Carrollian physics
… at the black hole horizon

based on arXiv: 1903.09654 with Charles Marteau

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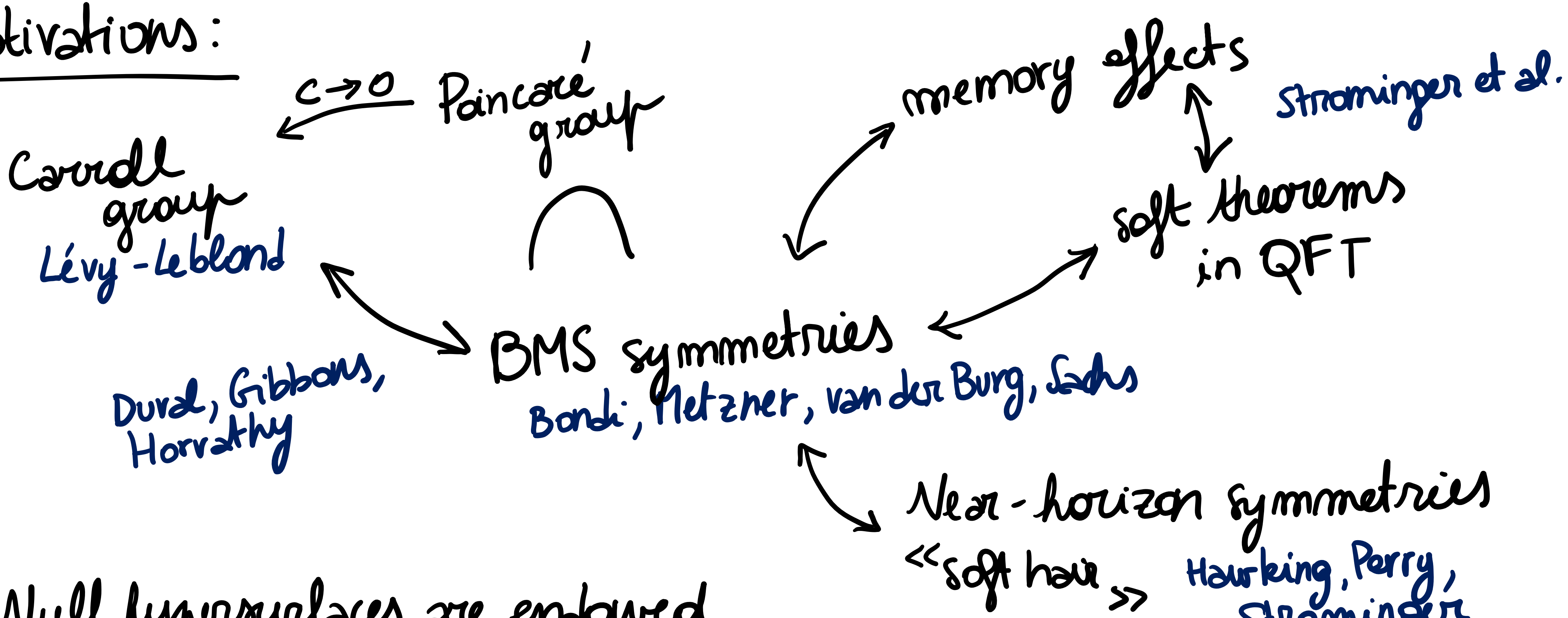
H2020- Marie Skłodowska-Curie "HoloBH – 746297"

September 3, 2020 – Non-Lorentzian Zoom Meeting

European Commission

Horizon 2020
European Union funding
for Research & Innovation

Motivations:



- Null hypersurfaces are endowed with a Carrollian structure

Hartong, Marteau, Ciambelli, Petropoulos, Bagchi, Petkou, Siampos, Bekaert, Morand, Henneaux
... many others

- So do black hole horizons!

Plan of the talk :

1. Geometry of a black hole horizon
2. Near-horizon limit as ultra-relativistic limit
3. Comments on conserved charges

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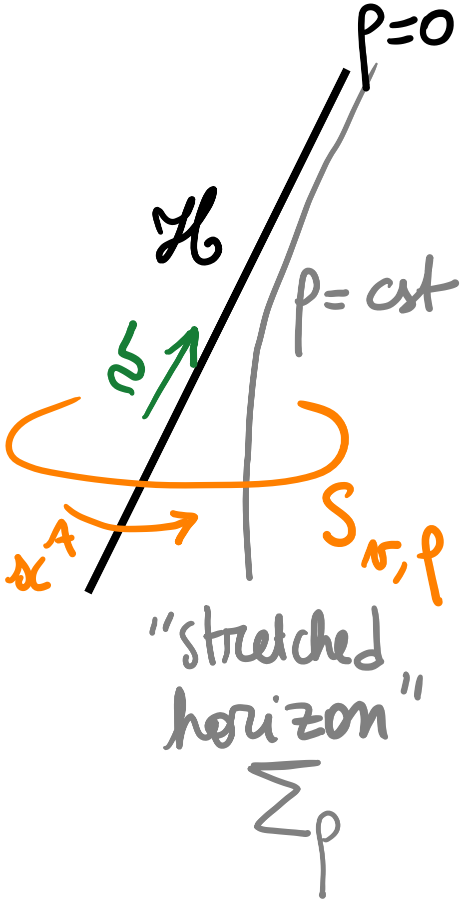
1. Geometry of a black hole horizon



D-dimensional spacetime

Coordinates $x^a = (v, \rho, x^A)$ $A = 3, \dots, D$

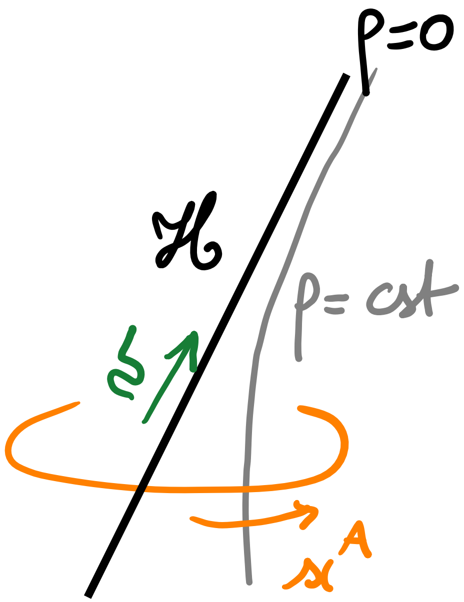
v: advanced null time

ρ: radial coordinate

x^A: parametrize the (D-2)-sphere $S_{v,\rho}$

The horizon $\mathcal{H}$ is located at $\rho = 0$ in these "null Gaussian" coordinates.

1. Geometry of a black hole horizon



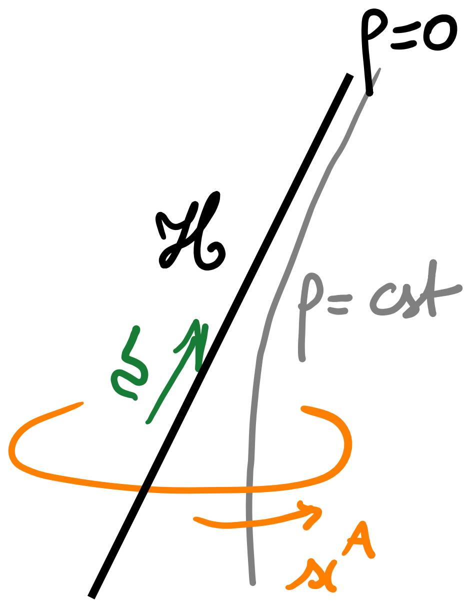
The near-horizon geometry is (expanding in small ρ):

$$ds^2 = -2\kappa\rho\, dv^2 + 2\, dv\, d\rho + 2\,\theta_A\, \rho\, dv\, dx^A + \Omega_{AB}\, dx^A dx^B + \underbrace{\dots}_{\text{subleading terms}}$$

$\kappa\,,\ \theta_A\,,\ \underbrace{\Omega_{AB}}_{\text{spatial metric (intrinsic geometry)}}$: functions of (v, x^A)

(raises/lowers spatial indices $A, B, \dots$)

1. Geometry of a black hole horizon

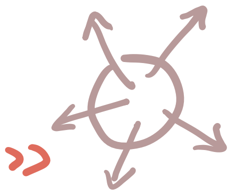


$$ds^2 = -2\kappa\rho\, dv^2 + 2\, dv\, d\rho + 2\,\theta_A\, \rho\, dv\, dx^A + \Omega_{AB}\, dx^A dx^B + \dots$$

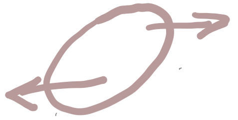
The horizon extrinsic geometry is given by

$$\begin{tikzcd} & \Theta = \Omega^{AB}\Sigma_{AB} \\ \Sigma_{AB} = \frac{1}{2}\partial_v \Omega_{AB} \arrow[ur, "\text{trace}"] \arrow[dr, "\text{traceless part}"'] & \\ & \sigma_{AB} = \frac{1}{2}\partial_v \Omega_{AB} - \frac{\Theta}{D-2}\Omega_{AB} \end{tikzcd}$$

“expansion”



“shear”



$$\omega_A = -\frac{1}{2}\theta_A$$ “twist”

κ : surface gravity $(\ell^b D_b \ell^a = \kappa \ell^a)$

$\uparrow$ normal to $\mathcal{H}$

Raychaudhuri & Damour equations

Projecting vacuum Einstein equations on the horizon gives 2 constraint equations :

- null Raychaudhuri eq. $\partial_v \Theta = \kappa \Theta - \frac{\Theta^2}{D-2} - \sigma_{AB}\sigma^{AB}$ (*)

  ↝ describes how the expansion Θ evolves along the null geodesic congruence

  ↝ key ingredient in proofs of singularity theorems

- (*) + null energy conditions $\Rightarrow$ $\Theta \geqslant 0$ (area always increases in time)

Raychaudhuri & Damour equations

Projecting vacuum Einstein equations on the horizon gives 2 constraint equations :

- Damour eq.

Levi-Civita connection associated to Ω_{AB}

$$(\partial_v + \text{\textcircled{H}})\,\theta_A + 2\,\nabla_A\left(K + \frac{D-3}{D-2}\,\text{\textcircled{H}}\right) - 2\,\nabla_B\,\sigma_A{}^B = 0$$

⇝ resembles a Navier-Stokes equation for a viscous fluid

$\nabla_A K$ ~ gradient of pressure

θ_A ~ impulsion density

σ_{AB} ~ shear , $\text{\textcircled{H}}$ ~ dilation

Near-horizon infinite-dimensional symmetries

[L.D., Giribet, González, Pino]

Vector fields $\xi = \xi^a \partial_a$ preserving the near-horizon form

$$ds^2 = -2\kappa \rho dv^2 + 2 dv d\rho + 2\theta_A \rho dv dx^A + \Omega_{AB} dx^A dx^B + \dots$$

are given by $\xi = f(v, x^A)\partial_v + Y^A(x^B)\partial_A + \dots$

(e.g. $\mathcal{L}_\xi g_{\rho\rho} \overset{!}{=} 0$

$\mathcal{L}_\xi g_{vv} \overset{!}{=} -2\rho\delta\kappa + O(\rho^2)$

etc.)

SUPERTRANSLATIONS

$v \to v + f(v, x^A)$

SUPERROTATIONS

(local v.s. global)

c.f. «BMS symmetries» of asymptotically flat spacetimes

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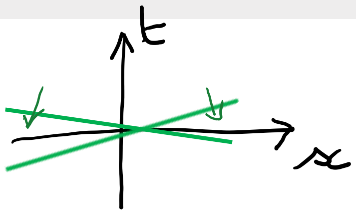
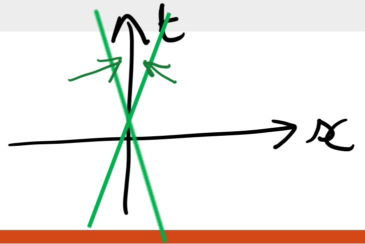
2. Near-horizon as an ultra-relativistic limit

(stands for Lewis Carroll)

	GALILEAN	CARROLLIAN
Limit from Poincaré	$c \to \infty$	$c \to 0$
Transformations	$\begin{cases} t' = t + b \\ \vec{x}' = R\vec{x} + \vec{v}t + \vec{a} \end{cases}$	$\begin{cases} t' = t + \vec{v}.R\vec{x} + b \\ \vec{x}' = R\vec{x} + \vec{a} \end{cases}$
Hypothesis	$\frac{\Delta x}{\Delta t} \ll 1$ (big timelike separations)	$\frac{\Delta x}{\Delta t} \gg 1$ (big spacelike separations)
Causality	YES (time is absolute)	NO (space is absolute)





2.1 Carrollian geometry

- It is the natural non-Riemannian geometry that ultra-relativistic theories couple to.

- [Ciambelli, Marteau, Petkou, Petropoulos, Siampos]

The $c \to 0$ limit of relativistic general-covariant theories is covariant under

Carrollian diffeomorphisms $$v' = v'(v, x^B)$$ $$x'^A = x'^A(x^B)$$

Infinitesimal version : $\xi = f(v, x)\partial_v + Y^A(x^B)\partial_A$

2.1 Carrollian geometry

The ultra-relativistic limit breaks the spacetime metric a into 3 independant data :

$$a = \begin{pmatrix} -c^2\alpha^2 & c^2\alpha b_A \\ c^2\alpha b_B & \Omega_{AB} - c^2 b_A b_B \end{pmatrix}_{\{dr, dx^A\}}$$

« Randers - Papapetrou parametrization »

α : « time lapse »

b_A : « temporal connection »

Ω_{AB} : « spatial metric »

define the Carrollian geometry (transform covariantly under Carrollian diffeos)

2.2. Near-horizon ↔ Carrollian geometry

[L.D., Marteau]

$$ds^2 = -2\kappa\rho\, dv^2 + 2\, dv\, d\rho + 2\,\theta_A\, \rho\, dv\, dx^A + \Omega_{AB}\, dx^A dx^B + \dots$$

$$\downarrow$$

$$ds^2_{\rho=cst} = \begin{pmatrix} -2\kappa\rho & \rho\,\theta_A \\ \rho\,\theta_B & \Omega_{AB} + \mathcal{O}(\rho) \end{pmatrix} \overset{!}{=} \begin{pmatrix} -\alpha^2 c^2 & c^2 \alpha\, b_A \\ c^2 \alpha\, b_B & \Omega_{AB} + \mathcal{O}(c^2) \end{pmatrix}$$

This naturally leads to the identifications

$$\boxed{c^2 = \rho \qquad \alpha = \sqrt{2\kappa} \qquad b_A = \frac{\theta_A}{\sqrt{2\kappa}}}$$

Hence, the near-horizon limit $\rho \to 0$ ↔ ultra-relativistic with virtual speed of light $c \to 0$

2.3 Horizon dynamics as ultra-relativistic conservation laws

Energy-momentum tensor ↔ Carrollian momenta [Ciambelli, Marteau]

$$T^{ij} \quad (i = v, A) \qquad \begin{cases} T^{vv} = c^{-3}\alpha^{-2}\mathcal{E} + O(c^{-1}) \\ T^{vA} = c^{-1}\alpha^{-1}\left(\pi^{A} - 2\, b_{B}\mathcal{A}^{AB}\right) + O(c) \\ T^{AB} = -2c^{-1}\mathcal{A}^{AB} + O(c) \end{cases}$$

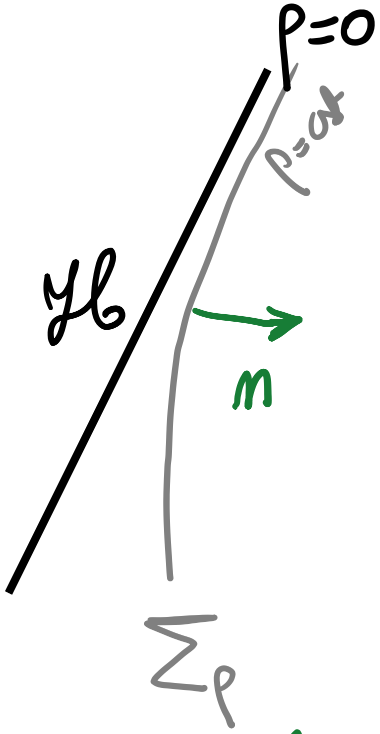
Interpretation of these quantities:

$\mathcal{E}$: energy density

π^{A} : heat current

$$\mathcal{A}^{AB} = -\frac{1}{2}\left(\overset{\text{pressure}}{\mathcal{P}}\,\Omega^{AB} - \overset{\text{dissipative tensor}}{\Xi^{AB}}\right)$$

2.3 Horizon dynamics as ultra-relativistic conservation laws



unit normal : $n = \frac{d\rho}{\sqrt{2\kappa\rho}}$

- Projector on Σ_ρ : $a_{ab} = g_{ab} - n_a n_b$
- Extrinsic curvature of Σ_ρ : $K^a{}_b = a^c{}_b D_c n^a$ ← connection w.r.t. g_{ab}

$K = K^a{}_a$ (trace)

« Membrane energy-momentum tensor »

$$T_{ab} = K a_{ab} - K_{ab}$$

2.3 Horizon dynamics as ultra-relativistic conservation laws

We computed the membrane stress-tensor and read-off the Carrollian momenta:

$$\mathcal{E} = \frac{\Theta}{\sqrt{2K}}$$

$$\mathcal{P} = \frac{-1}{\sqrt{2K}}\left(K + \frac{D-3}{D-2}\Theta - \frac{\partial_v K}{2K}\right)$$

$$\Xi_{AB} = \frac{-1}{\sqrt{2K}}\sigma_{AB}$$

$$\pi_A = -\frac{1}{2}\left(\frac{\partial_A K}{K} + \frac{\theta^B}{2K}\partial_v \Omega_{BA} + \frac{\theta_A}{2K^2}\partial_v K\right)$$

$$T^{vv} = c^{-3}\alpha^{-2}\mathcal{E} + O(c^{-1})$$

$$T^{vA} = c^{-1}\alpha^{-1}\left(\pi^A - 2b_B\mathcal{A}^{AB}\right) + O(c)$$

$$T^{AB} = -2c^{-1}\mathcal{A}^{AB} + O(c)$$

2.3 Horizon dynamics as ultra-relativistic conservation laws

Now, taking the ultra-relativistic limit of the energy-momentum conservation $\nabla_i T^{ij} = 0$, gives

$$\varphi_A = \alpha^{-1}(\partial_v b_A + \partial_A \alpha)$$

$$\beta = \alpha^{-1}\partial_v \ln\sqrt{\Omega}$$

$$(\alpha^{-1}\partial_v + \beta)\mathcal{E} - \mathcal{A}^{AB}\alpha^{-1}\partial_v \Omega_{AB} = 0$$

$$2(\hat{\nabla}_A + \varphi_A)\mathcal{A}^A{}_B - \mathcal{E}\varphi_B - (\alpha^{-1}\partial_v + \beta)\pi_B = 0$$

PLUG

$$\mathcal{E} = \frac{\Theta}{\sqrt{2K}}$$

$$\mathcal{P} = \frac{-1}{\sqrt{2K}}\left(K + \frac{D-3}{D-2}\Theta - \frac{\partial_v K}{2K}\right)$$

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$$\pi_A = -\frac{1}{2}\left(\frac{\partial_A K}{K} + \frac{\theta^B}{2K}\partial_v \Omega_{BA} + \frac{\theta_A}{2K^2}\partial_v K\right)$$

$$\alpha = \sqrt{2K}$$

$$b_A = \frac{\theta_A}{\sqrt{2K}}$$

[... lenghty computation...]

2.3 Horizon dynamics as ultra-relativistic conservation laws

$$(\alpha^{-1}\partial_v+\beta)\mathcal{E}-\mathcal{A}^{AB}\alpha^{-1}\partial_v\Omega_{AB}=0 \qquad (1)$$

$$2(\hat{\nabla}_A+\varphi_A)\mathcal{A}^A{}_B-\mathcal{E}\varphi_B-(\alpha^{-1}\partial_v+\beta)\pi_B=0 \qquad (2)$$

PLUG

$$\mathcal{E}=\frac{\Theta}{\sqrt{2\kappa}}$$

$$\mathcal{P}=\frac{-1}{\sqrt{2\kappa}}\left(\kappa+\frac{D-3}{D-2}\Theta-\frac{\partial_v\kappa}{2\kappa}\right)$$

$$\Xi_{AB}=\frac{-1}{\sqrt{2\kappa}}\sigma_{AB}$$

$$\pi_A=-\frac{1}{2}\left(\frac{\partial_A\kappa}{\kappa}+\frac{\theta^B}{2\kappa}\partial_v\Omega_{BA}+\frac{\theta_A}{2\kappa^2}\partial_v\kappa\right)$$

$$\alpha=\sqrt{2\kappa}$$

$$b_A=\frac{\theta_A}{\sqrt{2\kappa}}$$

Hence the dynamics of a black hole is mapped to ultra-relativistic conservation laws.

You finally get

[L.D, Marteau]

$$(1):\ \partial_v\Theta-\kappa\Theta+\frac{\Theta^2}{D-2}+\sigma_{AB}\sigma^{AB}=0 \qquad \text{(Raychaudhuri)}$$

$$(2)\ (\partial_v+\Theta)\theta_A+2\nabla_A\left(\kappa+\frac{D-3}{D-2}\Theta\right)-2\nabla_B\sigma^B{}_A=0 \qquad \text{(Damour)}$$

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- One can associate surface charges to asymptotic Killing vector fields via the covariant phase space formalism [Iyer-Wald-Zoupas, Barnich-Brandt].
- Supertranslation & superrotation charges at the horizon were computed in [L.D., Giribet, González, Pino].
- We mapped these charges with Carrollian charges that we constructed out of Carrollian Killing vector fields. [L.D., Marteau]

e.g. $$Q_\xi = \frac{-1}{16\pi G} \oint_{S^{D-2}} d^{D-2}x \sqrt{\Omega}\, Y^A \left(\theta_A + \frac{\partial_A \kappa}{\kappa}\right)$$: conserved!

↳ «superrotation charge» : generalizes the angular momentum for non-stationary black holes

with $\Omega_{AB}, \theta_A, \kappa$ fct of v & x^B

Conclusions

- The dynamics governing black hole horizons is encoded by ultra-relativistic (Carrollian) conservation laws.
- Our analysis clarifies several aspects of the «membrane paradigm» picture: * Damour-Navier-Stokes equation describes a Carrollian fluid (not Galilean, cf. mismatches).
  * Fluid quantities from the membrane stress-tensor had to be rescaled by hand ($\to \infty$ as $\rho \to 0$). [Price, Thorne, Macdonald]
- Fascinating features yet to be explored!

Thank you.