Entanglement entropy for warped conformal field theories

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Based on work with Luis Apolo, Bin Chen, Peng-Xiang Hao, Song He, Hongliang Jiang, Qiang Wen, Jianfei Xu, Junzhen Zheng and Yuan Zhong

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Motivation

What we have learned the most: AdS/CFT

What we need to know in the real world:

Non-AdS geometries

-Minkovski

-de Sitter

-Kerr

…

Non-CFTs

-Standard model

-condensed matter systems

-cold atom systems

…

In this talk: (W)AdS/WCFT

$WCFT$: 2d QFT without Lorentzian invariance

invariant under $x \to f(x), \quad y \to y + g(x)$

with conserved currents $T(x), \quad P(x)$

Hofman-Strominger
Deournay-Hartman-Hofman

On the canonical cylinder with $(\hat{x}, \hat{y}) \sim (\hat{x} + 2\pi, \hat{y})$, the warped conformal Killing vectors can be written as $l_m = e^{im\hat{x}}\partial_{\hat{x}}$, $p_n = e^{in\hat{x}}\partial_{\hat{y}}$

The conserved charges are $L_n = -\frac{1}{2\pi}\int d\hat{x}\, e^{in\hat{x}} T(\hat{x})$, $P_n = -\frac{1}{2\pi}\int d\hat{x}\, e^{in\hat{x}} P(\hat{x})$

Viraroso–Kac–Moody algebra

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n,-m}$$

$$[L_n, P_m] = -mP_{n+m}$$

$$[P_n, P_m] = \frac{kn}{2}\delta_{n,-m}$$

SL(2,R) x U(1) subalgebra : $L_1, L_{-1}, L_0, \; P_0$

Note: the algebra is Invariant under a spectral flow transformation $y \to y + \lambda\, x$, but the charges and hence the spectrum are not invariant.

Examples:

Chiral Liouville Gravity *Compère-WS-Strominger*

Weyl Fermion models *Hofman-Roiller*

Chiral boson models *Jessen*

The canonical cylinder with $(\hat{x},\hat{y}) \sim (\hat{x}+2\pi,\hat{y})$, $L_0^{vac} = -\frac{k\mu^2}{4} - \frac{c}{24}$, $P_0^{vac} = -\frac{ik\mu}{2}$

Useful for the holographic dictionary.

μ *is part of the definition of a WCFT*, crucial for mapping the BH entropy and EE.

The reference plane $z = e^{ix}, y' = \hat{y} - i\mu\hat{x}$, $(z, y') \sim \left(z\, e^{i2\pi}, y' - 2\pi\, i\, \mu\right)$, all vacuum charge vanishes.

Correlators can be determined by SL(2,R) x U(1) invariance. *WS-Xu*

i. e. $< \mathrm{O}(\hat{\mathrm{x}}, \mathrm{y}')\mathrm{O}(0,0) >= \mathrm{e}^{\mathrm{iqy}'} x^{-2h}$

Data of WCFT

- c, k
- Eigenvalues of L_0 *and* P_0 on the canonical circle $(\hat{x},\hat{y}) \sim (\hat{x}+2\pi,\hat{y})$
- Three point coefficients

Unitary WCFT:

$c > 0,\ k > 0,\ P_0\ \ real,\quad L_0 - \frac{P_0^2}{k}\ bounded$

Holographic WCFT

- $c \gg 1$
- $k < 0 \Rightarrow$ *states with imaginary charge* *Apolo-WS*
- a sparse spectrum/vacuum block dominance
- maximally chaotic *Apolo-WS-Xu-Zheng*

Holographic dualities for WCFT

$WAdS_3$/WCFT

- Asymptotic symmetry — *Detournay-Compere*
- BH entropy: $S_{BH} = S_{DHH}$ — *Deournay-Hartman-Hofman*
- Greybody factor — *WS-Xu*
- Holographic EE — *Castro-Hofman-Iqbal, WS-Wen-Xu, Apolo-Jiang-WS-Zhong*

AdS_3/WCFT

- Asymptotic symmetry — *Compere-WS-Strominger*
- BH entropy: $S_{BTZ} = S_{DHH}$
- 1-loop determinant — *Castro−Keeler−Szepietowski*
- Averaged 3 point coefficients — *WS-Xu*
- QNEC — *Detournay-Grumiller-Riegler-Vandermiers*

Lower spin gravity /WCFT

Hofman−Rollier

Holographic EE — *Castro-Hofman-Iqbal, Azeyanagi-Detournay-Riegler*

Kerr/WCFT

Aggarwal−Castro−Detournay

The AdS_3/WCFT dictionary

Compère-WS-Strominger,
Apolo-Song

The phase space of pure Einstein gravity under CSS boundary condition

$$\ell^2 \mathrm{ds}^2 = \frac{\mathrm{dr}^2}{\mathrm{r}^2} + \left(r du + \frac{\bar{L}}{r} dv\right)\left(r(dv + P'(u)du) + \frac{L(u)}{r} du\right),$$

$$(u, v) \sim (u + 2\pi, v + 2\pi), \qquad \bar{L} = const$$

- coordinates mapping: $u = \hat{x}, \quad v = \hat{x} + \frac{\hat{y}}{2\sqrt{-\frac{c\,L}{6\,k}}}$ the boundary is the canonical cylinder
- Asymptotic Killing vectors ⇔ warped conformal Killing vectors
- Asymptotic symmetry algebra ⇔ Virasoro-Kac-Moody algebra with $c = \frac{3\ell}{2G}$
- global AdS_3 ⇔ vacuum of WCFT on the canonical cylinder

$$L_0^{vac} = 0, \qquad P_0^{vac} = -\frac{ik\mu}{2}, \qquad \mu = \sqrt{-\frac{c}{6k}}$$

- BTZ black holes $L = \frac{c}{6} T_u^2,\ \bar{L} = \frac{c}{6} T_v^2,\ P'(u) = 0$ ⇔ $thermal\ circle\ \ \beta = \frac{\pi}{T_u},\ \bar{\beta} = 2\pi\mu(1 + \frac{T_v}{T_u})$

Modular covariance

Detournay-Hartman-Hofman

- Putting the theory on a torus with a `thermal circle' and a `spatial circle'

the canonical spatial circle : $(\hat{x}, \hat{y}) \sim (\hat{x} + 2\pi, \hat{y})$

the thermal circle : $(\hat{x}, \hat{y}) \sim (\hat{x} + i\beta, \hat{y} - i\bar{\beta})$

- Modular S-transformation:

1. locally an allowed symmetry transformation: warped conformal transformation

2. `thermal circle' $\xleftrightarrow{S-transformation}$ `spatial circle'

Note:
keep track of all anomalies and the vacuum charges

- Torus partition function

$$Z(\beta, \bar{\beta}) = \mathrm{Tr}(e^{-\beta L_0 + \bar{\beta} P_0}) = e^{k\frac{\bar{\beta}^2}{4\beta}} Z\left(\frac{4\pi^2}{\beta}, -\frac{2\pi i \bar{\beta}}{\beta}\right) .$$

A 'Cardy' entropy in the limit $\beta \to 0, S_{DHH} = -\frac{4\pi i}{k} P_0 P_0^{vac} + \sqrt{\left(\frac{c}{6}\left(L_0 - \frac{P_0^2}{k}\right)\right)}$

reproduces the Bekenstein-Hawking entropy for the WAdS black holes, as well as for the BTZ black holes

Modular covariance

- Putting the theory on a torus with a `thermal circle' and a `spatial circle'

  the canonical spatial circle : $(\hat{x},\hat{y}) \sim (\hat{x}+2\pi,\hat{y})$

  the thermal circle : $(\hat{x},\hat{y}) \sim (\hat{x}+i\beta,\hat{y}-i\bar{\beta})$

- Modular S-transformation:

  1. locally an allowed symmetry transformation: warped conformal transformation

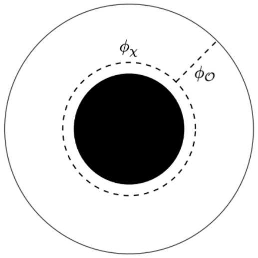
  2. `thermal circle' $\xleftrightarrow{S-transformation}$ `spatial circle'

- Torus 1pf $$\langle \mathcal{O}\rangle_{\beta,\bar{\beta}} = e^{k\frac{\bar{\beta}^2}{4\beta}} \left(\frac{\partial x'}{\partial x}\right)^{h_{\mathcal{O}}} \langle \mathcal{O}\rangle_{\frac{4\pi^2}{\beta},-\frac{2\pi i\bar{\beta}}{\beta}}\,.$$ *WS-Xu*

  averaged 3pf coefficients in WCFT ↔ tadpole diagram in the bulk

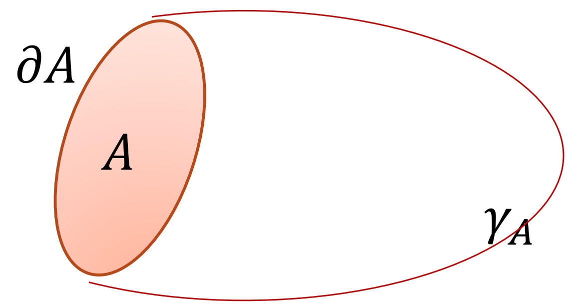
  in the heavy-heavy-light limit

  $$\overline{< i|O|i >} \quad \Leftrightarrow$$



Holographic entanglement entropy in AdS/CFT

Ryu-Takayanagi,
Hubeny-Rangamani-Takayanagi



$$S_A = \min \frac{\mathrm{Area}(\gamma_A)}{4\mathrm{G}}$$

γ_A: co-dimension 2 extremal surface homologous to ∂A

Well established in AdS/CFT, for Einstein Gravity

- Rindler method: *Casini-Huerta-Myers (CHM)*
- Generalized gravitational entropy
  Lewkowyzy-Madalcena, Dong-Lewkowyzy-Rangamani

Question: what is the holographic entanglement entropy in holographic dualities for non-AdS spacetimes?

- ✓ The setup: (W)AdS/WCFT
- ➢ The generalized Rindler method
- The swing surface proposal

similarity between entanglement entropy and thermal entropy

- Modular Hamiltonian $K_A = -\log \rho_A$ $\leftrightarrow$ Hamiltonian
- Renyi entropy $S_A{}^{(q)} = \frac{1}{(1-q)} Tr_A\, e^{-qK_A}$ $\leftrightarrow$ thermal partition function
- Entanglement entropy

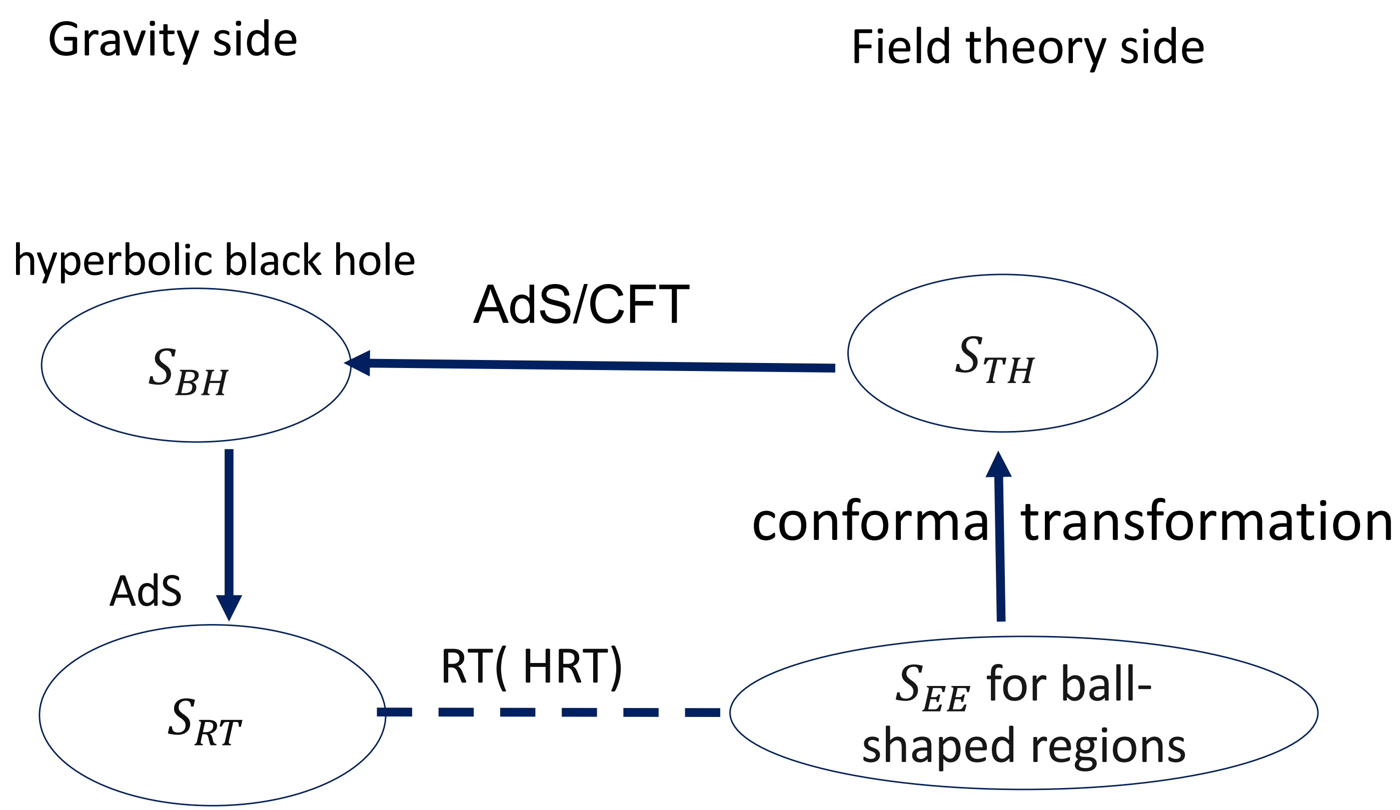
$$S_A = -Tr_A\,(\rho_A \log \rho_A) \quad \leftrightarrow \quad \text{thermal entropy}$$

In general, the modular Hamiltonian is non-local

However, for ball-shaped region in the CFT vacuum, the modular Hamiltonian is local and moreover generates a Rindler boost.

The Rindler method in AdS/CFT :
derivation of RT for ball-shaped subregions

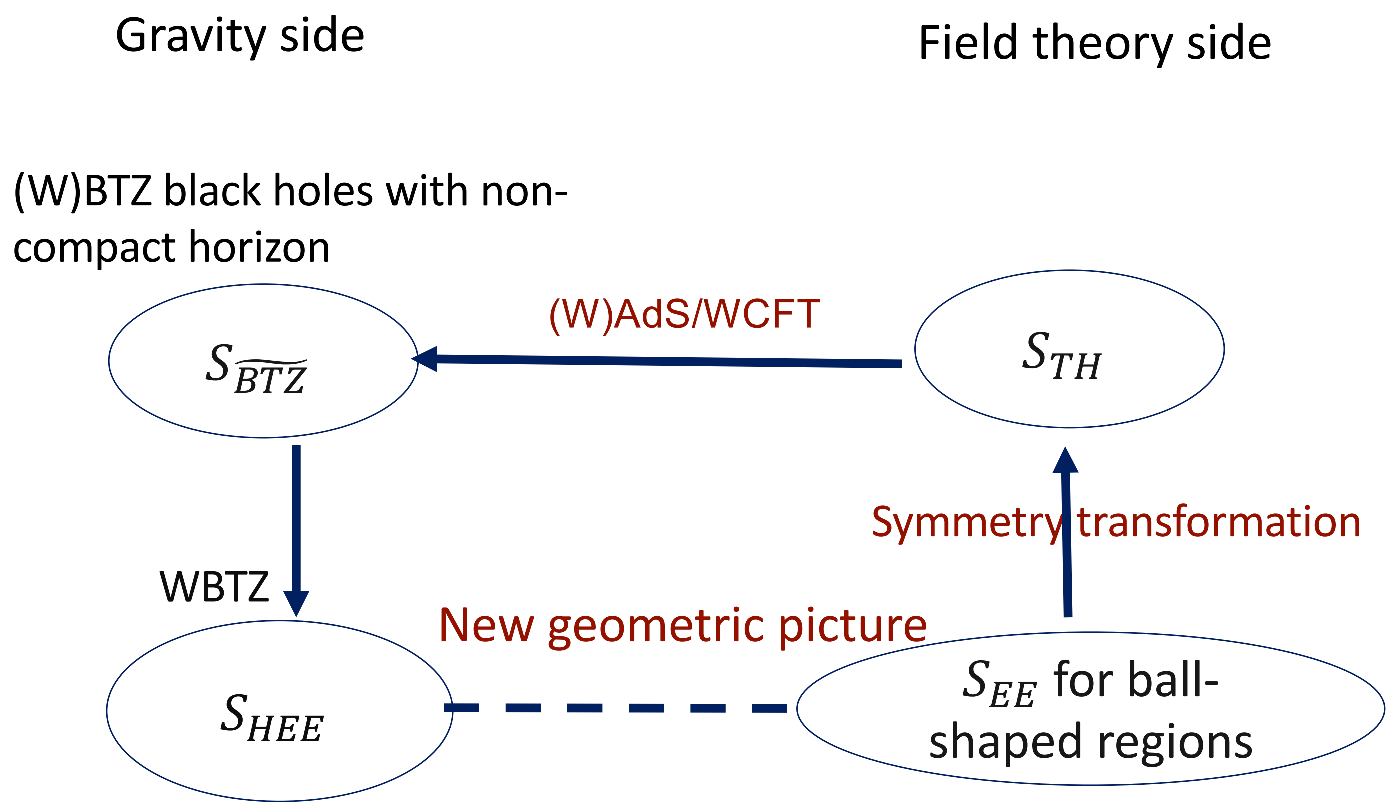
Casini-Huerta-Myers



Modular flow is generated by Rindler boost

The generalized Rindler method in non-AdS holography : derivation for ball-shaped subregions

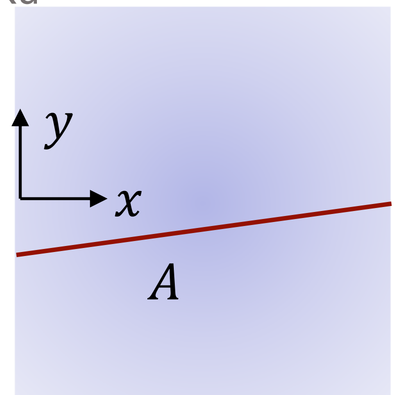
Castro-Hofman-Iqbal
WS-Wen-Xu
Jiang-WS-Wen



Modular flow is generated by Rindler boost

Entanglement entropy in WCFT from Rindler method

Castro-Hofman-Iqbal
WS-Wen-Xu



- EE on A *with end points at* $x_\pm = \pm\frac{l_x}{2}, y_\pm = \pm\frac{l_y}{2}$

- Rindler transformation on the cylinder

$$(x, y) \xrightarrow{\text{warped conformal transformation}} (\tilde{x}, \tilde{y})$$

$$\tanh\frac{\pi\tilde{x}}{\tilde{\beta}} = \frac{\tanh\frac{\pi\left(x-\frac{l_x}{2}\right)}{\beta}}{\tanh\frac{\pi l_x}{2\beta}}, \quad \tilde{y} + \left(\frac{\tilde{\bar{\beta}}}{\tilde{\beta}} - \frac{2\pi\mu}{\tilde{\beta}}\right)\tilde{x} = y + \left(\frac{\bar{\beta}}{\beta} - \frac{2\pi\mu}{\beta}\right) x \qquad D(A)$$

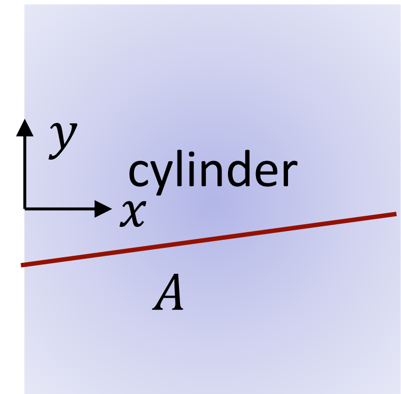
Warped conformal transformation: $D(A) \to \tilde{B}$
$\tilde{B}$ has a thermal identification $(\tilde{x}, \tilde{y}) \sim (\tilde{x} + i\tilde{\beta}, \tilde{y} - i\tilde{\bar{\beta}})$
Modular flow generator: $\zeta \equiv 2\pi\partial_\tau = \tilde{\beta}\partial_{\tilde{x}} + \tilde{\bar{\beta}}\partial_{\tilde{y}}$, $\quad e^{s\,\zeta}(\partial D) = \partial D$

- Entanglement entropy $S_{EE}(A) = S_{th}(\tilde{B}) = S_{DHH}(\tilde{B})$

$$= -\frac{\mu k}{2}\left(l_y + \frac{\bar{\beta}-2\pi\mu}{\beta}\, l_x\right) + \frac{c}{6}\ln\left(\frac{\beta}{\pi\epsilon}\sinh\frac{\pi l_x}{\beta}\right)$$

Entanglement entropy in WCFT from replica trick

Castro-Hofman-Iqbal
WS-Wen-Xu



$D(A)$

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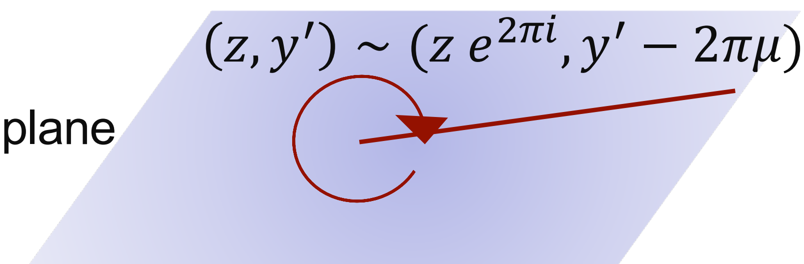
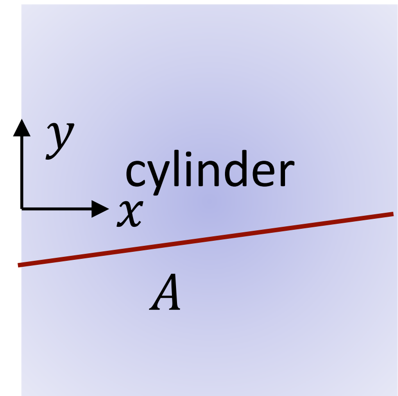
- Open and enlarge the circle circle on the reference plane



Castro-Hofman-Iqbal
WS-Wen-Xu

Entanglement entropy in WCFT from replica trick



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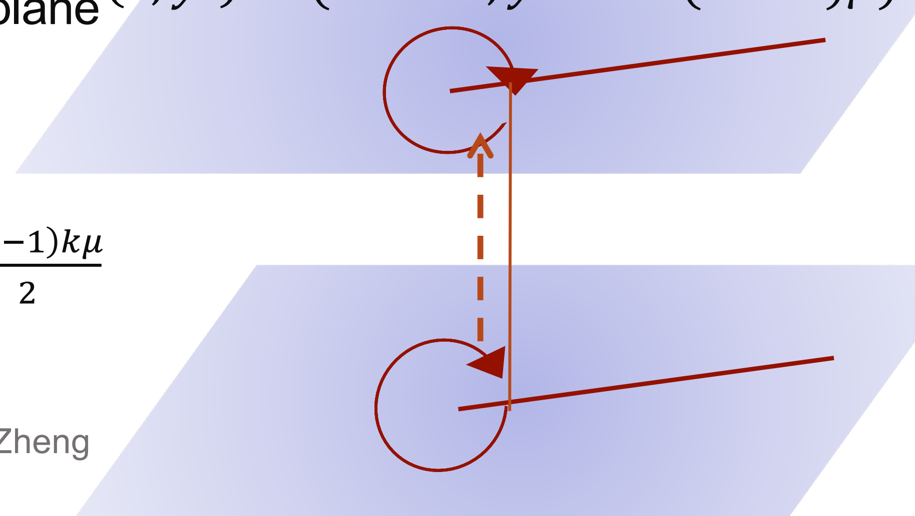
- Open and enlarge the circle on the reference plane

$$(z, y') \sim (z\, e^{n2\pi i}, y' - 2\pi(n-1)\mu)$$



- Twist operator $h_n = \frac{c}{24}\left(n - \frac{1}{n}\right) + \frac{q_n^2}{k}, \; q_n = -\frac{i(n-1)k\mu}{2}$
- EE on excited states and chaos Apolo-He-WS-Xu-Zheng
- Renyi mutual information Chen-Hao-WS

Apolo-Jiang-WS-Zhong

Modular Hamiltonian

- Modular flow generator is a vacuum symmetry generator which satisfies

$$\zeta = \sum_{i=-1}^{1} a_i \ell_i + \bar{a}_0 \bar{\ell}_0$$

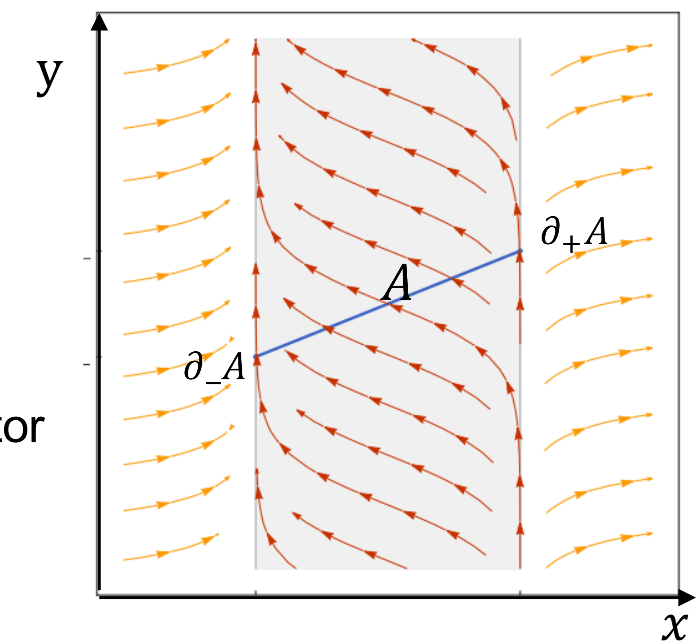
i) leaves the boundary of the causal domain ∂D invariant

ii) $e^{i\zeta}$ maps any point in D to itself, which is equivalent to the periodicity condition on the Rindler time $\tau \sim \tau + 2\pi i$

- Modular Hamiltonian is then given by the Noether charges

- For WCFT, one can explicitly check that the first law of entanglement entropy is satisfied on an excited state created by a local operator

$$\delta S_{\mathcal{A}}^{\psi} = \delta \langle \mathcal{H}_{mod} \rangle$$



Holographic entanglement entropy in (W)AdS/WCFT

WS-Wen-Xu

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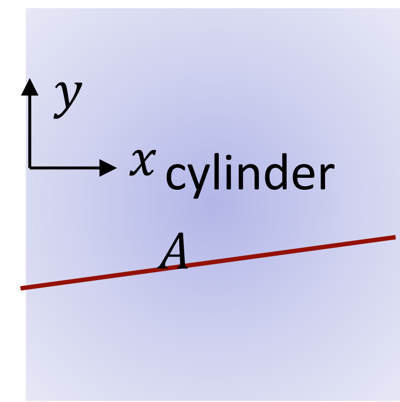


$D(A)$

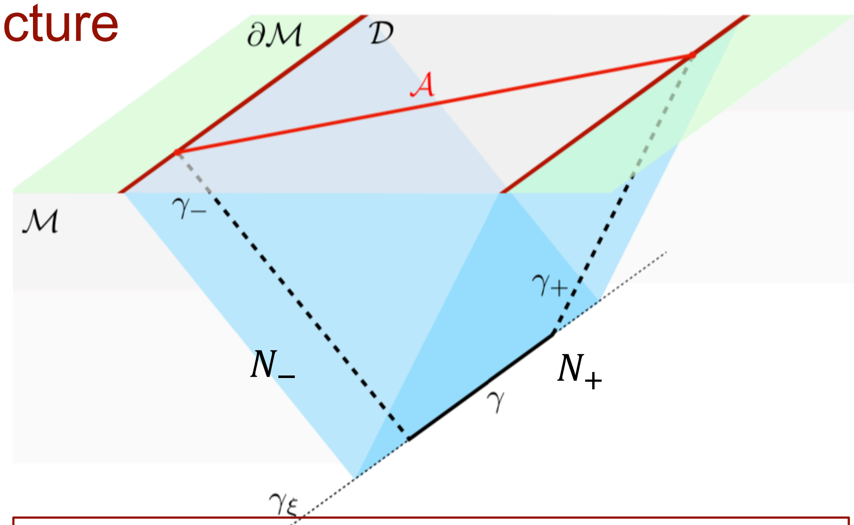
- Entanglement entropy $\quad S(A) = S_{th}(\tilde{B}) = S_{DHH}(\tilde{B})$

AdS/WCFT dictionary

Bulk calculation

- $\text{BTZ} \xrightarrow{coordinate\ transformation} \widetilde{BTZ}$
- $S_{HEE}(A) = S_{BH}(\widetilde{BTZ}) = S(A)$

Modular flow in the bulk and geometric picture



- The modular flow generator is mapped to a bulk Killing vector

$$\xi = \sum_i a_i H_i, \equiv 2\pi\partial_\tau, \quad \tau \sim \tau + 2\pi i$$

$$\xi|_{\partial\mathcal{M}} = \zeta$$

- bifurcating horizon $N_\pm$, with surface gravity $\kappa = 2\pi$
- the bifucating surface γ: $\xi|_\gamma = 0$
- ξ moves the ends points toward the bulk along null geodesics $\gamma_\pm$

A: a single interval at boundary

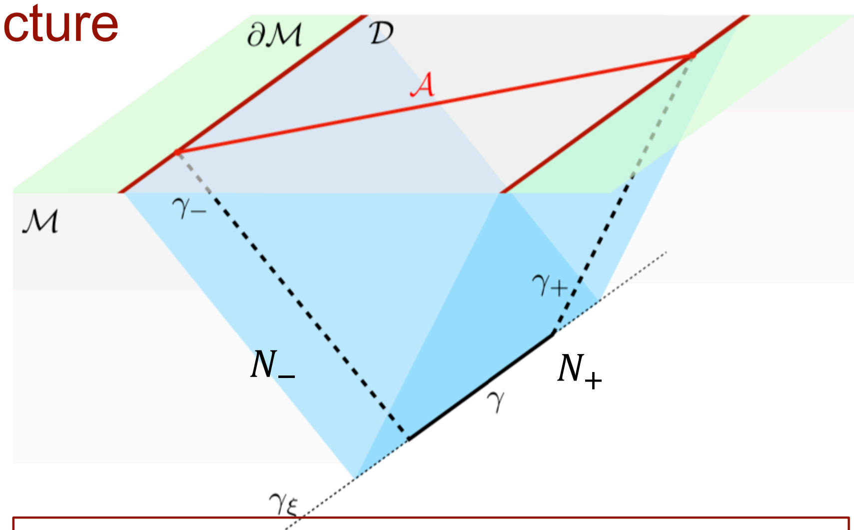
$$S_{HEE}(A) = \frac{\mathrm{L}(\gamma)}{4G} = \frac{\mathrm{L}(\gamma_A)}{4G}$$

Swing surface: $\gamma_A = \gamma \cup \gamma_+ \cup \gamma_-$

Ropes $\gamma_\pm$: null geodesics

Bench γ: spacelike geodesic

Modular flow in the bulk and geometric picture



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- bifurcating horizon $N_\pm$, with surface gravity $\kappa = 2\pi$
- the bifucating surface γ: $\xi|_\gamma = 0$
- ξ moves the ends points toward the bulk along null geodesics $\gamma_\pm$
- Gravitational charge evaluated on the interval $\mathcal{A}$ reproduces the modular Hamiltonian

$$\delta\mathcal{Q}^{\mathcal{A}}_\xi[\phi] \equiv \int_{\mathcal{A}} \chi_\xi[\delta\phi,\phi] = \int_{\gamma_{\mathcal{A}}} \chi_\xi[\delta\phi,\phi] \equiv \delta\mathcal{Q}^{\gamma_{\mathcal{A}}}_\xi[\phi]$$

$\delta\langle\mathcal{H}_\zeta\rangle$ ↔ $\delta\mathcal{Q}^{\mathcal{A}}_\xi[\phi]$; $\delta S_{\mathcal{A}}$ ↔ ? $\chi_\xi[\delta\phi,\phi]$ (over $\gamma_{\mathcal{A}}$)

A: a single interval at boundary

$$S_{HEE}(A) = \frac{L(\gamma)}{4G} = \frac{L(\gamma_A)}{4G}$$

Swing surface: $\gamma_A = \gamma \cup \gamma_+ \cup \gamma_-$

Ropes $\gamma_\pm$: null geodesics

Bench γ: spacelike geodesic

- ✓The setup: (W)AdS/WCFT
- ✓The generalized Rindler method
- ➢The swing surface proposal

Assumptions: holographic duality exists with the following properties:

Example: AdS3/CFT2 in Einstein gravity

1. The bulk theory of gravity admits a semiclassical description (in terms of Einstein gravity).

conformal symmetry

2. The field theory is invariant under a symmetry group G, and the vacuum state is invariant under a subgroup of G whose generators are denoted by h_i.

Brown-Henneaux boundary conditions *SL(2,R)x SL(2,R)*

3. Consistent boundary conditions exist such that the asymptotic symmetry group in the bulk agrees with G at the boundary.

Global AdS or Poincare AdS *SL(2,R)x SL(2,R)*

4. The bulk theory admits a special solution with Killing vectors H_i that reduce to the vacuum generators h_i at the boundary. This bulk geometry is identified with the vacuum state in the dual field theory.

5. The partition function in the bulk theory of gravity agrees with the partition function of the field theory at the boundary.

Rindler boost

6. A local modular Hamiltonian can be written down for ball-shaped regions (single intervals if d = 2) on the vacuum

(W)AdS/WCFT

1. The bulk theory of gravity admits a semiclassical description (in terms of Einstein gravity).

2. *WCFT* is field theory invariant under *warped symmetries*, and the vacuum state is invariant under the *SL(2,R) x U(1) subgroup* whose generators are denoted by $l_i, \bar{l}_0$.

3. *Under the CSS boundary conditions* asymptotically flat spacetimes have *waroed conformal symmetries*

4. *Global (W)AdS* is identified with the vacuum state in the dual field theory, The Killing vectors $L_i, \to l_i, \quad \bar{L}_0 \to \bar{l}_0$.

5. The partition function in the bulk theory of gravity agrees with the partition function of the field theory at the boundary.

6. A local modular Hamiltonian can be written down for single intervals on the vacuum. *Already shown!*

Holographic entanglement entropy for a class of holographic models

Apolo-Jiang-WS-Zhong

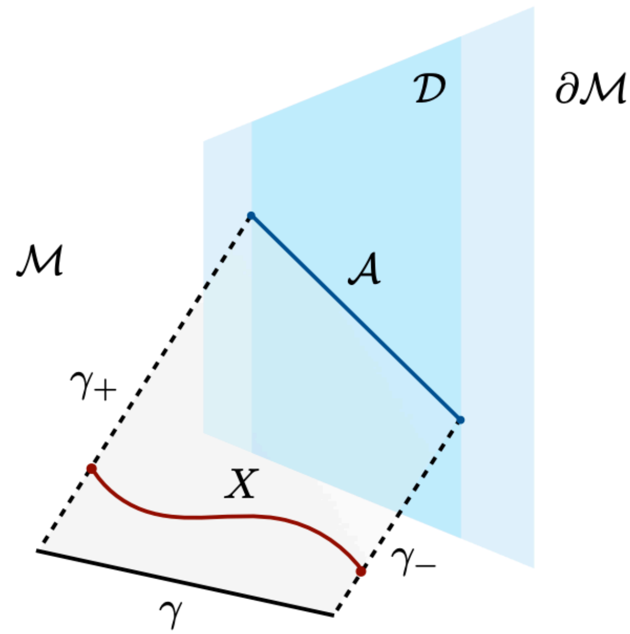
The swing surface proposal:

$$S_A = \min ext_{X_A \sim A} \frac{Area(X_A)}{4G},$$
$$X_A = X \cup_{p\in\partial A} \gamma_p$$

$swing\ surface$: $\gamma_A = \cup_{p\in\partial A} \gamma_p \cup \gamma$

ropes γ_p: null geodesics emanating from point $p \in \partial A$, tangent to the approximate modular flow near p at the boundary

bench γ: extremal surface between γ_p



The steps to find the swing surface:

(1) For each $p \in \partial A$, find the approximate modular flow generator ζ_p.

(2) For each $p \in \partial A$, find the null geodesic γ_p emanating from p whose tangent vector is an asymptotic Killing vector which reduces to ζ_p at the cutoff surface.

(3) Find the extremal surface spanning the region bounded by

$$\gamma_{b\partial} = \cup_{p \in \partial A} \gamma_p.$$

(4) If there are multiple extremal surfaces choose the minimal one.

Lewkowyzy-Maldacena
Dong-Lewkowyzy-Rangamani

Generalized gravitational entropy: general derivation of RT/HRT in AdS/CFT in Einstein gravity

- Near each point $p \in \partial A$, with approximate Rindler time

  $\zeta_p = 2\pi\partial_\tau$, $\zeta_p(p) = 0,\ \tau \sim \tau + 2\pi i$

- Replica trick: $\tau \sim \tau + 2\pi i\, n$, or equivalently $\tau_E \sim \tau_E + 2\pi\, n$

  p: fixed points of Z_n symmetry ⟷ fixed point of ζ_p

- toward the bulk: Consistent boundary conditions are required

  $\zeta_p = 2\pi\partial_\tau \longrightarrow \xi_p = 2\pi\partial_\tau$

  Usually $\mathrm{p} \in \partial A$ is automatically fixed point of the bulk vector ξ_p

  locally Rindler coordinates τ near the boundary end point p is directly continue to locally Rindler coordinates τ near γ_n (fixed points of bulk Z_n symmetry)

- When $\mathrm{n} = 1 + \epsilon$, linearized bulk EOM → $\gamma|_{n\to 1}$ is extremal surface
- Calculate the entropy

$$S_{\mathcal{A}} = n^2\partial_n I[\hat{g}_n]\big|_{n\to 1} \underset{on-shell}{=} \int_{\hat{\mathcal{M}}_n} \boldsymbol{d}\Theta[g, \partial_n \hat{g}_n]\big|_{n=1}. = \frac{Area\ (RT\ surface)}{4G}$$

Generalized gravitational entropy:
general derivation of swing surface in Einstein gravity

Apolo-Jiang-WS-Zhong

- Near each point $p \in \partial A$, with approximate Rindler time

  $\zeta_p = 2\pi\partial_\tau$, $\zeta_p(p) = 0$, $\tau \sim \tau + 2\pi i$

- Replica trick: $\tau \sim \tau + 2\pi i\, n$, or equivalently $\tau_E \sim \tau_E + 2\pi\, n$

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- toward the bulk: Consistent boundary conditions are required

  $\zeta_p = 2\pi\partial_\tau \longrightarrow \xi_p = 2\pi\partial_\tau$

Now $\mathrm{p} \in \partial A$ is NOT fixed point of the bulk vector ξ_p, instead ξ_p flows p along the null geodesic $\gamma_{(p)}$. Therefore the locally Rindler coordinates τ near the boundary end point p is first extended to the bulk along $\gamma_{(p)}$, and then further extended to locally Rindler coordinates τ near γ_n.

- When $\mathrm{n} = 1 + \epsilon$, linearized bulk EOM $\rightarrow \gamma|_{n\rightarrow 1}$ is extremal surface
- Calculate the entropy

$$S_{\mathcal{A}} = n^2\partial_n I[\hat{g}_n]\big|_{n\rightarrow 1} \underset{on-shell}{=} \int_{\hat{\mathcal{M}}_n} \boldsymbol{d}\Theta[g, \partial_n\hat{g}_n]\big|_{n=1} = \frac{\text{Area}(\gamma)}{4G} = \frac{\text{Area}(\gamma_{\mathcal{A}})}{4G}$$

- Entanglement 1st law

$$\delta\langle\mathcal{H}_\zeta\rangle \longleftrightarrow \delta\mathcal{Q}_\xi^{\mathcal{A}}[\phi] \equiv \int_{\mathcal{A}} \chi_\xi[\delta\phi, \phi] = \int_{\gamma_{\mathcal{A}}} \chi_\xi[\delta\phi, \phi] \equiv \delta\mathcal{Q}_\xi^{\gamma_{\mathcal{A}}}[\phi] \longleftrightarrow \delta S_{\mathcal{A}}$$

Summary

Question: what is the holographic entanglement entropy in holographic dualities for non-AdS spacetimes?

In this talk:

- Holographic EE in (W)AdS/WCFT
- The approach: assume holography exists,
  and use the generalized Rindler method/generalized gravitational entropy
- A new geometric picture: swing surface
- The swing surface proposal works for both (W)AdS/WCFT and flat holography in 3d

Thank You!