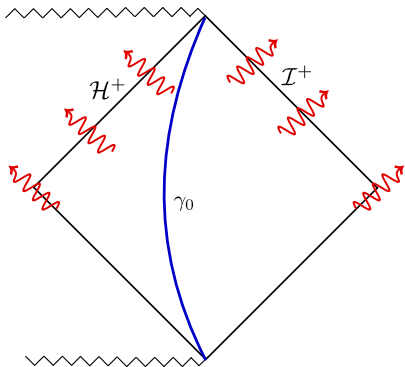


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- ② Lecture 2: the local problem: how to deal with small bodies
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Typical calculation at first order [Capra community]



- approximate the source orbit as a bound geodesic
- impose outgoing-wave BCs at $\mathcal{I}^+$ and $\mathcal{H}^+$
- unphysical on long timescales. Breaks down on dephasing time $t \sim M/\sqrt{\epsilon}$

Solving the first-order field equations [Capra community]

- directly solve Lorenz-gauge linearized Einstein equation for $h_{\mu\nu}^{(1)}$
 - advantage: most expressions for singular field are in Lorenz gauge. Easily separated into ℓm modes in Schwarzschild
 - disadvantage: not separable in Kerr
- solve Regge-Wheeler and Zerilli equations for master functions $\Psi_{\ell m}$

$$\Rightarrow h_{\mu\nu}^{(1)\ell m} \sim \partial \Psi_{\ell m}$$

- advantage: simple scalar ODEs
 - disadvantages: not defined in Kerr. Pathological singularities away from particle
- solve Teukolsky equation for curvature scalar $\psi_4^{\ell m \omega}$

$$\Rightarrow h_{\mu\nu}^{(1)\ell m \omega} \sim \partial \partial \int \int \int \int \psi_4^{\ell m \omega}$$

- advantage: fully separable into $\ell m \omega$ modes in Kerr
 - disadvantage: pathological singularities away from particle

- all time dependence in $h_{\mu\nu}$ follows from puncture's motion and black hole's evolution
- recall $(z^\mu, u^\mu) \rightarrow (\tilde{\varphi}_A, \tilde{J}^A)$. Define full set of system parameters $\mathcal{J}^A \sim (\tilde{J}^A, M, a)$

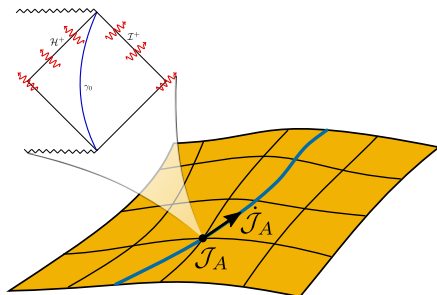
$$\begin{aligned}\frac{d\tilde{\varphi}_A}{dt} &= \Omega_A(\mathcal{J}^B) \\ \frac{d\mathcal{J}^A}{dt} &= \epsilon \tilde{G}_{(1)}^A(\mathcal{J}^B) + \epsilon^2 \tilde{G}_{(2)}^A(\mathcal{J}^B) + O(\epsilon^3)\end{aligned}$$

- treat $h_{\mu\nu}$ as function on extended manifold: in spacetime coords (t, x^i) ,

$$h_{\mu\nu} = \epsilon h_{\mu\nu}^{(1)}(\tilde{\varphi}_A, \mathcal{J}^A, x^i) + \epsilon^2 h_{\mu\nu}^{(2)}(\tilde{\varphi}_A, \mathcal{J}^A, x^i) + O(\epsilon^3)$$

- in Einstein equations, $\frac{\partial}{\partial t} = \Omega_A \frac{\partial}{\partial \tilde{\varphi}_A} + \frac{d\mathcal{J}^A}{dt} \frac{\partial}{\partial \mathcal{J}^A}$

Multiscale expansion [Miller & AP; AP & Wardell; Flanagan, Hinderer, Moxon, AP]



parameter space

- Fourier series:

$$h_{\mu\nu}^n = \sum_{k^A} h_{\mu\nu}^{n,\Omega_k}(\mathcal{J}^A, x^i) e^{-ik^A \varphi_A}$$

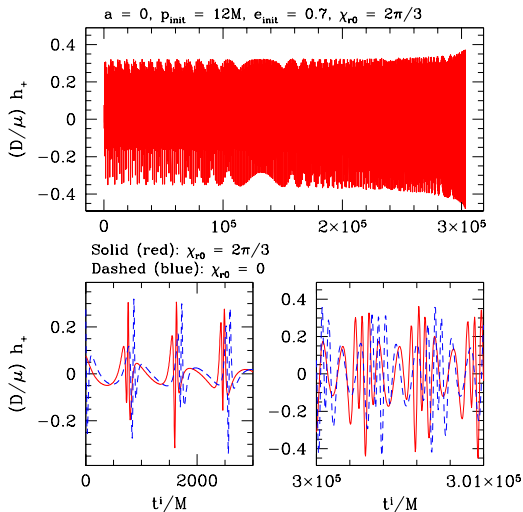
$$\Omega_k := k^A \Omega_A$$

- solve field equations for amplitudes $h_{\mu\nu}^{n,\Omega_k}$ on grid of $\mathcal{J}^A$ values

- millisecond waveform generation when combined with FastEMRIWaveforms tools [Katz, Chua, Speri, Warburton, Hughes]

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Adiabatic waveforms



[Hughes et al]

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Complete first-order self-force

- complete inspirals simulated in Schwarzschild using full F_1^μ (including spin force) [Warburton et al]
- and F_1^μ has been computed on generic orbits in Kerr [van de Meent]
- but still need F_2^μ for post-adiabatic inspiral

[image courtesy of Warburton]

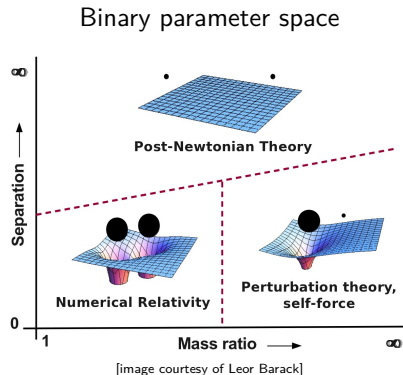
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[image courtesy of Warburton]

First-order results: improving other binary models

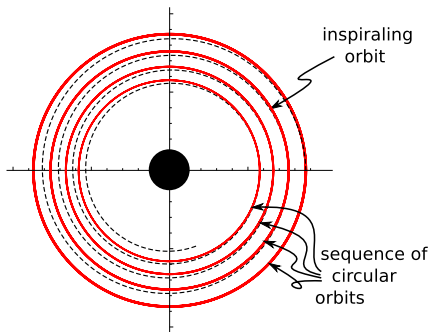
- PN and EOB models have been improved using data for effects of the self-force
- fixed high-order PN coefficients, completed 4PN, settled controversy at 4PN



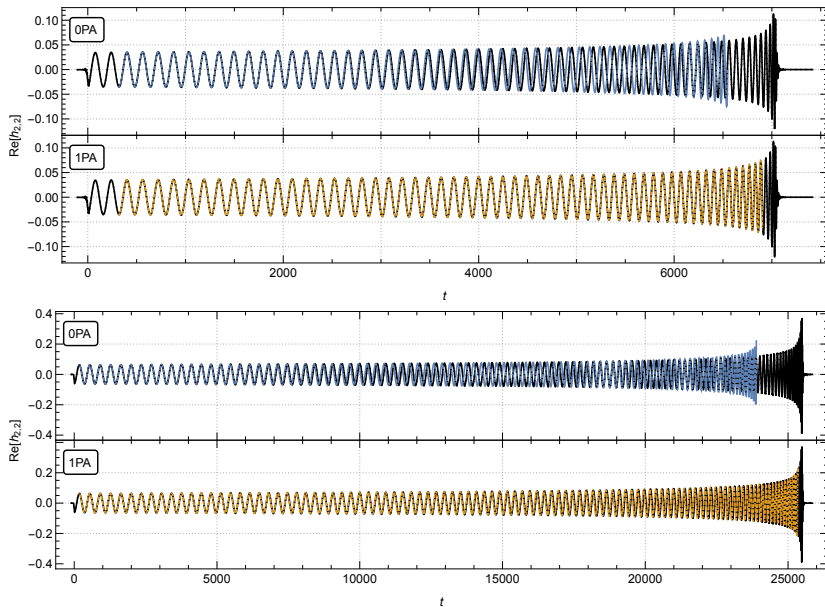
Complete 1PA for quasicircular orbits [AP, Warburton, Wardell 2013–]

- parameters: $\mathcal{J}^A = \{\Omega, M, a\}$, small a
- phase: $\frac{d\phi_p}{dt} = \Omega$
- $h_{\mu\nu}^{(n)} \sim \sum_{\ell m} h_{\mu\nu}^{(n)\ell m}(\mathcal{J}^A, r) e^{-im\phi_p} Y_{\ell m}$

- solve field equations for amplitudes $h_{\mu\nu}^{(n)\ell m}$



1PA waveforms [Wardell, AP, Warburton, Miller, Durkan, Le Tiec]



- self-force theory is currently the only viable method of modelling EMRIs
- surprisingly accurate even for comparable masses
 - use in LVK data analysis?
- can generate waveforms rapidly
- some work remains to populate parameter space for OPA waveforms
- a lot of recent work on spinning secondaries
- only have 1PA waveforms for quasicircular, nonspinning binaries