Soft theorems and massive recursion relations

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(#2005.08981 with A. Falkowski)

DESY.

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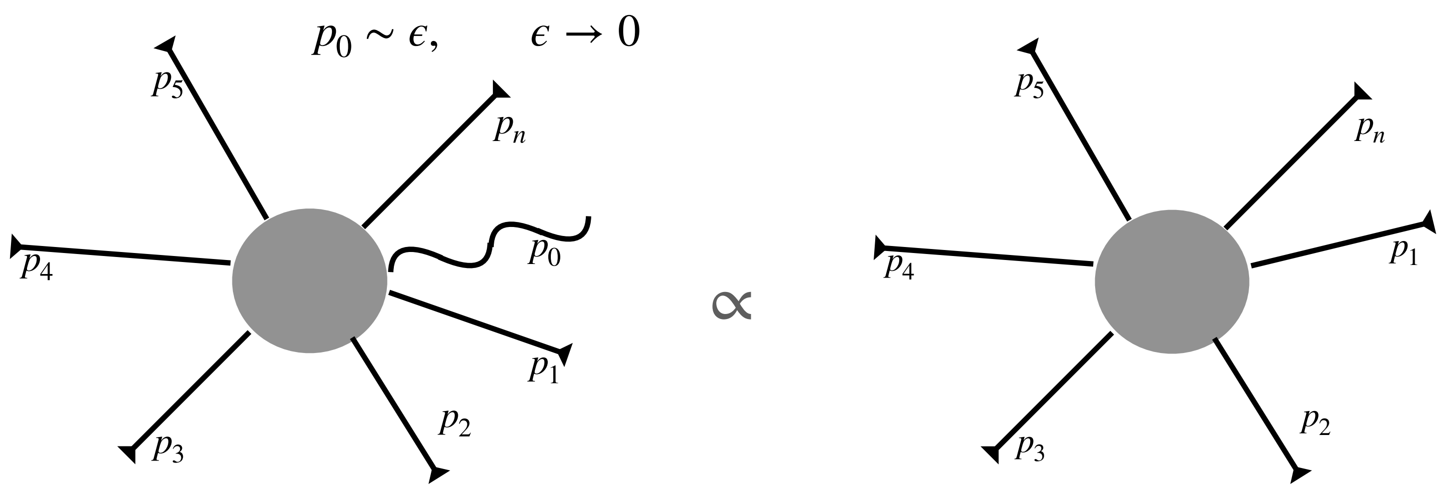
Soft Amplitudes



$$\mathcal{M}_{n+1}^{\pm 1} = \left(\frac{S_{\pm 1}^{(0)}}{\epsilon^2} + \frac{S_{\pm 1}^{(1)}}{\epsilon} \right) \mathcal{M}_n \,, \qquad \mathcal{M}_{n+1}^{\pm 2} = \left(\frac{S_{\pm 2}^{(0)}}{\epsilon^3} + \frac{S_{\pm 2}^{(1)}}{\epsilon^2} + \frac{S_{\pm 2}^{(2)}}{\epsilon} \right) \mathcal{M}_n .$$

- Connection with basic properties such as charge conservation and equivalence principle Weinberg '65 Elvang, Jones, Naculich '16
- Connection with asymptotic symmetries Strominger '17
- Connection with Adler zero and special EFTs Rodina '18 Cheung et al '16 Elvang et al '18 …

Soft Theorems via recursion relations

EFTs

Soft Graviton: $$M_{n+1} = \left(\frac{S^{(0)}}{\epsilon^3} + \boxed{\frac{S^{(1)}}{\epsilon^2} + \frac{S^{(2)}}{\epsilon}}\right) M_n + \boxed{\frac{\tilde{S}^{(2)}}{\epsilon}\tilde{M}_n}$$

Weinberg '64

Cachazo, Strominger '14

BCFW rec. relations

Elvang, Jones, Naculich '16

Soft + BCFW shift

assume only massless particles

Recursion relations using massive spinors?

What can we learn from soft theorems including masses?

Summary

1. Extend the soft+BCFW shift allowing massive particles;

2. Soft Theorems with massive spinors
    a. Gravitational multipole
    b. Soft Exponentiation

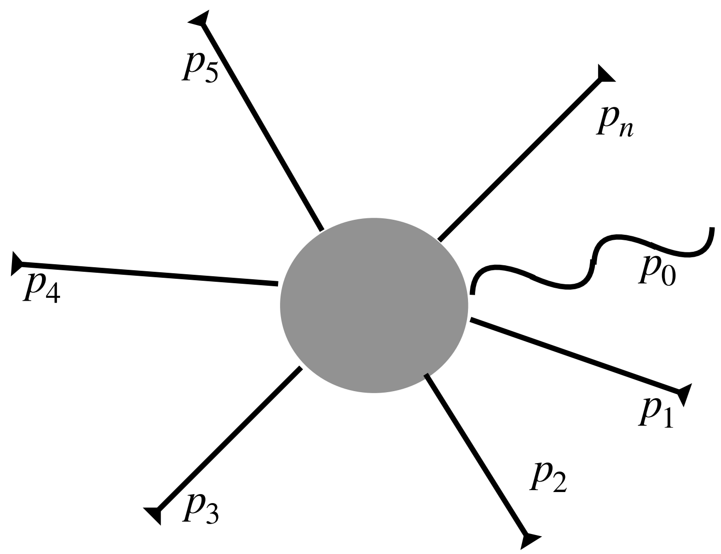
3. (Gravitational) Compton scattering and spurious poles

1. Massive recursion

$$p_0,\ p_1,\ p_2,\ \cdots\ p_{n-1},\ p_n$$

$$p_0^2 = 0, \quad p_i^2 = m_i^2, \quad i > 0$$

$$\sum_{j=0}^{n} p_j = 0$$



$$\hat{p}_0 = \epsilon p_0 - z q_0$$

$$\hat{p}_j = p_j - \frac{(\epsilon - 1)(p_0 p_k) - z(q_0 p_k)}{q_j p_k} q_j$$

$$\hat{p}_k = p_k - \frac{(\epsilon - 1)(p_0 p_j) - z(q_0 p_j)}{q_k p_j} q_k$$

ϵ is a small parameter controlling soft limit on the massless particle 0

z is a holomorphic variable and an auxiliary tool to derive recursion relations

$$q_0^2 = q_j^2 = q_k^2 = q_0 p_0 = q_j p_j = q_k p_k = 0$$ On-shell conditions

$$q_j p_0 = q_k p_0 = q_j q_k = q_0 q_j = q_0 q_k = 0$$ Momentum conservation

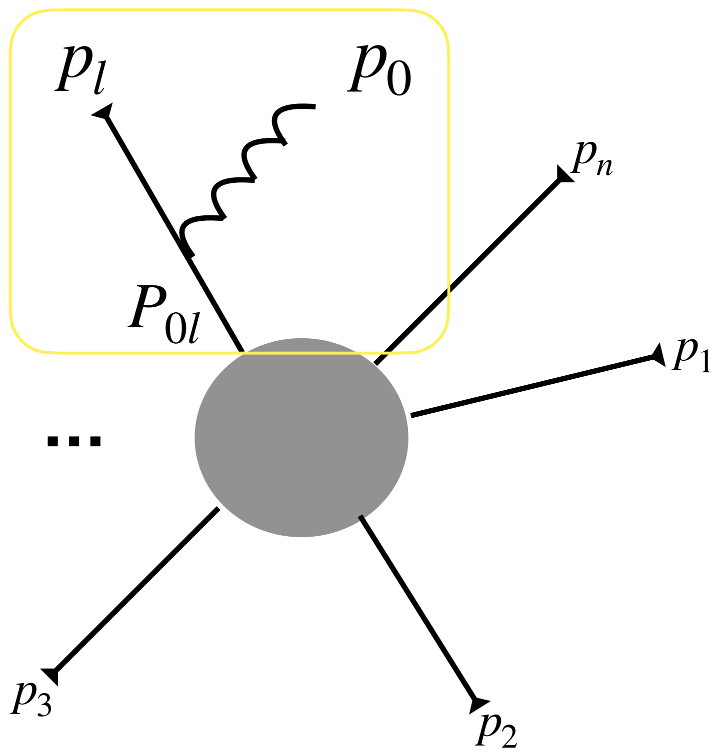
We can solve for the shift spinors **q** for the cases where the shifted particles are massive or massless (for all massless we recover the same shift as Elvang, Jones, Naculich)

$$q_0\sigma = y\tilde{\lambda}_0, \qquad q_j\sigma = p_j\sigma\tilde{\lambda}_0\tilde{\lambda}_0, \qquad q_k\sigma = p_k\sigma\tilde{\lambda}_0\tilde{\lambda}_0,$$

$$\{\mathbf{0}\bar{\mathbf{m}}\bar{\mathbf{m}}\}: \qquad \begin{aligned} \lambda_0^z &= \epsilon\lambda_0 - zy, \\ \tilde{\chi}_j^z &= \tilde{\chi}_j + \frac{(\epsilon-1)\langle 0p_k0] - z\langle yp_k0]}{[0p_jp_k0]}[\mathbf{j}0]\tilde{\lambda}_0, \\ \tilde{\chi}_k^z &= \tilde{\chi}_k - \frac{(\epsilon-1)\langle 0p_j0] - z\langle yp_j0]}{[0p_jp_k0]}[\mathbf{k}0]\tilde{\lambda}_0, \end{aligned}$$

$$\{\mathbf{0}\bar{\mathbf{m}}\bar{\mathbf{0}}\} \qquad \begin{aligned} \lambda_0^z &= \epsilon\lambda_0 - zy, \\ \tilde{\chi}_j^z &= \tilde{\chi}_j - \frac{(\epsilon-1)\langle k0\rangle - z\langle ky\rangle}{\langle kp_j0]}[\mathbf{j}\tilde{0}]\tilde{\lambda}_0, \\ \tilde{\lambda}_k^z &= \tilde{\lambda}_k - \frac{(\epsilon-1)\langle 0p_j0] - z\langle yp_j0]}{\langle kp_j0]}\tilde{\lambda}_0. \end{aligned}$$

(we use the massive spinor formalism of Arkani-Hamed, Huang and Huang, so the SU(2) little group indices are omitted and bold spinors are massive)



Poles related to emission of massless particle from external legs

$$\hat{P}_{0l}^2 \equiv (\hat{p}_0 + p_l)^2 = m_l^2 \leftrightarrow 2\hat{p}_0 p_l = 0 \leftrightarrow z_l = \epsilon \frac{p_0 p_l}{q_0 p_l}$$

$$\hat{M}^{z,\epsilon}(1 \ldots n 0)\,|_{\hat{P}_{0l}^2 \to m_l^2} = -\frac{\hat{M}^{z_l,\epsilon}(1 \ldots P_{0l} \ldots n)\hat{M}^{z_l,\epsilon}(-P_{0l} l 0)}{\hat{P}_{0l}^2 - m_l^2} = \frac{\hat{M}^{z_l,\epsilon}(1 \ldots P_{0l} \ldots n)\hat{M}^{z_l,\epsilon}(-P_{0l} l 0)}{2 q_0 p_l (z - z_l)}$$

$$\frac{\mathrm{Res}_{z \to z_l} \hat{M}^{z,\epsilon}(0 \ldots n)}{z_l} = \frac{1}{\epsilon} \frac{\hat{M}^{z_l,\epsilon}(1 \ldots P_{0l} \ldots n)\hat{M}^{z_l,\epsilon}(-P_{0l} l 0)}{2 p_0 p_l}$$

$$\boxed{\hat{M}^{0,\epsilon}(\hat{1} \ldots \hat{n}\, \hat{0}) = -\frac{1}{\epsilon} \sum_{l=1}^{n} \frac{\hat{M}^{z_l,\epsilon}(\hat{1} \ldots \hat{P}_{0l} \ldots \hat{n})\hat{M}^{z_l,\epsilon}(-\hat{P}_{0l} \hat{l} \hat{0})}{2 p_0 p_l} + \mathcal{O}(\epsilon^0)}$$

2. Massive Soft Theorems

Minimal gravity: $M(120^+) = -\frac{1}{M_{\text{Pl}}}\frac{\langle\zeta p_1 0]^2}{\langle 0\zeta\rangle^2}\left(\frac{\langle\mathbf{21}\rangle}{m}\right)^{2S}$

$$\{\mathbf{0}\bar{\mathbf{m}}\bar{\mathbf{m}}\}: \quad \begin{aligned} \lambda_0^z &= \epsilon\lambda_0 - zy, \\ \tilde{\chi}_j^z &= \tilde{\chi}_j + \frac{(\epsilon-1)\langle 0p_k0] - z\langle yp_k0]}{[0p_jp_k0]}[\mathbf{j}0]\tilde{\lambda}_0, \\ \tilde{\chi}_k^z &= \tilde{\chi}_k - \frac{(\epsilon-1)\langle 0p_j0] - z\langle yp_j0]}{[0p_jp_k0]}[\mathbf{k}0]\tilde{\lambda}_0, \end{aligned}$$

$$\hat{M}^{0,\epsilon}(\hat{1}\ldots\hat{n}\,\hat{0}) = -\frac{1}{\epsilon}\sum_{l=1}^{n}\frac{\hat{M}^{z_l,\epsilon}(\hat{1}\ldots\hat{P}_{0l}\ldots\hat{n})\boxed{\hat{M}^{z_l,\epsilon}(-\hat{P}_{0l}\hat{l}\hat{0})}}{2p_0p_l} + \mathcal{O}(\epsilon^0)$$

Expanding both sides in powers of ϵ

$$\hat{M}^{0,\epsilon}(1\ldots n0^+) = \left(\frac{S_{+2}^{(0)}}{\epsilon^3} + \frac{S_{+2}^{(1)}}{\epsilon^2} + \frac{S_{+2}^{(2)}}{\epsilon^1}\right)\hat{M}^{0,0}(1\ldots n) + \mathcal{O}\left(\frac{1}{\epsilon^0}\right)$$

Minimal gravity:

$$\hat{M}^{0,\epsilon}(1\ldots n0^+) = \left(\frac{S^{(0)}_{+2}}{\epsilon^3} + \frac{S^{(1)}_{+2}}{\epsilon^2} + \frac{S^{(2)}_{+2}}{\epsilon^1}\right)\hat{M}^{0,0}(1\ldots n) + \mathcal{O}\left(\frac{1}{\epsilon^0}\right)$$

$$S^{(0)}_{+2} = \frac{1}{M_{\mathrm{Pl}}}\sum_{l=1}^{n}\frac{\langle y p_l 0]^2}{\langle 0y\rangle^2(2p_0p_l)}$$

Independent of y due equivalence principle and momentum conservation

$$\frac{\partial}{\partial y}S^{(0)}_{+2} \sim \frac{y}{\langle y0\rangle^3}\sum_{l=1}^{n}\langle y p_l 0] = -\frac{y}{\langle y0\rangle^3}\langle y p_0 0] = 0$$

$$S^{(1)}_{+2} = \frac{1}{M_{\mathrm{Pl}}}\sum_{l=1}^{n}\frac{\langle y p_l 0]}{\langle 0y\rangle(2p_0p_l)}\tilde{\mathscr{D}}_l$$

Differential operator independent of y due angular momentum conservation

$$S^{(2)}_{+2} = \frac{1}{4M_{\mathrm{pl}}}\sum_{l=1}^{n}\frac{1}{p_0p_l}\tilde{\mathscr{D}}_l^2$$

Differential operator manifestly independent of y

$$\frac{\partial}{\partial y}S^{(1)}_{+2} \sim i\frac{(y\sigma^\mu\tilde{\lambda}_0)y}{\langle y0\rangle^3}p_0^\nu\sum_{l=0}^{n}J_l^{\mu\nu} = 0 \qquad J_l^{\mu\nu} \equiv \frac{i}{4}\left(\chi_l\sigma^\mu\bar{\sigma}^\nu\frac{\partial}{\partial\chi_l} - \chi_l\sigma^\nu\bar{\sigma}^\mu\frac{\partial}{\partial\chi_l}\right)$$

$$\tilde{\mathscr{D}}_l = [0\mathbf{l}][0\partial_{\mathbf{l}}] + \frac{[0p_kp_l0]}{[0p_jp_k0]}[0\mathbf{j}][0\partial_{\mathbf{j}}] - \frac{[0p_jp_l0]}{[0p_jp_k0]}[0\mathbf{k}][0\partial_{\mathbf{k}}]$$

Non-Minimal gravity:

Gravitational-dipole

$$M(120^+) = -\frac{1}{M_{\text{Pl}}}\frac{\langle\zeta p_1 0]^2}{\langle 0\zeta\rangle^2}\left\{\left(\frac{\langle\mathbf{21}\rangle}{m}\right)^{2S} + \boxed{\frac{d}{m^3}\frac{\langle\zeta p_1 0]}{\langle 0\zeta\rangle}\left(\frac{\langle\mathbf{21}\rangle}{m}\right)^{2S-1}\langle\mathbf{1}0\rangle\langle\mathbf{2}0\rangle} + \ldots\right\}$$

$$\hat{M}^{0,\epsilon}(1\ldots n0^+) = \left(\frac{S^{(0)}_{+2}}{\epsilon^3} + \frac{S^{(1)}_{+2}}{\epsilon^2}\right)\hat{M}^{0,0}(1\ldots n) + \mathcal{O}\left(\frac{1}{\epsilon^1}\right)$$

Not modified

$$S^{(1)}_{+2} = \frac{1}{M_{\text{Pl}}}\sum_{l=1}^{n}\frac{\langle y p_l 0]}{(2p_0p_l)\langle 0y\rangle}\left[\delta^{\vec{J}}_{\vec{L}}\tilde{\mathscr{D}}_l + \frac{d_l}{m_l^2}\delta^{J_2\ldots J_{2S_l}}_{L_2\ldots L_{2S_l}}(\tilde{\chi}_l^{J_1}\tilde{\lambda}_0)(\tilde{\chi}_{l\,L_1}\tilde{\lambda}_0)\right]$$

BUT

$$\frac{\partial}{\partial y}S^{(1)}_{+2} \sim \frac{y}{\langle y0\rangle^2}\delta^{J_2\ldots J_{2S_l}}_{L_2\ldots L_{2S_l}}\sum_{l=1}^{n}\frac{d_l}{m_l^2}(\tilde{\chi}_l^{J_1}\tilde{\lambda}_0)(\tilde{\chi}_{l\,L_1}\tilde{\lambda}_0) \neq 0.$$

Another way to show that gravitational dipole couplings are not consistent with unitarity, locality and Poincare invariance!

Non-Minimal gravity:

Do not affect soft theorems!

Gravitational-quadrupole

$$M(120^+) = -\frac{1}{M_{\rm Pl}}\frac{\langle \zeta p_1 0]^2}{\langle 0\zeta\rangle^2}\left\{\left(\frac{\langle \mathbf{21}\rangle}{m}\right)^{2S} + \frac{Q}{2m^6}\frac{\langle \zeta p_1 0]^2}{\langle 0\zeta\rangle^2}\left(\frac{\langle \mathbf{21}\rangle}{m}\right)^{2S-2}\langle \mathbf{1}0\rangle^2\langle \mathbf{2}0\rangle^2 + \dots\right\}$$

$$\hat{M}^{0,\epsilon}(1\dots n0^+) = \left(\frac{S^{(0)}_{+2}}{\epsilon^3} + \frac{S^{(1)}_{+2}}{\epsilon^2} + \frac{S^{(2)}_{+2}}{\epsilon^1}\right)\hat{M}^{0,0}(1\dots n) + \mathcal{O}\left(\frac{1}{\epsilon^0}\right)$$

Not modified

Not modified

$$S^{(2)}_{+2} = = \frac{1}{4M_{\rm Pl}}\sum_{l=1}^{n}\frac{1}{p_0 p_l}\left[\tilde{\mathscr{D}}_l^2\delta^{\vec{J}}_{\vec{L}} + \frac{Q_l}{m_l^2}(\tilde{\chi}_l^{J_1}\tilde{\lambda}_0)(\tilde{\chi}_l^{J_2}\tilde{\lambda}_0)(\tilde{\chi}_{l\,L_1}\tilde{\lambda}_0)(\tilde{\chi}_{l\,L_2}\tilde{\lambda}_0)\delta^{J_3\dots J_{2S_l}}_{L_3\dots L_{2S_l}}\right]$$

Non-trivial LG structure

$$\tilde{\mathcal{D}}_l = [0\mathbf{l}][0\partial_{\mathbf{l}}] + \frac{[0p_k p_l 0]}{[0p_j p_k 0]}[0\mathbf{j}][0\partial_{\mathbf{j}}] - \frac{[0p_j p_l 0]}{[0p_j p_k 0]}[0\mathbf{k}][0\partial_{\mathbf{k}}], \quad l \neq j, k,$$

Soft exponentiation:

(Extend the proof of He, Huang, Wen '14 valid only for the MHV sector)

$$\mathcal{M}(\mathbf{1}_X\mathbf{2}_{\bar{X}}3^h) = g_X \left(\frac{\langle\zeta p_1 3]}{[3\zeta]}\right)^h \frac{\langle\mathbf{21}\rangle^{2S}}{m^{2S-1+h}}$$

$$\boxed{\mathcal{M}^{0,\epsilon}(1\ldots n0^+) = \frac{1}{\epsilon^{1+h}\, m^{1+h}} \sum_{l=1}^{n} \frac{g_X\langle y p_l 0]^h}{(2p_0 p_l)\langle 0y\rangle^h} \exp\left(\epsilon \frac{\langle 0y\rangle}{\langle y p_l 0]} \tilde{\mathcal{D}}_l\right) \hat{\mathcal{M}}^{0,0}(1\ldots n) + \mathcal{O}(\epsilon^0).}$$

$$\tilde{\mathcal{D}}_l \to \tilde{\mathcal{D}}_l + \frac{S a_l}{q_l m}[0\mathbf{l}]^2 \qquad \tilde{\mathcal{D}}_l \to \tilde{\mathcal{D}}_l + \epsilon Q_l \frac{\langle y p_l 0]}{2\langle 0y\rangle}[0\mathbf{l}]^4$$

Electromagnetic dipole **Gravitational quadrupole**

$$\tilde{\mathcal{D}}_l = [0\mathbf{l}][0\partial_{\mathbf{l}}] + \frac{[0p_k p_l 0]}{[0p_j p_k 0]}[0\mathbf{j}][0\partial_{\mathbf{j}}] - \frac{[0p_j p_l 0]}{[0p_j p_k 0]}[0\mathbf{k}][0\partial_{\mathbf{k}}], \quad l \neq j, k,$$

3. Compton scattering

Emission from external legs accounts from all factorisation channels

$$\hat{\mathcal{M}}^{0,\epsilon}(\mathbf{1}_X\mathbf{2}_{\bar{X}}3_\gamma^{\pm}0_\gamma^{+}) = -\frac{1}{\epsilon^2}S_{+1}^{(0)}\hat{\mathcal{M}}^{0,0}(\mathbf{1}_X\mathbf{2}_{\bar{X}}3_\gamma^{\pm}) + B_\infty$$

Shift is not void!

$$S_{+1}^{(0)} = \sqrt{2}e\frac{\langle 3p_1 0][03]}{(2p_1p_0)(2p_2p_0)}$$

$$\begin{aligned}\hat{\mathcal{M}}^{0,0}(\mathbf{1}_X\mathbf{2}_{\bar{X}}3_\gamma^{-}) &= \sqrt{2}e\frac{\langle 3p_1 0]}{[30]}\left[-\frac{\langle\mathbf{13}\rangle[\mathbf{20}]+\langle\mathbf{23}\rangle[\mathbf{10}]}{\langle 3p_1 0]}\right]^{2S}\\ \hat{\mathcal{M}}^{0,0}(\mathbf{1}_X\mathbf{2}_{\bar{X}}3_\gamma^{+}) &= \sqrt{2}e\frac{[30]}{\langle 3p_1 0]}\frac{\langle\mathbf{21}\rangle^{2S}}{m^{2S-2}}.\end{aligned}$$

$$\begin{aligned}\hat{\mathcal{M}}^{0,\epsilon}(\mathbf{1}_X\mathbf{2}_{\bar{X}}3_\gamma^{-}0_\gamma^{+}) &= \frac{2e^2\langle 3p_1 0]^2}{\epsilon^2(2p_1p_0)(2p_2p_0)}\left[-\frac{\langle\mathbf{13}\rangle[\mathbf{20}]+\langle\mathbf{23}\rangle[\mathbf{10}]}{\langle 3p_1 0]}\right]^{2S} + B_\infty,\\ \hat{\mathcal{M}}^{0,\epsilon}(\mathbf{1}_X\mathbf{2}_{\bar{X}}3_\gamma^{+}0_\gamma^{+}) &= \frac{2e^2[30]^2}{\epsilon^2(2p_1p_0)(2p_2p_0)}\frac{\langle\mathbf{21}\rangle^{2S}}{m^{2S-2}} + B'_\infty.\end{aligned}$$

(we also compute the 5-points amplitudes with 3 photons/gravitons and 4-points in the presence of the dipole/quadrupole couplings)

We can also write the 3-point amplitudes in terms of the angular momentum

$$\mathcal{M}(\mathbf{1}_X \mathbf{2}_{\bar{X}} 3_\gamma^+) = \frac{2eq(p_1\epsilon_3^+)}{m^{2S}} \exp\left(-i\frac{\epsilon_3^{+\,\mu} p_3^\nu J_1^{\mu\nu}}{(p_1\epsilon_3^+)}\right)[\mathbf{21}]^{2S}$$

$\Big\downarrow$ **soft+BCFW massive recursion**

$$\hat{\mathcal{M}}^{0,\epsilon}(\mathbf{1}_X \mathbf{2}_{\bar{X}} 3_\gamma^- 0_\gamma^+) = (-1)^{2S}\frac{2q^2e^2\langle 3p_1 0]^2}{\epsilon^2 m^{2S}(t-m^2)(u-m^2)} \exp\left(-i\frac{\epsilon_3^{-\,\mu} p_3^\nu J_1^{\mu\nu}}{(p_1\epsilon_3^-)|_{\tilde{\zeta}=\tilde{\lambda}_0}}\right)\langle \mathbf{21}\rangle^{2S} + B_\infty$$

OR

$$\hat{\mathcal{M}}^{0,\epsilon}(\mathbf{1}_X \mathbf{2}_{\bar{X}} 3_\gamma^- 0_\gamma^+) = (-1)^{2S}\frac{2q^2e^2\langle 3p_1 0]^2}{\epsilon^2 m^{2S}(t-m^2)(u-m^2)} \exp\left(\frac{[30]\langle 3\mathbf{1}\rangle(\lambda_3\partial_{\chi_1})}{\langle 3p_1 0]}\right)\langle \mathbf{21}\rangle^{2S} + B_\infty$$

This illuminates one tiny detail in the derivation of A. Guevara, A. Ochirov, and J. Vines '18, where the choice of the reference spinor was made ad-hoc

Recursion relations do not solve the problem with spurious poles for higher-spins:

$$\mathcal{M}(\mathbf{1}_X\mathbf{2}_{\bar{X}}3_\gamma^-4_\gamma^+) = (-1)^{2S}\frac{2e^2\langle 3p_14]^2}{(t-m^2)(u-m^2)}\boxed{\left[\frac{\mathcal{A}+\mathcal{B}}{\langle 3p_14]}\right]^{2S}}$$

we need a non-zero boundary term to cancel it

Chung, Huang, Kim, Lee ‘18

The trick is to find the right variables:

$$\begin{aligned}\mathcal{A} &\equiv \langle\mathbf{13}\rangle[\mathbf{24}], \qquad \mathcal{B}\equiv\langle\mathbf{23}\rangle[\mathbf{14}], \qquad \mathcal{X}\equiv m\langle\mathbf{12}\rangle+m[\mathbf{12}]-\langle\mathbf{1}p_4\mathbf{2}]-\langle\mathbf{2}p_3\mathbf{1}]\\ \mathcal{Y} &\equiv (u-m^2)\mathcal{A}+(t-m^2)\mathcal{B}, \qquad \mathcal{Z}\equiv(u-m^2)\mathcal{A}-(t-m^2)\mathcal{B}.\end{aligned}$$

Useful identities:

$$\langle 3p_14]\mathcal{X}-\mathcal{Y}=2m^2\left(\mathcal{A}+\mathcal{B}\right), \qquad \mathcal{Z}=-\langle 3p_14]\left(\langle\mathbf{1}p_4\mathbf{2}]+\langle\mathbf{2}p_4\mathbf{1}]\right)$$

S=2

$$\begin{aligned}\mathcal{M}_{\rm IR} &= \frac{2e^2\langle 3p_14]^2}{(2m^2)^4(t-m^2)(u-m^2)}\left[\frac{\langle 3p_14]\mathcal{X}-\mathcal{Y}}{\langle 3p_14]}\right]^4\\ &= \frac{2e^2}{(2m^2)^4(t-m^2)(u-m^2)}\left[\frac{\mathcal{Y}^4}{\langle 3p_14]^2}-\frac{4\mathcal{X}\mathcal{Y}^3}{\langle 3p_14]}+\dots\right]\end{aligned}$$

No unphysical poles

S=2

$$\mathcal{M}_{\rm IR} = \frac{2e^2\langle 3p_1 4]^2}{(2m^2)^4(t-m^2)(u-m^2)}\left[\frac{\langle 3p_1 4]\mathcal{X}-\mathcal{Y}}{\langle 3p_1 4]}\right]^4$$
$$= \frac{2e^2}{(2m^2)^4(t-m^2)(u-m^2)}\left[\frac{\mathcal{Y}^4}{\langle 3p_1 4]^2}-\frac{4\mathcal{X}\mathcal{Y}^3}{\langle 3p_1 4]}+\dots\right]$$

$$\mathcal{M}_{\rm UV} = -\frac{2e^2}{(2m^2)^4(t-m^2)(u-m^2)}\left\{\frac{\mathcal{Y}^2(\mathcal{Y}^2-\mathcal{Z}^2)}{\langle 3p_1 4]^2}-\frac{4\mathcal{X}\mathcal{Y}(\mathcal{Y}^2-\mathcal{Z}^2)}{\langle 3p_1 4]}\right\}$$
$$= -\frac{e^2\mathcal{A}\mathcal{B}}{2m^8}\left\{\frac{\mathcal{Y}^2}{\langle 3p_1 4]^2}-\frac{4\mathcal{X}\mathcal{Y}}{\langle 3p_1 4]}\right\}+C,$$

$E \gg m$ as $\mathcal{O}(E^8/m^8)$

Cancel spurious poles without messing up with t and u-poles

Solution is not unique as pure contact terms can be present

See also recent discussion
M. Chiodaroli, H. Johansson, P. Pichini '21

Loss of perturbative unitarity at a cutoff scale around particle mass

However, it is possible to fix the contact term to get an improved UV behaviour

$$\mathcal{M}_{\rm UV} = -\frac{2e^2\mathcal{A}\mathcal{B}}{m^6}\left\{\frac{m^2(\mathcal{A}+\mathcal{B})^2}{\langle 3p_1 4]^2}+\frac{\mathcal{X}(\mathcal{A}+\mathcal{B})}{\langle 3p_1 4]}\right\} \sim \mathcal{O}(E^6/m^6)$$

$$\mathcal{M}_{\mathrm{UV}} = -\frac{2e^2}{(2m^2)^{2S}(t-m^2)(u-m^2)} \sum_{k=3}^{2S} \binom{2S}{k} \frac{(-\mathcal{X})^{2S-k}\mathcal{Y}\left[\mathcal{Y}^{k-1} - \{\mathcal{Y}||\mathcal{Z}\}\mathcal{Z}^{k-2}\right]}{\langle 3p_1 4]^{k-2}} \longrightarrow \mathcal{O}(E^{4S}/m^{4S})$$

One can always add a contact term such that the UV behaviour is improved to $\longrightarrow \mathcal{O}(E^{4S-2}/m^{4S-2})$

Maximal cutoff of an EFT with a particle of mass m, charge q and S>1:

$$\Lambda_{\max} \lesssim \left(\frac{4\pi}{|q|e}\right)^{\frac{1}{2S-1}}$$

M. Porrati, R. Rahman '09

Similar story follows for the gravitational Compton.

We also find a formula for a general boundary term that cancels the spurious poles for any spin.

We can still add a contact term and improve the UV behaviour, such that:

Maximal cutoff of an EFT with a particle of mass m and S>2:

$$\Lambda_{\max} \lesssim \left(4\pi\, M_{\mathrm{Pl}} m^{2S-2}\right)^{\frac{1}{2S-1}}$$

J. Bonifacio and K. Hinterbichler '18 , R. Rahman '09

Conclusions

- New massive soft+BCFW recursion relations
- Systematic study of soft theorems including massive states
- Connection with higher-spin amplitudes that play an important role in black hole dynamics
- Results for 5-points Compton for any spin and systematic way to build contact terms to cancel spurious poles

Thank you!