Non-Lorentzian Generalised Geometry: From DFT and ExFT to SNC and TNC

Chris Blair

Non-Lorentzian Zoom
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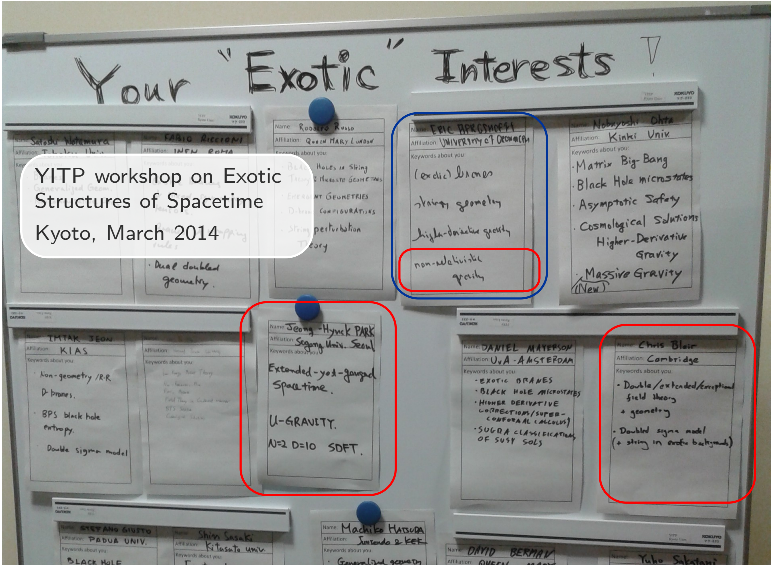
1908.00074
2002.12413

VRIJE UNIVERSITEIT BRUSSEL

Fonds Wetenschappelijk Onderzoek Vlaanderen
Opening new horizons

A pre-pandemic artefact



YITP workshop on Exotic Structures of Spacetime

Kyoto, March 2014

My motivations

Double field theory (DFT)/Exceptional Field theory (ExFT)

Unify metric + form fields

$\rightarrow$ generalised geometry

Use $O(D, D)$ or E_D (local) symmetries (not $GL(D)$)

$\rightarrow$ higher-dim origin of T-/U-duality symmetries

Applications in SUGRA

$\rightarrow$ flux compactifications, consistent truncations, higher-derivative corrections

Beyond SUGRA?

$\rightarrow$ winding coordinates, exotic geometries: T-/U-folds

Non-Lorentzian geometry in DFT/ExFT

Surprisingly contained in this formulation

[Lee, Park; Ko, Melby-Thompson, Meyer, Park; Morand, Park;...]

Naively singular limits well-defined via "extended" geometry

A DFT cheatsheet

Given: D-dimensional metric $g_{\mu\nu}$, two-form $B_{\mu\nu}$, dilaton ϕ

Generalised vectors $\Lambda^M = (v^\mu, \lambda_\mu)$

$GL(D)$ diffeos

Gauge $B \to B + d\lambda$

$O(D,D)$ metric

$$\eta_{MN} = \begin{pmatrix} 0 & \delta_\mu^\nu \\ \delta_\nu^\mu & 0 \end{pmatrix}$$

Generalised metric and dilaton

$$\mathcal{H}_{MN} = \begin{pmatrix} g - Bg^{-1}B & Bg^{-1} \\ -g^{-1}B & g^{-1} \end{pmatrix} \in \frac{O(D,D)}{O(1,D-1) \times O(1,D-1)}$$

$$e^{-2d} = e^{-2\phi}\sqrt{|g|}$$

$\Rightarrow \mathcal{H}\eta^{-1}\mathcal{H} = \eta$

Generalised diffeomorphisms

$$\mathcal{L}_\Lambda V^M = \Lambda^N \partial_N V^M - V^N \partial_N \Lambda^M + \eta^{MN}\eta_{PQ}\partial_N \Lambda^P V^Q$$

$$\eta^{MN}\partial_M \otimes \partial_N = 0 \Leftrightarrow \partial_M = (\partial_\mu, 0)$$

[Siegel]

[Hohm, Hull, Zwiebach;...]

Generalised metrics for non-Lorentzian geometry

Example: Gomis-Ooguri

$$ds^2 = c^2 \eta_{ab} dX^a dX^b + \delta_{ij} dX^i dX^j \quad B_{ab} = c^2 \epsilon_{ab} \quad a = 0,1 \quad i = 2,\dots,8$$

$$\mathcal{H}_{MN} = \begin{pmatrix} g - Bg^{-1}B & Bg^{-1} \\ -g^{-1}B & g^{-1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & {\epsilon_a}^b & 0 \\ 0 & \delta_{ij} & 0 & 0 \\ {\epsilon_b}^a & 0 & c^{-2}\eta^{ab} & 0 \\ 0 & 0 & 0 & \delta^{ij} \end{pmatrix}$$

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$$\mathcal{H}_{MN} = \begin{pmatrix} g - Bg^{-1}B & Bg^{-1} \\ -g^{-1}B & g^{-1} \end{pmatrix} \overset{c^2 \to \infty}{=} \begin{pmatrix} 0 & 0 & \epsilon_a{}^b & 0 \\ 0 & \delta_{ij} & 0 & 0 \\ \epsilon_b{}^a & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta^{ij} \end{pmatrix}$$

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Non-Riemannian parametrisation [Morand, Park]

$$\mathcal{H}_{MN}\Big|_{\text{non-rel}} = \begin{pmatrix} \delta_\mu^\rho & B_{\mu\rho} \\ 0 & \delta_\rho^\mu \end{pmatrix} \begin{pmatrix} K_{\rho\sigma} & x_\rho y^\sigma - \bar{x}_\rho \bar{y}^\sigma \\ y^\rho x_\sigma - \bar{y}^\rho \bar{x}_\sigma & H^{\rho\sigma} \end{pmatrix} \begin{pmatrix} \delta_\nu^\sigma & 0 \\ -B_{\sigma\nu} & \delta_\sigma^\nu \end{pmatrix}$$

Degenerate metrics plus zero vectors:

$$\begin{aligned} H^{\mu\nu} x_\nu = H^{\mu\nu} \bar{x}_\nu = 0\,, \quad x_\mu y^\mu = \bar{x}_\mu \bar{y}^\mu = 1 \\ K_{\mu\nu} y^\nu = K_{\mu\nu} \bar{y}^\nu = 0\,, \quad x_\mu \bar{y}^\mu = \bar{x}_\mu y^\mu = 0 \end{aligned} \qquad H^{\mu\rho} K_{\rho\nu} + x_\nu y^\mu + \bar{x}_\nu \bar{y}^\mu = \delta_\nu^\mu$$

$$e^{-2d} = e^{-2\phi} \sqrt{x_{\mu_1} \bar{x}_{\mu_2} x_{\nu_1} \bar{x}_{\nu_2} \epsilon^{\mu_1 \cdots \mu_D} \epsilon^{\nu_1 \cdots \nu_D} K_{\mu_3 \nu_3} \dots K_{\mu_D \nu_D}}$$

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e.g. Gomis-Ooguri:

$$K_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \delta_{ij} \end{pmatrix} \quad y^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad \bar{y}^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

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More generally: n x_μ and $\bar{n}$ $\bar{x}_\mu$

e.g. $n = D-1, \bar{n} = 0$: Carroll

Let's try this in worldsheet actions

Worldsheet (Hamiltonian)

$$S = \int d^2\sigma \, \dot{X}^\mu P_\mu - \frac{1}{2} e \begin{pmatrix} X'^\mu & P_\mu \end{pmatrix} \mathcal{H}_{MN} \begin{pmatrix} X'^\nu \\ P_\nu \end{pmatrix} - \frac{1}{2} u \begin{pmatrix} X'^\mu & P_\mu \end{pmatrix} \eta_{MN} \begin{pmatrix} X'^\nu \\ P_\nu \end{pmatrix}$$

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Worldsheet (doubled target space)

$\tilde{X}'_\mu = P_\mu$

$X^M = (X^\mu, \tilde{X}_\mu)$

$$S = \int d^2\sigma\, \frac{1}{2}\dot{X}^M \eta_{MN} X'^N - \frac{1}{2} e X'^M \mathcal{H}_{MN} X'^N - \frac{1}{2} u X'^M \eta_{MN} X'^N$$

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The essential feature of non-relativistic parametrisations

$$\frac{1}{2} \begin{pmatrix} X' & P \end{pmatrix} \mathcal{H}_{MN} \begin{pmatrix} X' \\ P \end{pmatrix} = \frac{1}{2} H^{\mu\nu} P_\mu P_\nu + \dots$$

$\Rightarrow$ can't integrate out momenta in x_μ, $\bar{x}_\mu$ directions

$\Rightarrow$ $y^\mu P_\mu, \bar{y}^\mu P_\mu$ become "Lagrange multipliers" for longitudinal directions:

$$\mathcal{L} \sim (y^\rho P_\rho) x_\mu D_- X^\mu + (\bar{y}^\rho P_\rho) \bar{x}_\mu D_+ X^\mu$$

$$D_\pm = \partial_0 - u\partial_1 \pm e\partial_1$$

Matching worldsheets

Using non-relativistic parametrisation (covariant result)

$$S = \int d^2\sigma - \frac{1}{2}\sqrt{-\gamma}\gamma^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu K_{\mu\nu} - \frac{1}{2}\epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}$$
$$+ \beta_\alpha x_\mu(\sqrt{-\gamma}\gamma^{\alpha\beta} - \epsilon^{\alpha\beta})\partial_\beta X^\mu$$
$$+ \bar\beta_\alpha \bar x_\mu(-\sqrt{-\gamma}\gamma^{\alpha\beta} - \epsilon^{\alpha\beta})\partial_\beta X^\mu$$

DFT to SNC dictionary:

$$K_{\mu\nu} = H_{\mu\nu} - 2x_{(\mu}\bar m_{\nu)} - 2\bar x_{(\mu} m_{\nu)} \qquad \beta_\alpha = \lambda_\alpha + \bar m_\mu(\delta_\alpha^\gamma + \tfrac{1}{\sqrt{-\gamma}}\epsilon_\alpha{}^\gamma)\partial_\gamma X^\mu$$
$$B_{\mu\nu} = B^{\text{SNC}}_{\mu\nu} - 2x_{[\mu}\bar m_{\nu]} + 2\bar x_{[\mu} m_{\nu]} \qquad \bar\beta_\alpha = \bar\lambda_\alpha - m_\mu(\delta_\alpha^\gamma - \tfrac{1}{\sqrt{-\gamma}}\epsilon_\alpha{}^\gamma)\partial_\gamma X^\mu$$

DFT to TNC includes e.g. $x_\mu = \frac{1}{\sqrt{2}}(\tau_i, 1)$, $\bar x_\mu = \frac{1}{\sqrt{2}}(\tau_i, -1)$ [CB, Berman, Otsuki; CB 1908] and matches SNC to TNC c.f. [Harmark et al]

$$\beta_\alpha \sim y^\mu(\partial_\alpha \tilde X_\mu + \dots) \quad \bar\beta_\alpha \sim \bar y^\mu(\partial_\alpha \tilde X_\mu + \dots)$$

How to elaborate on this?

Spacetime eom?

Expect: generalised diffeo invariance / beta functional $\rightarrow$ background field equations for $\mathcal{H}_{MN}$, d, regardless of parametrisation

BUT $\exists$ variations $\delta\mathcal{H}_{MN}$ which do not preserve non-rel nature of param [Cho, Park]

Similar issues in deriving beta-functionals of non-rel string $\rightarrow$ geometric constraints? [Gomis et al; Gallegos et al; Bergshoeff et al; talk by Zinnato in 1st NL Zoom]

Goal: insert non-rel parametrisations into DFT descriptions of:

1) 10-dim type II and heterotic SUGRA
2) lower-dim gauged SUGRA (generalised Scherk-Schwarz reductions)
3) SUSY worldsheet actions

$\rightarrow$ different approaches, e.g. superspace [Hull; Hackett-Jones, Moutsopoulos],
Green-Schwarz [Park] $\rightarrow$ Gomis-Ooguri
Hamiltonian [CB, Malek, Routh]

4) isometries/conserved charges [CB, Oling, Park, in progress]

Worldsheet SUSY: Left and right see different spaces

1) Double bosons: $(X^\mu, P_\mu) \to (X^\mu, \tilde{X}_\mu)$, fermions are their own momenta $\to$ do not double

2) Worldsheet fermions: $\tilde{\psi}^A \leftrightarrow O(1,D-1)_L$, $\psi^{\bar{A}} \leftrightarrow O(1,D-1)_R$

$$\mathcal{H}_{MN} \in \frac{O(D,D)}{O(1,D-1)\times O(1,D-1)}$$

3) Left/right *worldsheet* fermions see different *spacetime* vielbeins
Normally: identify via compensating Lorentz transform $\Lambda \sim e_L e_R^{-1}$
Non-rel: cannot identify left/right local tangent spaces

$\Rightarrow$ <u>Non-relativistic geometry</u> $\sim$ <u>relativistic non-geometry</u>

4) $\psi^{\bar{A}}, \tilde{\psi}^A$ couple to left/right projected generalised vielbeins

$$\mathcal{H}_{MN} = \begin{pmatrix} V_M{}^A & \bar{V}_M{}^{\bar{A}} \end{pmatrix} \begin{pmatrix} \boldsymbol{h}_{AB} & 0 \\ 0 & \bar{\boldsymbol{h}}_{\bar{A}\bar{B}} \end{pmatrix} \begin{pmatrix} V_N{}^B \\ \bar{V}_N{}^{\bar{B}} \end{pmatrix} \qquad \begin{aligned} V_M{}^A &= \eta_{MP}\mathcal{H}^{PN} V_N{}^A \\ \bar{V}_M{}^{\bar{A}} &= -\eta_{MP}\mathcal{H}^{PN}\bar{V}_N{}^{\bar{A}} \end{aligned}$$

$$-\bar{V}_M{}^{\bar{A}} V_N{}^{\bar{B}} \bar{\boldsymbol{h}}_{\bar{A}\bar{B}} = \bar{P}_{MN} \equiv \tfrac{1}{2}(\eta_{MN} - \mathcal{H}_{MN}) \quad V_M{}^A V_N{}^B \boldsymbol{h}_{AB} = P_{MN} \equiv \tfrac{1}{2}(\eta_{MN} + \mathcal{H}_{MN})$$

$h_{AB}, \bar{h}_{\bar{A}\bar{B}}$ are separate copies of Minkowski metric, $A = 0, \dots, D-1$, $\bar{A} = 0, \dots, D-1$

We can write down the locally SUSY worldsheet action

The doubled RNS action [CB, Malek, Routh]

$$S = \int d^2\sigma \, \frac{1}{2}\dot{X}^M \eta_{MN} X'^N - \frac{i}{2}\left(\psi^{\bar{A}}\dot{\psi}^{\bar{B}}\bar{\boldsymbol{h}}_{\bar{A}\bar{B}} + \tilde{\psi}^A\dot{\tilde{\psi}}^B \boldsymbol{h}_{AB}\right) - (e-u)\mathcal{H} - (e+u)\tilde{\mathcal{H}} - \underbrace{i\xi\mathcal{Q} - i\tilde{\xi}\tilde{\mathcal{Q}}}_{\text{Worldsheet gravitino}}$$

Constraints: $\mathcal{H}, \tilde{\mathcal{H}}$ generate worldsheet diffeos, $\mathcal{Q}, \tilde{\mathcal{Q}}$ worldsheet SUSY

$$-\sqrt{2}\mathcal{Q} = X'^M \eta_{MN} \bar{V}^N{}_{\bar{A}}\psi^{\bar{A}} + \frac{i}{2}\bar{V}^M{}_{\bar{C}}\omega_{M\bar{A}\bar{B}}\psi^{\bar{A}}\psi^{\bar{B}}\psi^{\bar{C}} + \frac{i}{2}\bar{V}^M{}_{\bar{C}}\tilde{\omega}_{MAB}\tilde{\psi}^A\tilde{\psi}^B\psi^{\bar{C}}$$

$$\sqrt{2}\tilde{\mathcal{Q}} = X'^M \eta_{MN} V^N{}_A\tilde{\psi}^A + \frac{i}{2}V^M{}_C\omega_{M\bar{A}\bar{B}}\psi^{\bar{A}}\psi^{\bar{B}}\tilde{\psi}^C + \frac{i}{2}V^M{}_C\tilde{\omega}_{MAB}\tilde{\psi}^A\tilde{\psi}^B\tilde{\psi}^C$$

$$\omega_{M\bar{A}\bar{B}} = \bar{V}_{P\bar{A}}\partial_M \bar{V}^P{}_{\bar{B}} + \Gamma_{MN}{}^P \bar{V}_{P\bar{A}}\bar{V}^N{}_{\bar{B}} \quad \tilde{\omega}_{MAB} = V_{PA}\partial_M V^P{}_B + \Gamma_{MN}{}^P V_{PA} V^N{}_B$$

Connection: $\nabla\mathcal{H} = 0$, $\nabla\eta = 0$, $\nabla e^{-2d} = 0$, generalised torsion free
not unique [Siegel] [Jeon, Lee, Park] [Hohm, Zwiebach] but quantities in action are fixed

A worldsheet supersymmetric Newton-Cartan string [CB 1908]

1) Insert NC parametrisation of V_{MA}, $\bar{V}_{M\bar{A}}$ into action
Automatically: Hamiltonian formulation, SUSY transformations
2) Integrate out P_μ as before
Lagrangian, local SUSY, curved background, full order in fermions
(ψ^3: torsions, ψ^4: curvatures)

e.g. focus on longitudinal dofs $(y^\mu P_\mu, x_\mu X^\mu, \psi^0, \psi^1)$,
$(\bar{y}^\mu P_\mu, \bar{x}_\mu X^\mu, \tilde{\psi}^0, \tilde{\psi}^1)$:

$$\mathcal{L} \supset y^\nu P_\nu \left(x_\mu D_- X^\mu + \tfrac{ie}{\sqrt{2}}(f_0 - f_1) \right) + \bar{y}^\nu P_\nu \left(\bar{x}_\mu D_+ X^\mu + \tfrac{ie}{\sqrt{2}}(\bar{f}_0 - \bar{f}_1) \right)$$
$$- \frac{i}{2}(-\tilde{\psi}^0 D_- \tilde{\psi}^0 + \tilde{\psi}^1 D_- \tilde{\psi}^1) - \frac{i}{2}(-\psi^0 D_+ \psi^0 + \psi^1 D_+ \psi^1)$$

$$f_C \equiv V^M{}_C \omega_{M\bar{A}\bar{B}} \psi^{\bar{A}} \psi^{\bar{B}} + \tfrac{\tilde{\xi}}{\sqrt{2}e} \tilde{\psi}_C \quad \bar{f}_{\bar{C}} \equiv \bar{V}^M{}_{\bar{C}} \tilde{\omega}_{MAB} \tilde{\psi}^A \tilde{\psi}^B - \tfrac{\xi}{\sqrt{2}e} \psi_{\bar{C}}$$

$$f_0 - f_1 = \sqrt{2} \bar{h}^\mu{}_{\bar{m}} \psi^{\bar{m}} (\bar{h}^\nu{}_{\bar{m}} \psi^{\bar{n}} + \bar{y}^\nu (\psi^0 + \psi^1)) \partial_{[\mu} x_{\nu]} - \tfrac{\tilde{\xi}}{\sqrt{2}e} (\tilde{\psi}^0 + \tilde{\psi}^1)$$

$$\bar{f}_0 - \bar{f}_1 = \sqrt{2} h^\mu{}_m \tilde{\psi}^m (h^\nu{}_n \tilde{\psi}^n + y^\nu (\tilde{\psi}^0 + \tilde{\psi}^1)) \partial_{[\mu} \bar{x}_{\nu]} + \tfrac{\xi}{\sqrt{2}e} (\psi^0 + \psi^1)$$

$m, \bar{m} = 2, \ldots, D$ (transverse flat)

As below, so above: ExFT and non-Lorentzian M-theory

Double field theory
$\mathcal{H}_{MN} \in \frac{O(D,D)}{O(D)\times O(D)}$, $O(D,D)$-compatible generalised diffeomorphisms
(10-dim max/half-max) SUGRA, worldsheet actions

As below, so above: ExFT and non-Lorentzian M-theory

Exceptional field theory — See recent review [Berman, CB]

$\mathcal{M}_{MN} \in \frac{E_{D(D)}}{H_D}$, $E_{D(D)}$-compatible generalised diffeomorphisms

(11- and 10-dim max) SUGRA, (to an extent) worldvolume actions

$E_{3(3)} = \mathrm{SL}(3) \times \mathrm{SL}(2),\ E_{4(4)} = \mathrm{SL}(5),\ E_{5(5)} = \mathrm{SO}(5,5),\ E_{6(6)}, E_{7(7)}, E_{8(8)}, \dots$

Double field theory

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Example: $\mathrm{SL}(5)$ $D = 4$, metric g_{ij}, three-form, $C^i = \frac{1}{6}\epsilon^{ijkl}C_{jkl}$

$$\mathcal{M}_{MN} = |g|^{1/10} \begin{pmatrix} |g|^{-1/2} g_{ij} & |g|^{-1/2} g_{ik} C^k \\ |g|^{-1/2} g_{jk} C^k & -|g|^{1/2} + |g|^{-1/2} g_{kl} C^k C^l \end{pmatrix} \in \frac{\mathrm{SL}(5)}{\mathrm{SO}(2,3)}$$

Non-Riemannian parametrisations [Berman, CB, Otsuki]

$$\mathcal{M}_{MN} = \begin{pmatrix} k_{ij} & X_i \\ X_j & \varphi \end{pmatrix} \qquad \varphi \det k - \frac{1}{6} X_{i_1} X_{j_1} \epsilon^{i_1 \cdots i_4} \epsilon^{j_1 \cdots j_4} k_{i_2 j_2} k_{i_3 j_3} k_{i_4 j_4} = 1$$

$\det k = 0 \Rightarrow$ no invertible spacetime metric
More carefully: $\mathcal{M}^{ij}\mathcal{M}^{55} - \mathcal{M}^{i5}\mathcal{M}^{j5}$ is rank $1 \Rightarrow 3$ non-rel directions
Describes Gomis-Ooguri style limit of M2 $\rightarrow$ membranes? c.f. [Kluson], some comments in [CB 2002], also NC embeddings

thanks for listening!