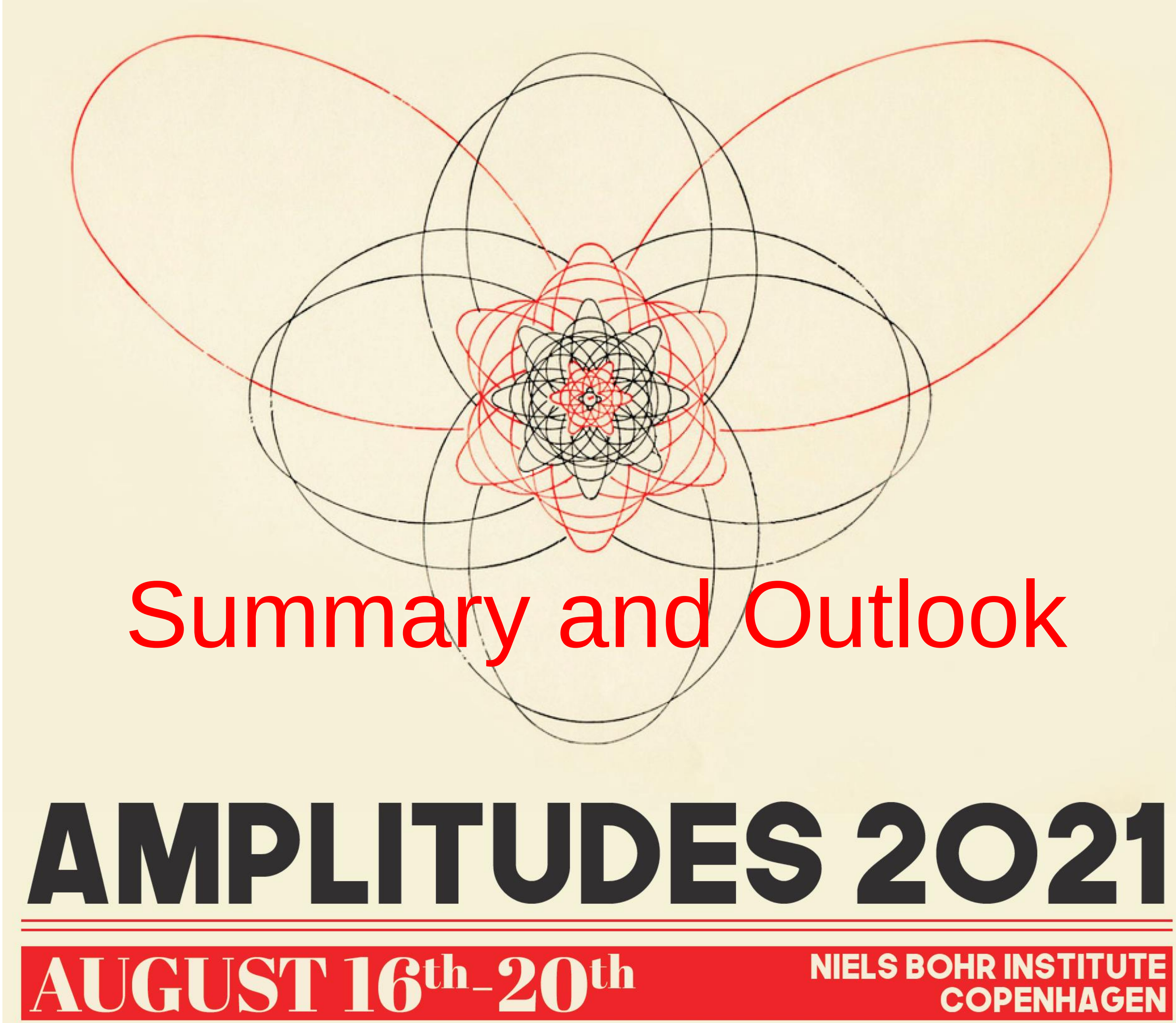
Summary and Outlook



Lance Dixon

SLAC
NATIONAL ACCELERATOR LABORATORY

What we missed

[According to Weatherman Jake]

Jake Bourjaily

Poul Damgaard

Matthias Wilhelm

Michèle Levi

Matt von Hippel

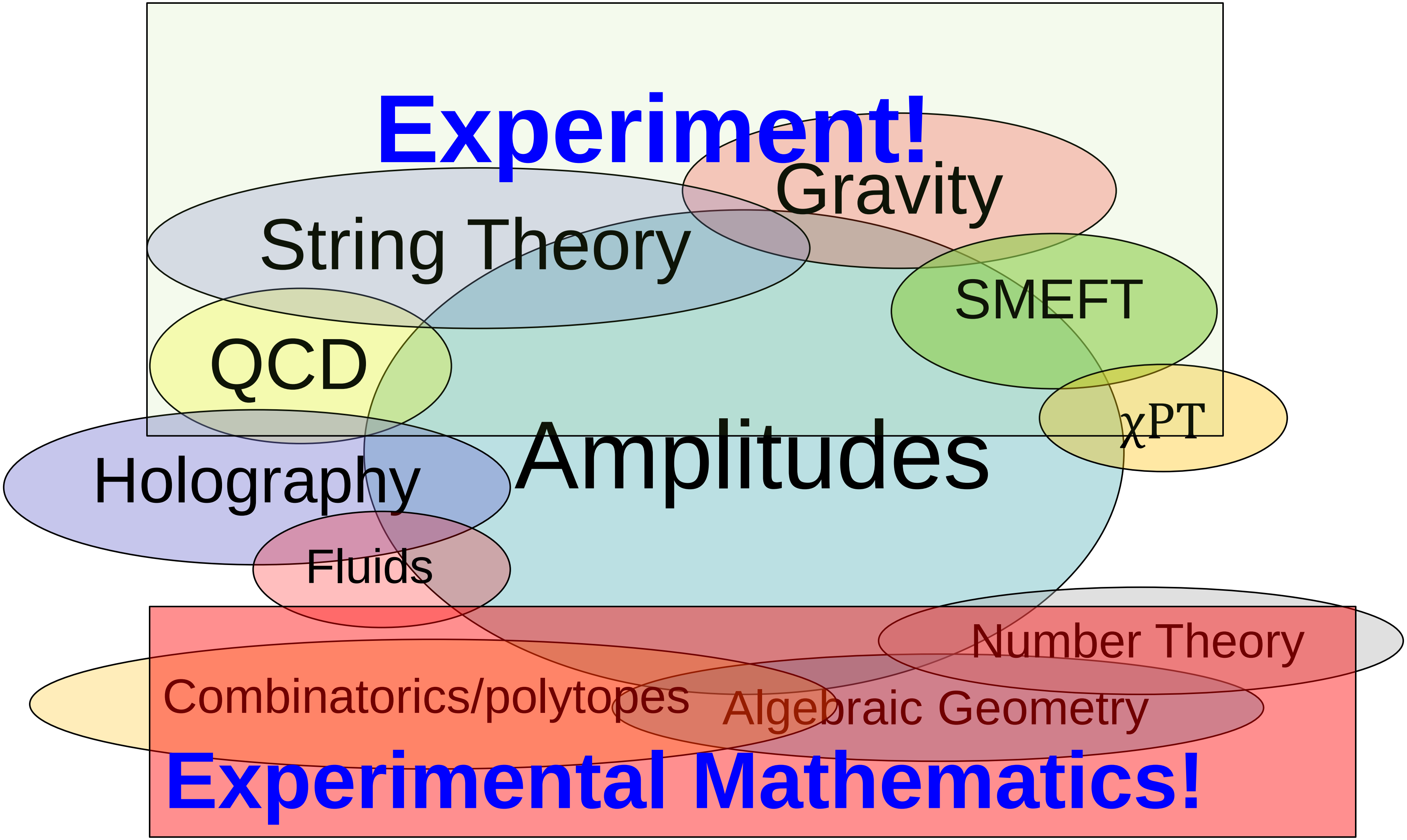
+ many students!

Emil Bjerrum-Bohr

Andrew McLeod

Apologies in advance

If I don't cover your favorite topic or speaker



Experiment!
Gravity
String Theory
SMEFT
QCD
χPT
Amplitudes
Holography
Fluids
Number Theory
Combinatorics/polytopes
Algebraic Geometry
Experimental Mathematics!

Those who explore an unknown world are travelers without a map: the map is the result of the exploration. The position of their destination is not known to them, and the direct path that leads to it is not yet made.

Hideki Yukawa

Steven Weinberg

May 3, 1933 – July 23, 2021

He gave us SU(2) x U(1)

A MODEL OF LEPTONS*

Steven Weinberg†
Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 17 October 1967)

Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interactions. What could be more natural than to unite[1] these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences by imagining that the symmetries relating the weak and electromagnetic interactions are exact symmetries of the Lagrangian but are bro-

and on a right-handed singlet

$$R \equiv [\tfrac{1}{2}(1-\gamma_5)]e. \qquad (2)$$

The largest group that leaves invariant the kinematic terms $-\bar{L}\gamma^\mu \partial_\mu L - \bar{R}\gamma^\mu \partial_\mu R$ of the Lagrangian consists of the electronic isospin $\vec{T}$ acting on L, plus the numbers N_L, N_R of left- and right-handed electron-type leptons. As far as we know, two of these symmetries are entirely unbroken: the charge $Q = T_3 - N_R - \frac{1}{2}N_L$, and the electron number $N = N_R + N_L$. But the

He also gave us EFTs

- Current algebra, soft pions phrased in S matrix language
- Weinberg → non-linear σ model Lagrangian

"the assumptions of S-matrix theory, supplemented by chiral invariance, were indeed all that are needed at low energy, but the most convenient way of implementing these assumptions in actual calculations was by good old quantum field theory, which the S-matrix theorists had hoped to supplant"

- Now of course we like to understand it all again in terms of S matrix language
  talks by Shadmi, Machado, Huang, Caron Huot, Rosen,…

And soft graviton theorems

PHYSICAL REVIEW VOLUME 140, NUMBER 2B 25 OCTOBER 1965

Infrared Photons and Gravitons*

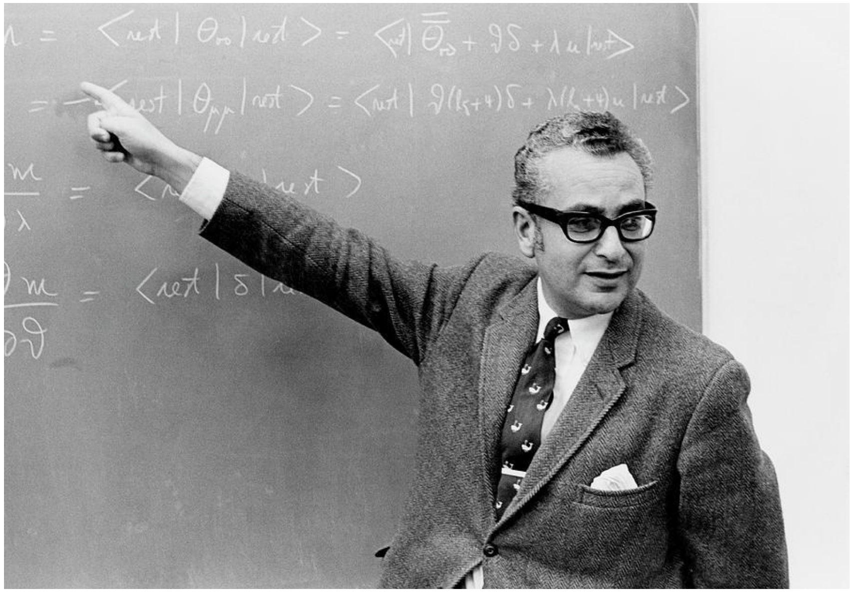
STEVEN WEINBERG†
Department of Physics, University of California, Berkeley, California
(Received 1 June 1965)

It is shown that the infrared divergences arising in the quantum theory of gravitation can be removed by the familiar methods used in quantum electrodynamics. An additional divergence appears when infrared photons or gravitons are emitted from noninfrared external lines of zero mass, but it is proved that for infrared gravitons this divergence cancels in the sum of all such diagrams. (The cancellation does not occur in massless electrodynamics.) The formula derived for graviton bremsstrahlung is then used to estimate the gravitational radiation emitted during thermal collisions in the sun, and we find this to be a stronger source of gravitational radiation (though still very weak) than classical sources such as planetary motion. We also verify the conjecture of Dalitz that divergences in the Coulomb-scattering Born series may be summed to an innocuous phase factor, and we show how this result may be extended to processes involving arbitrary numbers of relativistic or nonrelativistic particles with arbitrary spin.

talks by Puhm, Heissenberg, Machado, …

Murray Gell-Mann

Sept. 15, 1929 – May 24, 2019



Physics Today

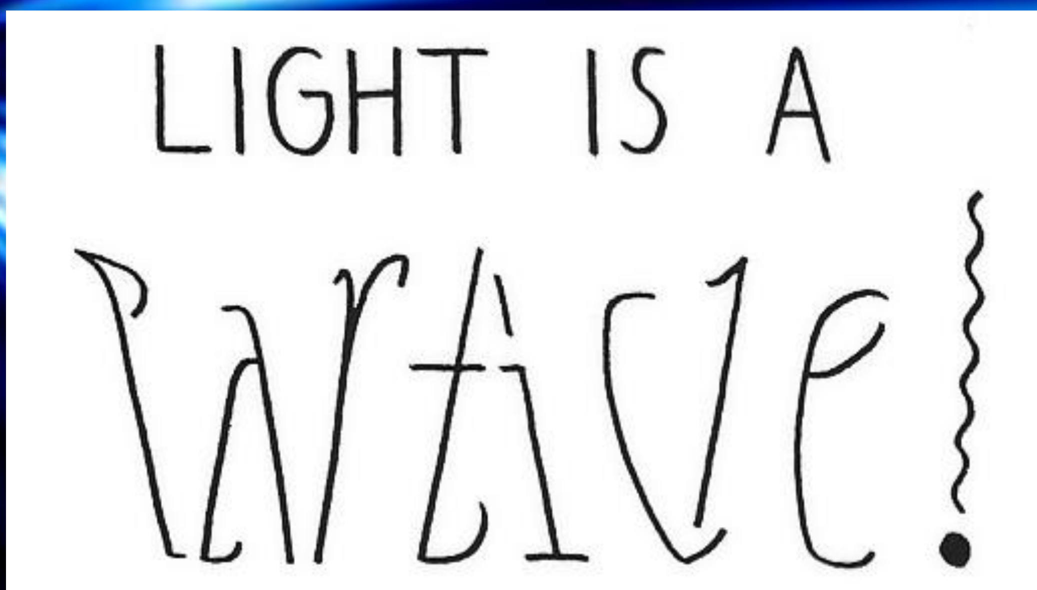
He gave us SU(3), not once but twice

→ He would have loved the gauged biadjoint scalar theory!

Niels Bohr


I would like to bid all of you welcome to the conference Amplitudes,
in this year of 2021.

wave-particle duality and classical correspondence



Critical to “scattering amplitudes for LIGO”
Damour talk

phys.org

Richard Feynman

- He gave us Feynman diagrams (post Stückelberg!)

which we often try hard to avoid

- But also $i\varepsilon$ – relativistic causality

[talks by Mizera, Caron Huot,…]

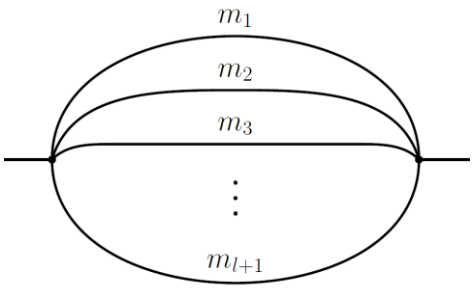
- You can sometimes run

(to Euclidean regions of planar theories and
positive regions of Grassmannians)
but you can't hide from $i\varepsilon$!

And Feynman Integrals!

- The gift that keeps giving!
- Talks by Langer, Klemm, Mizera, Badger, Volovich, Zeng, Weinzierl, Brown, Zhang, Signorile-Signorile, Duhr, Britto
- Under the hood: Mistlberger, Vita, Damour, Heissenberg, Solon
- → Feynman polytopes [Arkani-Hamed]

Algebraic geometry of [banana] integrals



	$l=(n+1)$-loop banana integrals in $D=2$ dimensions	Calabi-Yau (CY) geometry
1	Maximal cut integrals in $D=2$ dimensions	$(n,0)$-form periods of CY manifolds or CY motives
2	Dimensionless ratios $z_i = {m_i}^2/p^2$	Unobstructed complex moduli of M_n, or equivalently Kähler moduli of the mirror W_n
3	Integrand-basis for maximal cuts of master integrals in $D=2$ dimensions	Middle (hyper) cohomology $H^n(M_n)$ of M_n
4	Quadratic relations among maximal cut integrals	Quadratic relations from Griffiths transversality
5	Integration-by-parts (IBP) reduction	Griffiths reduction method
6	Complete set of differential operators annihilating a given maximal cut in $D=2$ dimensions	Homogeneous Picard-Fuchs differential ideal (PFI) / Gauss-Manin (GM) connection
7	(Non-)maximal cut contours	(Relative) homology of CY geometry $H_n(M_n)$ $(H_{n+1}(F_{n+1}, \partial\sigma_{n+1}))$
8	Contributions from subtopologies to the differential equations	Extensions of the PFI or the GM connection
9	Full banana integrals in $D=2$ dimensions	Chain integrals in CY geometry or extensions of Calabi-Yau motive
10	Degenerate kinematics (e.g., $m_i^2=0$ or $p^2/m_i^2 \to 0$)	Critical divisors of the moduli space
11	Large-momentum regime $p^2 \gg m_i^2$	Point of maximal unipotent monodromy & $\widehat{\Gamma}$-classes of W_n
12	General logarithmic degenerations	Limiting mixed Hodge structure from monodromy weight filtration
13	Analytic structure and analytic continuation	Monodromy of the CY motive and its extension
14	Special values of the integrals for special values of the z_i	Reducibility of Galois action & L-function values
15	(Generalized?) modularity of Feynman integrals	Global $\mathrm{O}(\boldsymbol{\Sigma},\mathbb{Z})$-monodromy, integrality of mirror map & instantons expansion

Klemm and Duhr talks

Because Calabi-Yau(s) associated with a multi-loop Feynman integral are often singular, better to associate them with (family of) CY motives, compatible with Hodge structure and a Galois group action.

The elliptic double box

Zhang talk

- Expressed explicitly

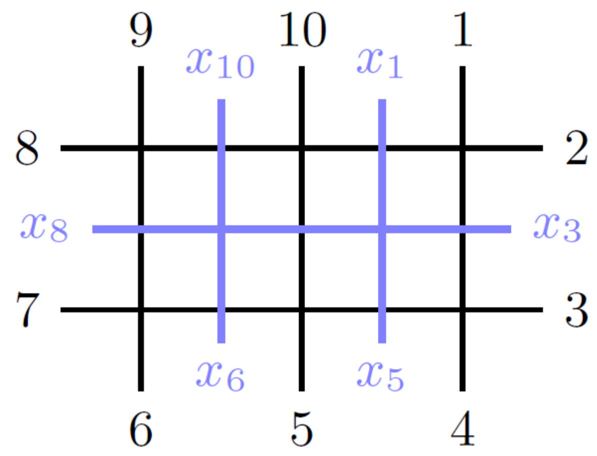
in terms of elliptic polylogs

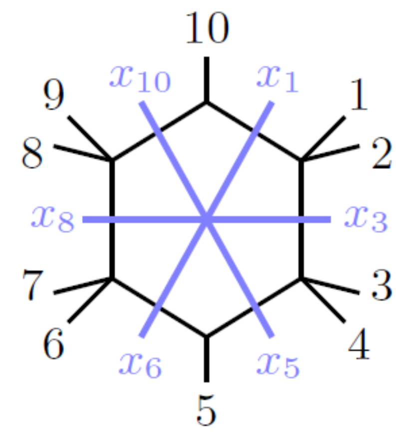
- Differential equation

sourced by one-loop D=6 hexagon

→ Symbol has elliptic letters only

in last 2 entries

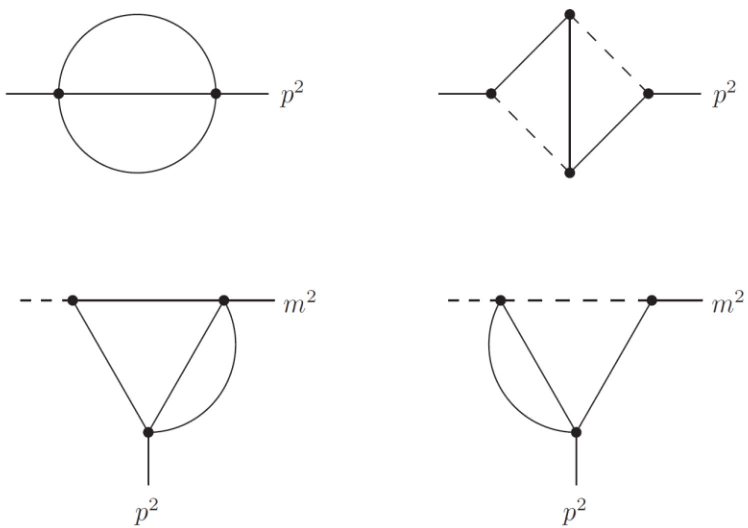




Modular properties of elliptic integrals

Examples of elliptic Feynman integrals depending on one kinematic
$x = \frac{-p^2}{m^2}$ (solid internal lines of mass m, dashed lines massless):

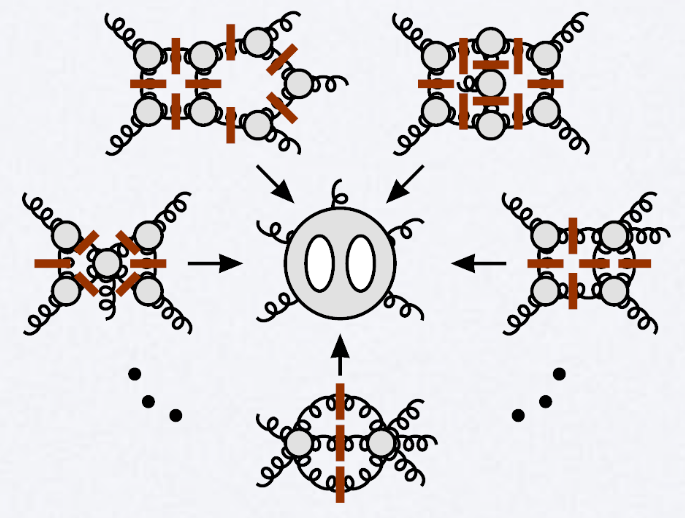
Weinzierl talk



- For numerical purposes, handy to use modular transformations to move $q = e^{2\pi i\tau}$ into fundamental region for modular group, so $|q| < 0.0043$

- Have to be careful about accompanying fibre transformations when doing this.

Unitarity carving knives

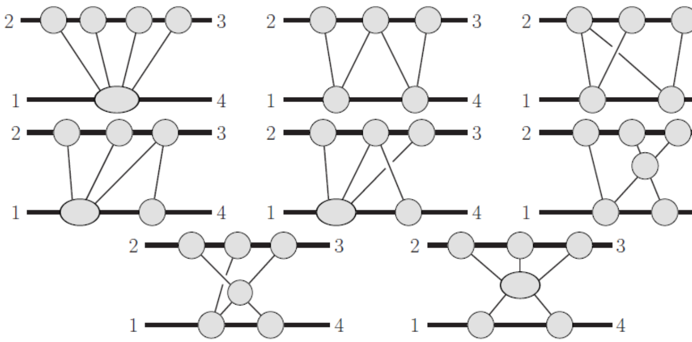


Used to only be good (in Amplitudes) for slicing up integrands. and it still is

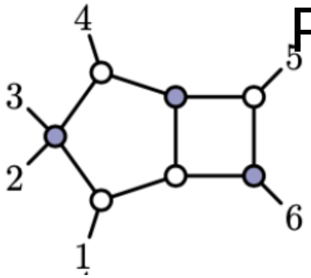
talk by Badger

and Solon

and Langer



Prescriptive variant uses chiral traces in numerator



$$-[\![p_1,b,c,p_4]\!]\,[\![e,f,p_6,d]\!]$$

Natural objects: chiral traces

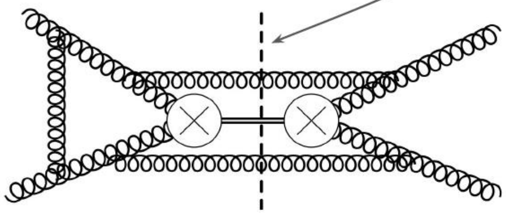
$$[\![a_1,a_2,b_1,b_2,\cdot,\cdot,c_1,c_2]\!] := \left[(a_1\cdot a_2)^{\alpha}{}_{\beta}(b_1\cdot b_2)^{\beta}{}_{\gamma}\cdots(c_1\cdot c_2)^{\delta}{}_{\alpha}\right]$$

$$= \mathrm{tr}_+(a_1,a_2,b_1,b_2,\ldots,c_1,c_2)$$

Also carves in reverse

$$\delta_+(p^2) \sim \lim_{\varepsilon\to 0}\left[\frac{1}{p^2+i\varepsilon} - \frac{1}{p^2-i\varepsilon}\right]$$

See phase space constraints as
"cut" propagators

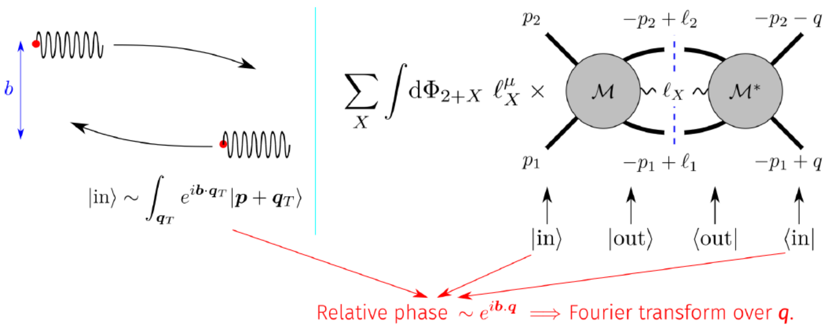


Vita talk

Zeng talk

Radiated momentum from KMOC formalism

[Herrmann, Parra-Martinez, Ruf, MZ, '21 (PRL)]. Follows formalism of [Kosower, Maybee, O'Connel, '18]



Phase space integrand from **cutting** virtual integrand. Integration by **reverse unitarity.**

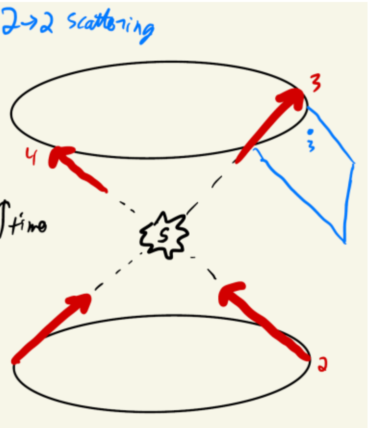
Signorile-Signorile talk

$$I^{(gg)}_{S_{56}} = \int [\mathrm{d}k_5]\,[\mathrm{d}k_6]\,\theta(E_{\max}-E_5)\,\theta(E_5-E_6)\,I^{(gg)(56)}_{12}\big(k_1,k_2,k_5,k_6\big)$$

$$[\,\mathrm{d}f_i\,] = \frac{\mathrm{d}^d k_i}{(2\pi)^d}\,(2\pi)\,\delta_+(k_i^2)$$

$$E_5 = E_{\max}\,\xi \qquad E_6 = E_{\max}\,\xi\, z \qquad 0<\xi<1\,,0<z<1$$

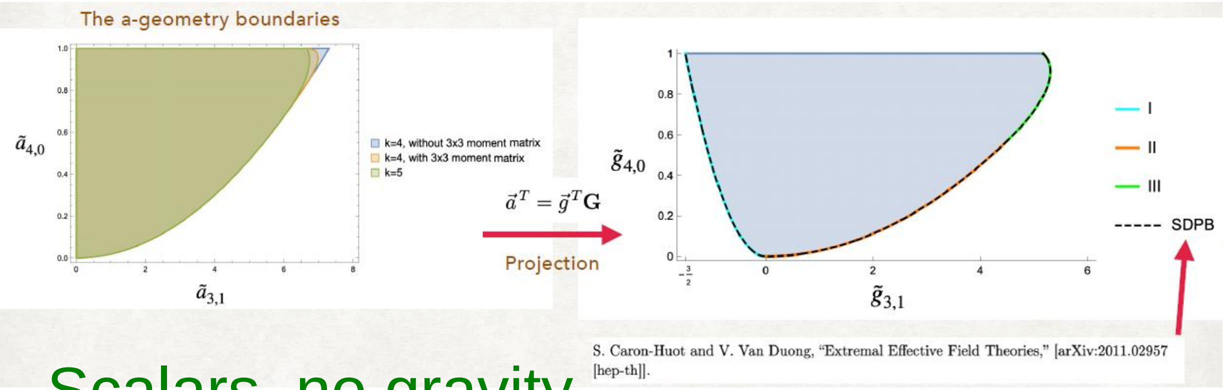
Reverse unitarity: **map phase space integrals onto loop integrals** *[Anastasiou, Melnikov 0207004]*

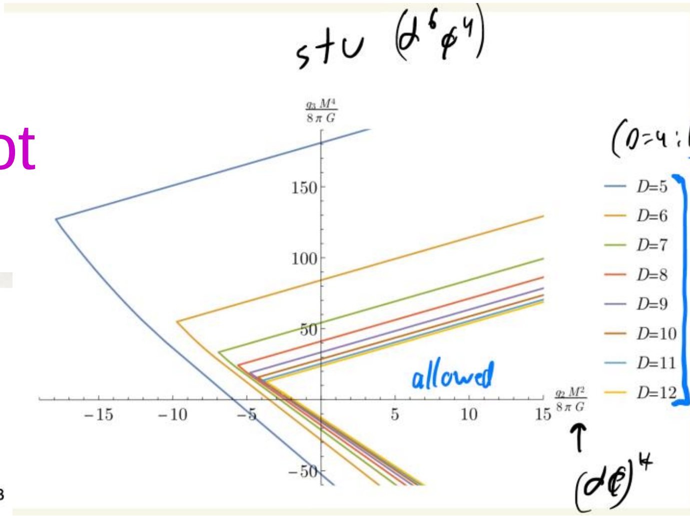


Now also really excellent tool (along with causality, crossing symmetry, Hankel matrices, linear programming, …)

for carving up space of EFTs

talks by Huang & Caron-Huot





Scalars with gravity

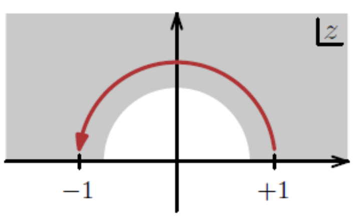
Scalars, no gravity

Beautiful proof of crossing symmetry

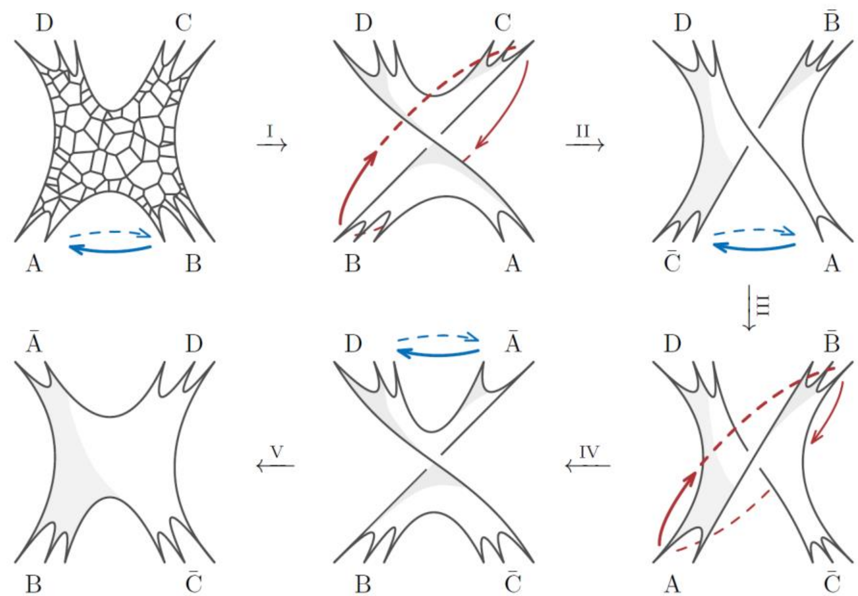
- Got to track those $i\varepsilon$ factors carefully!

Mizera talk

$$
\begin{aligned}
p_1^\mu &= (\ \ p_1^+,\ \ p_1^-,\ \vec{p}_1) \\
p_2^\mu &= (\ \ z p_2^+,\ \ \tfrac{1}{z} p_2^-,\ \vec{p}_2) \\
p_3^\mu &= (-z p_2^+, -\tfrac{1}{z} p_2^-,\ \vec{p}_3) \\
p_4^\mu &= (-\ p_1^+,\ -\ p_1^-,\ \vec{p}_4)
\end{aligned}
$$



(preserves on-shell conditions and mom. cons.)



- Perturbative, large N, but otherwise very general

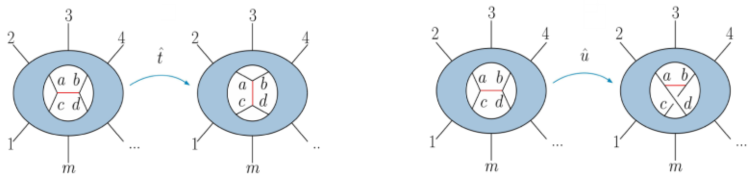
Double copy / color kinematics duality

- Began with KLT and a slogan:

  Gravity = YM^2

- Then became graphical [BCJ (2008, 2010); Vasquez-Holm talk]

$$n_i - n_j = n_k \quad \Leftrightarrow \quad c_i - c_j = c_k,$$
$$n_i \to -n_i \quad \Leftrightarrow \quad c_i \to -c_i$$



- Now connects a much broader web of theories
- Underlies much of the motivation for the “scattering amplitudes for LIGO” program [Zeng, Solon talks]

Generalized frameworks

Elvang talk [Carrasco talk related]

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

$$\underbrace{\mathbf{1} = \mathbf{1} \otimes \mathbf{1}\,.}_{\text{KLT Bootstrap Equation}}$$

What are the rules for generalizing the KLT kernel?

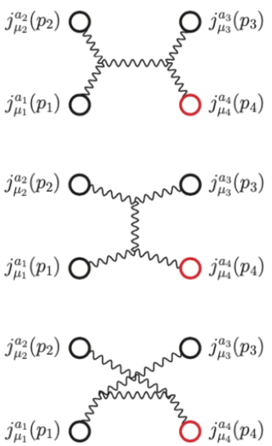
New solutions for higher dimension operators found by solving the KLT bootstrap equations:

FT KLT	YM: $\mathrm{tr}\,F^2$ ✓	$\mathrm{tr}\,F^3$ ✓	$\mathrm{tr}\,F^4$ 1✗	$\mathrm{tr}\,D^2F^4$ 1✓1✗	$\mathrm{tr}\,D^4F^4$ 1✓2✗	...
Gen. KLT	YM: $\mathrm{tr}\,F^2$ ✓	$\mathrm{tr}\,F^3$ ✓	$\mathrm{tr}\,F^4$ 1✓	$\mathrm{tr}\,D^2F^4$ 1✓1✗	$\mathrm{tr}\,D^4F^4$ 1✓2✓	...

Opens new avenues for color-kinematics / double copy

Color-kinematics duality derived from field equations for field strengths

Cheung talk

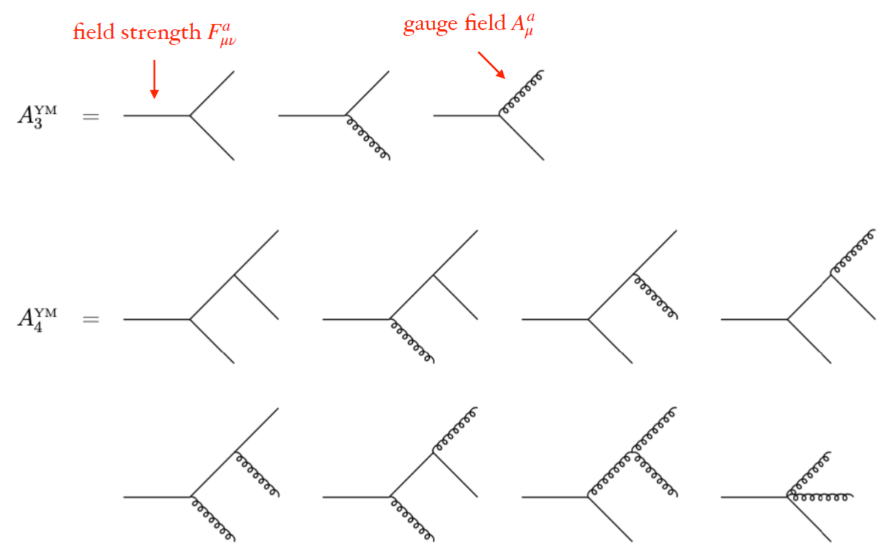


$$n_s = p_1^{\mu_2}(p_1+p_2)^{\mu_3}\eta^{\mu_1\mu_4} + p_2^{\mu_1}p_3^{\mu_2}\eta^{\mu_3\mu_4} - \{1 \leftrightarrow 2\}$$

$$n_t = p_2^{\mu_3}(p_2+p_3)^{\mu_1}\eta^{\mu_2\mu_4} + p_3^{\mu_2}p_1^{\mu_3}\eta^{\mu_1\mu_4} - \{2 \leftrightarrow 3\}$$

$$n_u = p_3^{\mu_1}(p_3+p_1)^{\mu_2}\eta^{\mu_3\mu_4} + p_1^{\mu_3}p_2^{\mu_1}\eta^{\mu_2\mu_4} - \{3 \leftrightarrow 1\}$$

$\otimes$ NLSM



GBAS $\otimes$ F^3 = YM

- **Strikingly new perspective!**
- **Can it be applied to loops??**

Double copy with vector masses

Paranjape talk

For $\mathcal{M}_n = \mathcal{A}_n[\alpha] m_n[\alpha|\beta]^{-1} \mathcal{A}_n[\beta]$ to be local,

1. KLT kernel has spurious poles but these are canceled by $\mathcal{A}_n$ in the final double-copy $\mathcal{M}_n$
2. KLT kernel has no spurious poles when the masses are carefully tuned: the "minimal rank" condition

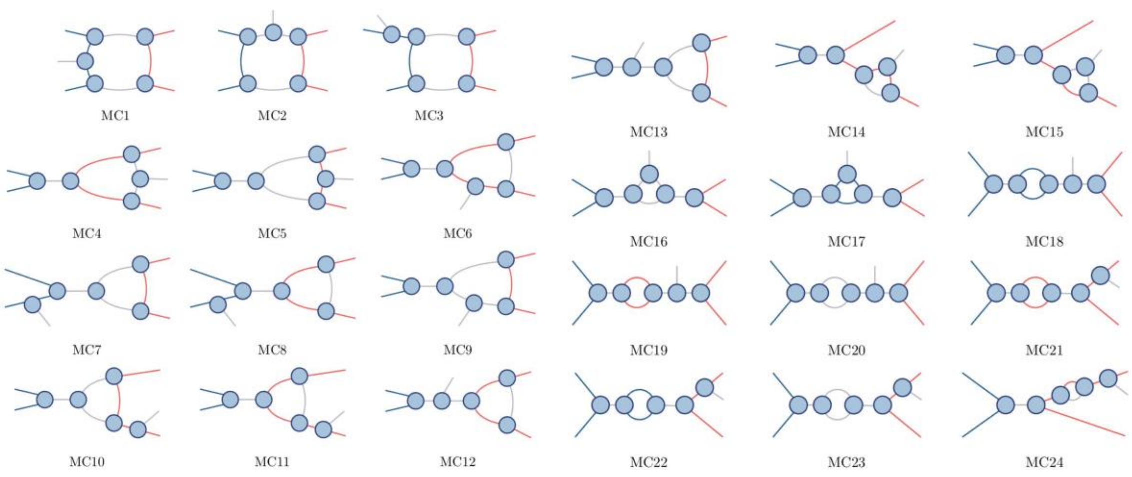
- Option one can give dRGT massive gravity, but not at 5 points?
- Option 2 satisfied for toroidal Kaluza-Klein compactifications, but are there other solutions?

Double copy with scalar masses

Vasquez-Holm talk

- At loop level, 5-points, motivated by BH inspiral with radiation
- Apply maximal cuts, remove dilaton-axion "by hand"

Projective double copy at loop-level

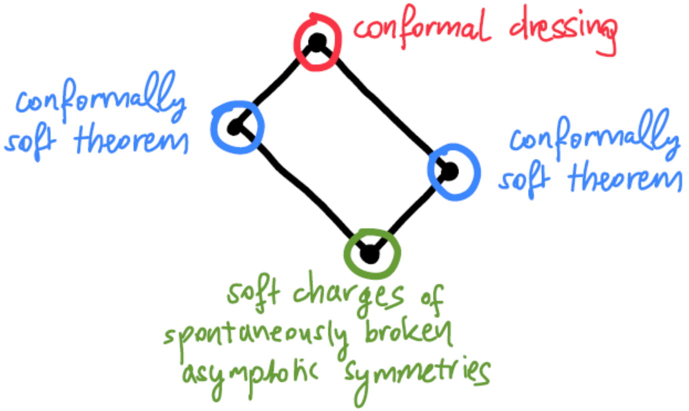


Double copy in the sky

Puhm talk

CELESTIAL DIAMONDS:

Natural structure in celestial CFT that unifies the discussion of ***soft sector****:*



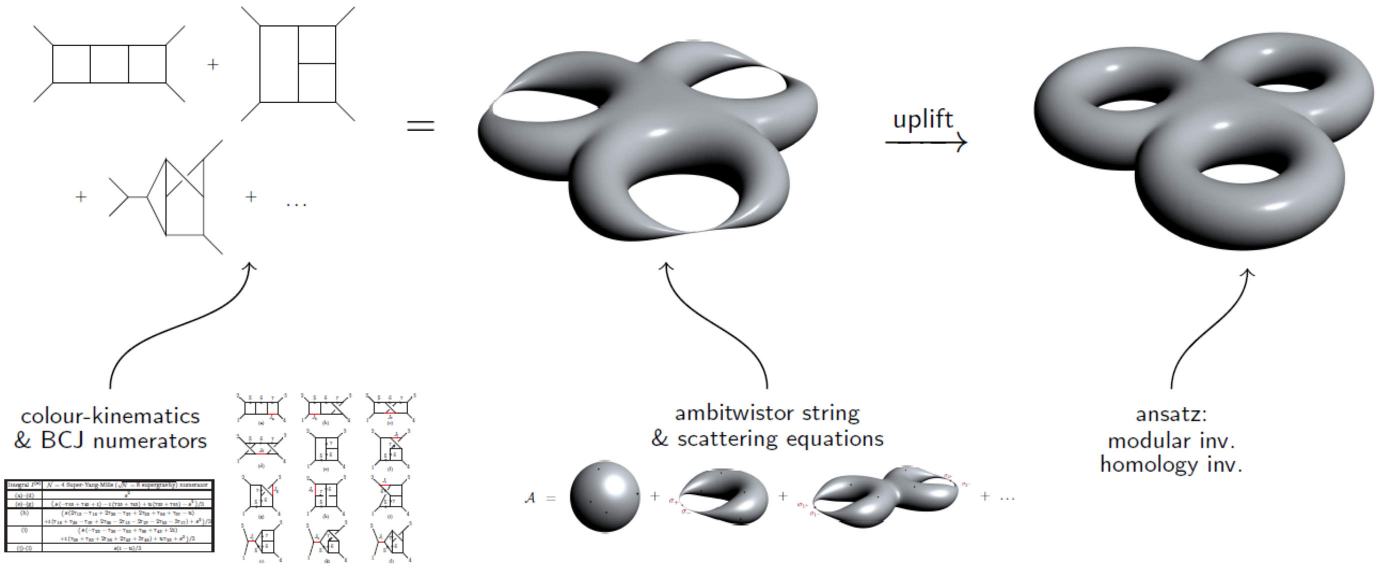
Appearance of *celestial double copy* and *shockwaves* in the diamond.

Soft behavior also analyzed in 2d "sky" in Kalyanapuram talk and in massive case in Machado talk

String amplitudes from FT?

▶ Strategy for importing field theory results to superstring

Geyer talk



⇒ proposal for 4-pt 3-loop superstring amplitude

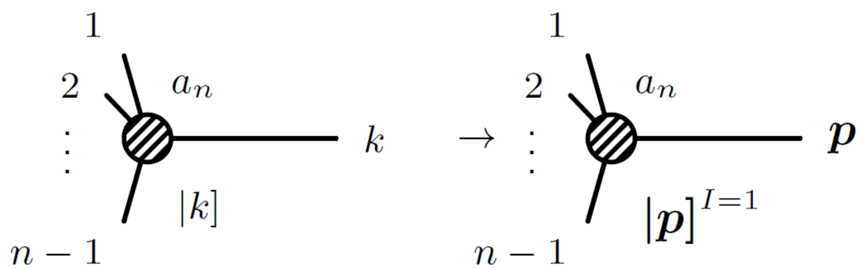
- Works through 2 loops.
- Would be incredibly fascinating if FT contains enough information to reconstruct ST!

On-shell SMEFTing

Shadmi talk

- New arena where Amplitudes meets experiment!
- Both massless and massive versions of EFT; latter leverages Arkani-Hamed, Huang, Huang massive spinor technology

bolding as higgsing



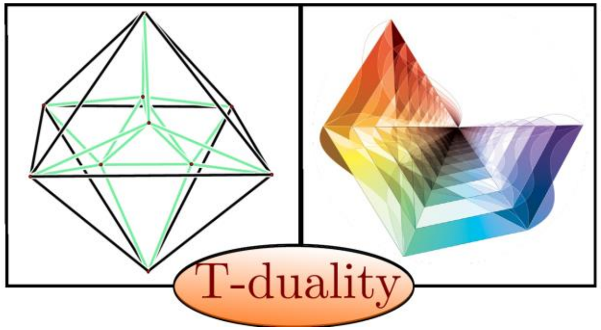
massless EFT → massive EFT

Geometry of amplitudes

- Talks by Parisi, Trnka, Arkani-Hamed

Parisi talk

the Hypersimplex



the Amplituhedron

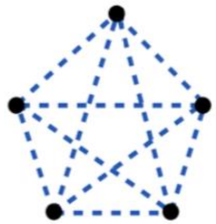
- $\mathrm{tr}[\phi^3]$ now looks like string theory (but with more, infinite-dimensional, polytopes)!

Arkani-Hamed lecture

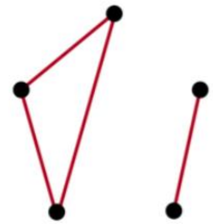
Turn a positive into a negative (geometry)

Trnka talk

$$\Omega(\mathcal{A}_L) =$$



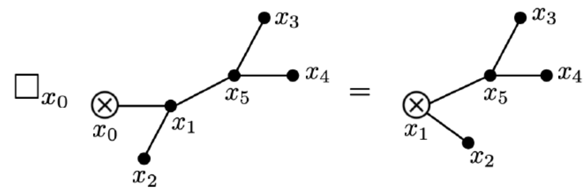
$$= \sum_{\text{all } G} (-1)^{E(G)}$$



sum over all graphs

This is IR finite function of one cross ratio

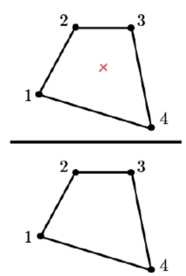
$$\mathcal{F}(g,z) \qquad\qquad z = \frac{\langle AB_0 12\rangle\langle AB_0 34\rangle}{\langle AB_0 14\rangle\langle AB_0 23\rangle}$$



This object is also natural from the dual Wilson loop picture

(Alday, Buchbinder, Tseytlin, 2011) (Englund, Roiban, 2011)

$$\frac{\langle W_F(x_1,x_2,x_3,x_4)\mathcal{L}(x_0)\rangle}{\langle W_F(x_1,x_2,x_3,x_4)\rangle} = \frac{1}{\pi^2}\frac{x_{13}^2 x_{24}^2}{x_{10}^2 x_{20}^2 x_{30}^2 x_{40}^2} F(g;z)$$



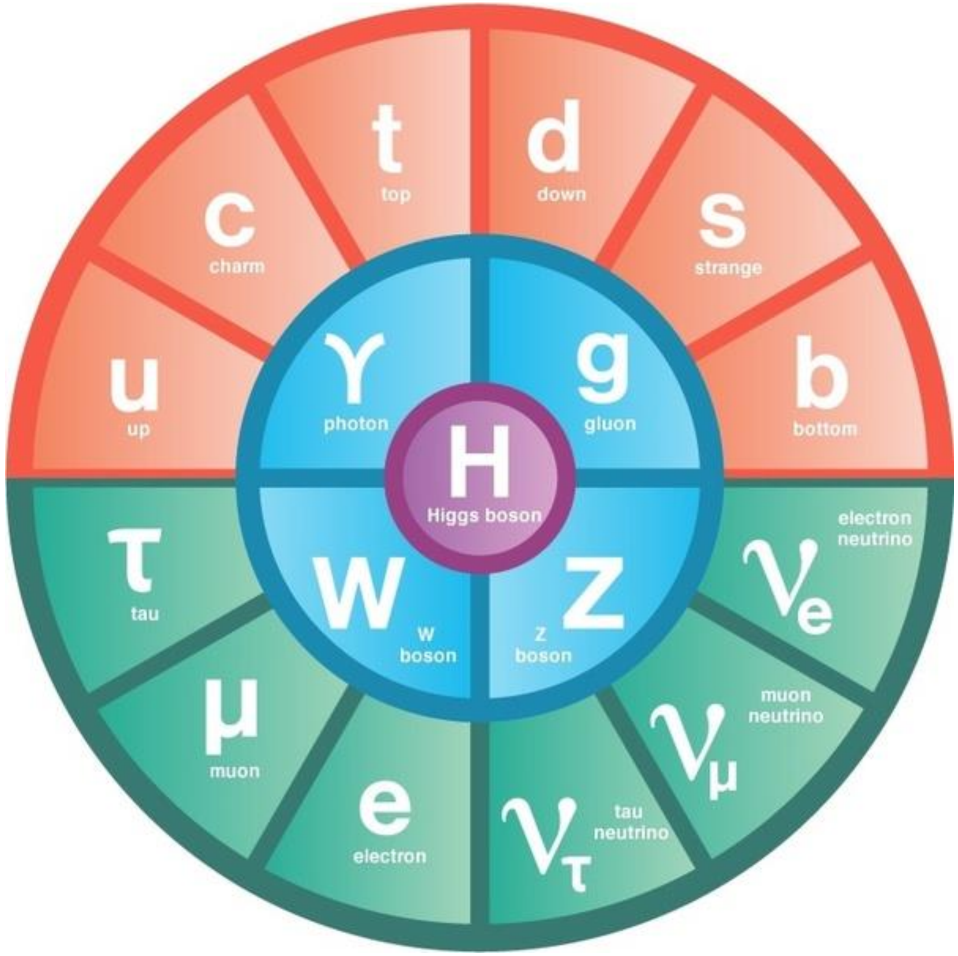
Wilson loop with Lagrangian insertion at point x_0

$$F = -g^2\mathcal{F}$$

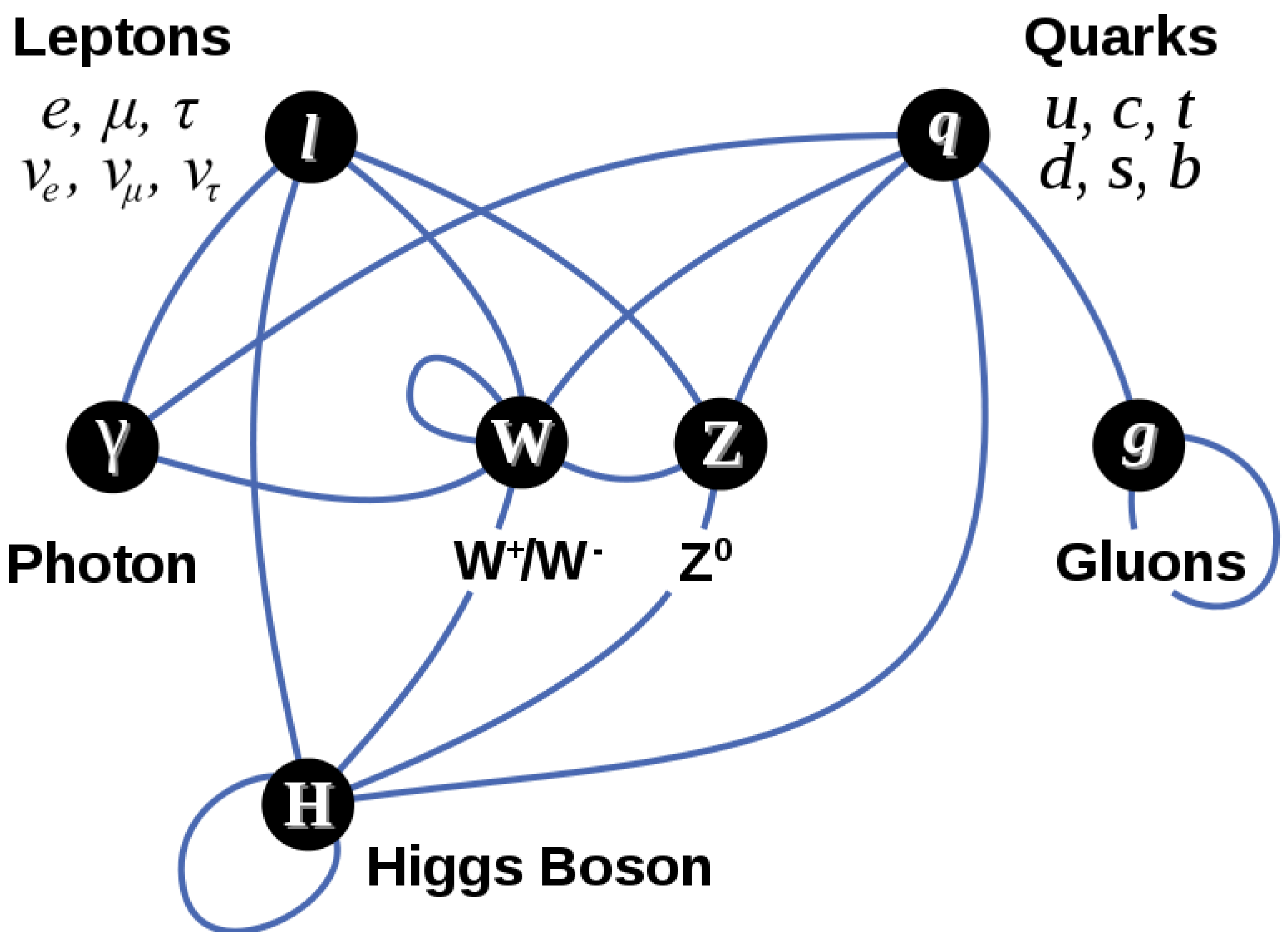
$$\mathcal{F}_{\text{tree}}(g,z) = \frac{A^2}{g^2}\frac{z^A}{(z^A+1)^2}\,, \quad \text{where} \quad \frac{A}{2g\cos\frac{\pi A}{2}} = 1$$

World's Higgs factory (among other things)

Testing the Standard Model



Higgs talks to almost everyone



…but not equally

Profiling the Higgs boson

- Since Higgs is so different from any other particle, need to test whether it is **exactly as predicted by SM**
- **Higgs could be portal to new physics**

- Measure its properties
  - mass (not predicted by SM)
  - spin (0, got that right!)
  - how often it's produced (σ)
  - branching fractions (Br_i) to all final states, a.k.a. its couplings

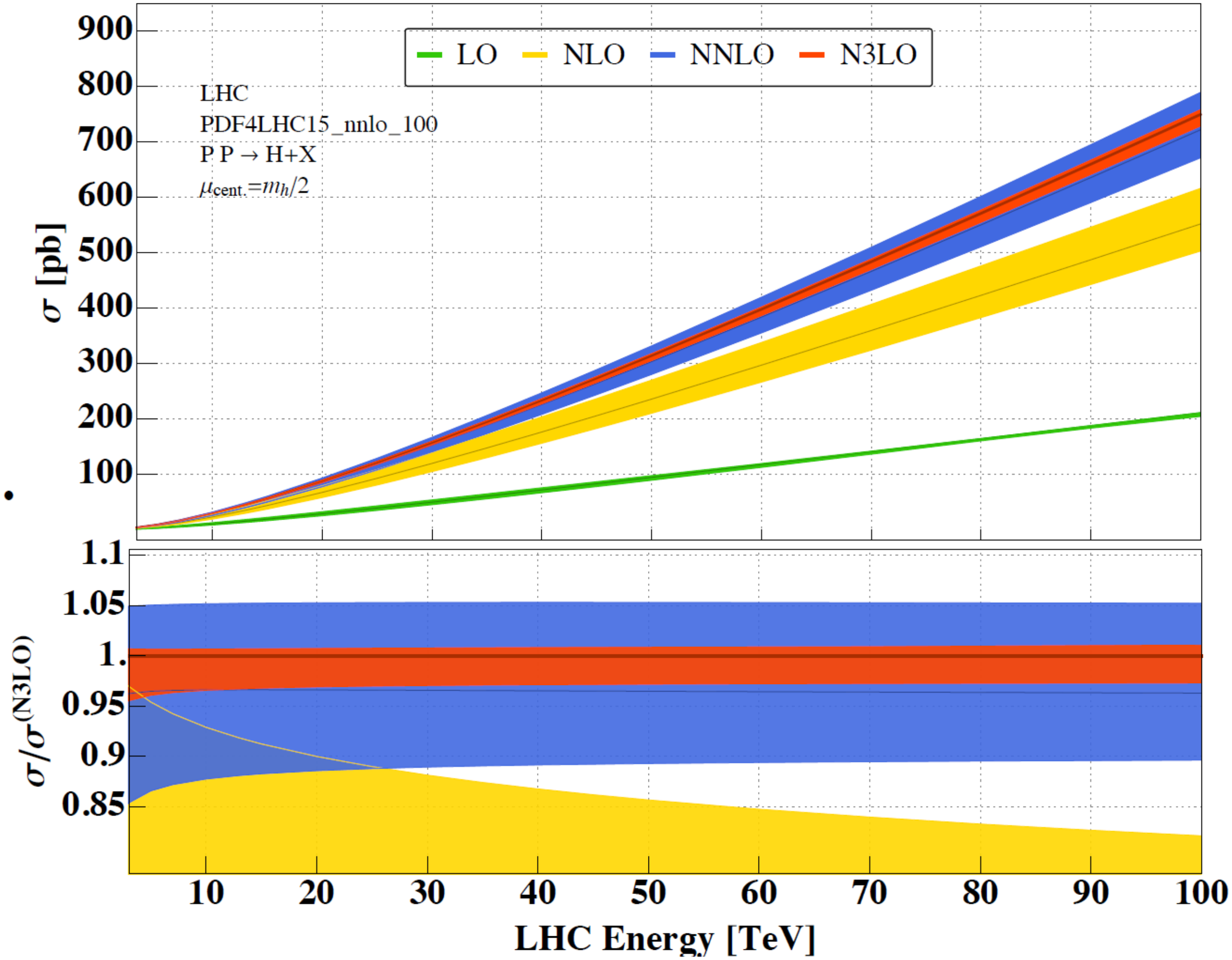
But at LHC can only measure

$$\sigma \times \mathrm{Br}_i$$

→ **Extreme importance of** σ

gg → *H* total cross section at N3LO

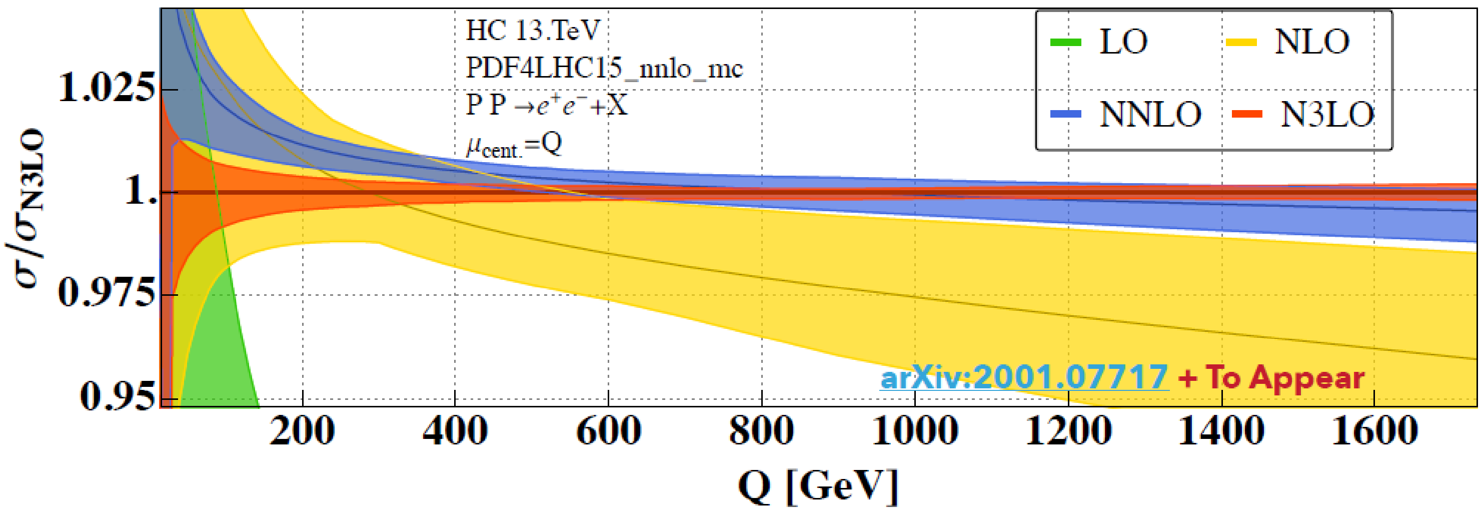
Mistlberger talk



New: "Parton luminosity monitor" now at same order

Mistlberger talk

$$P\,P \rightarrow e^+ e^- + X$$

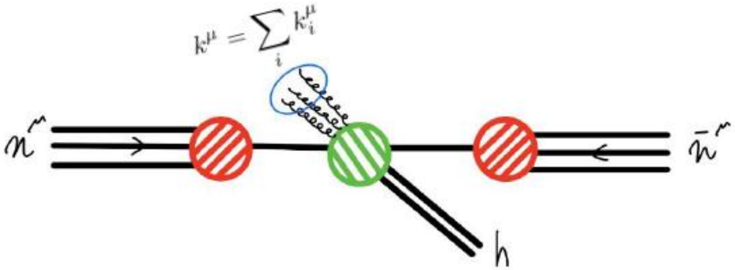


- Remarkable sub-% precision!
- Here N3LO PDF evolution may also be important,

and definitely including experimental cuts (see below)

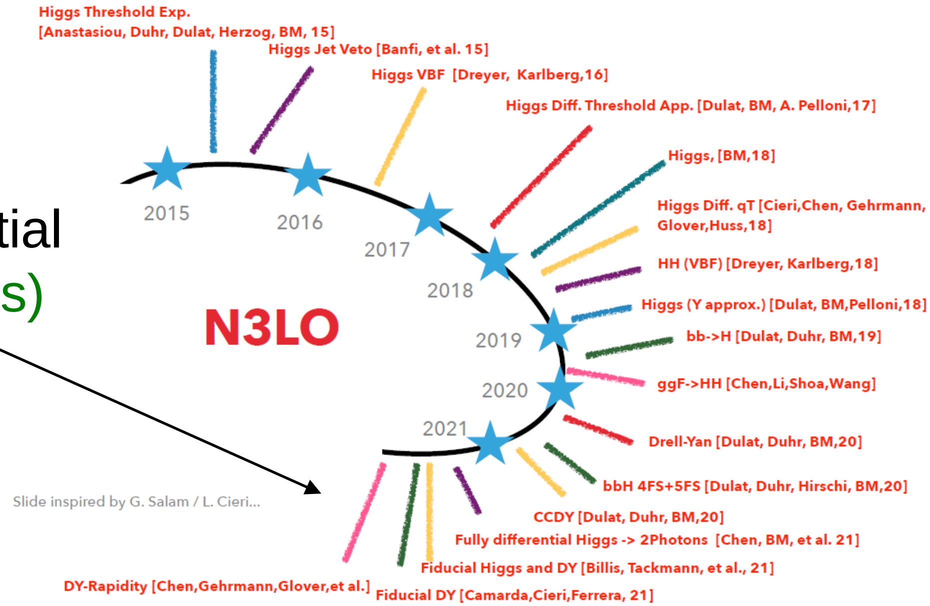
Collinear expansions of cross sections

Vita talk



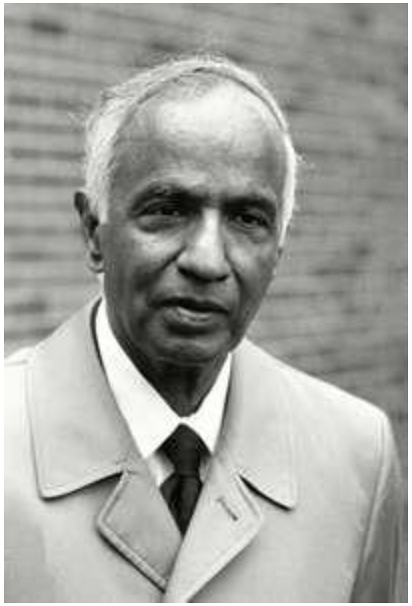
All partons radiated collinear to beam.

- Many applications, including energy-energy-correlator in back-to-back region
- But also: Make N3LO more differential (i.e. implement actual experimental cuts)
- Enabling the next steps in the N3LO revolution



“The black holes of nature are **the most perfect macroscopic objects** there are in the universe: the only elements in their construction are our concepts of space and time.”

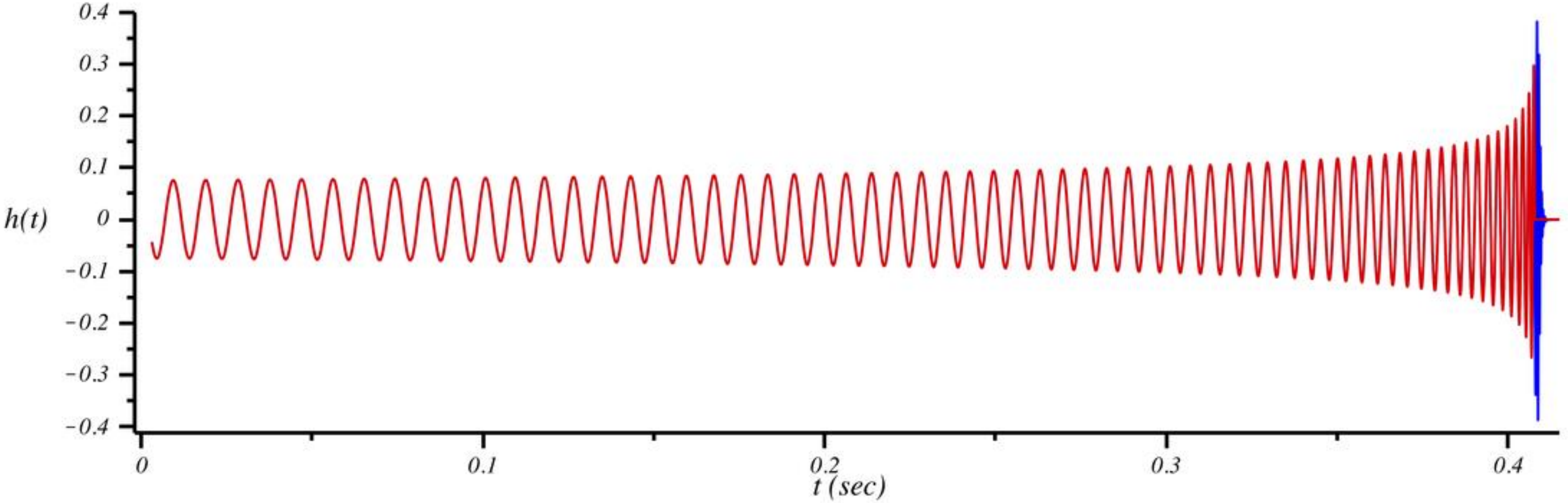
- S. Chandrasekhar (1979)

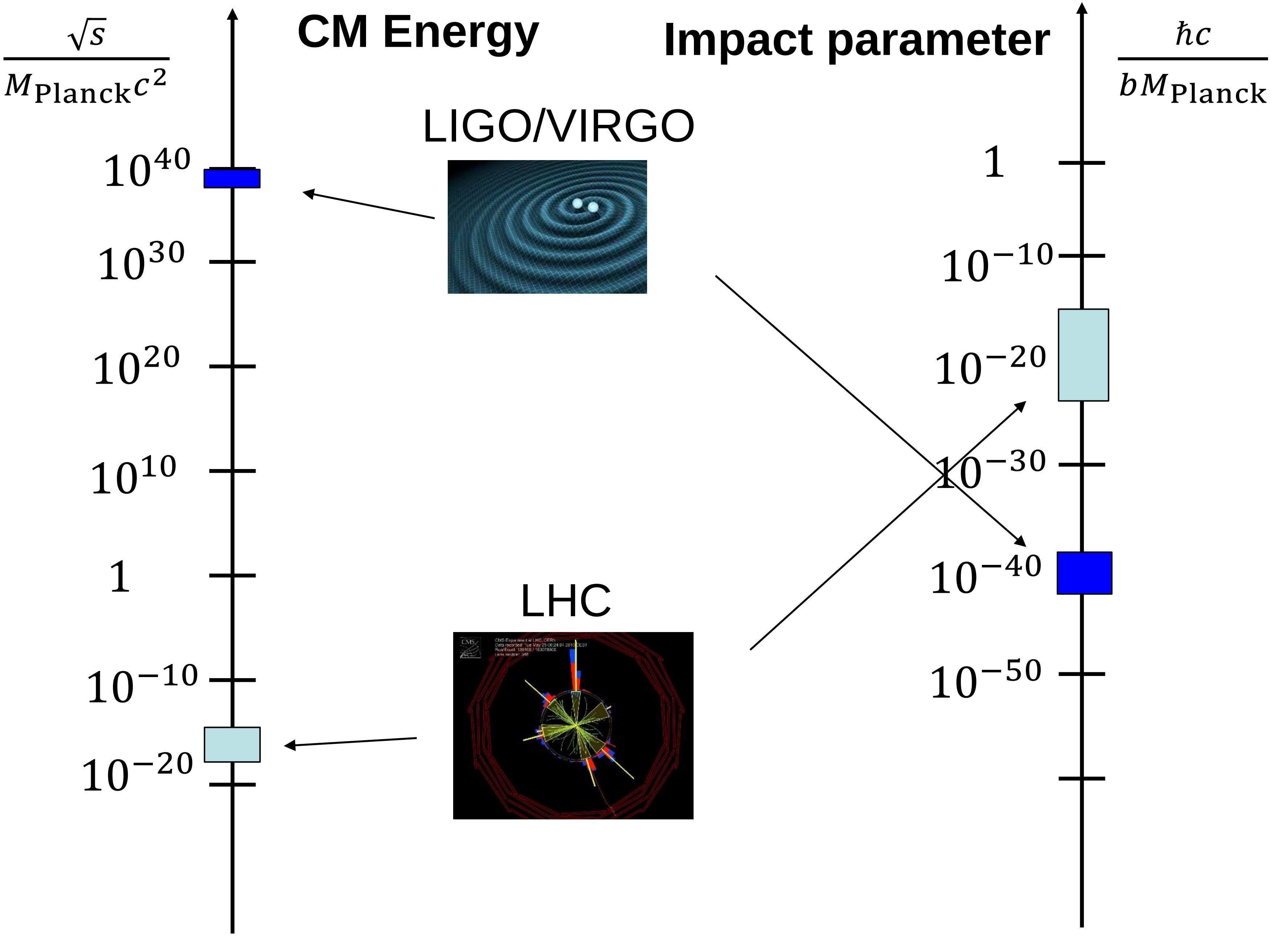


Scattering amplitudes are in many ways **the most perfect microscopic objects** we can imagine, smooth functions of the external data and little else. Remarkable that we can use them to glean insight about BH’s.

talks by Zeng, Goldberger, Damour, Heissenberg, Solon

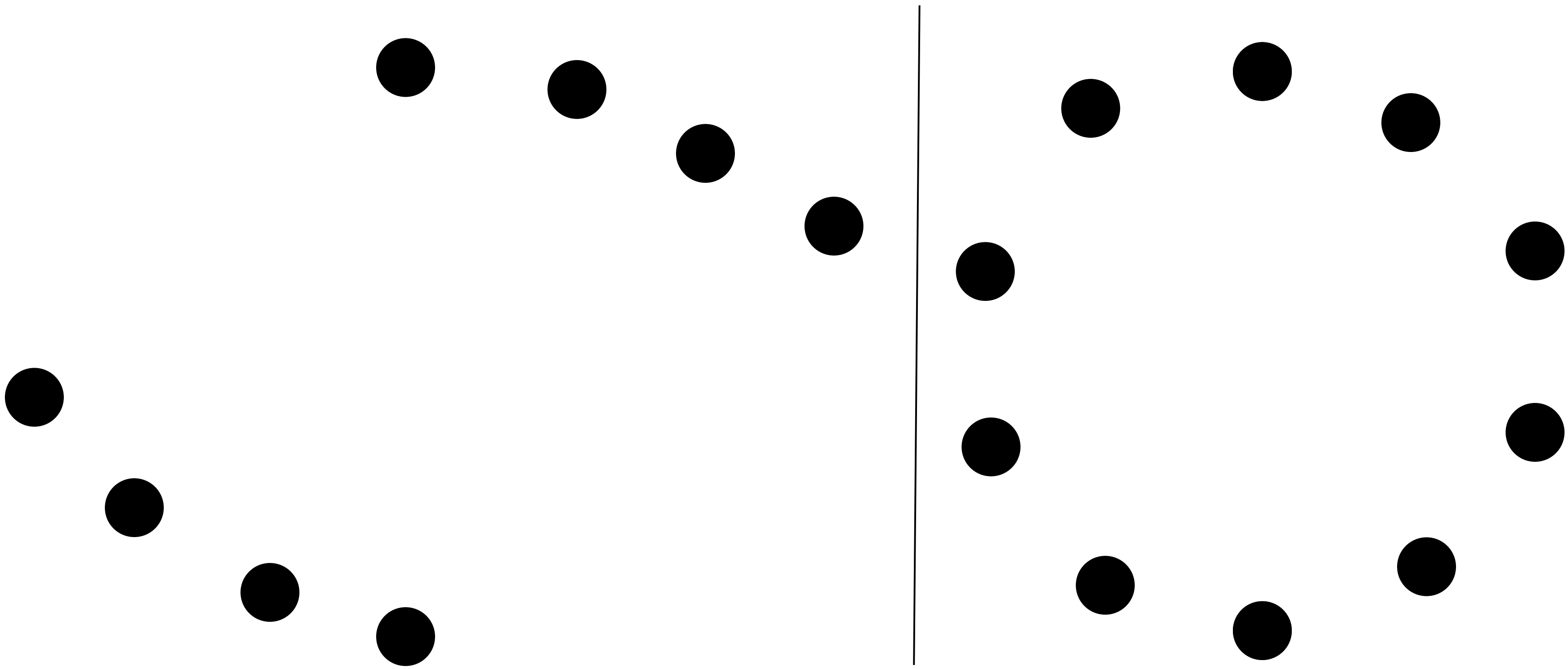
Binary inspiral





CM Energy
Impact parameter
$\frac{\sqrt{s}}{M_{\text{Planck}}c^2}$
$\frac{\hbar c}{b M_{\text{Planck}}}$
LIGO/VIRGO
LHC
10^{40}
10^{30}
10^{20}
10^{10}
1
10^{-10}
10^{-20}
1
10^{-10}
10^{-20}
10^{-30}
10^{-40}
10^{-50}

Black hole scattering vs. inspiral



- Related by "analytic continuation around $r = \infty$"
- Accomplish with effective Hamiltonian, e.g. Cheung, Rothstein, Solon, 1808.02489
- Or more directly in terms of trajectories Kälin, Porto, 1910.03008, 1911.09130
- Extremely useful for understanding issues in the bound state too Damour

Remarkable confluence of different approaches at high orders

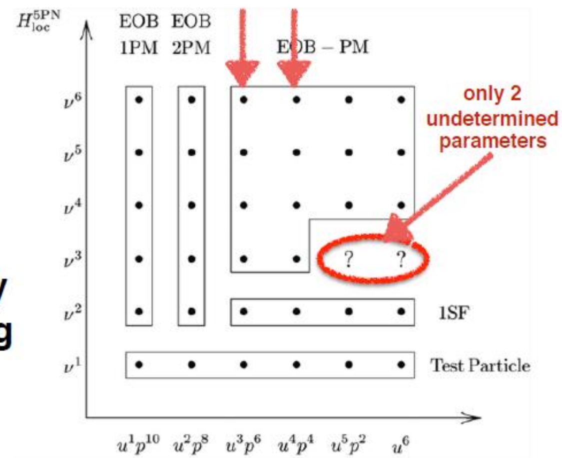
Novel Approach to Binary Dynamics: Application to the Fifth Post-Newtonian Level

Damour talk

Donato Bini,[1,2] Thibault Damour,[3] and Andrea Geralico[1]

Tutti Frutti: combine several efficient, complementary tools:

PN
PM
MPM
EOB
SF
Delaunay averaging



Step 1: Use MPM + EFT to separate off the nonlocal part

\+ worldline approaches — Porto et al.

\+ on-shell approaches — talks by Zeng, Solon

First contributions of "amplitudes" to LIGO physics

	1687	1938	1972 -1985	2001	2014	2020, 2010.13672, Blümlein, Maier, Marquard, Schäfer
	0PN	1PN	2PN	3PN	4PN	5PN

non-potential terms still to do

1PM (1956): $(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\, G$

2PM (1960): $(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots)\, G^2$

3PM (2019 Z. Bern et al., 1901.04404): $(1 + v^2 + v^4 + v^6 + v^8 + \cdots)\, G^3$

4PM (2021 Z. Bern et al., 2101.07254): $(1 + v^2 + \boxed{v^4} + v^6 + \cdots)\, G^4$

2019-2020

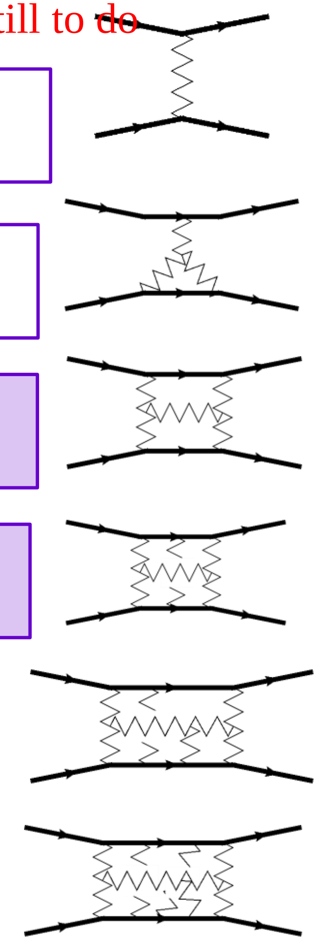
5PM: $(1 + v^2 + v^4 + \cdots)\, G^5$

almost

6PM: $(\boxed{1} + v^2 + \cdots)\, G^6$

2019

Bini, Damour, Geralico, 1909.02375



4PM new state of art for PM

Zeng, Solon talks

$$\mathcal{M}_4(\boldsymbol{q}) = G^4 M^7 \nu^2 |\boldsymbol{q}| \left(\frac{\boldsymbol{q}^2}{4^{\frac{1}{3}} \tilde{\mu}^2} \right)^{-3\epsilon} \pi^2 \left[\mathcal{M}_4^{\mathrm{p}} + \nu \left(\frac{\mathcal{M}_4^{\mathrm{t}}}{\epsilon} + \mathcal{M}_4^{\mathrm{f}} \right) \right]$$

$$\mathcal{M}_4^{\mathrm{p}} = -\frac{35\left(1 - 18\sigma^2 + 33\sigma^4\right)}{8\left(\sigma^2 - 1\right)} \qquad \mathcal{M}_4^{\mathrm{t}} = h_1 + h_2 \log\left(\tfrac{\sigma+1}{2}\right) + h_3 \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}}$$

$$\begin{aligned}
\mathcal{M}_4^{\mathrm{f}} = {} & h_4 + h_5 \log\left(\tfrac{\sigma+1}{2}\right) + h_6 \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} + h_7 \log(\sigma) - h_2 \frac{2\pi^2}{3} + h_8 \frac{\operatorname{arccosh}^2(\sigma)}{\sigma^2 - 1} + h_9 \left[\mathrm{Li}_2\left(\tfrac{1-\sigma}{2}\right) + \frac{1}{2} \log^2\left(\tfrac{\sigma+1}{2}\right) \right] \\
& + h_{10} \left[\mathrm{Li}_2\left(\tfrac{1-\sigma}{2}\right) - \frac{\pi^2}{6} \right] + h_{11} \left[\mathrm{Li}_2\left(\tfrac{1-\sigma}{1+\sigma}\right) - \mathrm{Li}_2\left(\tfrac{\sigma-1}{\sigma+1}\right) + \frac{\pi^2}{3} \right] + h_2 \frac{2\sigma(2\sigma^2-3)}{(\sigma^2-1)^{3/2}} \left[\mathrm{Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - \mathrm{Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) \right] \\
& + \frac{2h_3}{\sqrt{\sigma^2-1}} \left[\mathrm{Li}_2\left(1-\sigma-\sqrt{\sigma^2-1}\right) - \mathrm{Li}_2\left(1-\sigma+\sqrt{\sigma^2-1}\right) + 5\mathrm{Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - 5\mathrm{Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) + 2\log\left(\tfrac{\sigma+1}{2}\right) \operatorname{arccosh}(\sigma) \right] \\
& + h_{12} \mathrm{K}^2\left(\tfrac{\sigma-1}{\sigma+1}\right) + h_{13} \mathrm{K}\left(\tfrac{\sigma-1}{\sigma+1}\right) \mathrm{E}\left(\tfrac{\sigma-1}{\sigma+1}\right) + h_{14} \mathrm{E}^2\left(\tfrac{\sigma-1}{\sigma+1}\right),
\end{aligned}$$

elliptic integrals!

non-local “tail” still missing

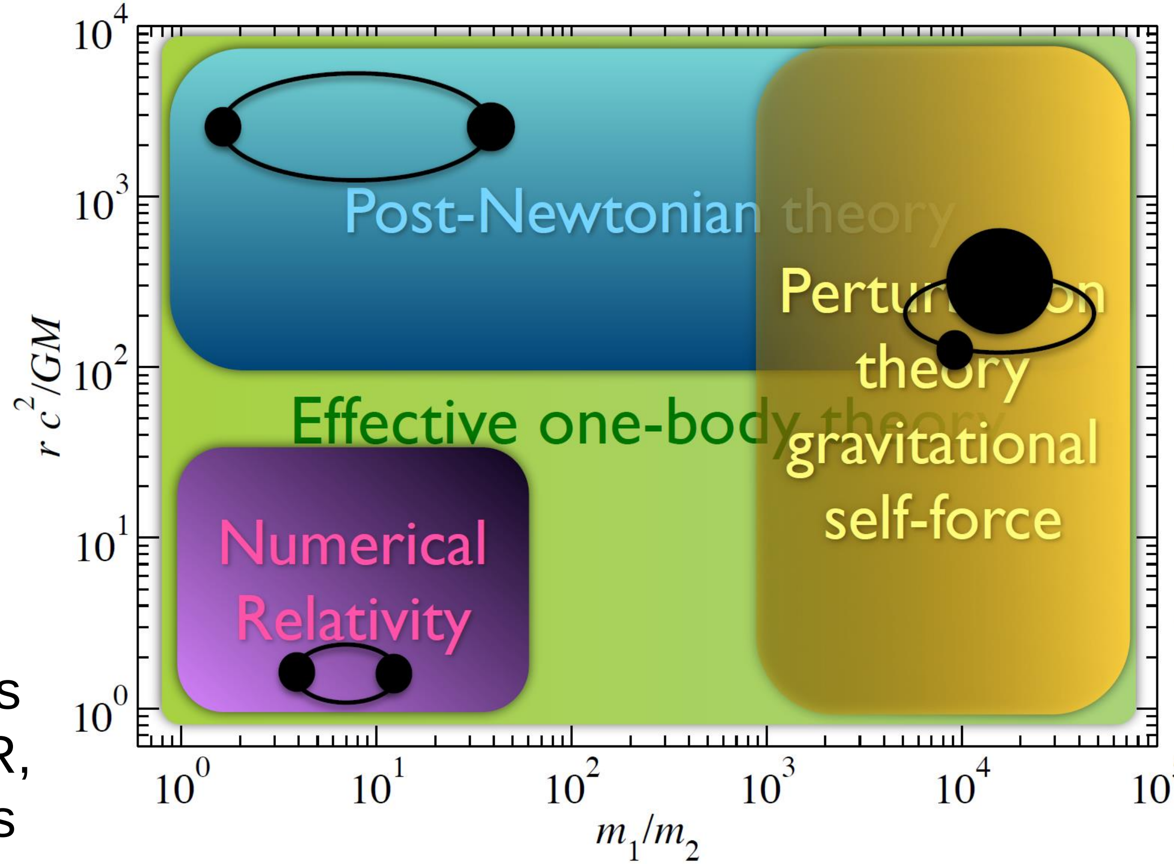
Effective one-body approach

Inspired by properties of bound states in QFT.

Interpolates information from various sources, including PN and PM expansions, test particle limit $m_1 \ll m_2$, and numerical relativity results.

Provides accurate gravitational wave templates very close in, faster than NR, allowing many combinations of initial masses and spins.

Buonanno, Damour, 9811091



Buonanno, Satyaprakash, 1410.7832

4PM (sans tail)
now competes with previous EOB!

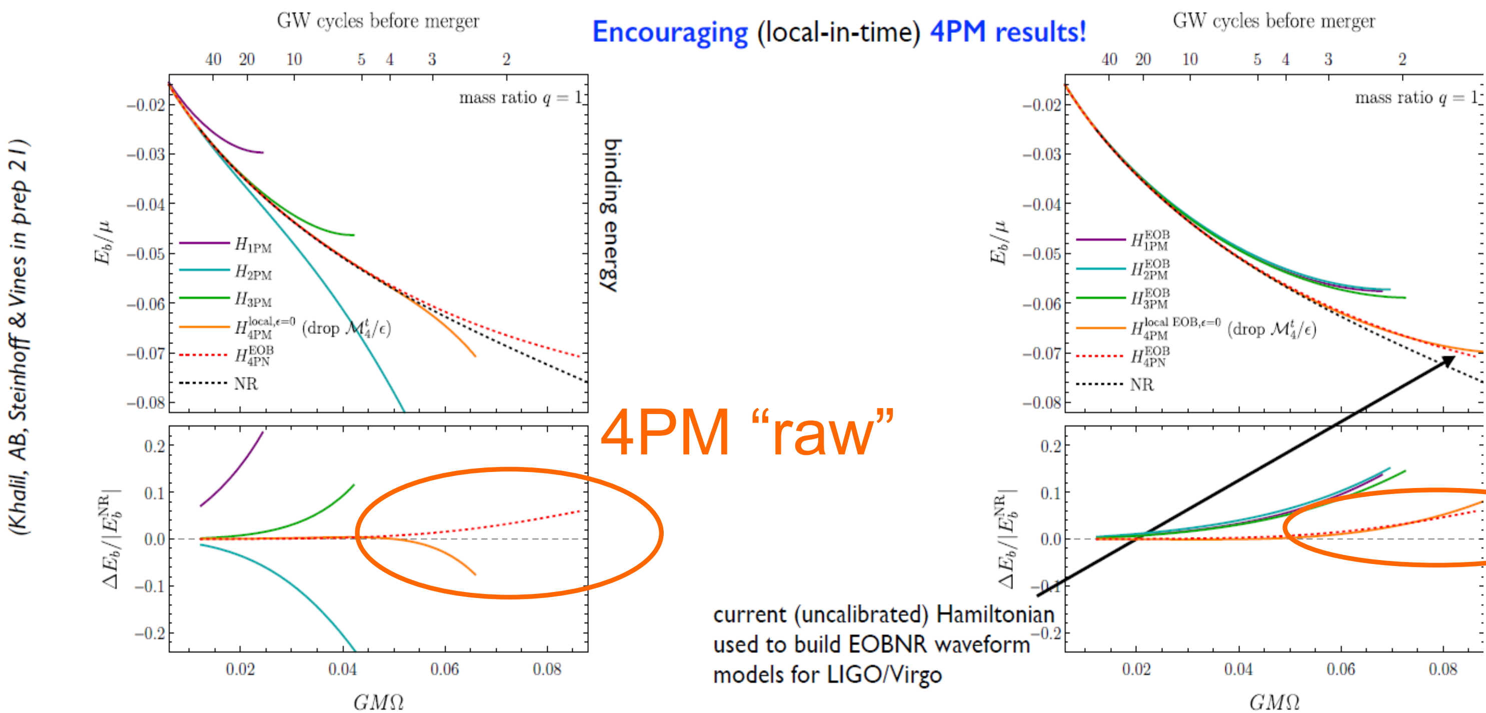
Solon talk

NEW! from Alessandra Buonanno's recent talk at GGI

Crucial to **push PM** calculations at **higher order,** and **resum them** in EOB formalism.

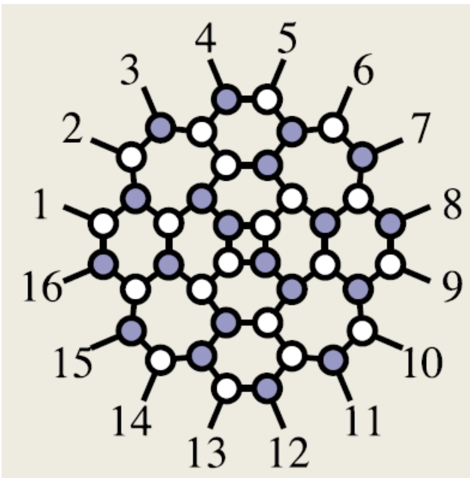
(Damour 19, Antonelli, AB, Steinhoff, van de Meent & Vines 19, Khalil, AB, Steinhoff & Vines in prep 21)

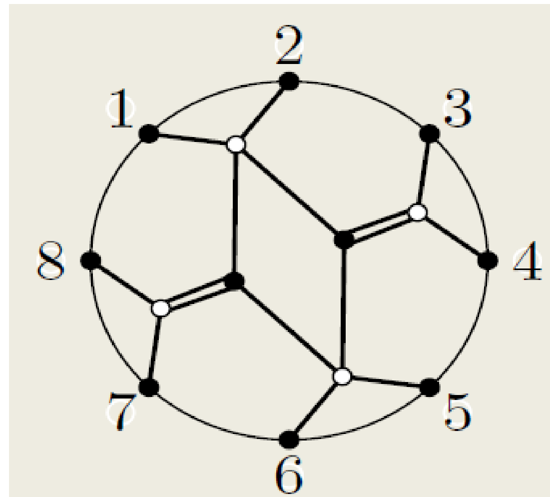
Encouraging (local-in-time) **4PM results!**



4PM informs EOB

Planar N=4 SYM symbol letters from plabic graphs & tensor diagrams



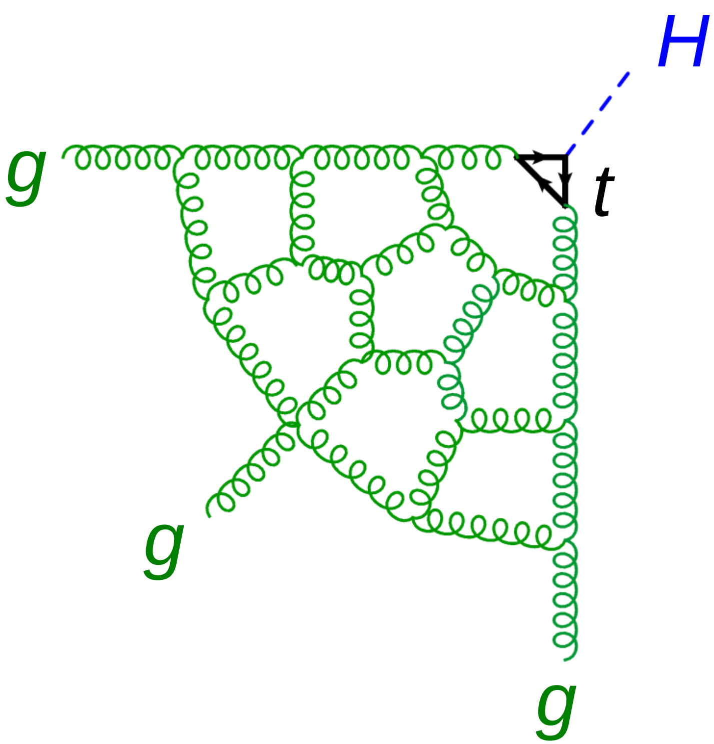


Volovich talk

- Almost arborizable TDs can be "resummed" to give square root letters seen in $\bar{Q}$ computations
- Anastasia was also "rapporteur" for 3 new orbits of n=8 letters found at 3 loops

Zhang poster

Bootstrapping “Higgs” amplitudes and planar N=4 SYM form factors



- As $\boldsymbol{m_{top}} \to \infty$, integrate out top quark to get operator $H\, G_{\mu\nu}^{a} G^{\mu\nu\, a}$ (stress tensor supermultiplet in N=4)
- Higgs amplitudes equivalent to matrix elements of this operator with multiple gluons: “form factors”
- *Hgg* Sudakov form factor is “too simple”, no kinematic dependence beyond overall $(-s_{12})^{-L\epsilon}$
- *Hggg* is “just right”, depends on 2 dimensionless ratios

Hggg kinematics is two-dimensional

$$k_1 + k_2 + k_3 = -k_H$$

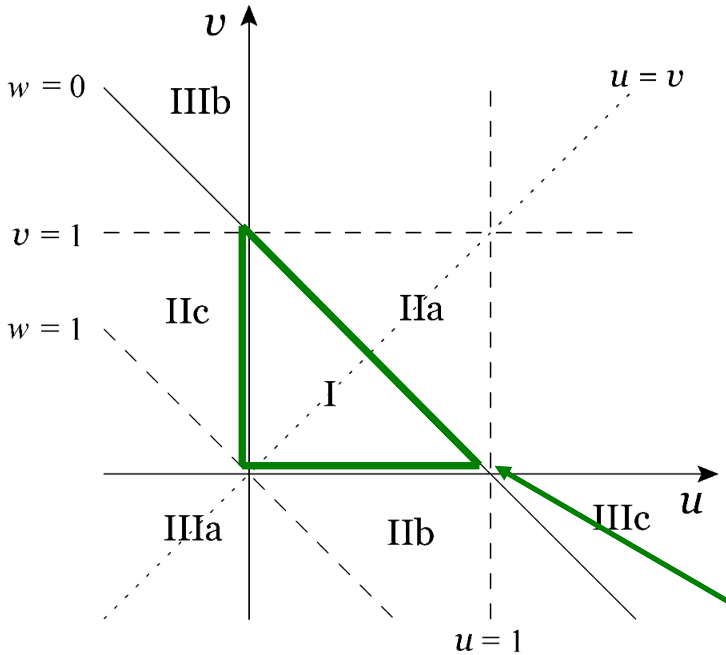
$$s_{123} = s_{12} + s_{23} + s_{31} = m_H^2$$

$$s_{ij} = (k_i + k_j)^2 \qquad k_i^2 = 0$$

$$u = \frac{s_{12}}{s_{123}} \qquad v = \frac{s_{23}}{s_{123}} \qquad w = \frac{s_{31}}{s_{123}}$$

not cross ratios!

$$u + v + w = 1$$



I = decay / Euclidean

IIa,b,c = scattering / spacelike operator

IIIa,b,c = scattering / timelike operator

Boundary data, near-collinear limit from the form factor OPE:
Sever, Tumanov, Wilhelm, 2009.11297, 2105.13367

2d HPLs

Gehrmann, Remiddi, hep-ph/0008287

Space graded by weight n. Every function F obeys:

$$\boxed{\begin{aligned}\frac{\partial F(u,v)}{\partial u} &= \frac{F^u}{u} - \frac{F^w}{1-u-v} - \frac{F^{1-u}}{1-u} + \frac{F^{1-w}}{u+v} \\ \frac{\partial F(u,v)}{\partial v} &= \frac{F^v}{v} - \frac{F^w}{1-u-v} - \frac{F^{1-v}}{1-v} + \frac{F^{1-w}}{u+v}\end{aligned}}$$

$$w = 1-u-v$$

where $F^u, F^v, F^w, F^{1-u}, F^{1-v}, F^{1-w}$ are weight n-1 2d HPLs.

Symbol alphabet: $\{u, v, w, 1-u, 1-v, 1-w\}$

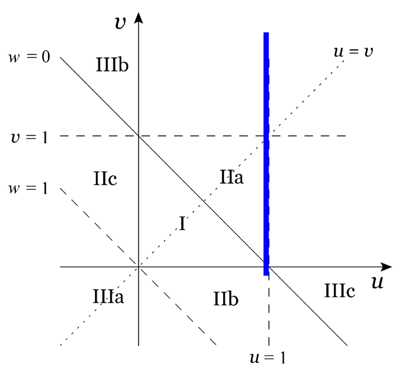
To bootstrap *Hggg* amplitude, find as small a subspace of 2d HPLs as possible, construct it to high weight.

Works like magic!

LD, A. McLeod, M. Wilhelm, 2012.12286
+ Ö. Gürdoğan, in preparation

3,4,5 loops
6,7,8 loops

4 hours of MITP Amplitude Games lectures:
https://indico.mitp.uni-mainz.de/event/204/timetable/#20210726



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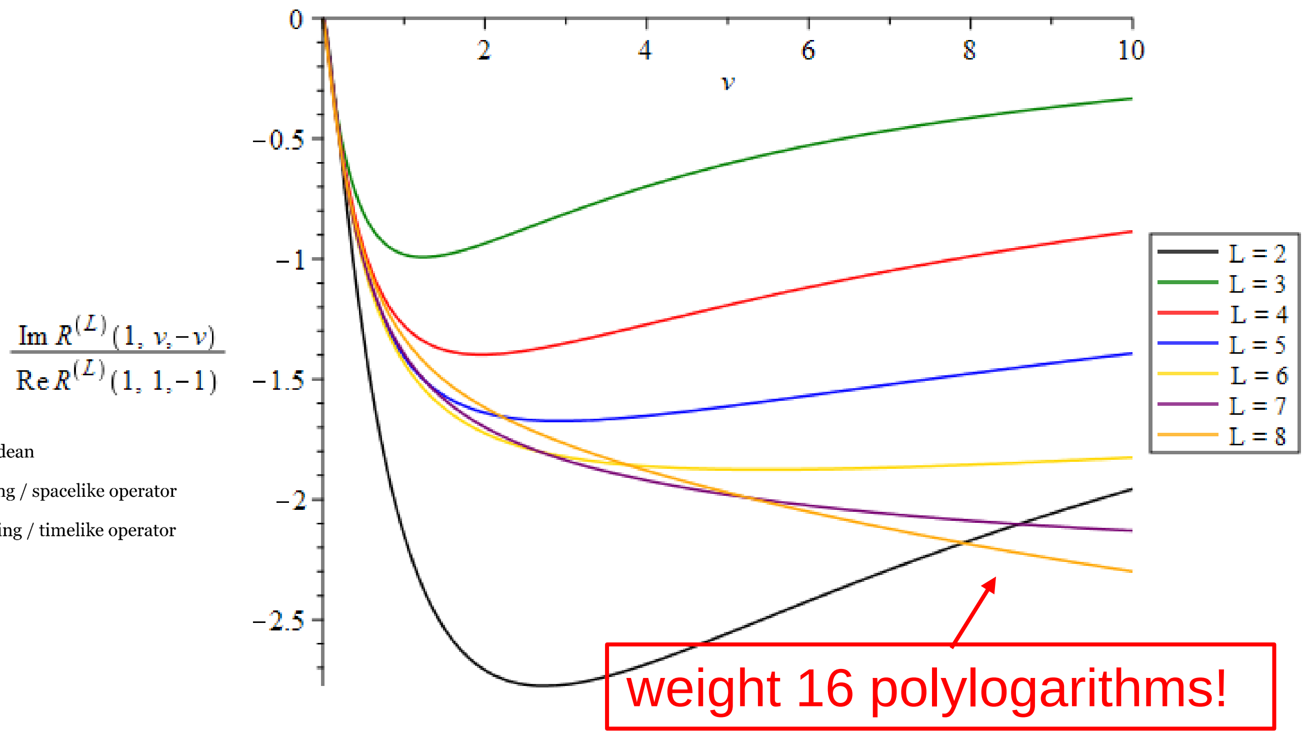
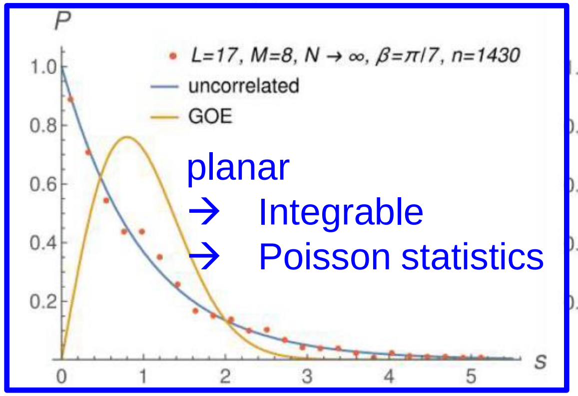


Short Segue:
statistics of anomalous dimensions

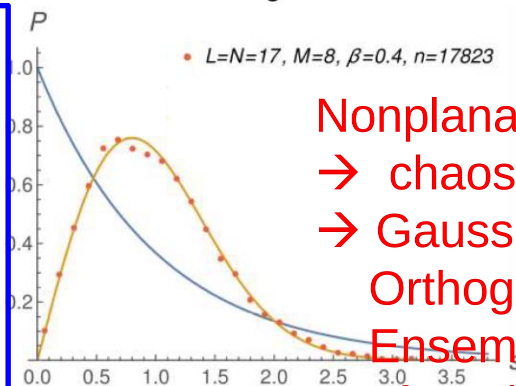
Integrability and chaos in SYM theories

Spiering talk

- Results for β-deformed $\mathcal{N} = 4$ SYM theory

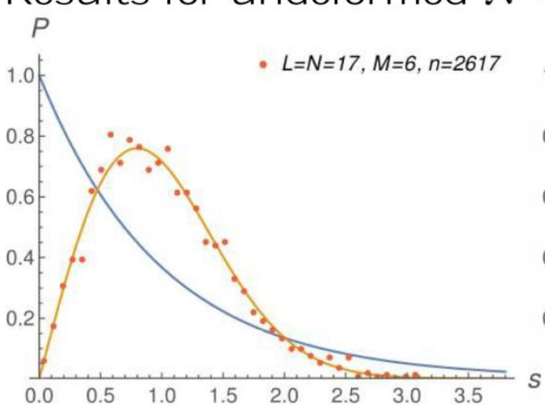


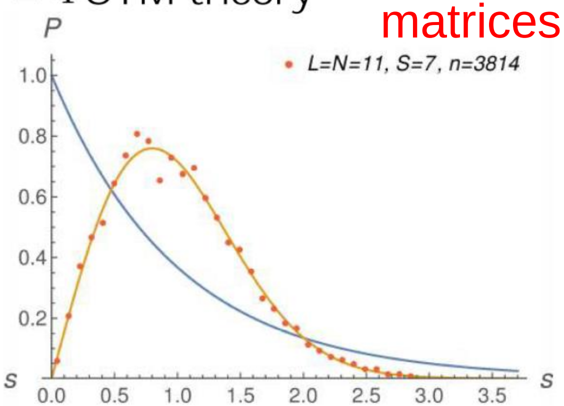
planar
→ Integrable
→ Poisson statistics



Nonplanar
→ chaos
→ Gaussian Orthogonal Ensemble of random matrices

- Results for undeformed $\mathcal{N} = 4$ SYM theory





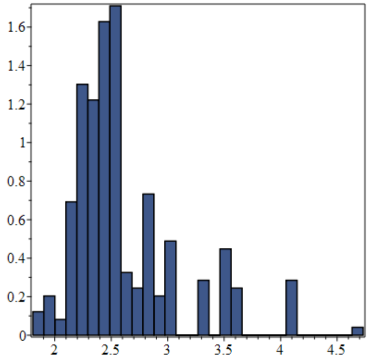
planar spectrum
~ Poisson distribution
↓
integrability

non-planar spectrum
~ RMT (GOE)
↓
quantum chaos

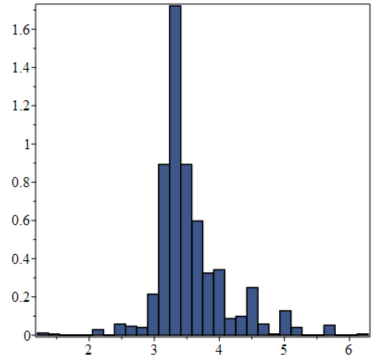
S = nearest neighbor spacing

Statistics of the *Hggg* planar amplitude: $\log_{10}$ of HPL coefficients (all integers)

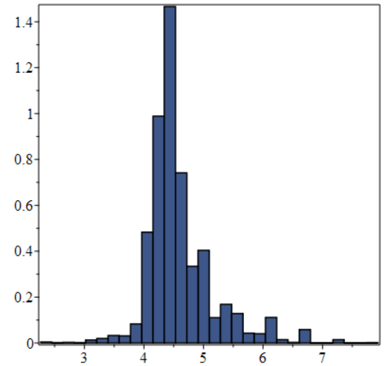
$v \to \infty$ line



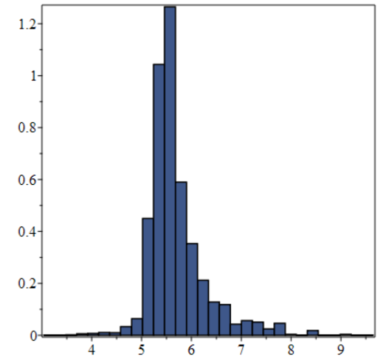
$L = 5$



$L = 6$

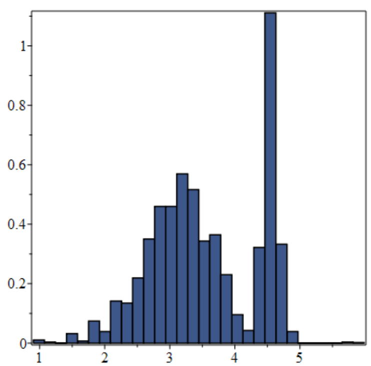


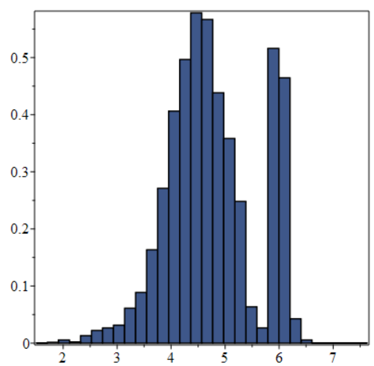
$L = 7$

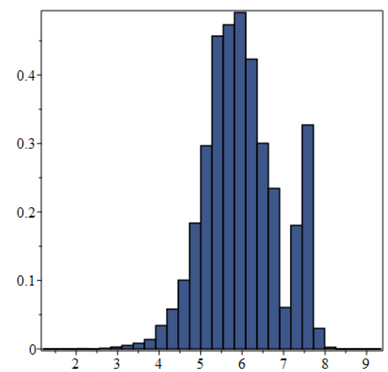


$L = 8$

$u = 1$ line







first attempt crashed the cluster

Grist for a machine to learn $L = 9$?

Predictions for the future

- Totally magical and highly nontrivial results will be found, rivaling even the ones presented this year!
- There will be 3 new -hedra discovered by 2022, to go along with the amplitu-, cluster-, surface-, binary-, and halohedra discussed this week
- Zvi will finally lose a bottle of wine
- The "loss function" for total magicality will be discovered, enabling deep learning to put us all out of business.

Some more thank yous

To first responders & medical personnel

Public health officials

Vaccine manufacturers

To all of you and (almost) everyone else for taking vaccines and other precautions for over 17 months

Thank you (again) from Greta

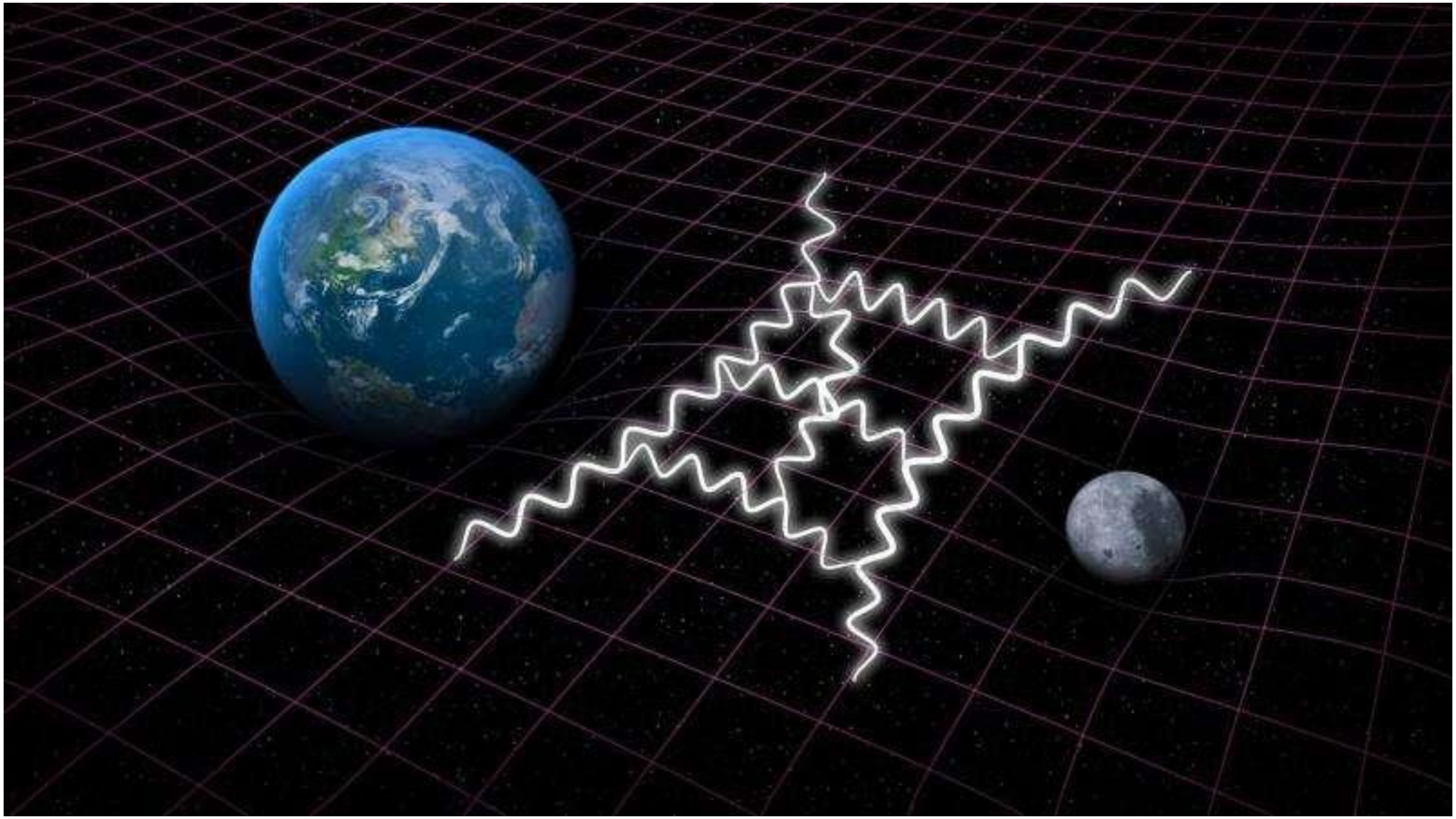
Thank you for not flying
to Amplitudes also this year!
We’ll talk again next year
in Prague.

Prague 2022!

Thanks in advance, Jara and Karol!

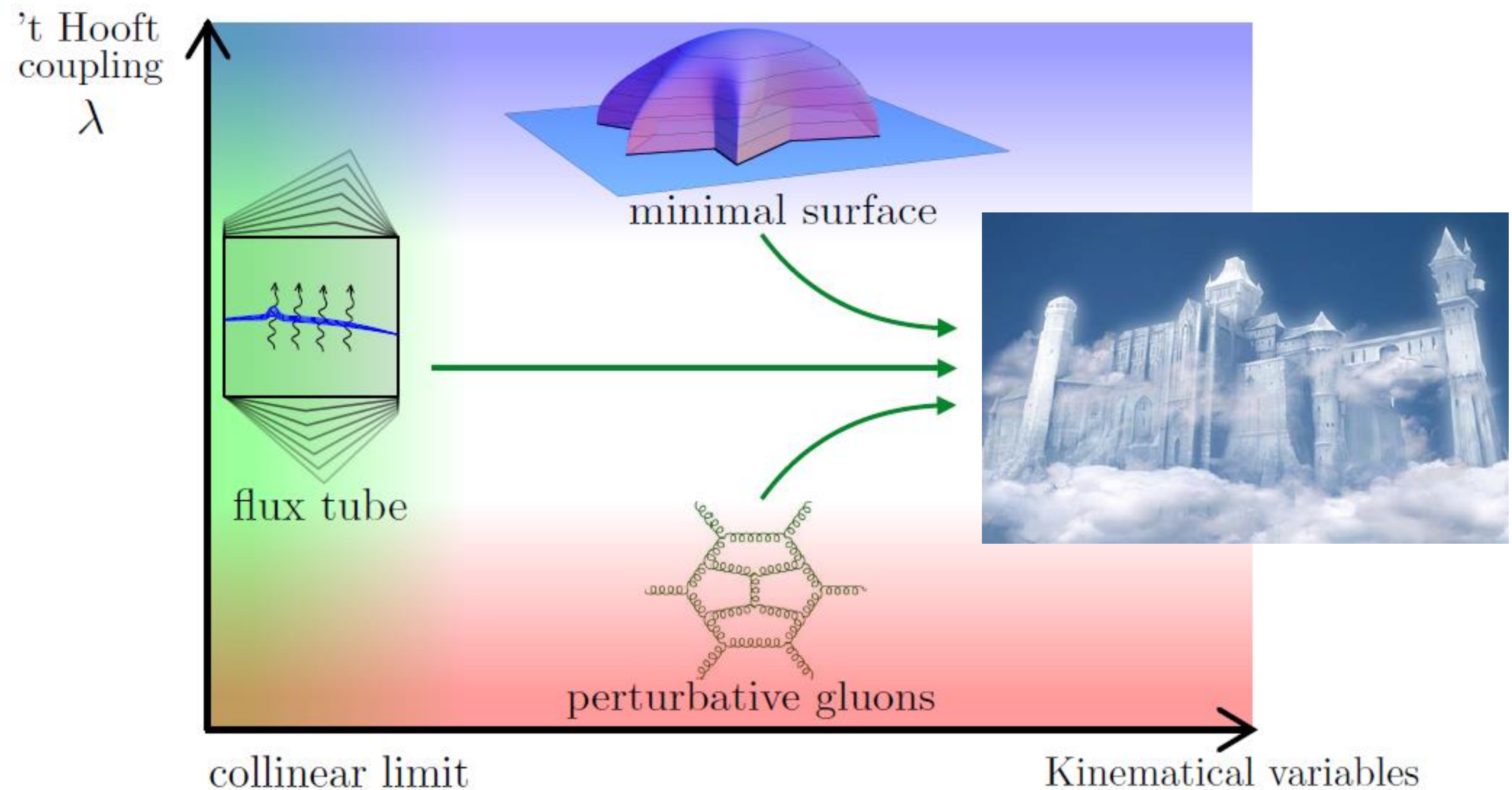
Extra Slides

Thanks for your attention!



Solving for Planar N=4 SYM Amplitudes

Images: A. Sever, N. Arkani-Hamed



Different routes to perturbative amplitudes

Draw all Feynman graphs G_i

↓

Evaluate all Feynman rules: I_i

↓

Perform all loop integrations: A_i

↓

$$A = \sum_i A_i$$

Evaluate all unitarity cuts C_α

↓

Construct **local**: integrand I

↓

Perform all loop integrations: A_α

↓

$$A = \sum_\alpha A_\alpha$$

Bootstrap: Guess $A = \sum_m c_m F_m$

F_m known functions

$c_m \in \mathbb{Q}$ unknown constants

↓

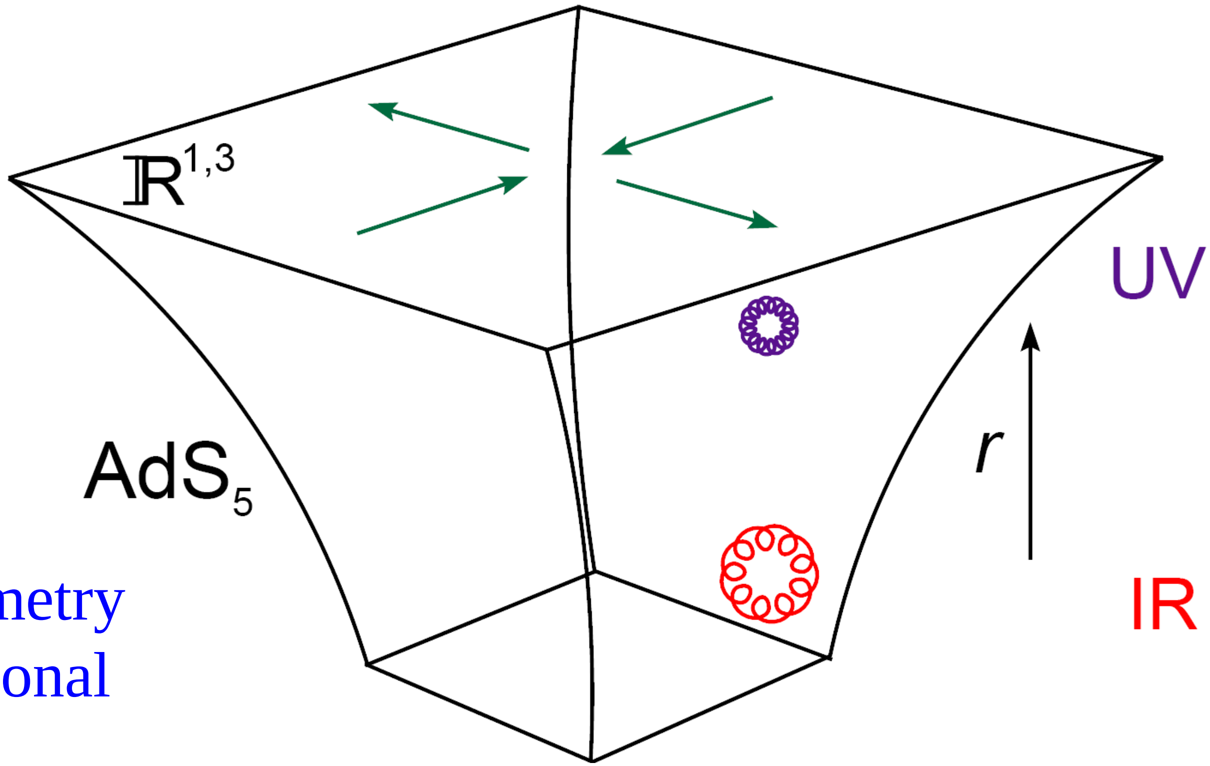
Solve constraints, linear equations for $c_m \to r_m$

↓

$$A = \sum_m r_m F_m$$

Dual conformal invariance from AdS/CFT + T-duality

Alday, Maldacena, 0705.0303



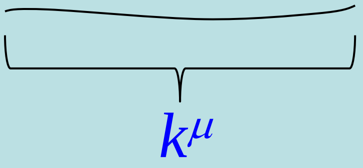
SO(4,2) isometry
of 5 dimensional
space-time
←→ 4d conformal symmetry

T-duality symmetry of string theory

Alday, Maldacena, 0705.0303

- Exchanges string world-sheet variables σ, τ
- $X^\mu(\tau,\sigma) = x^\mu + k^\mu \tau$ + oscillators

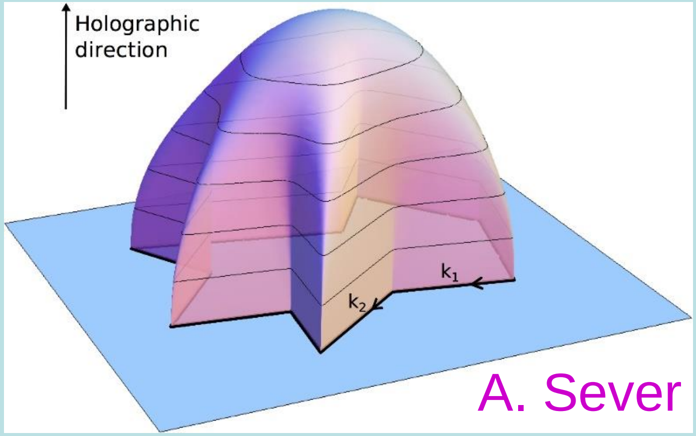
$\rightarrow$ $X^\mu(\tau,\sigma) = x^\mu + k^\mu \sigma$ + oscillators



- Strong coupling limit of planar N=4 SYM

is semi-classical limit of string theory: world-sheet stretches tight around minimal area surface in AdS.

- Boundary determined by momenta of external states: light-like polygon with null edges = momenta k^μ



A. Sever