gravity

Started from curiosities in 4 loop $N = 4$ integrands!

If we can arrange:

$$c_i + c_j + c_j = 0 \quad \Leftrightarrow \quad n_i + n_j + n_j = 0$$

gauge theory

$$\mathcal{A}_n^{\text{tree}}(1,2,3,\ldots,n) = \sum_i \frac{n_i c_i}{(\prod_j p_j^2)_i}$$

$$\tilde{\mathcal{A}}_n^{\text{tree}}(1,2,3,\ldots,n) = \sum_i \frac{\tilde{n}_i c_i}{(\prod_j p_j^2)_i}$$

gravity double copy

$$-iM_n^{\text{tree}}(1,2,3,\ldots,n) = \sum_i \frac{n_i \tilde{n}_i}{(\prod_j p_j^2)_i}$$

Seems to also hold at loop level but still no proof. In some cases extends to classical solutions.

Extremely important to track down origin of curiosities. Choose wisely.

Summary Comments

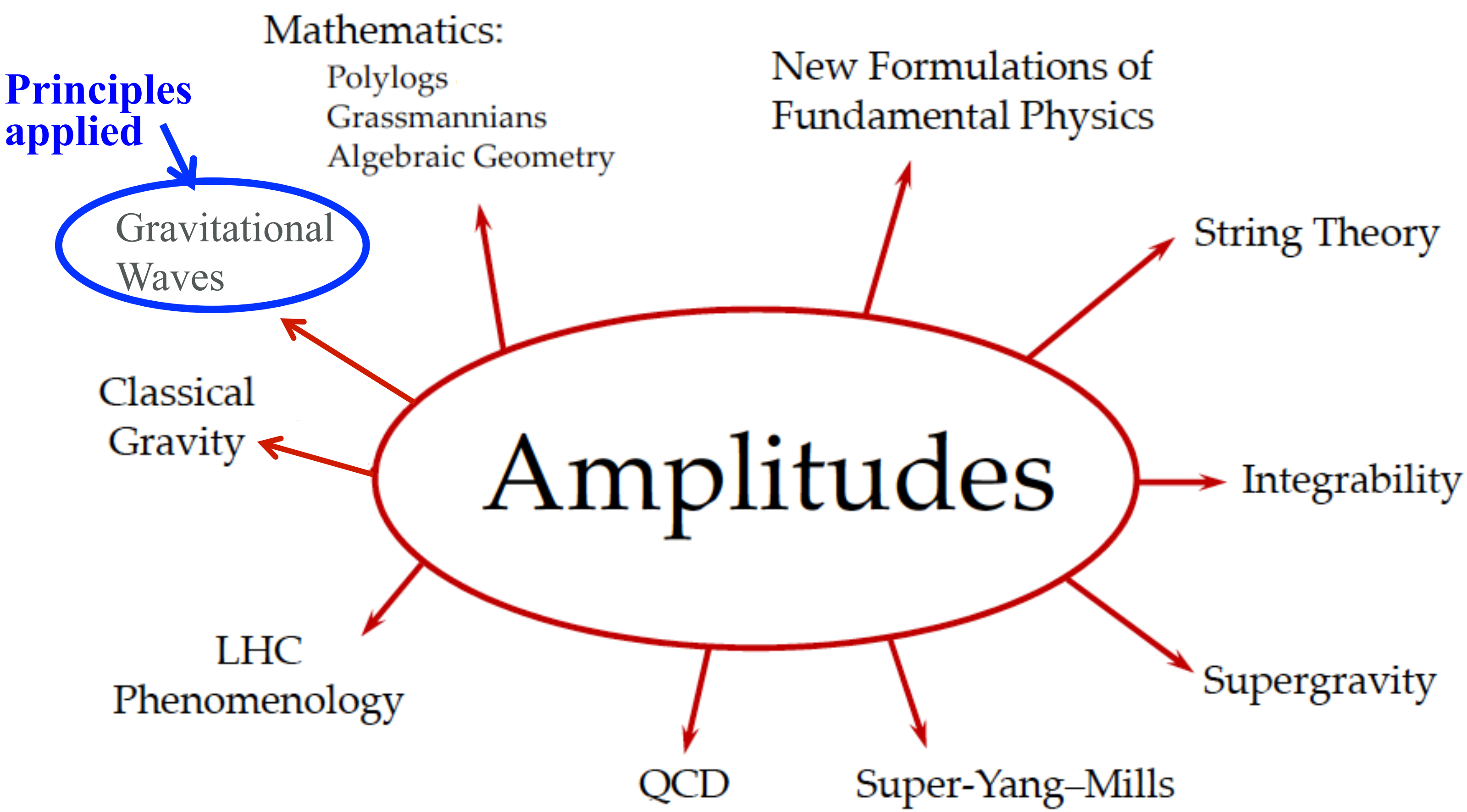
"A method is more important than a discovery, since the right method will lead to new and even more important discoveries." *—Lev Landau*

Past success came from following principles:

1) **Amplitudes are smarter than we are. Use data as a guide.**
2) **Take inspiration from real world physics.**
3) **Understand curiosities.**
4) **Find connection to other subfields. Very important!**
5) **Search for new structures.**
6) **Make bold guesses (but be careful and back up with calculations).**
7) **Push the state of the art.**

Looking Forward to New Directions

Looking at the broad range of talks, our field has blossomed in many directions.



Two new directions which fit well with principles

1. **SMEFT** — **Talk from Shadmi**
2. **Bounds on EFT Coefficients** — **Talks from Caron-Huot, Huang, de Rham**

Standard Model EFT

Buchmuller and Wyler; Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek

See Shadmi's talk

- **Difficult to find BSM physics at the LHC.**
- **Need open mind: quantified by constructing EFT's for BSM physics.**
- **For good reason SMEFT and its cousin HEFT are becoming more popular.**

$$\mathcal{L}_{\mathrm{SM}} + \Delta\mathcal{L} \qquad \Delta\mathcal{L} = \frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)}$$

For example dimension 6 operators

At dimension 6: 59 independent operators, not including flavor indices

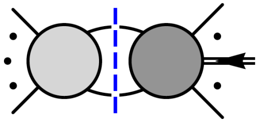
$$F^3, \quad \phi^2 F^2, \quad D^2\phi^4, \quad \psi^4,$$ etc

Basic idea is simple: Parametrize new physics using EFTs

Anomalous Dimension Matrix Zeros in SMEFT

Unexpected zeros in one-loop anomalous dimension matrix
Followed a pattern analogous to susy.
Explanations from helicity and angular momentum.

Alonso, Jenkins, Manohar, Trott;
Elias-Miro, Espinoza, Pamarol
Cheung and Shen
Jaing, Shu, Xiao, Zheng



operator insertion (columns); renormalized operator (rows)

	$(w,\bar w)$	F^3 $(0,6)$	$F^2\phi^2$ $(2,6)$	$F\psi^2\phi$ $(2,6)$	ψ^4 $(2,6)$	$\psi^2\phi^3$ $(4,6)$	$\bar F^3$ $(6,0)$	$\bar F^2\phi^2$ $(6,2)$	$\bar F\bar\psi^2\phi$ $(6,2)$	$\bar\psi^4$ $(6,2)$	$\bar\psi^2\phi^3$ $(6,4)$	$\bar\psi^2\psi^2$ $(4,4)$	$\bar\psi\psi\phi^2 D$ $(4,4)$	ϕ^4D^2 $(4,4)$	ϕ^6 $(6,6)$
F^3	$(0,6)$			×	×	×			×	×	×	×	×	×	×
$F^2\phi^2$	$(2,6)$				×	×				×	×	×			×
$F\psi^2\phi$	$(2,6)$									×				×	×
ψ^4	$(2,6)$	×	×			×	×	×	×	×	×	y^2		×	×
$\psi^2\phi^3$	$(4,6)$	$\times^*$									y^2				×
$\bar F^3$	$(6,0)$			×	×	×			×	×	×	×	×	×	×
$\bar F^2\phi^2$	$(6,2)$				×	×				×	×	×			×
$\bar F\bar\psi^2\phi$	$(6,2)$				×									×	×
$\bar\psi^4$	$(6,2)$	×	×	×	×	×	×	×			×	$\bar y^2$		×	×
$\bar\psi^2\phi^3$	$(6,4)$					$\bar y^2$	$\times^*$								×
$\bar\psi^2\psi^2$	$(4,4)$		×		$\bar y^2$	×		×		y^2	×			×	×
$\bar\psi\psi\phi^2 D$	$(4,4)$														×
ϕ^4D^2	$(4,4)$				×					×		×			×
ϕ^6	$(6,6)$	$\times^*$		×	×		$\times^*$		×	×		×			

On-shell methods explain the structure

Grey are zeros. x means trivial zero.

from Cheung and Shen

- **One-loop anomalous dimension matrix has a surprising number of zeros!**
- **Amazingly new zero appear at all orders even though Feynman diagrams exist.**

ZB, Parra-Martinez and Sawyer

Opportunity: Beside helping with cross-section calculations uncover new structures

EFT Coefficient Bounds

Consider gravity EFT:

See talks from Caron-Huot, Huang, de Rham

$$S_{\rm EFT} = \int d^4x\sqrt{-g}\left[-\frac{2}{\kappa^2}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{2\beta_\phi}{\kappa^2}\phi\, C + \frac{8}{\kappa^3}\frac{\beta_{R^3}}{3!}R^3 + \frac{2\beta_{R^4}}{\kappa^4}C^2 + \frac{2\tilde\beta_{R^4}}{\kappa^4}\tilde C^2 + \ldots\right]$$

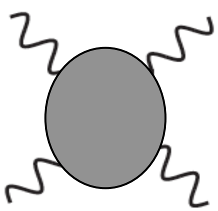
$$R^3 \equiv R^{\mu\nu\kappa\lambda}R_{\kappa\lambda\alpha\gamma}R^{\alpha\gamma}{}_{\mu\nu} \qquad C \equiv R^{\mu\nu\kappa\lambda}R_{\mu\nu\kappa\lambda} \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

What EFT coefficients are physically allowed?

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi (2006)

Many others: Bellazzini, Camanho, Caron-Huot, Chaing, ChandrasekaranCheung, de Rham, Dubovsky, Edelstein, Huang, Huang, Li, Maldecena, Mazac, Melville, Miro, Nicolis, Rastelli, Rattazzi, Remmen, Riembau, Riva, Shahbazi-Moghaddam, Rodina, Simmons-Duffin, Tolley, Van Duong, Weng, Zhiboedov, Zhou, etc.

To study bounds want to look at a gauge-invariant quantities:



2 to 2 scattering amplitudes

- **Avoids issues with field redefinitions and gauge fixing.**
- **Unitarity, crossing and dispersion relations natural tools.**

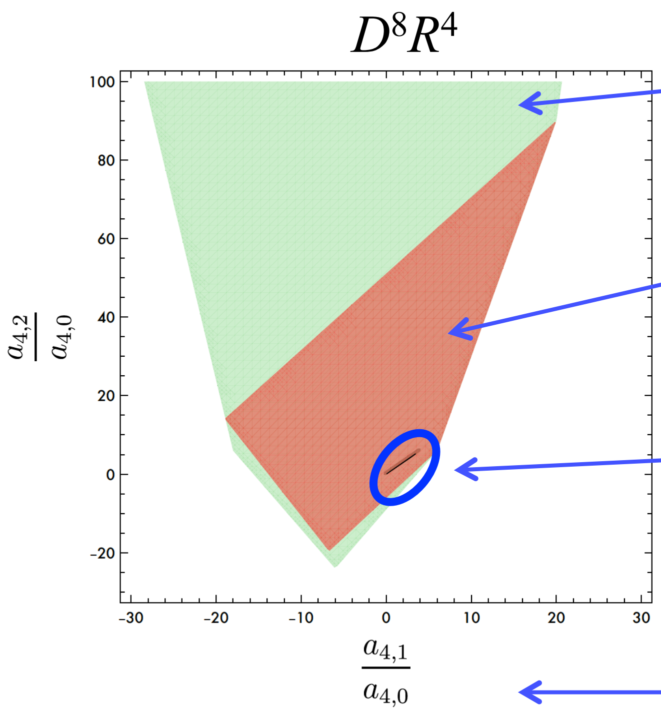
An opportunity: Scattering amplitudes are the natural language

A Curiosity: Tiny Theory Islands

ZB, Kosmopoulos, Zhiboedov

See talks from Caron-Huot, Huang, de Rham

Consider a 4 graviton amplitude in an EFT



Green region from unitarity and positivity.

Reddish region is two sided bound from unitarity and crossing.

A really tiny black region is the observed physical theory island for multiple EFT examples from string and field theory

Essentially ratio of Wilson coefficients.

Evidence from explicit examples suggests that physically sensible weakly coupled gravity EFTs all lie on tiny islands.

Is this a little curiosity or is this of fundamental importance?

Summary Comments

"A method is more important than a discovery, since the right method will lead to new and even more important discoveries." *—Lev Landau*

Past success came from following principles:

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7) **Push the state of the art.**

Following these principles, I am very confident the field will continue to thrive for many years to come.