Nonrelativistic Open String Theory

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Collaborations

1. **Nonrelativistic Open Strings and Yang-Mills Theory (2020.xxxxx)**

2. **A Triality for Relativistic, Nonrelativistic and Noncommutative Open Strings (2020.xxxxx)**

with Jaume Gomis, Matthew Yu (Perimeter Institute)

Open Strings in Nonrelativistic String Theory

- **longitudinal D-brane: noncommutative open string (NCOS)**
  [Seiberg, Susskind, Toumbas '00] [Gopakumar, Maldacena, Minwalla, Strominger '01]
- **transverse D-brane: nonrelativistic open string (NROS)**
  [Danielsson, Guijosa, Kruczenski '01]

We will focus on NROS.

gauge theories from NROS?

- **Galilean electrodynamics**
- **Galilean Yang-Mills**
- **Galilean DBI**
- ...

Nonrelativistic Closed Strings

- **2D foliation**

$$X^\mu = (X^A, X^{A'}) \qquad A = 0,1 \qquad A' = 2, \cdots d$$

- **light-cone coordinates**
  - **spacetime** $X = X^0 + X^1 \quad \overline{X} = X^0 - X^1$
  - **worldsheet** $\partial = \partial_\sigma - i\partial_\tau \quad \overline{\partial} = \partial_\sigma + i\partial_\tau$

- **nonrel. string theory** [Gomis, Ooguri '00] [Danielsson, Guijosa, Kruczenski '00]

$$S_0 = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \left(\partial_\alpha X^{A'} \partial^\alpha X^{A'} + \lambda \, \overline{\partial} X + \overline{\lambda} \, \partial \overline{X} \right)$$

- **closed string spectrum** $p_0 = \frac{1}{wR}\left(\alpha' p^{A'} p^{A'} + N + \overline{N}\right)$, $N - \overline{N} = wK$

Open String Sector

- **transverse sector is in form the same as in rel. string theory**
- **boundary conditions in longitudinal sector?**
- **to be concrete**
  - **boundary at** $\sigma = 0$
  - $\partial_\sigma X^{A'}\big|_{\partial\Sigma} = 0$ **(N)**

Noncommutative Open String Theory (NCOS)

- **Neumann in X^A**
- **nonzero electric field**

$$\frac{E}{2\pi\alpha'} \int_{\partial\Sigma} d\tau \, i \, X^0 \, \partial_\tau X^1$$

- **noncommutativity** $[X^A(\tau), X^B(\tau')] \sim iE^{-1} \, \epsilon^{AB} sgn(\tau - \tau')$
- **relativistic open string spectrum**

[Gomis, Ooguri '00]

Nonrelativistic Open String Theory (NROS)

- **Dirichlet in X^1 and Neumann in X^0**
- **boundary variation of the action**

$$\delta S = \frac{1}{2\pi\alpha'}\int_{\partial\Sigma} d\tau \left[\delta X^{A'}\partial_\sigma X^{A'} + \tfrac{1}{2}(\lambda+\overline{\lambda})\,\delta X^0 + \tfrac{1}{2}(\lambda-\overline{\lambda})\,\delta X^1\right]$$

- **Dirichlet boundary condition:** $\delta X^1 = 0$ **(D)**
- **varying X^0:** $\lambda + \overline{\lambda} = 0$
- **open string spectrum** $\quad p_0 = \frac{1}{wR}\left(\alpha' p^{A'}p^{A'} + \frac{1}{2}N\right)$
- **global spacetime symmetries on D-brane:**

$$\delta X^{A'} = \Xi^{A'} + \Lambda^{A'}X^0 - \Lambda^{A'}{}_{B'}X^{B'} \qquad \delta\lambda = -\Lambda_{A'}\partial X^{A'}$$

$$\delta X^0 = \Xi^0 \qquad \delta X^1 = 0 \qquad \delta\overline{\lambda} = -\Lambda_{A'}\overline{\partial}X^{A'}$$

$\Longrightarrow$ **Bargmann algebra**

Open String Vertex Operators in NROS

- **independent vertex operators on D-brane:**

$$V = \frac{1}{2\pi\alpha'}\int_{\partial\Sigma} d\tau \left(\lambda\, N + A_0\, \partial_\tau X^0 + A_{A'}\partial_\tau X^{A'}\right)$$

- $U(1)$ **gauge transformation:** $\delta_\epsilon A_0 = \partial_0 \epsilon$ **,** $\delta_\epsilon A_{A'} = \partial_{A'}\epsilon$

- **invariant under Bargmann:**

$$N \to N \qquad A_0 \to A_0 - \Lambda^{A'} A_{A'} \qquad A^{A'} \to A^{A'} - \Lambda^{A'} N + \Lambda^{A'B'} A_{B'}$$

BRST Invariance (Zero Winding)

- **BRST invariance** $0 = [Q, V] = -\frac{\alpha'}{4} \int_{\partial\Sigma} dz\, \partial_z c\, \mathcal{E}(z)$

$$\mathcal{E}(z) = \frac{1}{2}\left(\partial^{A'}\partial_{A'} N\right)\lambda - \left(\partial_0^2 N - \partial^{A'} E_{A'}\right)\partial X^0 - \left(\partial_0 \partial_{A'} N + \partial^{B'} F_{B'A'}\right)\partial X^{A'}$$

- **e.o.m.**

$$\partial^{A'}\partial_{A'} N = 0 \qquad \partial_0^2 N - \partial^{A'} E_{A'} = 0 \qquad \partial_{A'}\partial_0 N + \partial^{B'} F_{B'A'} = 0$$

Galilean Electrodynamics

- **Galilean electrodynamics (GED) on D-brane**

$$S = -\frac{1}{g^2}\int dX^0\, dX^{A'} \left(\tfrac{1}{2}\,\partial_0 N\,\partial_0 N - E_{A'}\partial_{A'} N - \tfrac{1}{4} F_{A'B'}F_{A'B'}\right)$$

- **in the literature, GED from null reduction and nonrel. limit**

  [Santos, de Montigny, Khanna, Santana '04] [Bergshoeff, Rosseel, Zojer '15]

  [Festuccia, Hansen, Hartong, Obers '16] ...

- **quantum properties** [Chapman, Di Pietro, Grosvenor, ZY, to appear]

Winding Modes

- **open string vertex operator with a fixed winding number** w

$$V^w(z) = \int dz \left[\tfrac{1}{2} N^w \lambda(z) - A_0^w \, \partial_z X^0(z) - A_{A'}^w \partial_z X^{A'}(z) \right] e^{iq \int^z dz' \lambda(z')}$$

where $q = wR/(4\alpha')$

- **define** $X^r(z) \equiv \frac{1}{2} \int^z dz' \lambda(z)$, $A_r = -N$, **and** $\mathcal{A} = (r, 0, A')$

$$A_{\mathcal{A}}(X^{\mathcal{B}}) = \sum_w A_{\mathcal{A}}^w(X^0, X^{A'}) \, e^{2iqX^r}$$

- **BRST invariance** $\sum_w [Q, V^w(z)] = 0$ **implies**

$$G^{\mathcal{BC}} \partial_{\mathcal{B}} F_{\mathcal{CA}} = 0 \qquad G^{\mathcal{AB}} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \delta^{A'B'} \end{pmatrix}$$

- $d - 2$ **propagating d.o.f. with** $p_0 = \frac{\alpha'}{wR} p_{A'} p_{A'}$

Coinciding D-branes

- M **coinciding D-branes; zero winding**
- **adjoint gauge field** $(A_0\,, A_{A'})$ **in** $U(M)$
- **Wilson line coupled to adjoint scalar** N

$$W = \mathbf{tr}\,\mathcal{P}\exp\left[\frac{1}{2\pi\alpha'}\int_{\partial\Sigma} d\tau\,\Big(\lambda N + iA_0\,\partial_\tau X^0 + iA_{A'}\,\partial_\tau X^{A'}\Big)\right]$$

- **At the lowest order in field strength: Galilean Yang-Mills**

$$S_{\mathbf{YM}} = -\frac{1}{g_{\mathbf{YM}}^2}\int dX^0\,dX^{A'}\,\mathbf{tr}\Big(\tfrac{1}{2}\,D_0 N\,D_0 N - E_{A'}\,D_{A'}\,N - \tfrac{1}{4}\,F_{A'B'}\,F_{A'B'}\Big)$$

Curved Backgrounds (Abelian)

- **target space coordinates** $X^\mu \quad \mu = 0, 1, \cdots d-1$
- **D-brane worldvolume coordinates** $Y^i \quad i = 0, 1, \cdots d-2$
- **embedding of D-brane in target space** $X^\mu\big|_{\partial\Sigma} = f^\mu(Y^i)$
- **Dirichlet nonlinear sigma model on flat worldsheet**

$$S = \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma \Big(\partial X^\mu\,\overline{\partial} X^\nu \left(H_{\mu\nu} + B_{\mu\nu}\right) + \lambda\,\overline{\partial} X^\mu \tau_\mu + \overline{\lambda}\,\partial X^\mu\,\overline{\tau}_\mu\Big)$$

$$+ \frac{1}{2\pi\alpha'}\int_{\partial\Sigma} d\tau \left[\tfrac{1}{2} N(Y)\left(\lambda - \overline{\lambda}\right) + i\,A_i(Y)\,\partial_\tau Y^i\right]$$

$$H_{\mu\nu} = E_\mu{}^{A'} E_\nu{}^{A'} + \left(\tau_\mu{}^A m_\nu{}^B + \tau_\nu{}^A m_\mu{}^B\right)\eta_{AB}$$

- **dilaton only contributes classically at the lowest order in** α'

Spontaneous Symmetry Breaking: Unbroken Phase

- **boundary conditions** ($t_i{}^A = \tau_\mu{}^A \partial_i f^\mu$, $\mathcal{F}_{ij} = \partial_i f^\mu B_{\mu\nu} \partial_j f^\nu + F_{ij}$)

$$N(Y) = 0 \qquad \partial_\sigma X^\mu \, \tau_\mu{}^A = i\, \epsilon^A{}_B \, \partial_\tau Y^i \, t_i{}^B$$

$$\partial_i f^\mu \, H_{\mu\nu} \, \partial_\sigma X^\nu + i \mathcal{F}_{ij} \, \partial_\tau X^j + \tfrac{1}{2} \left(\lambda \, t_i + \overline{\lambda} \, \overline{t}_i \right) = 0$$

- **string Newton-Cartan symmetry**

$$\delta \tau_\mu{}^A = \Lambda \, \epsilon^A{}_B \, \tau_\mu{}^B \qquad \delta E_\mu{}^{A'} = -\Lambda_A{}^{A'} \tau_\mu{}^A + \Lambda^{A'}{}_{B'} E_\mu{}^{B'}$$

$$\delta m_\mu{}^A = D_\mu \sigma^A + \Lambda \, \epsilon^A{}_B \, m_\mu{}^B + \Lambda^{AA'} E_\mu{}^{A'} - \tau_\mu{}^B \, \sigma^A{}_B \, , \quad \sigma^A{}_A = 0$$

$$\delta A_i = -\epsilon_{AB} \, \sigma^A \, t_i{}^B$$

Spontaneous Symmetry Breaking: Broken Phase

- **coordinates adapted to the submanifold** $X^\mu = (y, Y^i)$
- **for NROS**

$$\tau_y{}^0 = E_y{}^{A'} = 0 \qquad \tau_y{}^1 \neq 0$$

- **uniformly broken phase**

$$\langle f^\mu(Y^i)\rangle = f_0^\mu(Y^i) \qquad f_0^y = y_0 \qquad f_0^i = Y^i$$

- **Nambu-Goldstone boson** $N(Y^i)$

$$f^y(Y^i) = y_0 + N(Y^i) \qquad f^i(Y^i) = Y^i$$

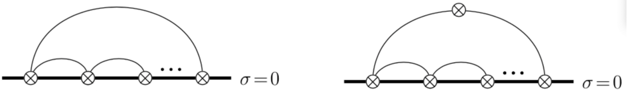
- **Bargmann symmetry on D-brane**

$$\delta E_i{}^{A'} = \Lambda^{A'}\tau_i{}^0 + \Lambda^{A'}{}_{B'}E_i{}^{B'} \qquad \delta m_i{}^0 = \partial_i\sigma^0 + \Lambda^{A'}E_i{}^{A'}$$

$$\delta A_i = -\sigma^0\left(\tau_i{}^1 + \partial_i N\right) \qquad \delta m_y{}^0 = \partial_y\sigma^0$$

Boundary Beta-Functions and D-brane Action

- **beta-functions for N and A_i**



$$\beta(N) = \alpha' \left[J^{ij}\, K^{\mu}{}_{ij}\, \tau_{\mu}{}^{1} + t_{A'}{}^{1} \left(\tfrac{1}{2} J^{ia}\, \mathscr{F}_{a}{}^{j}\, \mathscr{H}_{ijA'} - \partial_{A'} \Phi \right) \right]$$

$$\beta_k(A) = \alpha' \Big\{ J^{ri}\, \nabla_k\, t_i{}^{1} + J^{ij} \Big[\nabla_{(i} \mathscr{F}_{j)k} + \left(K^{\mu}{}_{ij}\, \mathscr{B}_{\mu k} + K^{\mu}{}_{k(i} \mathscr{B}_{j)\mu} \right) \Big]$$
$$- \left(\tfrac{1}{2} J^{ia}\, \mathscr{F}_{a}{}^{j}\, \mathscr{H}_{ij\rho} - \partial_{\rho} \Phi \right) \left(\epsilon^{A}{}_{B}\, \tau^{\rho}{}_{A}\, t_k{}^{B} - E^{\rho}{}_{A'}\, \mathscr{F}_{A'k} \right) \Big\}$$

where $\mathscr{F}_{ij} \equiv \mathscr{B}_{ij} + F_{ij}$ **,** $\mathscr{B}_{\mu\nu} = B_{\mu\nu} + 2\, m_{[\mu}{}^{A}\, \tau_{\nu]}{}^{B} \epsilon_{AB}$ **and**

$$J^{ab} = \left(G_{ab} - \mathscr{F}_{ac}\, \mathscr{F}^{c}{}_{b} \right)^{-1}, \quad G_{ab} = \begin{pmatrix} 0 & t_j{}^{0} \\ t_i{}^{0} & e_{ij} \end{pmatrix}, \quad \mathscr{F}_{ab} = \begin{pmatrix} 0 & t_j{}^{1} \\ -t_i{}^{1} & \mathscr{F}_{ij} \end{pmatrix}$$

- **worldvolume action for D-brane**

$$S_{\text{brane}} \propto \int d^{d-1} Y\, e^{-\Phi} \sqrt{ - \det \begin{pmatrix} 0 & \partial_j f^{\mu}\, \tau_{\mu} \\ \partial_i f^{\mu}\, \overline{\tau}_{\mu} & \partial_i f^{\mu}\, \partial_j f^{\nu} \left(H_{\mu\nu} + B_{\mu\nu} \right) + F_{ij} \end{pmatrix} }$$

Outlooks

- **nonabelian generalization of Galilean DBI**
- **SUSY and analog of Dirac's equation?**
- **quantum properties: coupling GED/GYM to fermions?**
- **black hole/brane, AdS/CFT...**

Thank you!