Universiteit van Amsterdam

# Renormalization of Galilean Electrodynamics

GED

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Based on 2007.03033 w/

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Non Lorentzian Zoom Meeting #3 - 8 July 2020

# Outline

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Symmetries

The GED Action

## Part II - Quantum Aspects

Non-Renormalization Theorems

Beta Functions

## Part III - SUSY Generalization

## Summary & Outlook

(in progress w/)

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# Motivation

Modern condensed matter approach is based on emergent symmetries and field content

Emergent Symmetries: <u>Lifshitz scaling</u>

$$t \to \lambda^z t,\; x^i \to \lambda\, x^i$$

used for strange metallic phases

<u>Schrödinger symmetry</u> - NR conformal symmetry
fermions at unitarity (e.g., ultra-cold atoms)

Emergent fields: <u>Emergent gauge fields</u> - anyons -
using a Chern-Simons term

Which <u>Schrödinger invariant gauge theories</u> can be constructed?

# Historical Note

1973 - Le Bellac and Lévy-Leblond Galilean Maxwell?

- Electric/Magnetic limits
  Hard to combine - $\epsilon_0 \mu_0 c^2 = 1$

2004 - Santos, Montigny, Khanna, Santana

- Combine using auxiliary field (from null reduction)

2016 - Festuccia, Hansen, Hartong, Obers

- Obtain from non-relativistic limit of Maxwell+Scalar
- Study the symmetries & couple to curved background

What about quantum properties?

# Symmetries

$$a_i \to a_i$$
$$a_t \to a_t - v^i a_i$$

**Galilean boosts:**

$$t \to t \qquad x^i \to x^i + v^i t$$

$$\partial_t \to \partial_t - v^i \partial_i \qquad \partial_i \to \partial_i$$

**Gauge Invariance $(a_t, a_i)$:**

$$a_t \to a_t + \partial_t \Lambda \qquad a_i \to a_i + \partial_i \Lambda$$

(And of course also time/space translations, rotations)

**Schrödinger Symmetry (NR conformal):**

dilations with z=2: $t \to \lambda^2 t$, $x^i \to \lambda\, x^i$
and one special-conformal transformation

# Galilean Electrodynamics Action

With the previous symmetries - only magnetic action

$$\mathcal{L} = -\frac{1}{4} f_{ij} f^{ij}$$

$$f_{ij} \equiv \partial_i a_j - \partial_j a_i$$
$$E_i \equiv \partial_t a_i - \partial_i a_t$$

add another scalar φ - unique possibility:

$$\mathcal{L}_{\mathrm{GED}} = \frac{1}{2}(\partial_t \varphi)^2 + E^i \partial_i \varphi - \frac{1}{4} f^{ij} f_{ij}$$

(with modified boost transformations of the gauge field

$$a_i \to a_i + v_i \varphi \quad a_t \to a_t - v^i a_i + \frac{1}{2} v_i^2 \varphi \quad \varphi \to \varphi)$$

does not transform under gauge

<u>Comments:</u>

- can be obtained from null reduction/non-relativistic limit
- gauge fields are non-dynamical (instantaneous mediators)

# Galilean Electrodynamics Action

add Matter - a Schrödinger Scalar σ (2+1 dimensions):

$$S_{\mathrm{GED}} = \int dt d^2\mathbf{x} \left[ \frac{1}{2}(\partial_t \varphi)^2 + E^i \partial_i \varphi - \frac{1}{4} f^{ij} f_{ij} + \frac{i}{2}(\overline{\sigma} D_t \sigma - \sigma D_t \overline{\sigma}) - \frac{1}{2M} D_i \overline{\sigma} D_i \sigma \right]$$

where $M \equiv \Omega - e\varphi$

and $D_t \sigma = (\partial_t - iea_t)\sigma \,, D_i \sigma = (\partial_i - iea_i)\sigma$

Comments:

- More symmetries in gauge sector, but with matter just Schrödinger

$[t] = -2, \quad [x^i] = -1; \quad [a_t] = 2, \quad [a_i] = 1, \quad [\varphi] = 0, \quad [\sigma] = 1$

dimensionless

- Infinite series of interactions: $\frac{1}{2M} = \frac{1}{2\Omega}\left(1 + \frac{e\varphi}{\Omega} + \left(\frac{e\varphi}{\Omega}\right)^2 + \cdots \right)$

# Quantum Corrections?

Yes! The most general set of marginal corrections that is formed:

$$S_{\mathrm{sGED}} = \int dt d^2\mathbf{x} \left[\frac{1}{2}(\partial_t \varphi)^2 + E^i \partial_i \varphi - \frac{1}{4} f^{ij} f_{ij} + \frac{i}{2}(\overline{\sigma} D_t \sigma - \sigma D_t \overline{\sigma}) - \frac{1}{2M} D_i \overline{\sigma} D^i \sigma \right.$$

$$\left. + \mathcal{J}[M] \partial_i M \partial^i M \overline{\sigma} \sigma - \frac{1}{4} \lambda \mathcal{V}[M] (\overline{\sigma} \sigma)^2 - \mathcal{E}[M] (\partial_i \partial^i M - e^2 \overline{\sigma} \sigma) \overline{\sigma} \sigma \right]$$

Taylor expansion in $M \equiv \Omega - e\varphi$,

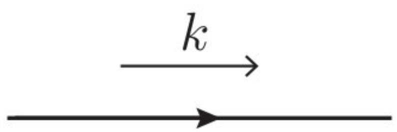
e.g., $\mathcal{J}[M] = \mathcal{J}_0 + \mathcal{J}_1 e\varphi + \mathcal{J}_2 (e\varphi)^2 + \cdots$

Handle the infinitely many couplings using the background field method (similar to 2d sigma-models)

$$M = M_0 + \delta M, \qquad M_0 = \Omega - e\varphi_0, \qquad \delta M = -e\delta\varphi$$

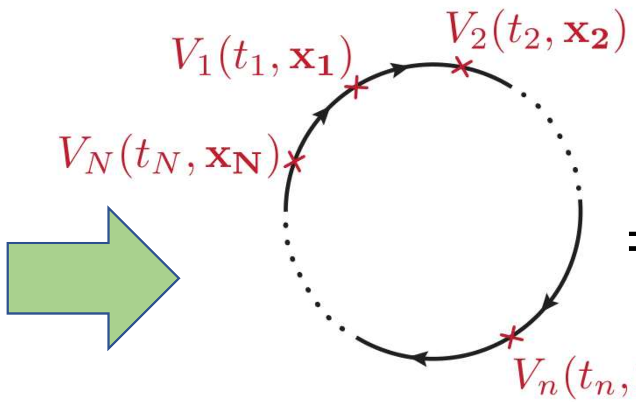
# Non-Renormalization Theorems

**Closed scalar loops vanish!**




$$= \langle \sigma(k)\overline{\sigma}(-k)\rangle = \frac{i}{\omega - \frac{\mathbf{k}^2}{2M_0} + i\epsilon}$$

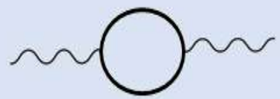
or $\langle \sigma(\boldsymbol{x},t)\overline{\sigma}(\boldsymbol{x}',t')\rangle \propto \Theta(t-t')$




$= 0$

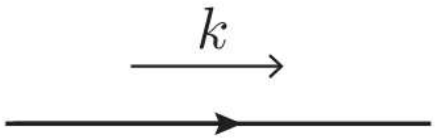
**Consequences (@any loop order):**

1. no wavefunction renormalization for the gauge fields



2. e does not get renormalized (gauge invariance: $D_\mu = \partial_\mu - i\, e\, A_\mu$)

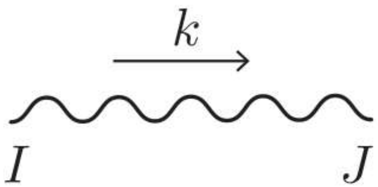
# Feynman Rules




$$= \langle \sigma(k)\bar{\sigma}(-k)\rangle = D_\sigma(k) \equiv \frac{i}{\omega - \frac{\mathbf{k}^2}{2M_0} + i\epsilon}$$

Convenient to define $\mathcal{A}_I \equiv (\delta\varphi, a_t, a_i)$, $\quad i \in \{1,2\}$

Add Galilean covariant gauge fixing term $-\frac{1}{2\xi}(\partial_t \delta\varphi + \partial_i a_i)^2$




$$= \langle \mathcal{A}_I(k)\mathcal{A}_J(-k)\rangle = \mathcal{D}_{IJ}(k)$$

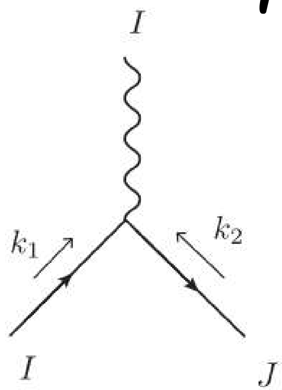
$$\equiv -\frac{i}{\mathbf{k}^2}\left[\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \delta_{ij} \end{pmatrix} - \frac{(1-\xi)}{\mathbf{k}^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \omega^2 & -\omega \mathbf{k}_j \\ 0 & -\omega \mathbf{k}_i & \mathbf{k}_i \mathbf{k}_j \end{pmatrix}\right]$$

1. Non dynamical
2. No $\langle \delta\varphi \, \delta\varphi \rangle$ propagator

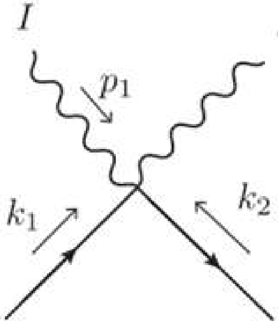
# Feynman Rules

Note that the vertices and propagators depend on $M_0$.

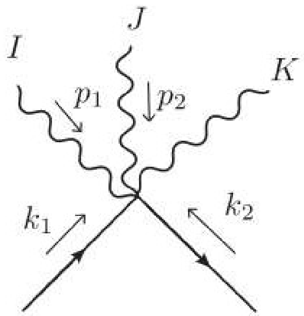
This saves us the need to compute infinitely many Feynman diagrams



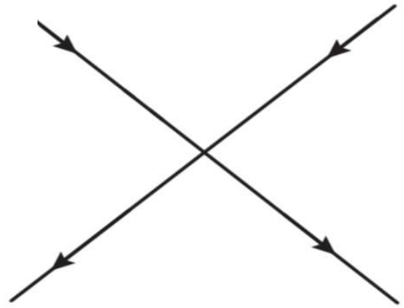

$$= ie \begin{pmatrix} \frac{1}{2M_0^2}\mathbf{k}_1 \cdot \mathbf{k}_2 - \mathcal{E}(M_0)(\mathbf{k}_1 + \mathbf{k}_2)^2 \\ 1 \\ \frac{1}{2M_0}(\mathbf{k}_1 - \mathbf{k}_2)^i \end{pmatrix}$$



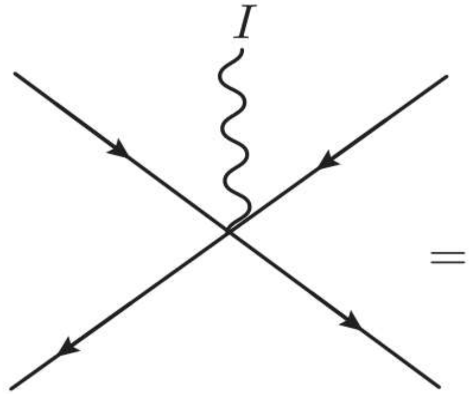

$$= \frac{ie^2}{M_0} \begin{pmatrix} \frac{\mathbf{k}_1\cdot\mathbf{k}_2}{M_0^2} & 0 & \frac{(\mathbf{k}_1-\mathbf{k}_2)_j}{2M_0} \\ 0 & 0 & 0 \\ \frac{(\mathbf{k}_1-\mathbf{k}_2)_i}{2M_0} & 0 & -\delta_{ij} \end{pmatrix} + ie^2\left(\mathcal{E}'(M_0)(\mathbf{p}_1^2 + \mathbf{p}_2^2) - 2\mathcal{J}(M_0)\mathbf{p}_1 \cdot \mathbf{p}_2\right)\delta^{I\varphi}\delta$$




= Something long that depends on: $\mathcal{E}''(M_0)$ and $\mathcal{J}'(M_0)$



$$= -i\lambda \mathcal{V}(M_0) + 4ie^2\,\mathcal{E}(M_0)$$



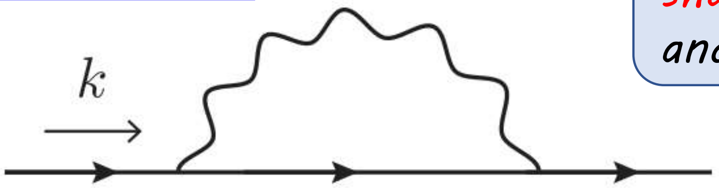

$$= i\lambda\, e\, \mathcal{V}'(M_0) - 4ie^3\,\mathcal{E}'(M_0)$$

# Renormalization of GED

## Wave function renormalization

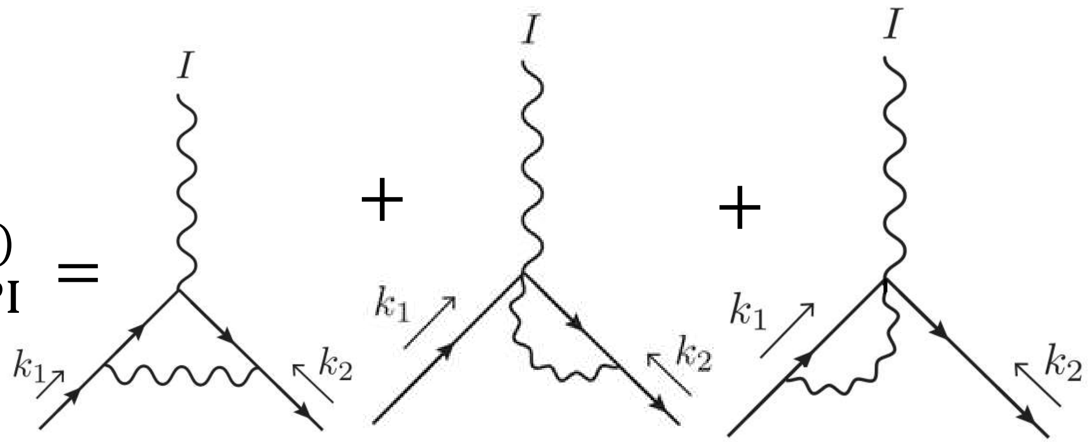
We regulate the diagrams with
sharp UV cutoff $\Lambda$
and sharp IR cutoff $\mu$

$$\langle \sigma_B(k)\overline{\sigma}_B(-k)\rangle^{(1)}_{\mathrm{1PI}} =$$




## Renormalization for $\mathcal{E}(M)$

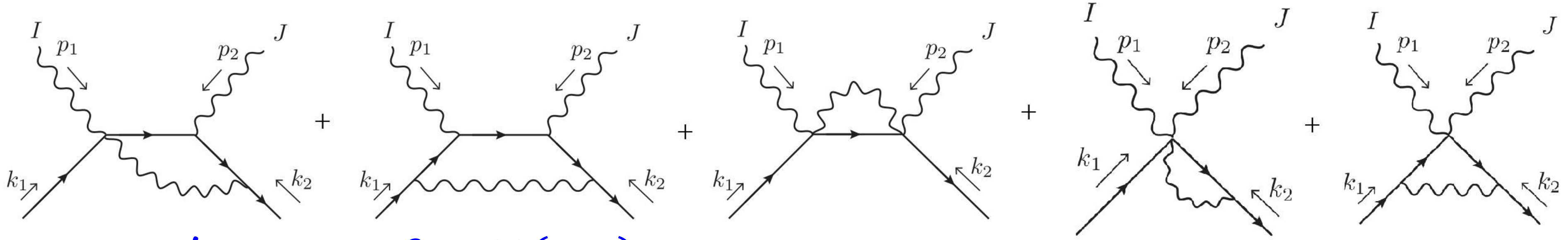
$$\langle \sigma_B(k_1)\overline{\sigma}_B(k_2)\mathcal{A}^I(-k_1-k_2)\rangle^{(1)}_{\mathrm{1PI}} =$$



# Renormalization of GED

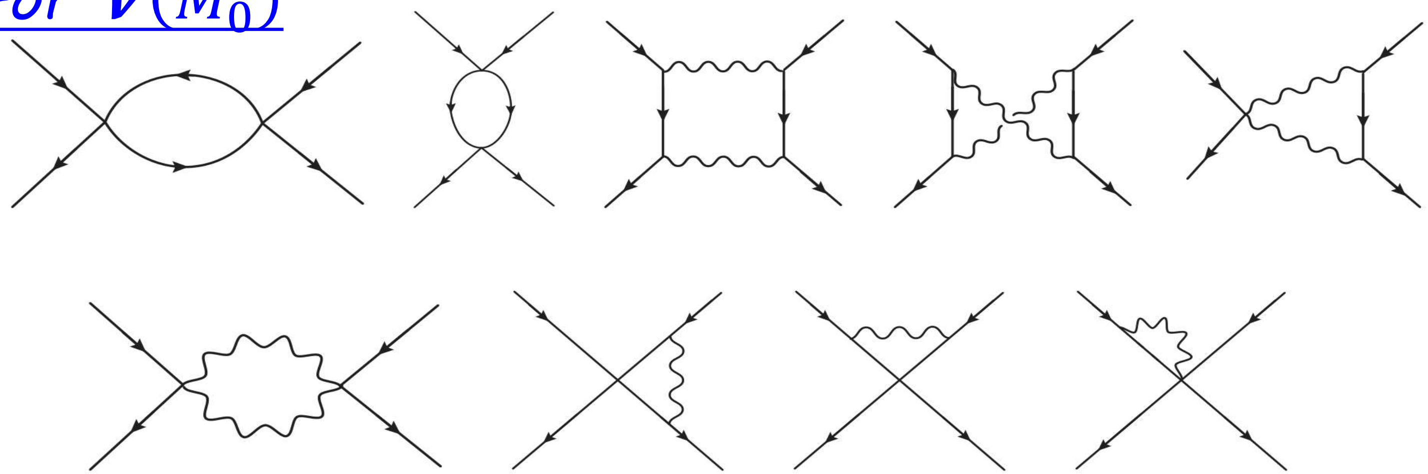
## Renormalization for $\mathcal{J}(M_0)$

$$\langle \sigma_B(k_1)\overline{\sigma}_B(k_2)\mathcal{A}^I(p_1)\mathcal{A}^J(p_2)\rangle^{(1)}_{\text{1PI}} =$$




## Renormalization for $\mathcal{V}(M_0)$

$$\langle \sigma_B\overline{\sigma}_B\sigma_B\overline{\sigma}_B\rangle^{(1)}_{\text{1PI}} =$$



# Beta Function and Fixed Points

## Beta Function(al)s (understand as a series)

$\mathcal{J}[M] = \mathcal{J}_0 + \mathcal{J}_1 e\varphi + \mathcal{J}_2 (e\varphi)^2 + \cdots$

$\beta_{\mathcal{J}[M]} = \beta_{\mathcal{J}_0} + \beta_{\mathcal{J}_1} e\varphi + \beta_{\mathcal{J}_2} (e\varphi)^2 + \cdots$

$\beta_{\mathcal{J}_n} \sim \# \, \mathcal{J}_{n+1} + \#$

$$\beta_{\mathcal{E}[M]} = \frac{e^2}{2\pi}\left(\mathcal{E}'[M] + \frac{1}{4M^3}\right) \qquad \beta_{\mathcal{J}[M]} = \frac{e^2}{2\pi}\left(\mathcal{J}'[M] + \frac{5}{8M^4}\right)$$

$$\beta_{\lambda\mathcal{V}[M]} = \frac{e^4}{2\pi}\left(4\mathcal{J}[M] + \frac{5}{8M^3}\right) + \frac{\lambda e^2}{2\pi}\left(2\mathcal{V}'[M] + \frac{\mathcal{V}[M]}{2M}\right) + \frac{\lambda^2}{4\pi} M\mathcal{V}[M]^2$$

## Fixed points (solve differential equations):

Special example: $\mathcal{E}[M] = \frac{1}{8M^2} + \mathcal{E}_0, \quad \mathcal{J}[M] = \frac{5}{24M^3} + \mathcal{J}_0, \quad \lambda\mathcal{V}[M] = \frac{21 \pm 4\sqrt{21}}{6M^2} e^2$

Something complicated

3 integration constants –

manifold of fixed points with 4 parameters

Fixed points are Schrödinger invariant

# Comments

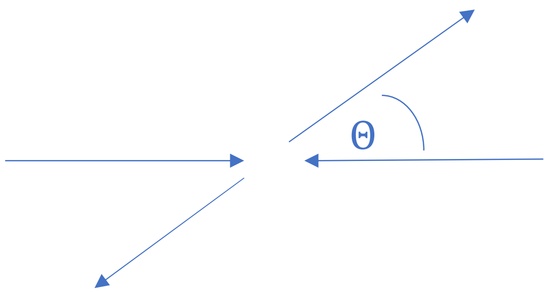
1. Conformal manifolds are special!

2. Infinitely many relevant deformations - $\varphi^n \bar{\sigma}\sigma$
But generalizations (SUSY, Flavors, YM) might get rid of them?

3. Statistics – bosons or fermions?

4. IR divergences and soft theorems

5. 2-2 $\sigma$ scattering in the center of mass frame:




$$i\mathcal{M}_{\text{tree}} = -i\lambda\mathcal{V}[M_0] - \frac{ie^2}{2M_0^2} \cdot \frac{1+3\cos^2\theta}{\sin^2\theta}$$

6. What is the relevant quantum mechanics?

w/ Yaron Oz and Avia Raviv Moshe

# SUSY GED

## Null reduction of SUSY gauge action

gaugino

$$\mathcal{S} = \int d^2x dt \Big( -i\sqrt{2}\lambda_1^* \partial_t \lambda_1 - i\lambda_1^*(\partial_1 - i\partial_2)\lambda_2 - i\lambda_2^*(\partial_1 + i\partial_2)\lambda_1$$
$$- \frac{1}{2} f^{12} f_{12} + \partial_i \varphi E^i + \frac{1}{2}(\partial_t \varphi)^2 + \frac{1}{2} D^2 \Big)$$

Auxiliary field

$$\delta\varphi = \epsilon_1^* \lambda_1 + \lambda_1^* \epsilon_1$$
$$\delta a_t = -\epsilon_2^* \lambda_2 - \lambda_2^* \epsilon_2$$
$$\delta a_i = -\frac{1}{\sqrt{2}} \left( \epsilon^\dagger \bar{\sigma}_i \lambda + \lambda^\dagger \bar{\sigma}^i \epsilon \right)$$
$$\delta D = i(\epsilon_1^* \partial_t \lambda_1 - \partial_t \lambda_1^* \epsilon_1) + \frac{i}{\sqrt{2}} \left( \partial_i \lambda^\dagger \bar{\sigma}^i \epsilon - \epsilon^\dagger \bar{\sigma}^i \partial_i \lambda \right)$$
$$\delta\lambda_1 = \frac{1}{\sqrt{2}} \epsilon_1 D + \frac{i}{\sqrt{2}} \epsilon_1 \partial_t \varphi + \frac{1}{\sqrt{2}} \epsilon_1 f_{12} + i\epsilon_2 (\partial_1 - i\partial_2)\varphi$$
$$\delta\lambda_2 = \frac{1}{\sqrt{2}} \epsilon_2 D - \frac{i}{\sqrt{2}} \epsilon_2 \partial_t \varphi - \frac{1}{\sqrt{2}} \epsilon_2 f_{12} + i\epsilon_1 (E_1 + E_2) \Big)$$

Holomorphic structure!

Study on curved NC geometry

Natural since condition to preserve SUSY + one R charge is the existence of a null killing vector - which also permits null reduction

# Summary and outlook

We studied the quantum properties of Galilean electrodynamics using the background field method

Manifold of fixed points where NR symmetry is preserved (this is very hard to get outside SUSY without giving up unitarity or locality)

Here we saw that also giving up boost invariance is an option

Various possible generalization might make our theory closer to the lab

**Lots to explore!**

# Thank you!