Double Copy and Higher-Spin Amplitudes

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Based on recent work with:

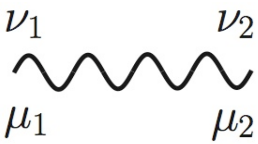
Marco Chiodaroli, Paolo Pichini [2107.14779]

Outline

- Motivation and review of the double copy
- Application to PN calculations
- Scattering amplitudes for Kerr
- EFTs underlying the low-spin Kerr amplitudes
- Higher-spin amplitudes → see talk by Pichini !
- Conclusion

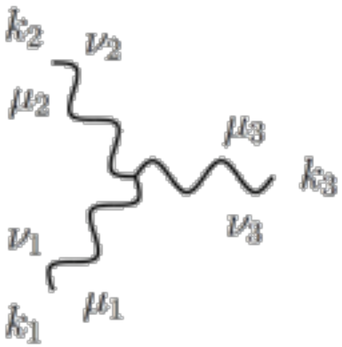
Perturbative Einstein gravity (textbook)

$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{g}R, \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$



$$= \frac{1}{2}\left[\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \frac{2}{D-2}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\right]\frac{i}{p^2+i\epsilon}$$

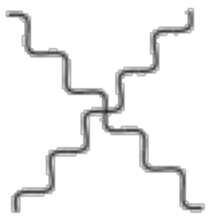
de Donder gauge



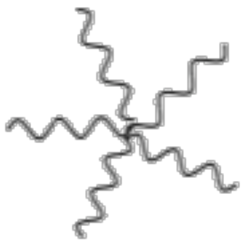
$$\begin{aligned}
= \mathrm{sym}\Big[&-\frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}) - \frac{1}{2}P_6(k_{1\mu_1}k_{1\nu_2}\eta_{\mu_1\nu_1}\eta_{\mu_3\nu_3}) + \frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\eta_{\mu_3\nu_3})\\
&+P_6(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3}) + 2P_3(k_{1\mu_2}k_{1\nu_3}\eta_{\mu_1\nu_1}\eta_{\nu_2\mu_3}) - P_3(k_{1\nu_2}k_{2\mu_1}\eta_{\nu_1\mu_1}\eta_{\mu_3\nu_3})\\
&+P_3(k_{1\mu_3}k_{2\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + P_6(k_{1\mu_3}k_{1\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + 2P_6(k_{1\mu_2}k_{2\nu_3}\eta_{\nu_2\mu_1}\eta_{\nu_1\mu_3})\\
&+2P_3(k_{1\mu_2}k_{2\mu_1}\eta_{\nu_2\mu_3}\eta_{\nu_3\nu_1}) - 2P_3(k_1\cdot k_2\eta_{\nu_1\mu_2}\eta_{\nu_2\mu_3}\eta_{\nu_3\mu_1})\Big]
\end{aligned}$$

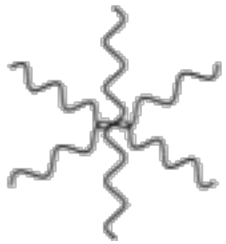
After symmetrization
$\sim$ 100 terms !

higher order vertices...



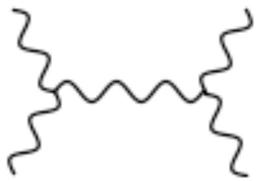
$\sim 10^3$ terms



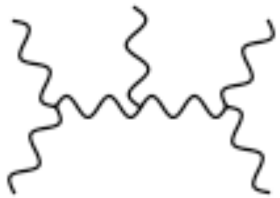


. . .

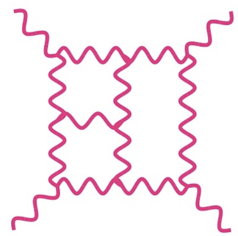
complicated diagrams:



$\sim 10^4$ terms



$\sim 10^7$ terms



$\sim 10^{21}$ terms

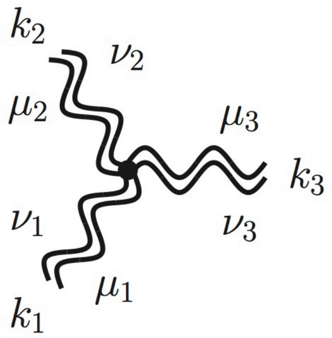
On-shell simplifications

Graviton plane wave: $\varepsilon^{\mu}(p)\varepsilon^{\nu}(p)\, e^{ip\cdot x}$

Yang-Mills polarization

$$|\text{spin } 2\rangle \sim |\text{spin } 1\rangle \otimes |\text{spin } 1\rangle$$

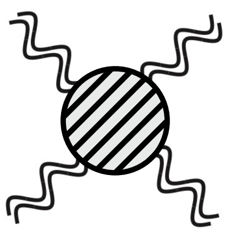
On-shell 3-graviton vertex:



$$= \Big(\eta_{\mu_1\mu_2}(k_1-k_2)_{\mu_3} + \text{cyclic}\Big)\Big(\eta_{\nu_1\nu_2}(k_1-k_2)_{\nu_3} + \text{cyclic}\Big)$$

Yang-Mills vertex

Gravity scattering amplitude:



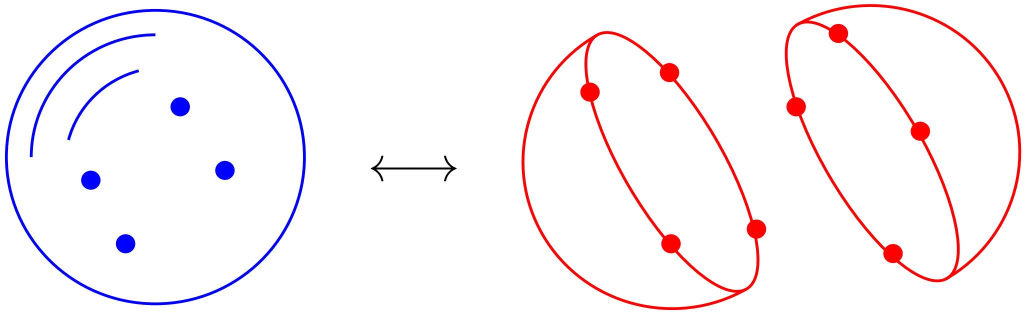
Yang-Mills amplitude

$$M^{\text{GR}}_{\text{tree}}(1,2,3,4) = \frac{st}{u}\Big[A^{\text{YM}}_{\text{tree}}(1,2,3,4)\Big]^2$$

Gravity processes = “squares” of gauge theory ones

Kawai-Lewellen-Tye Relations ('86)

closed string ~ (left open string) × (right open string)



$$\mathcal{A}_n(\sigma) = \int_{z_{\sigma(1)}<\cdots<z_{\sigma(n)}} \frac{dz_1\cdots dz_n}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{R}))} \prod_{i<j} |z_{ij}|^{\alpha' k_i\cdot k_j} \exp\Big[\sum_{i<j} \frac{e_i\cdot e_j}{(z_i-z_j)^2} + \frac{k_{[i}\cdot e_{j]}}{z_i-z_j}\Big]\Big|_{\text{multi-linear}}$$

KLT relations → closed string amplitudes

Bern, Dixon, Perelstein, Rozowsky

$$\mathcal{M}_n = \sum_{\sigma,\rho}^{(n-3)!} \mathcal{A}_n(\sigma)\, \mathcal{S}_{\alpha'}[\sigma|\rho]\, \widetilde{\mathcal{A}}_n(\rho)$$

$S[\sigma|\rho]$ poly. of $s_{ij} = (p_i+p_j)^2$

$\mathcal{S}_{\alpha'}[\sigma|\rho]$ poly. of $\sin(\pi\alpha' s_{ij})$

Bjerrum-Bohr, Damgaard, Søndergaard, Vanhove

Field theory limit $\Rightarrow$

$$M_n = \sum_{\sigma,\rho}^{(n-3)!} A_n(\sigma)\, S[\sigma|\rho]\, \widetilde{A}_n(\rho)$$

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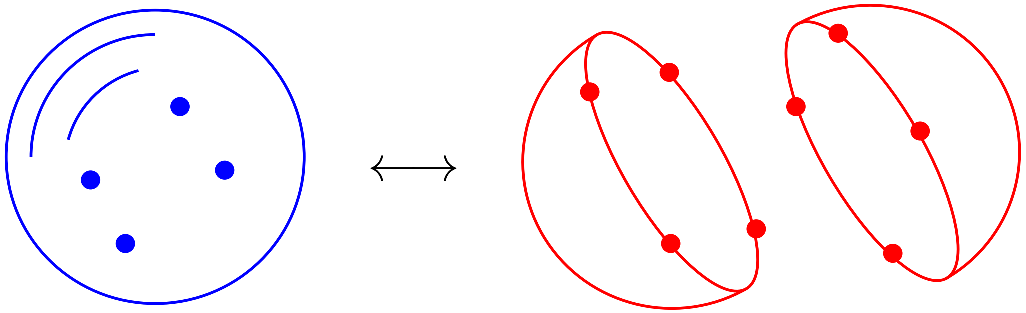
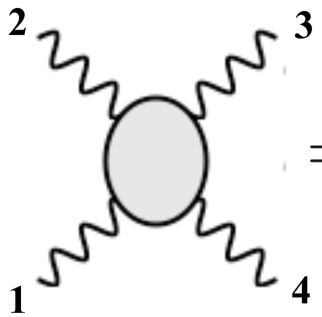


Field theory limit $\Rightarrow$ $$M_n = \sum_{\sigma,\rho}^{(n-3)!} A_n(\sigma)\, S[\sigma|\rho]\, \widetilde{A}_n(\rho)$$

Double copy: $$(\text{gravity}) = (\text{gauge}) \otimes \widetilde{(\text{gauge})}$$

Color-kinematics duality

Consider Yang-Mills 4p tree amplitude:



$$= \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

color factors: $c_s = f^{a_1 a_2 b} f^{b a_3 a_4}$

kinematic numerators:

$$n_s = \Big[(\varepsilon_1 \cdot \varepsilon_2) p_1^\mu + 2(\varepsilon_1 \cdot p_2)\varepsilon_2^\mu - (1 \leftrightarrow 2)\Big]\Big[(\varepsilon_3 \cdot \varepsilon_4) p_{3\mu} + 2(\varepsilon_3 \cdot p_4)\varepsilon_{4\mu} - (3 \leftrightarrow 4)\Big]$$
$$+ s\big[(\varepsilon_1 \cdot \varepsilon_3)(\varepsilon_2 \cdot \varepsilon_4) - (\varepsilon_1 \cdot \varepsilon_4)(\varepsilon_2 \cdot \varepsilon_3)\big]\,,$$

consider gauge transformation $\delta A_\mu = \partial_\mu \phi$

$$n_s\Big|_{\varepsilon_4 \to p_4} = s\Big[(\varepsilon_1 \cdot \varepsilon_2)\big((\varepsilon_3 \cdot p_2) - (\varepsilon_3 \cdot p_1)\big) + \mathrm{cyclic}(1,2,3)\Big] \equiv s\,\alpha(\varepsilon, p)$$

(individual diagrams not gauge inv.)

Color-kinematics duality

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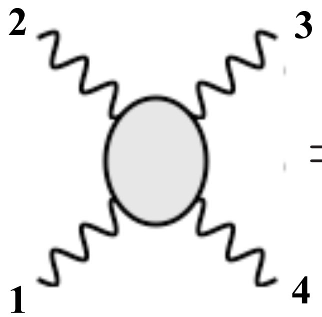

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consider gauge transformation $\delta A_\mu = \partial_\mu \phi$

$$\left.\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right|_{\varepsilon_4 \to p_4} = \underbrace{(c_s + c_t + c_u)}_{=0}\,\alpha(\varepsilon, p)$$

= 0 Jacobi identity

Color-kinematics duality

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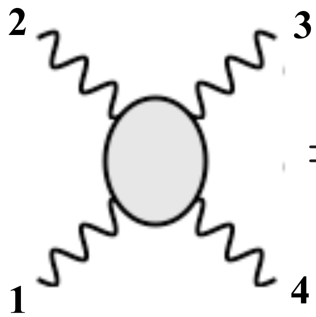

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$$+ s\Big[(\varepsilon_1 \cdot \varepsilon_3)(\varepsilon_2 \cdot \varepsilon_4) - (\varepsilon_1 \cdot \varepsilon_4)(\varepsilon_2 \cdot \varepsilon_3)\Big]\,,$$

$c_s + c_t + c_u = 0$ Jacobi Id. (gauge invariance)

$\Leftrightarrow$

$n_s + n_t + n_u = 0$ kinematic Jacobi Id. (diffeomorphism inv.)

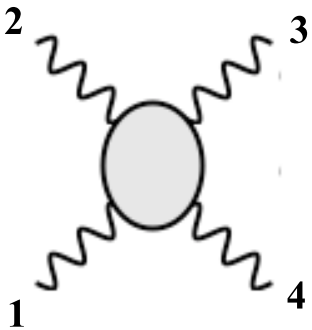
BCJ ('08)

Double copy

Color and kinematics are dual...

$$c_s + c_t + c_u = 0 \qquad \Leftrightarrow \qquad n_s + n_t + n_u = 0$$

...replace color by kinematics $\quad c_i \to n_i \quad$ BCJ double copy



$$= \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

← gravity ampl.

Properties of ampl:
- spin-2 scattering $\qquad \varepsilon_{\mu\nu} = \varepsilon_\mu \varepsilon_\nu$
- 2-derivative interactions $\qquad \partial_\mu \to \partial_\mu \partial_\nu$
- diffeomorphism inv. $\qquad \delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

$$\left. \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u} \right|_{\varepsilon_4^{\mu\nu} \to p_4^\mu \varepsilon_4^\nu + p_4^\nu \varepsilon_4^\mu} = 2(n_s + n_t + n_u)\, \alpha(\varepsilon, p) = 0$$

General multiplicity and loop order

Gauge theories are controlled by a hidden kinematic Lie algebra

→ Amplitude represented by cubic graphs:

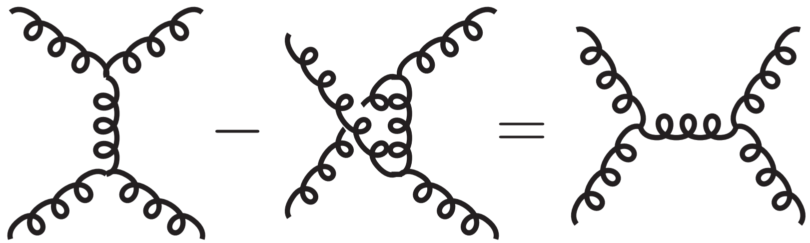
numerators

color factors

$$\mathcal{A}_m^{(L)} = \sum_{i \in \Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

propagators

Color & kinematic numerators satisfy same relations:



Jacobi identity

$$f^{dac} f^{cbe} - f^{dbc} f^{cae} = f^{abc} f^{dce}$$

$$n_i \quad - \quad n_j \quad = \quad n_k$$

General multiplicity and loop order

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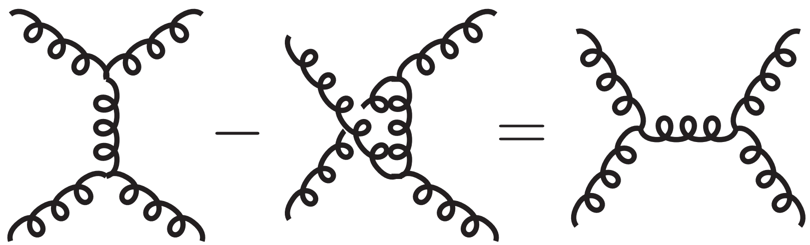

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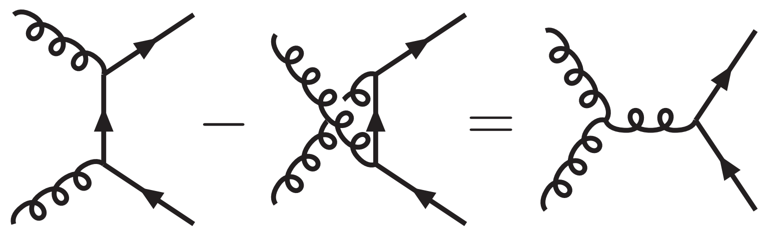


commutation identity

$$T^a T^b - T^b T^a = f^{abc} T^c$$

$$n_i \quad - \quad n_j \quad = \quad n_k$$

color	c_i
⬍	⬍
kinematics	n_i

Bern, Carrasco, HJ

Gravity as a double copy

Gravity amplitudes obtained by replacing color with kinematics

$$\mathcal{A}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

double copy

$$\mathcal{M}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

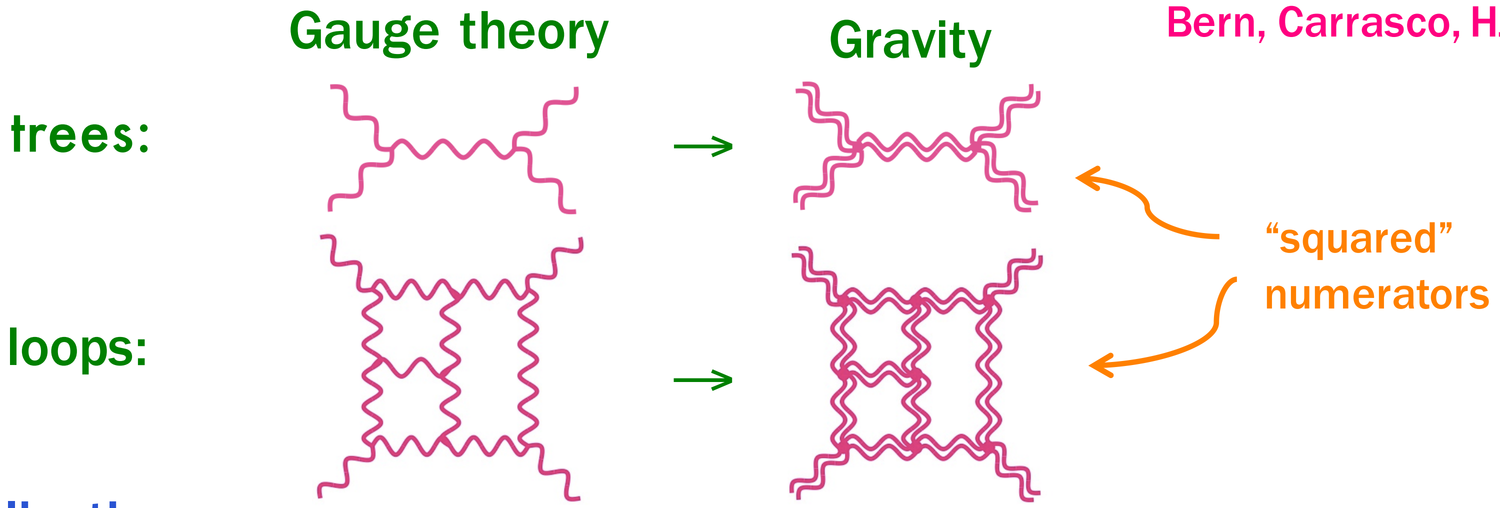
- $(\text{pure YM}) \otimes (\text{pure YM}) = \text{GR} + \phi + B^{\mu\nu}$
- $\text{QCD} \otimes \text{QCD} = \text{GR} + \text{matter}$ (Einstein-Maxwell)
- $(\text{YM}) \otimes (\text{YM} + \phi^3) = \text{GR} + \text{YM}$ (Einstein-Yang-Mills)

and many more...

$\longrightarrow$ $(\text{gauge sym}) \otimes (\text{gauge sym}) = \text{diffeo sym}$

Generality of double copy

Bern, Carrasco, HJ ('10)



Some generalizations:

→ Theories not truncations of max *SUGRA* HJ, Ochirov; Chiodaroli, Gunaydin, Roiban

→ Theories with fundamental matter HJ, Ochirov; Chiodaroli, Gunaydin, Roiban

→ Spontaneously broken theories Chiodaroli, Gunaydin, HJ, Roiban

→ Form factors Boels, Kniehl, Tarasov, Yang

→ Gravity off-shell symmetries from YM Anastasiou, Borsten, Duff, Hughes, Nagy

→ Classical (black hole) solutions Luna, Monteiro, O'Connell, White; Ridgway, Wise; Goldberger,...

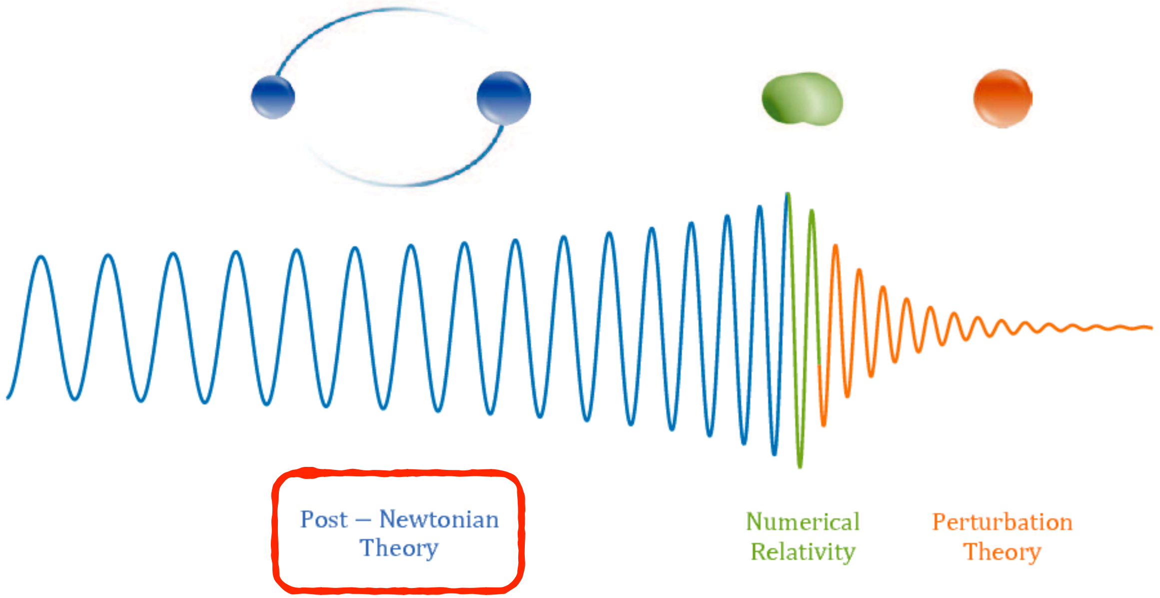
→ Amplitudes in curved background Adamo, Casali, Mason, Nekovar, Alday, Zhou, Roiban, Teng,...

→ CHY scattering eqs, twistor strings Cachazo, He, Yuan, Skinner, Mason, Geyer, Adamo, Monteiro,..

→ New double copies for string theory Mafra, Schlotterer, Stieberger, Taylor, Broedel, Carrasco... ... Azevedo, Marco Chiodaroli, HJ, Schlotterer

→ Conformal gravity HJ, Nohle

Double copy and gravitational waves



Explicit PM calculations done using double copy:

Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove ('18)
Bern, Cheung, Roiban, Shen, Solon, Zeng ('19)+ Ruf, Parra-Martinez ('21)
Brandhuber, Chen, Travaglini, Wen (21)

Some methods developed for PM calc. using double copy:

Bjerrum-Bohr, Cristofoli, Damgaard, Gomez+Brown;
Cristofoli, Gonzo, Kosower, O'Connell;
Maybee, O'Connell, Vines; Luna, Nicholson, O'Connell, White; …

See Kosower's lectures!

GR + non-spinning matter

Brandhuber, Chen, HJ, Travaglini, Wen

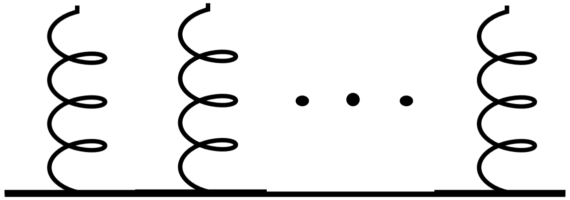
Exact formula tree-level GR:

$$M_n = \sum_{\Gamma} \frac{\mathcal{N}^2(\Gamma)}{D_\Gamma}$$

(all cubic graphs Γ)

Numerator function:

$$\mathcal{N}(\phi_0, 1, 2, \ldots, n{-}1, \phi_n) \;= \sum_{\tau \in \mathrm{OP}(2,\ldots,n-1)} \frac{v\cdot F_{1\tau_1}\cdot V_{\tau_2}\cdot F_{\tau_2}\cdots V_{\tau_r}\cdot F_{\tau_r}}{(n{-}2)\, v\cdot p_1\, v\cdot p_{1\tau_1}\cdots\, v\cdot p_{1\tau_1\tau_2\cdots\tau_r}}$$



$$F^{\mu\nu}_{\sigma} = (F_{\sigma_1}\cdot F_{\sigma_2}\cdots F_{\sigma_s})^{\mu\nu}$$

$$V^{\mu\nu}_{\tau_i} = p^{\mu}_{1\tau_1\cdots\tau_{i-1}\cap 1\ldots\tau_{i[1]}}\; v^{\nu}$$

comes from heavy-mass limit of non-spining particle

Similar formulas: Edison, Teng;
Mangan, Cheung;
Bjerrum-Bohr, Damgaard, Sondergaad, Vanhove

Schwarzschild BH ?

Scattering amplitudes for Kerr BH

Recap of massive spinor helicity

Following AHH we bold massive spinors, and symmetrize little group

$$|\,\mathbf{i}\,\rangle \equiv |i^a\rangle z_{i,a}\,, \qquad |\,\mathbf{i}\,] \equiv |i^a] z_{i,a}$$

Analytic fn's of the spinors can now be constructed

$$\langle \mathbf{12} \rangle^{2s} = \text{degree-}4s \text{ polynomial in } (z_1^a, z_2^a)$$

Massive polarizations are null vectors Chiodaroli, HJ, Pichini

$$\varepsilon_i^\mu = \frac{\langle \mathbf{i}|\sigma^\mu|\mathbf{i}]}{\sqrt{2}m_i} = \frac{[\mathbf{i}|\bar{\sigma}^\mu|\mathbf{i}\rangle}{\sqrt{2}m_i} = (z_i^1)^2 \varepsilon^\mu_{i,-} - \sqrt{2} z_i^1 z_i^2 \varepsilon^\mu_{i,L} - (z_i^2)^2 \varepsilon^\mu_{i,+}$$

Guarantees that higher-spin states are symmetric, transverse and traceless

$$\varepsilon_i^{\mu_1 \mu_2 \cdots \mu_s} \equiv \varepsilon_i^{\mu_1} \varepsilon_i^{\mu_2} \cdots \varepsilon_i^{\mu_s} = \text{degree-}2s \text{ polynomial in } z_i^a$$

(and gamma-traceless for fermions)

AHH amplitudes $\leftrightarrow$ Kerr BH?

Arkani-Hamed, Huang, Huang wrote down natural higher-spin amplitudes:

Gauge th 3pt:

$$A(1\phi^s, 2\bar{\phi}^s, 3A^+) = mx\frac{\langle \mathbf{12}\rangle^{2s}}{m^{2s}}\,, \qquad A(1\phi^s, 2\bar{\phi}^s, 3A^-) = \frac{m}{x}\frac{[\mathbf{12}]^{2s}}{m^{2s}}$$

Gravity 3pt:

$$M(1\phi^s, 2\bar{\phi}^s, 3h^+) = im^2x^2\frac{\langle \mathbf{12}\rangle^{2s}}{m^{2s}}\,, \qquad M(1\phi^s, 2\bar{\phi}^s, 3h^-) = i\frac{m^2}{x^2}\frac{[\mathbf{12}]^{2s}}{m^{2s}}$$

Shown to reproduce Kerr by: Guevara, Ochirov, Vines

Gravity Compton ampl.
via BCFW recursion ?

$$M(1\phi^s, 2\bar{\phi}^s, 3h^+, 4h^+) = i\frac{\langle \mathbf{12}\rangle^{2s}[34]^4}{m^{2s-4}s_{12}t_{13}t_{14}}$$

$$M(1\phi^s, 2\bar{\phi}^s, 3h^-, 4h^+) = i\frac{[4|p_1|3\rangle^{4-2s}([4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle)^{2s}}{s_{12}t_{13}t_{14}}$$

spurious pole for $s > 2$

What EFTs give the AHH amplitudes ?

Rewrite the 3pt AHH amplitudes on covariant form → identify theory

Chiodaroli, HJ, Pichini;
HJ, Ochirov

1) introduce generating series, e.g.

$$\sum_{s=0}^{\infty} A(1\phi^s, 2\bar{\phi}^s, 3A^+) = \frac{mx}{1 - \frac{\langle \mathbf{12} \rangle^2}{m^2}}$$

2) rewrite covariantly (for both helicity sectors):

$$\sum_{s=0}^{\infty} A(1\phi^s, 2\bar{\phi}^s, 3A) = A_{\phi\phi A} + \frac{A_{WWA} - (\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_2)^2 A_{\phi\phi A}}{(1 + \boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_2)^2 + \frac{2}{m^2} \boldsymbol{\varepsilon}_1 \cdot p_2 \, \boldsymbol{\varepsilon}_2 \cdot p_1}$$

$$A_{\phi\phi A} \equiv i\sqrt{2}\, \varepsilon_3 \cdot p_1 \,, \quad A_{WWA} \equiv i\sqrt{2}\, (\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_2\, \varepsilon_3 \cdot p_2 + \boldsymbol{\varepsilon}_2 \cdot \varepsilon_3\, \boldsymbol{\varepsilon}_1 \cdot p_3 + \varepsilon_3 \cdot \boldsymbol{\varepsilon}_1\, \boldsymbol{\varepsilon}_2 \cdot p_1)$$

$s = 0$ & $s = 1/2$ minimally coupled scalar & fermion

$s = 1$ W-boson $s = 3/2$ charged/massive gravitino

EFTs for AHH 3pt gravity amplitudes?

Are related to the gauge th. ones via KLT

$$M(1\phi^s, 2\bar{\phi}^s, 3h^\pm) = iA(1\phi^{s_\mathrm{L}}, 2\bar{\phi}^{s_\mathrm{L}}, 3A^\pm)A(1\phi^{s_\mathrm{R}}, 2\bar{\phi}^{s_\mathrm{R}}, 3A^\pm)$$

Works for any decomposition: $s = s_\mathrm{L} + s_\mathrm{R}$

Preferred decomposition $s = 1 + (s-1)$ give fewest derivatives :

$$\sum_{2s=0}^{\infty} M(1\phi^s, 2\bar{\phi}^s, 3h) = M_{0\oplus 1/2} + A_{WWA}\Big(A_{0\oplus 1/2} + \frac{A_{1\oplus 3/2} - (\varepsilon_1\cdot\varepsilon_2)^2 A_{0\oplus 1/2}}{(1+\varepsilon_1\cdot\varepsilon_2)^2 + \frac{2}{m^2}\varepsilon_1\cdot p_2\,\varepsilon_2\cdot p_1}\Big)$$

From double-copy structure, we can infer:

$s=0,\ s=1/2,\ s=1,\ s=3/2$ minimally-coupled matter (Proca th, massive gravitino)

$s=2$ Kaluza-Klein graviton

Also works for Compton, and higher-point amplitudes (Lagrangians known)

Summary of EFTs

The AHH amplitudes for $s \leq 2$ admit double copies to any multiplicity

$$(\mathrm{YM} + \mathrm{scalar}) \otimes (\mathrm{YM} + \mathrm{scalar}) = (\mathrm{GR} + \mathrm{scalar})$$

$$(\mathrm{YM} + \mathrm{scalar}) \otimes (\mathrm{YM} + \mathrm{fermion}) = (\mathrm{GR} + \mathrm{fermion})$$

$$(\mathrm{YM} + \mathrm{scalar}) \otimes (\mathrm{YM} + \text{W-boson}) = (\mathrm{GR} + \mathrm{Proca})$$

$$(\mathrm{YM} + \text{W-boson}) \otimes (\mathrm{YM} + \mathrm{fermion}) = (\mathrm{GR} + \text{massive gravitino})$$

$$(\mathrm{YM} + \text{W-boson}) \otimes (\mathrm{YM} + \text{W-boson}) = (\mathrm{GR} + \text{massive KK graviton})$$

Lagrangians unique: have no non-minimal terms beyond cubic order in fields

Can be used for $(S^\mu)^{\leq 4}$ PM/PN calculations. see Levi's lectures

Compton $(S^\mu)^4$ yet to be confirmed via other methods (BHPT, worldline).

What special about the EFTs ?

The $s \leq 1$ gauge theories and $s \leq 2$ gravities admit a massless limit

and all states that carries vector indices acquires a gauge symmetry

$$s = 1 \quad \text{(YM + W-boson)} \rightarrow \text{non-abelian gauge symmetry}$$

$$s = 3/2 \quad \text{(GR + massive gravitino)} \rightarrow \text{supersymmetry}$$

$$s = 2 \quad \text{(GR +massive KK graviton)} \rightarrow \text{General covariance}$$

See Pichini's talk for the higher-spin continuation !

(Note: we only study amplitudes with 2 massive states, and *n*-2 massless in which case the enlarged theories consistently truncate)

Conclusions

- Studied double copies for massive $s \leq 2$ states coupled to GR
- All can be written in terms gauge theories with massive spin $s \leq 1$
- For Compton amplitudes with $s = 5/2$ things are more complicated
- Obtained EFT Lagrangians to all order in the fields.
- Do the amplitudes match Kerr black holes ?
- Further work is needed for spin-3 and higher amplitudes.
- Outlook: results should be useful for simplifying PN/PM calculations!

Extra Slides

Which gauge theories obey C-K duality

- Pure $\mathcal{N}$=0,1,2,4 super-Yang-Mills (any dimension) — Bern, Carrasco, HJ ('08) Bjerrum-Bohr, Damgaard, Vanhove; Stieberger; Feng et al. Mafra, Schlotterer, etc ('08-'11)
- Self-dual Yang-Mills theory O'Connell, Monteiro ('11)
- Heterotic string theory Stieberger, Taylor ('14)
- Yang-Mills + F^3 theory Broedel, Dixon ('12)
- QCD, super-QCD, higher-dim QCD HJ, Ochirov ('15); Kälin, Mogull ('17)
- Generic matter coupled to $\mathcal{N}$= 0,1,2,4 super-Yang-Mills — Chiodaroli, Gunaydin, Roiban; HJ, Ochirov ('14)
- Spontaneously broken $\mathcal{N}$= 0,2,4 SYM Chiodaroli, Gunaydin, HJ, Roiban ('15)
- Yang-Mills + scalar ϕ^3 theory Chiodaroli, Gunaydin, HJ, Roiban ('14)
- Bi-adjoint scalar ϕ^3 theory — Bern, de Freitas, Wong ('99), Bern, Dennen, Huang; Du, Feng, Fu; Bjerrum-Bohr, Damgaard, Monteiro, O'Connell
- NLSM/Chiral Lagrangian Chen, Du ('13)
- *D*=3 BLG theory (Chern-Simons-matter) — Bargheer, He, McLoughlin; Huang, HJ, Lee ('12 -'13)
- (Non-)Abelian Z-theory Carrasco, Mafra, Schlotterer ('16)
- Dim-6 gauge theories: $(DF)^2 + F^3 + \ldots$ HJ, Nohle ('17)

Which gravity theories are double copies

- Pure $\mathcal{N}$=4,5,6,8 supergravity (2 < D < 11) KLT ('86), Bern, Carrasco, HJ ('08 -'10)
- Einstein gravity and pure $\mathcal{N}$=1,2,3 supergravity HJ, Ochirov ('14)
- *D*=6 pure $\mathcal{N}$=(1,1) and $\mathcal{N}$=(2,0) supergravity HJ, Kälin, Mogull ('17)
- Self-dual gravity O'Connell, Monteiro ('11)
- Closed string theories Mafra, Schlotterer, Stieberger ('11); Stieberger, Taylor ('14)
- Einstein + R^3 theory Broedel, Dixon ('12)
- Abelian matter coupled to supergravity
  - Carrasco, Chiodaroli, Gunaydin, Roiban ('12)
  - HJ, Ochirov ('14 - '15)
- Magical sugra, homogeneous sugra Chiodaroli, Gunaydin, HJ, Roiban ('15)
- (S)YM coupled to (super)gravity Chiodaroli, Gunaydin, HJ, Roiban ('14)
- Spontaneously broken YM-Einstein gravity Chiodaroli, Gunaydin, HJ, Roiban ('15)
- *D*=3 supergravity (BLG Chern-Simons-matter theory)^2
  - Bargheer, He, McLoughlin;
  - Huang, HJ, Lee ('12 -'13)
- Born-Infeld, DBI, Galileon theories (CHY form) Cachazo, He, Yuan ('14)
- $\mathcal{N}$=0,1,2,4 conformal supergravity HJ, Nohle ('17)