

The NS-NS Branes of Non-relativistic String Theory

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*work in progress with
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Veldhuis*

Non-relativistic gravity (in the bulk)

Gomis, Ooguri (2000); Danielsson, Guijosa, Kruczenski (2000)

Gomis, Gomis, Kamimura (2005)

Bagchi, Gopakumar (2009)

Non-relativistic String Theory

NR strings couple to a curved spacetime, a Kalb Ramond two form and dilaton.

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- String Newton-Cartan (SNC) theory

Gomis, Gomis, Kamimura; Andringa, Bergshoeff, Gomis, de Roo; Bergshoeff, Gomis, Yan

- Torsional Newton-Cartan (TNC) theory

Harmark, Hartong, Obers
hear talk by Natale Zinnato

Classically these two approaches converge to the same theory in the presence of Kalb-Ramond field and assuming the longitudinal spatial isometry.

Harmark, Hartong, Menculini, Obers, Oling

NR Particle

$$\hat{A} = (0, A') : \begin{cases} 0 & \text{is longitudinal to worldline} \\ A' & \text{is transverse to worldline} \end{cases}$$

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NC background fields

$$\{\tau_\mu \equiv \tau_\mu^0, m_\mu \equiv m_\mu^0, E_\mu^{A'}\},$$

$$\mu = (0, \dots, D-1), A' = (1, \dots, D-1),$$

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$$\{\tau_\mu^A, m_\mu^A, E_\mu^{A'}\}$$

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?

NR string sigma model

$$S_{\text{Pol.}} = -\frac{T}{2} \int d^2\sigma \left[\sqrt{-h} h^{\alpha\beta} \partial_\alpha x^\mu \partial_\beta x^\nu H_{\mu\nu} + \varepsilon^{\alpha\beta} (\lambda e_\alpha \tau_\mu + \bar{\lambda} \bar{e}_\alpha \bar{\tau}_\mu) \partial_\beta x^\mu \right]$$
$$-\frac{T}{2} \int d^2\sigma \varepsilon^{\alpha\beta} \partial_\alpha x^\mu \partial_\beta x^\nu B_{\mu\nu}$$

$$\delta\tau_\mu^A = 0,$$

$$\delta E_\mu^{A'} = -\lambda_{A'}^{A'} \tau_\mu^A,$$

$$\delta m_\mu^A = D_\mu(\omega) \lambda^A + \lambda^A_{A'} E_\mu^{A'} + \sigma^A_B \tau_\mu^B.$$

$$D_{[\mu}(\omega) \tau_{\nu]}^A = 0$$

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$$D_{[\mu}(\omega) \tau_{\nu]}^A = 0 \rightarrow \varepsilon_C^{(A} \tau_{[\mu}^{B)} \partial_\nu \tau_\rho]^C = 0.$$

Stückelberg Symmetries

$$H_{\mu\nu} \rightarrow H'_{\mu\nu} = H_{\mu\nu} - (C_{\mu}{}^A \tau_{\nu}{}^B + C_{\nu}{}^A \tau_{\mu}{}^B) \eta_{AB},$$

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$$\mathcal{S}_{\text{SNG}} = -\frac{T}{2} \int d^2\sigma \left[\sqrt{-\tau} \tau^{\alpha\beta} \partial_\alpha x^\mu \partial_\beta x^\nu E_\mu^{A'} E_\nu^{B'} \delta_{A'B'} + \varepsilon^{\alpha\beta} \partial_\alpha x^\mu \partial_\beta x^\nu \mathcal{B}_{\mu\nu} \right]$$

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galilean boost?

Relativistic T-duality

$$\hat{S}_{Rel} \sim T \int d^2\sigma [\sqrt{-h} h^{\alpha\beta} \partial_\alpha x^\mu \partial_\beta x^\nu \hat{G}_{\mu\nu}(x) + \varepsilon^{\alpha\beta} \partial_\alpha x^\mu \partial_\beta x^\nu \hat{B}_{\mu\nu}(x)]$$

Assume an isometry v with $\mu = (v, i)$

Buscher's T-duality rules are given by [Buscher; Roček, Verlinde](#)

$$\tilde{G}_{vv} = \frac{1}{\hat{G}_{vv}},$$

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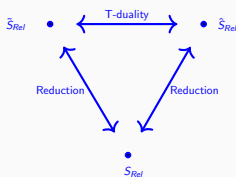
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T-Duality in Non-relativistic String Theory

For the purpose of our discussion, we distinguish two distinct T-duality rules:

Gomis, Ouguri, Gomis; Gomis, Kamimura

Bergshoeff, Gomis, Yan; Bergshoeff, Rosseel, S, Yan

- Longitudinal Spatial T-duality
- Transverse T-duality

NR T-duality

- The **transverse** T-dual of a non-relativistic string theory gives a non-relativistic string theory in a T-dual background like in relativistic T-duality.

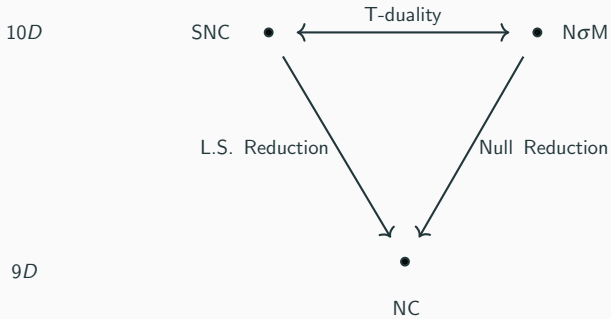


NR T-duality

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- Along a longitudinal spatial direction, T-duality maps **nonrelativistic** string theory on a SNC background to **relativistic** string theory on a Lorentzian geometry with a **null** isometry direction and $\partial_{[i} \tilde{G}_{j]v} = \partial_{[i} \tilde{B}_{j]v} = 0$.



Solutions that satisfy $\tilde{G}_{\nu\nu} = \partial_{[i}\tilde{G}_{j]\nu} = \partial_{[i}\tilde{B}_{j]\nu} = 0$ are

- pp-wave solution

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- KK monopole

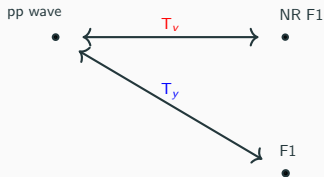
Relativistic Solutions vs Non-relativistic Solutions I

PP-wave solution

$$ds_{pp}^2 = 2du dv + K du^2 + dz_{(8)}^2,$$

$$\hat{B} = 0,$$

$$e^{\hat{\phi}} = g_s.$$



$$F = 1 + \frac{K(z_{(8)})}{2}, \quad v + u \propto y$$

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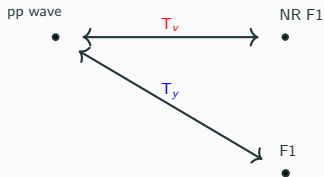
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NR fundamental string solution

$$\tau_{\mu}^A = \delta_{\mu}^A, \quad E_{\mu}^{A'} = \delta_{\mu}^{A'},$$

$$\mathcal{B}_{01} = K(z_{(8)}), \quad e^{\Phi} = g_s.$$



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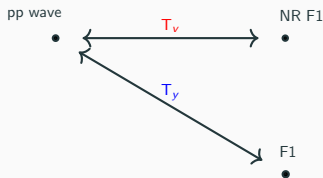
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Fundamental String solution F1

$$ds_{\text{F1}}^2 = F^{-1}(-dt^2 + dy^2) + dz_{(8)}^2,$$

$$\tilde{B}_{01} = (1 - F^{-1}),$$

$$e^{\tilde{\Phi}} = g_s F^{-1/2},$$

$$F = 1 + \frac{K(z_{(8)})}{2}, \quad v + u \propto y$$

NS5 brane solution

$$ds_{\text{NS5}}^2 = -dt^2 + dy^2 + dy_{(4)}^2 + F dz_{(4)}^2,$$
$$(d\tilde{B})_{mnp} = \frac{1}{6} \varepsilon_{mnp}{}^r \partial_r F, \quad e^{\tilde{\Phi}} = g_s F^{1/2},$$

Relativistic Solutions

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KK monopole

$$ds_{\text{KK5}}^2 = -dt^2 + dy^2 + dy_{(4)}^2 + F dz_{(3)}^2 + F^{-1} \left(dz + A_m(z_{(3)}) dz_{(3)}^m \right)^2,$$

$$\text{with } \partial_{[m} A_{n]} = \varepsilon_{mnp} \partial^p F.$$

$$t + y \propto v$$

$$\partial_{[i} \tilde{G}_{j]v} = \partial_{[i} \tilde{B}_{j]v} = 0$$

Relativistic vs Non-relativistic Solutions II

NR NS5



NR KK5

NS5

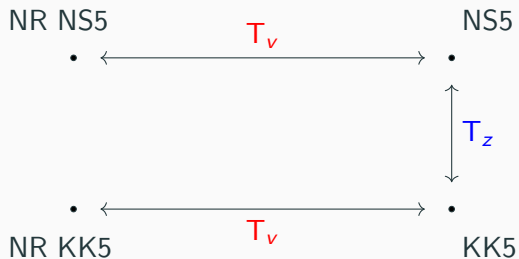


T_z

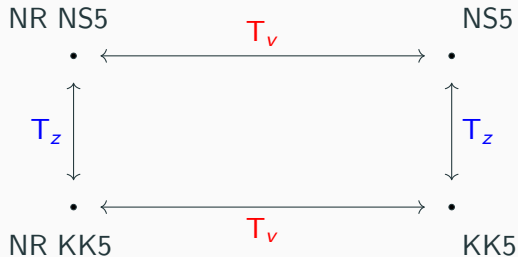


KK5

Relativistic vs Non-relativistic Solutions II



Relativistic vs Non-relativistic Solutions II



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hear talk by Ziqi Yan

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hear talk by Ziqi Yan
- Does intersecting brane give a higher dimensional viewpoint to some black holes in four and five dimensions as in relativistic case? Strong non-relativistic gravity?

Thank you!