From Quantum Field Theory to Real-World Gravity

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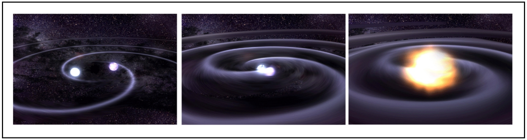
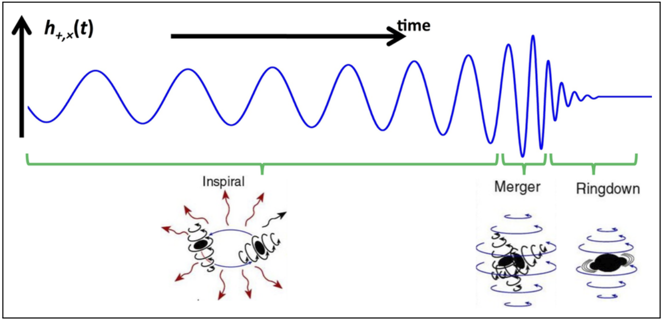
UK Research and Innovation

GW signal from compact binaries



3 phases in the life of a compact binary



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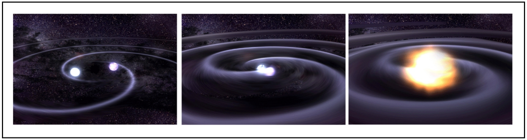
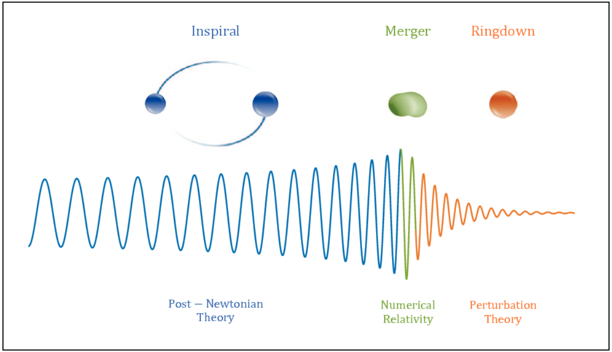
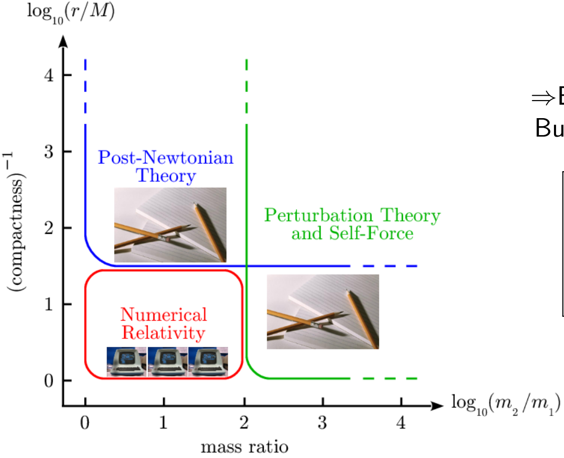


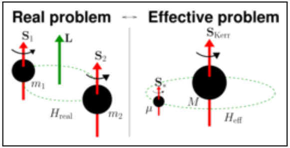
3 phases in the life of a compact binary



Theory of GW templates



⇒Effective One-Body (EOB), Buonanno and Damour 1999

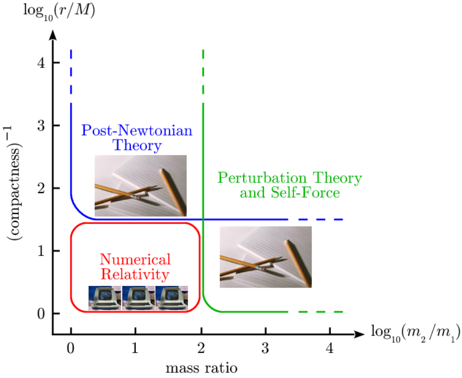


⇒Numerical Relativity breakthrough, Pretorius 2005

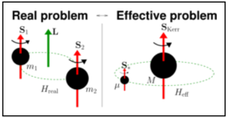
Detection by matched filtering

- High demand on accurate theoretical waveform templates
- Modeling of waveforms currently uses PN parameters of up to 6th PN order! Up to 10% possible discrepancy w simulations at 5PN [nPN$\equiv v^{2n}$]

Theory of GW templates



⇒Effective One-Body (EOB)
Buonanno & Damour 1999



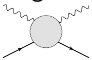
- Increasing influx of real-world GW data
  ⇒ PN gravity is key for theoretical GW data → EFTs of PN Gravity
- Underlying Science: Informs on strong gravity, QFT ↔ Gravity
- Can we get insight on the graviton Compton amplitude with $s \geq 5/2$ from PN gravity? [Arkani-Hamed, Huang, Huang; 2017]

State of the Art in PN Gravity

State of the Art for Generic Compact Binary Dynamics

l \ n	(N^0)LO	$\mathrm{N}^{(1)}$LO	N^2LO	N^3LO	N^4LO	N^5LO
S^0	1	0	3	0	25	0
S^1	2	7	32	**174**		
S^2	2	2	**18**	**52**		
S^3	4	**24**				
S^4	3	**5**				

- (n, l) entry at $n + l + \text{Parity}(l)/2$ PN order
- A measure for loop computational scale:
  $n =$ highest n-loop graphs at N^nLO, $l =$ highest multipole moment S^l
- Gray area corresponds to gravitational Compton scattering

  with $s \geq 3/2$ since classical $S^l \leftrightarrow$ quantum $s = l/2$
- All (but top right ones) are derived in the public EFTofPNG code:
  https://github.com/miche-levi/pncbc-eftofpng

State of the Art

State of the Art for Generic Compact Binary Dynamics

l \ n	$(N^0)LO$	$N^{(1)}LO$	N^2LO	N^3LO	N^4LO	N^5LO
S^0	++	++	++	++	++	+
S^1	++	++	++	+		
S^2	++	++	+	+		
S^3	++	+				
S^4	++	+				

- (n, l) entry at $n + l + \text{Parity}(l)/2$ PN order
- $++$ = fully done/verified; $+$ = partial/not verified
- Even l easier than odd l; Also in particular at $l = 0 \to n$ odd easier
- As of 2PN – UV dependence essential to complete accuracy
- At 4PN all sectors fully verified except (n,l)=(2,2) [Levi+ 2015]
- At 4.5PN & 5PN – NO sector is currently fully done/verified!

EFTs are Universal

Levi, Rept. Prog. Phys. 2020

There is a Hierarchy of Scales

1. r_s, scale of internal structure, $r_s \sim m$
2. r, orbital separation scale, $\dfrac{r_s}{r} \sim v^2$
3. λ, radiation wavelength scale, $\dfrac{r}{\lambda} \sim v$

$v \ll 1 \to nPN \equiv v^{2n}$ correction in classical gravity to Newtonian gravity

Multistage strategy for EFTs of inspiraling binaries

[Goldberger & Rothstein 2006]

1. One-Particle EFT
2. EFT of a Composite Particle
3. Effective Theory of Dynamical Multipoles

It's a multiscale!

Setup of EFTs is Universal

Bottom-Up and/or Top-Down Strategies

Two generic procedures to construct Effective Field Theories:

1. Top-Down, perturbative theory in high resolution/close-up is known, so we reduce resolution by systematically removing extra pixels. Feynman formalism/technology enables that.
   – *"Wilsonian approach", Wilson 1971-1974*

2. Bottom-Up, no perturbative theory for the system in close-up, so we look from afar, and gradually zoom in. This is done by identifying the Degrees Of Freedom and Symmetries.
   – *"Decoupling theorem", Appelquist & Carazzone 1975*

One-Particle EFT

1st Stage Remove scale r_S, i.e. internal structure of compact object

In the full theory we only have a vacuum gravitational field:

$$S\left[g_{\mu\nu}\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{g}R\left[g_{\mu\nu}\right]$$

"Integrate out" strong field modes $g^s_{\mu\nu}$, $g_{\mu\nu} \equiv g^s_{\mu\nu} + \bar{g}_{\mu\nu}$ via bottom-up approach:

$$S_{\text{eff}}\left[\bar{g}_{\mu\nu}, y^\mu(\sigma), e^\mu_A(\sigma)\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}R\left[\bar{g}_{\mu\nu}(x)\right] + \underbrace{\sum_{i=1}^{\infty} C_i(r_s)\int d\sigma \mathcal{O}_i(\sigma)}_{\equiv S_{pp}(\sigma)\ \text{with Wilson coefficients}}$$

The operators $\mathcal{O}_i(\sigma)$ must respect the symmetries that pertain at low energies.

$$S_{\text{eff}}[\bar{g}_{\mu\nu}, y^\mu] = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}R\left[\bar{g}_{\mu\nu}(x)\right] - \int m d\sigma + \underbrace{c_{5\text{PN}}\int d\sigma\left(R_{\mu\alpha\nu\beta}\dot{y}^\alpha\dot{y}^\beta\right)^2 + \cdots}_{\text{finite size effects}}$$

EFT of Composite Particle

2nd Stage Remove orbital scale r of binary, namely separation between objects, via the top-down approach:

$$\bar{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \underbrace{H_{\mu\nu}}_{\text{orbital}} + \underbrace{\widetilde{h}_{\mu\nu}}_{\text{radiation}}$$

$$\partial_t H_{\mu\nu} \sim \frac{v}{r} H_{\mu\nu}, \quad \partial_i H_{\mu\nu} \sim \frac{1}{r} H_{\mu\nu}, \quad \partial_\rho \widetilde{h}_{\mu\nu} \sim \frac{v}{r} \widetilde{h}_{\mu\nu}$$

$$S_{\text{eff}}\left[\bar{g}_{\mu\nu}, y_1^\mu, y_2^\mu, e_{(1)}{}_A^\mu, e_{(2)}{}_A^\mu\right] = -\frac{1}{16\pi G}\int d^4x \sqrt{\bar{g}}\, R\left[\bar{g}_{\mu\nu}\right] + S_{\text{pp}}(\sigma_1) + S_{\text{pp}}(\sigma_2)$$

Integrate out orbital field modes - in this classical context - only tree level

$$\Rightarrow e^{iS_{\text{eff(composite)}}\left[\widetilde{h}_{\mu\nu}, y^\mu, e^\mu_{(Comp)A}\right]} \equiv \int \mathcal{D}H_{\mu\nu}\; e^{iS_{\text{eff}}\left[\bar{g}_{\mu\nu}, y_1^\mu, y_2^\mu, e_{(1)}{}_A^\mu, e_{(2)}{}_A^\mu\right]}$$

Stop here for effective action strictly in conservative sector, that is **WITHOUT** any remaining (orbital scale) field modes

EFTs of Extended Gravitating Objects

$$S_{\text{eff}} = S_{\text{g}}[g_{\mu\nu}] + \sum_{a=1}^{2} S_{\text{pp}}(\lambda_a); \quad S_{pp}(\lambda_a) = \sum_{i=1}^{\infty} C_i(r_s) \int d\lambda_a \mathcal{O}_i(\lambda_a)$$

$$S_{\text{g}}[g_{\mu\nu}] = -\frac{1}{16\pi G_d} \int d^{d+1}x \sqrt{g}\, R + \frac{1}{32\pi G_d} \int d^{d+1}x \sqrt{g}\, g_{\mu\nu} \Gamma^{\mu}\Gamma^{\nu},$$

$$G_d \equiv G_N \left(\sqrt{4\pi e^{\gamma}}\, R_0 \right)^{d-3},$$

To facilitate computations in PN: [Kol & Smolkin 2008]

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \equiv e^{2\phi} \left(dt - A_i dx^i \right)^2 - e^{-\frac{2}{d-2}\phi} \gamma_{ij} dx^i dx^j,$$

$\langle\, \phi(x_1) \;\; \phi(x_2)\, \rangle =$ $= \dfrac{16\pi\, G_d}{c_d} \cdot \delta(t_1 - t_2) \displaystyle\int_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{x}_1 - \vec{x}_2)}}{\vec{k}^2},$

$\langle A_i(x_1)\; A_j(x_2) \rangle =$ $= -16\pi\, G_d \cdot \delta(t_1 - t_2) \displaystyle\int_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{x}_1 - \vec{x}_2)}}{\vec{k}^2}\, \delta_{ij}.$

Spinning Particle: DOFs

[**ML**, Rept. Prog. Phys. 2020]

Assume isolated object has no intrinsic permanent multipoles beyond mass (monopole) and spin (dipole)

1. *Gravitational field*
   - Metric $g_{\mu\nu}(x)$
   - Tetrad field $\eta^{ab}\tilde{e}_a{}^{\mu}(x)\tilde{e}_b{}^{\nu}(x) = g^{\mu\nu}(x)$

2. *Particle Coordinate*

   $y^{\mu}(\sigma)$ function of arbitrary affine parameter σ
   Particle worldline position does not in general coincide with object's 'center'

3. *Particle rotating DOFs*

   Worldline tetrad, $\eta^{AB}e_A{}^{\mu}(\sigma)e_B{}^{\nu}(\sigma) = g^{\mu\nu}$

   $\Rightarrow$ Angular velocity $\Omega^{\mu\nu}(\sigma) \equiv e^{\mu}_A \frac{De^{A\nu}}{D\sigma}$ + conjugate spin $S_{\mu\nu}(\sigma)$

   $\Rightarrow$ Lorentz matrices $\eta^{AB}\Lambda_A{}^a(\sigma)\Lambda_B{}^b(\sigma) = \eta^{ab}$ + conjugate local spin $S_{ab}(\sigma)$

Where is our spinning "particle"?

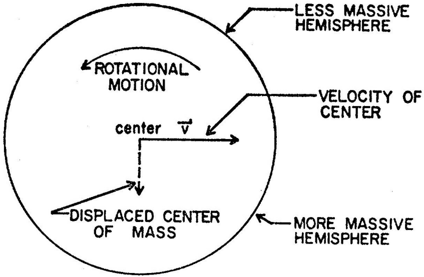
Spin is NOT 0-size!

- Special Relativity already requires a minimal finite measure, S/Mc, for the rotational velocity not to exceed the speed limit!
- Also in General Relativity, where this is the ring singularity of Kerr

In Newtonian physics a unique notion of a center, with the 3 nice properties:

1. 3-vector
2. Forms canonical pair together with the total momentum
3. Center of spatial mass distribution

No unique "center" in relativistic physics!



Fleming 1965

Spinning Particle: Symmetries

[**ML**, Rept. Prog. Phys. 2020]

1. General coordinate invariance, and parity invariance
2. Worldline reparametrization invariance
3. Internal Lorentz invariance of local frame field
4. SO(3) invariance of "body-fixed" spatial triad
5. Spin gauge invariance, that is invariance under choice of completion of "body-fixed" spatial triad through timelike vector

Spin as Extra Particle DOF

Effective Action of Spinning Particle

- $u^\mu \equiv dy^\mu/d\sigma$, $\Omega^{\mu\nu} \equiv e^\mu_A \frac{De^{A\nu}}{D\sigma}$ $\Rightarrow L_{\text{pp}}\left[\bar{g}_{\mu\nu}, u_\mu, \Omega^{\mu\nu}\right]$

  [Hanson & Regge 1974, Bailey & Israel 1975]

- $S_{\mu\nu} \equiv -2\frac{\partial L}{\partial \Omega^{\mu\nu}}$ spin as further particle DOF – classical source

  [...Levi+ JHEP 2015]

$$\Rightarrow S_{\text{pp}}(\sigma) = \int d\sigma \left[-p_\mu u^\mu - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{\text{NMC}}\left[\bar{g}_{\mu\nu}\left(y^\mu\right), u^\mu, S_{\mu\nu}\right]\right]$$

For EFT of spin – gauge of both rotational DOFs should be fixed at level of one-particle action

- This form implicitly assumes initial "covariant gauge":

$$e^\mu_{[0]} = \frac{p^\mu}{\sqrt{p^2}}, \quad S_{\mu\nu}p^\nu = 0$$

[Tulczyjew 1959]

- Linear momentum $p_\mu \equiv -\frac{\partial L}{\partial u^\mu} = m\frac{u^\mu}{\sqrt{u^2}} + \mathcal{O}(RS^2)$

EFT of Spinning Particle

Effective Action of Spinning Particle

- $u^\mu \equiv dy^\mu/d\sigma$, $\Omega^{\mu\nu} \equiv e^\mu_A \frac{De^{A\nu}}{D\sigma} \Rightarrow L_{\rm pp}\left[\bar{g}_{\mu\nu}, u_\mu, \Omega^{\mu\nu}\right]$
  [Hanson & Regge 1974, Bailey & Israel 1975]
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  [...Levi+ JHEP 2015]

$$\Rightarrow S_{\rm pp}(\sigma) = \int d\sigma \left[-p_\mu u^\mu - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{\rm NMC}\left[\bar{g}_{\mu\nu}\left(y^\mu\right), u^\mu, S_{\mu\nu}\right]\right]$$

Theory challenges tackled [...Levi+ JHEP 2015, Levi Rept. Prog. Phys. 2020]

1. Relativistic spin has a minimal finite measure S/M
   $\rightarrow$ Clashes with the EFT/point-particle viewpoint
   $\Rightarrow$ Introduce "gauge freedom" in choice of rotational variables
2. Fix non-minimal coupling part of the action, $L_{\rm NMC}$

Introduce Gauge Freedom in Tetrad & Spin

[**ML** & Steinhoff, JHEP 2015]

Introduce gauge freedom into tetrad

Transform from a gauge condition

$$e_{A\mu} q^{\mu} = \eta_{[0]A} \Leftrightarrow e_{[0]\mu} = q_{\mu}$$

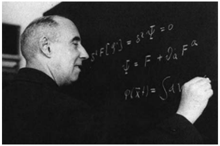
to

$$\hat{e}_{A\mu} w^{\mu} = \eta_{[0]A} \Leftrightarrow \hat{e}_{[0]\mu} = w_{\mu}$$

with a boost-like transformation in covariant form

$$\hat{e}^{A\mu} = L^{\mu}{}_{\nu}(w, q) e^{A\nu}$$

with q_μ, w_μ timelike unit 4-vectors



Ernst Stueckelberg

Generic gauge for the tetrad entails the generic "SSC"

$$\hat{e}_{[0]\mu} = w_{\mu} \quad \Rightarrow \quad \hat{S}^{\mu\nu}\left(p_{\nu} + \sqrt{p^2} w_{\nu}\right) = 0$$

Extra term in Minimal Coupling

[**ML** & Steinhoff, JHEP 2015]

$$\Rightarrow \hat{S}^{\mu\nu} = S^{\mu\nu} - \delta z^{\mu} p^{\nu} + \delta z^{\nu} p^{\mu}, \qquad \delta z^{\mu} p_{\mu} = 0$$

⇒ Extra term in action appears!

- From minimal coupling

$$\frac{1}{2} S_{\mu\nu} \Omega^{\mu\nu} = \frac{1}{2} \hat{S}_{\mu\nu} \hat{\Omega}^{\mu\nu} + \frac{\hat{S}^{\mu\rho} p_{\rho}}{p^2} \frac{D p_{\mu}}{D\sigma}$$

- Extra term with covariant derivative of momentum, contributes to finite size effects, yet carries no Wilson coefficient
- As of LO with spin, to all orders in spin!
- Essentially Thomas precession (later recovered as "Hilbert space matching")

- We transform between spin variables by projecting onto the hypersurface orthogonal to p_{μ}

$$S_{\mu\nu} = \hat{S}_{\mu\nu} - \frac{\hat{S}_{\mu\rho} p^{\rho} p_{\nu}}{p^2} + \frac{\hat{S}_{\nu\rho} p^{\rho} p_{\mu}}{p^2}$$

Why Generalized Canonical Gauge?

Here are some of the obvious reasons to use it:

1. Allows to disentangle DOFs in EFT and land on well-defined effective action
2. Standard procedure to land on Hamiltonian, similar to non-spinning sectors
3. Essential for EOB framework – needed to generate waveforms
4. Direct and simple derivation of physical EOMs for position and spin
5. Enables most stringent consistency check of Poincaré algebra of invariants
6. Natural classical treatment to be promoted to QFT

Non-minimal coupling: Under construction

[**Levi**+, JHEP 2014, JHEP 2015]

Spin-induced higher multipoles

- Consider the spin vector similar to the Pauli-Lubanski pseudovector

$$S^{\mu} \equiv *S^{\mu\nu}\frac{p_{\nu}}{\sqrt{p^2}} \simeq *S^{\mu\nu}\frac{u_{\nu}}{\sqrt{u^2}}, \qquad *S_{\alpha\beta} \equiv \frac{1}{2}\epsilon_{\alpha\beta\mu\nu}S^{\mu\nu}$$

$$\Rightarrow\ S_{\mu}S^{\mu} = -\frac{1}{2}S_{\mu\nu}S^{\mu\nu} \equiv -S^2$$

- Consider dependence of higher powers of spin:

$$S^{\alpha}{}_{\mu}S^{\mu}{}_{\beta} = -S^{\alpha}S_{\beta} - S^2\left(\delta^{\alpha}{}_{\beta} - \frac{u^{\alpha}u_{\beta}}{u^2}\right)$$

$$S^{\alpha}{}_{\mu}S^{\mu}{}_{\nu}S^{\nu}{}_{\beta} = -S^2 S^{\alpha}{}_{\beta}$$

$$\Rightarrow X(X+iS)(X-iS) = 0$$

$\Rightarrow$ Independent combinations: S^{α}, $S^{\alpha}{}_{\mu}S^{\mu\beta} \sim S^{\alpha}S^{\beta}$...

Non-minimal coupling: Under construction

[**Levi**+, JHEP 2014, JHEP 2015]

Spin-induced higher multipoles

- Considering body-fixed frame: Spin multipoles are SO(3) irreps tensors
- Recall we start from "covariant gauge": ${e_{[0]}}^{\mu} = u^{\mu}/\sqrt{u^2}$, ${e_{[i]}}^{\mu} u_{\mu} = 0$

$\Rightarrow$ Spin-induced higher multipoles are symmetric, traceless,
and spatial, constant tensors in body-fixed frame

Curvature

- Electric component $E_{\mu\nu} \equiv R_{\mu\alpha\nu\beta} u^{\alpha} u^{\beta}$
- Magnetic component $B_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\alpha\beta\gamma\mu} {R^{\alpha\beta}}_{\delta\nu} u^{\gamma} u^{\delta}$

Field is vacuum solution at LO, properties of Riemann, Bianchi identities
$\Rightarrow$ E/B are symmetric, traceless, and orthogonal to u^{μ},
also when projected to body-fixed frame, where they are spatial

Building blocks $\sim$ Riemann components • Spin-induced multipoles

Even/odd spin-induced multipoles couple to even/odd parity electric/magnetic curvature components, and their covariant derivatives

Leading Non-Minimal Couplings to All Orders in Spin

[**Levi**+, JHEP 2014, JHEP 2015]

Key: Consider classical spin vector similar to Pauli-Lubanski vector

→ Massive spinor-helicity, Arkani-Hamed+ 2017 – resonates with this form

New Wilson coefficients of linear-in-curvature couplings → "Love numbers":

$$L_{\rm NMC} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}$$

Leading - linear in curvature - spin couplings up to 5PN order

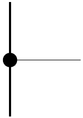
- $L_{ES^2} = -\frac{C_{ES^2}}{2m} \frac{E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu$, Quadrupole @2PN
- $L_{BS^3} = -\frac{C_{BS^3}}{6m^2} \frac{D_\lambda B_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda$, Octupole @3.5PN
- $L_{ES^4} = \frac{C_{ES^4}}{24m^3} \frac{D_\lambda D_\kappa E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda S^\kappa$, Hexadecapole @4PN

Kaluza-Klein decomposition of field

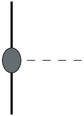
Reduction over time dimension à la Kaluza-Klein

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \equiv e^{2\phi}(dt - A_i dx^i)^2 - e^{-2\phi}\gamma_{ij} dx^i dx^j$$

- $\phi, A_i, \gamma_{ij} \equiv \delta_{ij} + \sigma_{ij}$, KK fields



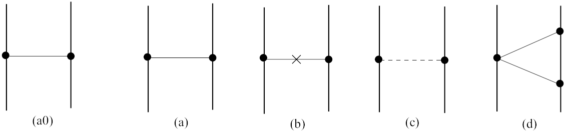
- Newtonian potential scalar ϕ

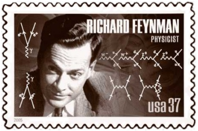


- Gravitomagnetic vector A_i
- Hierarchy in coupling to mass and to spin
- Advantageous, preferable e.g. over Lorentz covariant, ADM decompositions...

LO sectors beyond Newtonian

Feynman graphs of non-spinning sector to 1PN order

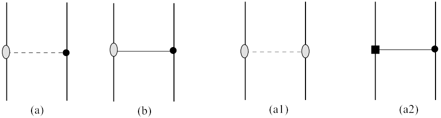




Newton
.

One-loop diagram – absent from 1PN
with KK parametrization of field

LO Feynman diagrams with spin – to S^2

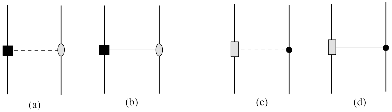


Spin-Orbit Spin-Spin

LO $S^3 + S^4$

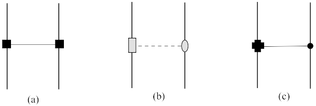
[**Levi**+, JHEP 2014]

Feynman diagrams of LO **cubic** in spin sector



- On left pair – quadrupole-dipole, on right – octupole-monopole
- Note analogy of each pair with LO spin-orbit

Feynman diagrams of LO **quartic** in spin sector

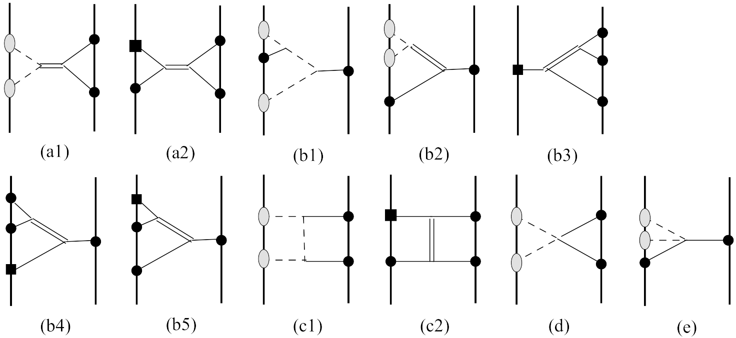


- On left and right – quadrupole-quadrupole and hexadecapole-monopole
  Each is analogous to LO spin-squared
- In middle – octupole-dipole analogous to LO spin1-spin2

NNLO S^2 sector

[**ML** & Steinhoff, 2015]

Feynman diagrams of order G^3 with 2 loops

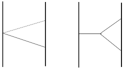


- Recall at N^{n}LO – n-loop graphs are realized, in particular with spin!
- Five 2-loop topologies actually fall into 3 kinds
- I (or H rotated) topology (c1,c2) – is the leading nasty one, i.e. IBP needed!

Graph Topologies up to 2-Loop

[Rept. Prog. Phys. 2020, **Levi+** + JHEPx2 2020]

Single topology at $O(G)$:
One-graviton exchange.

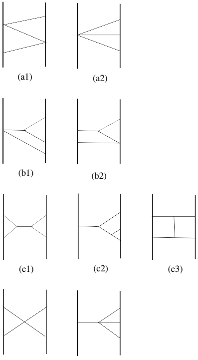


(a) (b)

Topologies at $O(G^2)$:
(a) Two-graviton exchange;
(b) Cubic self-interaction
$\equiv$ One-loop topology.

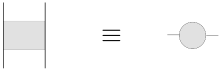
$$\int_{\vec{p}_1} \frac{e^{i\vec{p}_1\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{p}_1^2} \int_{\vec{p}_2} \frac{e^{i\vec{p}_2\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{p}_2^2},$$

$$p_1 + p_2 \to p, \quad p_2 \to k_1, \quad \to \int_{\vec{p}} e^{i\vec{p}\cdot(\vec{x}_1-\vec{x}_2)} \int_{\vec{k}_1} \frac{1}{\vec{k}_1^2(\vec{p}-\vec{k}_1)^2}$$



(a1) (a2) (b1) (b2) (c1) (c2) (c3) (d1) (d2)

Topologies at $O(G^3)$

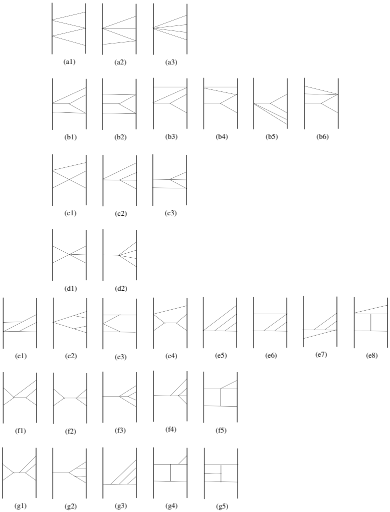


Standard QFT multi-loops:
n-loop master integrals and IBPs (Integration By Parts)
`EFTofPNG` code
[**Levi+** 2017, 2020,...]

A topology at G^{n+1} is rank r, when r basic n-loop integral types form its n-loop integral

Graph Topologies at G^4=up to 3-Loop

[**Levi**+ 2020]



Topologies at $O(G^4)$

At G^n the loop order n_L

$$n_L \equiv 2n - \sum_{i=1}^{n+1} m_i$$

with m_i gravitons on insertion i

“Only” 12 topologies are 3-loop out of total 32 at $O(G^4)$ [rows d,f,g]

Complete N^{3}LO S^2

Discovering first quadratic-in-curvature couplings

[**Levi**+ 2020; Kim, **ML**, Yin, 2021]

Graph distribution in N^{3}LO S^2 sector in a total of 1121 - from linear in R

Order in G	1	2	3	4
No. of graphs	19	251	688	163

Do we have more contributions beyond linear in curvature? Yes, at G^2!

Integration and Scalability

- Building on publicly-available `EFTofPNG` code [**ML** & Steinhoff 2017] https://github.com/miche-levi/pncbc-eftofpng
- Higher-rank graphs reduced using IBP method, e.g. 83 at G^3, 31 at G^4
- Upgrade using projection method for integrand numerators as high as rank-8
- Upgrade from IBP "by hand" to algorithmic IBP – our variation of Laporta

Extending Non-Minimal Action with Spin

Extending effective action beyond linear-in-curvature

[**Levi**+ 2020, JHEP 2021]

$$
\begin{aligned}
L_{\text{NMC(R}^2\text{)}} = {} & C_{E^2} \frac{E_{\alpha\beta} E^{\alpha\beta}}{\sqrt{u^2}^3} + C_{B^2} \frac{B_{\alpha\beta} B^{\alpha\beta}}{\sqrt{u^2}^3} + \dots \\
& + C_{E^2S^2} S^\mu S^\nu \frac{E_{\mu\alpha} E_\nu^\alpha}{\sqrt{u^2}^3} + C_{B^2S^2} S^\mu S^\nu \frac{B_{\mu\alpha} B_\nu^\alpha}{\sqrt{u^2}^3} \\
& + C_{E^2S^4} S^\mu S^\nu S^\kappa S^\rho \frac{E_{\mu\nu} E_{\kappa\rho}}{\sqrt{u^2}^3} + C_{B^2S^4} S^\mu S^\nu S^\kappa S^\rho \frac{B_{\mu\nu} B_{\kappa\rho}}{\sqrt{u^2}^3} \\
& + C_{\nabla EBS} S^\mu \frac{D_\mu E_{\alpha\beta} B^{\alpha\beta}}{\sqrt{u^2}^3} + C_{E\nabla BS} S^\mu \frac{E_{\alpha\beta} D_\mu B^{\alpha\beta}}{\sqrt{u^2}^3} \\
& + C_{\nabla EBS^3} S^\mu S^\nu S^\kappa \frac{D_\kappa E_{\mu\alpha} B_\nu^\alpha}{\sqrt{u^2}^3} + C_{E\nabla BS^3} S^\mu S^\nu S^\kappa \frac{E_{\mu\alpha} D_\kappa B_\nu^\alpha}{\sqrt{u^2}^3} \\
& + C_{(\nabla E)^2S^2} S^\mu S^\nu \frac{D_\mu E_{\alpha\beta} D_\nu E^{\alpha\beta}}{\sqrt{u^2}^3} + C_{(\nabla B)^2S^2} S^\mu S^\nu \frac{D_\mu B_{\alpha\beta} D_\nu B^{\alpha\beta}}{\sqrt{u^2}^3} \\
& + C_{(\nabla E)^2S^4} S^\mu S^\nu S^\kappa S^\rho \frac{D_\kappa E_{\mu\alpha} D_\rho E_\nu^\alpha}{\sqrt{u^2}^3} + C_{(\nabla B)^2S^4} S^\mu S^\nu S^\kappa S^\rho \frac{D_\kappa B_{\mu\alpha} D_\rho B_\nu^\alpha}{\sqrt{u^2}^3} + \dots,
\end{aligned}
$$

- New (unstudied) Wilson coefficients
- Are there any redundant terms ("on-shell operators") here? $C_{E^2S^2}$ is not!

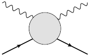
Nonlinear Higher-in-Spin Sectors

What is the nature of massive particles of $s > 2$?

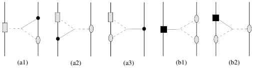
- Gravitational interaction with spins $\leftrightarrow$ Scattering of graviton and massive spin

  $$\boxed{\text{Classical } S^l \leftrightarrow \text{Quantum } s = l/2}$$

- Insight on Compton scattering of graviton and massive higher-spin $s \geq 5/2$

  [Arkani-Hamed+ 2017] 

NLO S^3, S^4 [**Levi+**, Teng, JHEP 2021 x 2, + Morales in prep.]



- Graphs with "elementary" worldline-graviton couplings up to 1-loop
- Some worldline-graviton couplings become quite intricate and subtle, new "composite" multipoles in terms of "elementary" spin multipoles
- Operators quadratic-in-curvature at NLO S^4

QFT for Real-World Gravity

Levi Rept. Prog. Phys. 2020
Levi+ 2x 2020, 2x JHEP 2021, 2022 + in prep. + Kim, Morales, Yin
My Public Webpage: DEFYING GRAVITY

- Real-world scalability:
  - EFT of gravitating spinning objects - self-contained framework
    $\Rightarrow$ Direct derivation of useful & physical quantities
    $\Rightarrow$ Self-consistency checks
  - Precision frontier with spins being pushed to 5PN order!
  - Continuous development of public computational tools
    $\rightarrow$ EFTofPNG code [CQG Highlights 2017, upgrades...]

- Fundamental lessons:
  - PN gravity informs us about gravity in general
  - Extend effective theories to new effects, theories of gravity...
  - Possible insights for graviton Compton amplitude with higher spins?
  - Can QFT advances further simplify computations, or even enable analytical predictions to capture the strong gravity regime of the GW signal?