Carroll symmetry and the Friedmann equations

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Based on work (in progress) with:
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Plan for today:

Carroll fluids

Curved space and hydrostatic partition function

Friedmann equations

Carroll boost

Carroll boost versus other boosts

Carroll boost: $t' = t - b_i x^i\,, \quad x'^i \to x^i\,.$

Generator: $L_i \equiv \frac{1}{c} x^i \partial_t + ct\partial_i\,, \quad C_i \equiv cL_i \to x^i \partial_t\,.$

$$[P_i, C_j] = \delta_{ij} H$$
$$[H, C_j] = 0$$

Velocity: $v'^i = \frac{dx'^i}{dt'} = \frac{v^i}{1 - b_i v^i}\,.$

$$n^i \equiv \frac{v^i}{|\vec{v}|}\,, \qquad n'^i = n^i\,.$$

Seminal work on Carroll: [Levy-Leblond '65][Bacry, Levy-Leblond'68]

Looking at Carroll through fluid

Fluid can be studied in absence of specific microscopics

Just consider symmetries and thermodynamics

Perfect uncharged relativistic fluid: $$T^\mu{}_\nu = \frac{\tilde{\mathcal{E}}+P}{c^2}U^\mu U_\nu + P\delta^\mu{}_\nu \qquad U^\mu U_\mu = -c^2$$

Relativistic boost Ward identity: $$\partial_\mu\left(T^\mu{}_\nu L_i^\nu\right) = 0 \quad \rightarrow \quad \frac{1}{c}T^i{}_0 + cT^0{}_i = 0$$

Carroll boost Ward identity: $$\partial_\mu\left(T^\mu{}_\nu C_i^\nu\right) = 0 \quad \rightarrow \quad T^i{}_0 = 0$$

Covariant transformation Carroll EMT: $$T'^0{}_0 = T^0{}_0 - b_i T^i{}_0\,, \qquad T'^i{}_j = T^i{}_j + b_j T^i{}_0\,,$$
$$T'^i{}_0 = T^i{}_0\,, \qquad T'^0{}_i = T^0{}_i + b_i T^0{}_0 - b_i b_j T^j{}_0 - b_j T^j{}_i\,,$$

Generalized mass density: $T^0{}_i =: \rho v^i$ Total energy: $\mathcal{E} = \tilde{\mathcal{E}} + \rho v^2$

Related work on Carrol fluids: [de Boer, Hartong, Obers, WS, Vandoren'17][Ciambelli, Marteau, Petkou, Petropoulos, Siampos'18x2]
[Campoleoni, Ciambelli, Marteau, Petropoulos, Siampos'18][Poovuttikul, WS'19][Ciambelli, Marteau, Petropoulos, Ruzziconi'20]

Resting Carroll

Relativistic EMT explicitly:

$$T^0{}_0 = -(\tilde{\mathcal{E}} + P)\gamma^2 + P \qquad T^i{}_0 = -(\tilde{\mathcal{E}} + P)v^i\gamma^2$$

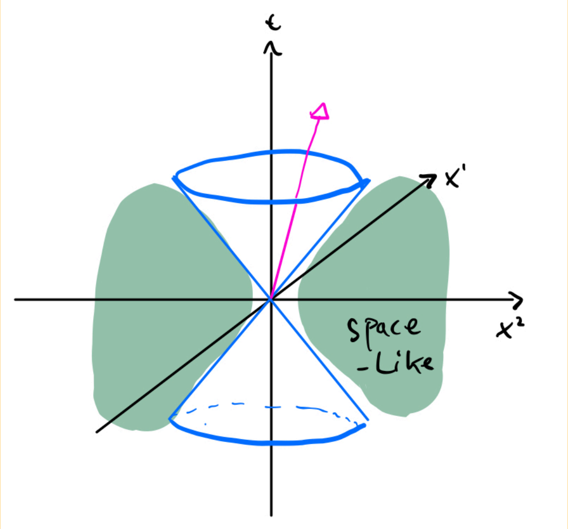
$$T^0{}_j = \frac{\tilde{\mathcal{E}} + P}{c^2} v^j \gamma^2 \qquad T^i{}_j = \frac{\tilde{\mathcal{E}} + P}{c^2} v^i v^j \gamma^2 + P\delta^i{}_j$$

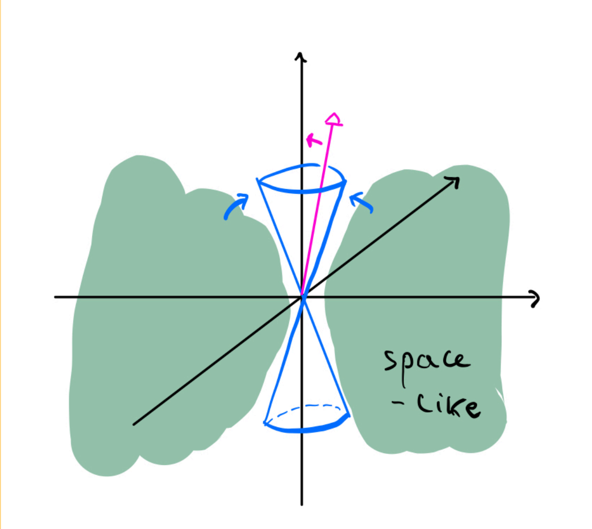
$$\gamma^2 = \frac{1}{1 - v^2/c^2}$$

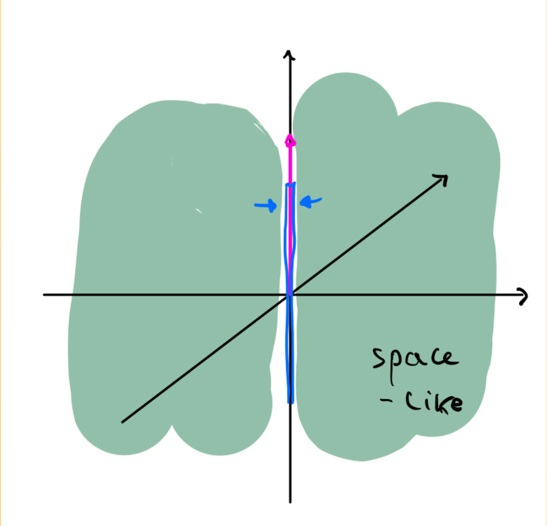
Generalized mass density:

$$T^0{}_j =: \rho v^j$$

Velocity must vanish quicker than c: (v ~ c^a)







Carroll EMT (rest frame):

$$T^0{}_0 = P \qquad T^i{}_0 = 0$$

$$T^0{}_j = 0 \qquad T^i{}_j = P\delta^i{}_j$$

Moving Carroll

Relativistic EMT explicitly:

$$T^0{}_0 = -(\tilde{\mathcal{E}} + P)\gamma^2 + P \qquad T^i{}_0 = -(\tilde{\mathcal{E}} + P)v^i\gamma^2$$

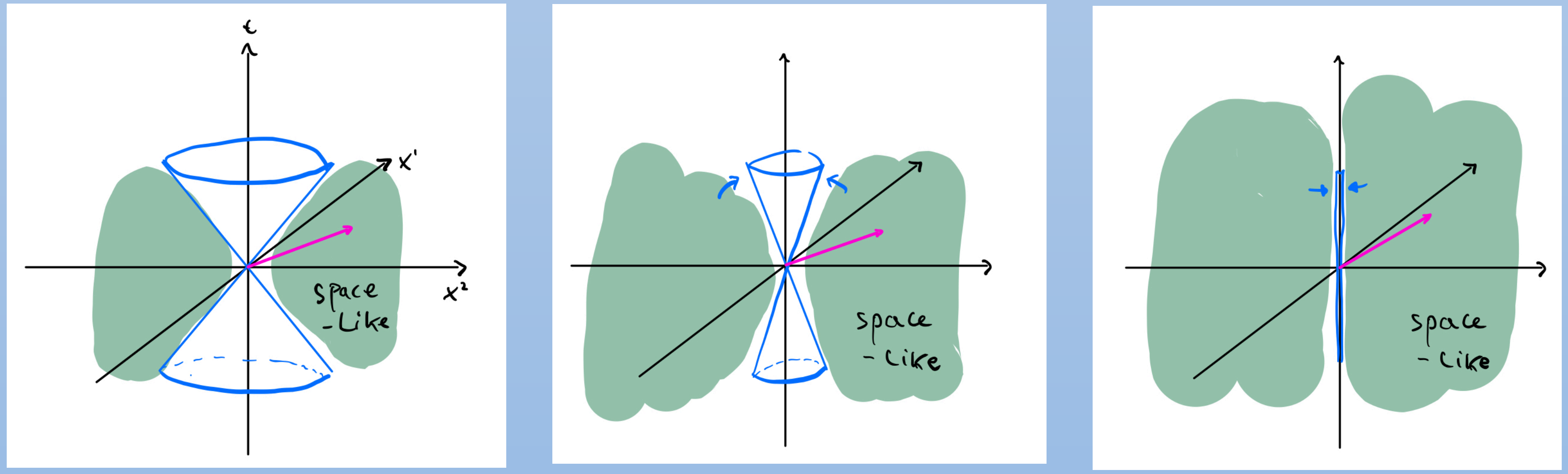
$$T^0{}_j = \frac{\tilde{\mathcal{E}} + P}{c^2} v^j\gamma^2 \qquad T^i{}_j = \frac{\tilde{\mathcal{E}} + P}{c^2} v^i v^j \gamma^2 + P\delta^i{}_j$$

$$\gamma^2 = \frac{1}{1 - v^2/c^2}$$

Generalized mass density:

$$T^0{}_j =: \rho v^j$$

Velocity must be greater than c: (v > c)



Carroll EMT (restless frame):

$$T^0{}_0 = P \qquad T^i{}_0 = 0$$

$$T^0{}_j = \rho v^j \qquad T^i{}_j = \rho v^i v^j + P\delta^i{}_j$$

hydrostatic partition function + curved space

Using curved spacetime to compute perfect fluid EMT

In passing insight into NL geometry

Thermal partition function on weakly curved stationary background: $\mathcal{Z} = \mathrm{Tr}\left[e^{-H/T}\right]$

Time translation symmetry implies timelike Killing vector: $\mathcal{L}_\beta g_{\mu\nu} = 0$

Preferred choice of temperature and velocity: $\beta^\mu = \frac{U^\mu}{\gamma T}$ $\quad g_{\mu\nu}\beta^\mu\beta^\nu = -\frac{c^2}{\gamma^2 T^2}$ $\quad g_{\mu\nu}U^\mu U^\nu = -c^2$

Leading order hydrostatic partition function action: $-i\log\mathcal{Z} = S_{HPF} = \int d^{3+1}x\sqrt{-g}P(T) + \mathcal{O}(\partial^1)$

Metric variation + thermo identities: $\delta S_{HPF} \sim \sqrt{-g}\left[\frac{\tilde{\mathcal{E}}+P}{c^2}U^\mu U^\nu + P g^{\mu\nu}\right]\delta g_{\mu\nu}$

Seminal HPF work: [Banerjee, Bhattacharya, Bhattacharyya, Jain, Minwalla, Sharma'12][Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom'12]

a closer look into NL geometry + HPF

Thermal partition function on weakly curved stationary background: $\mathcal{Z} = \mathrm{Tr}\left[e^{-H/T}\right]$

Split in metric: $g_{\mu\nu} = -c^2\tau_\mu\tau_\nu + h_{\mu\nu}$

In flat space: $\tau_\mu = \delta^0{}_\mu$ $\quad h_{\mu\nu} = \mathrm{diag}(0,1,1,1)$

Geometric identification temperature and velocity:
(relativistic case)

$$g_{\mu\nu}\beta^\mu\beta^\nu = -\frac{c^2}{\gamma^2 T^2} \qquad g_{\mu\nu}U^\mu U^\nu = -c^2$$

Carroll limit |v| < c (rest frame)	Carroll limit |v| > c (restless frame)
$\gamma^2 \to 1 \qquad (\tau_\mu\beta^\mu)^2 = \frac{1}{T^2} \qquad h_{\mu\nu}\beta^\mu\beta^\nu = 0$	$\gamma^2 \to -\frac{c^2}{v^2} \qquad (\tau_\mu\beta^\mu)^2 = 0 \qquad h_{\mu\nu}\beta^\mu\beta^\nu = \frac{v^2}{T^2}$

Variation wrt geometry: $\delta S_{\mathrm{HPF}} \sim -T^\mu\delta\tau_\mu + \frac{1}{2}T^{\mu\nu}\delta h_{\mu\nu}$

Relation to EMT: $T^\mu{}_\nu = -T^\mu\tau_\nu + T^{\mu\rho}h_{\rho\nu}$

Carroll and the Friedmann equations

FLRW metric: $ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega_\kappa^2\right]$

Einstein equations: $G^\mu{}_\nu = T^\mu{}_\nu$

General perfect fluid at rest:

$$T^0{}_0 = -\mathcal{E} \qquad T^i{}_0 = 0$$
$$T^0{}_j = 0 \qquad T^i{}_j = P\delta^i{}_j$$

Equation of state: $P = w\mathcal{E}$

Conservation of energy yields: $\mathcal{E} \sim a(t)^{-3(1+w)}$

Matter	Radiaton	Vacuum energy
w=0	w=1/3	w=-1

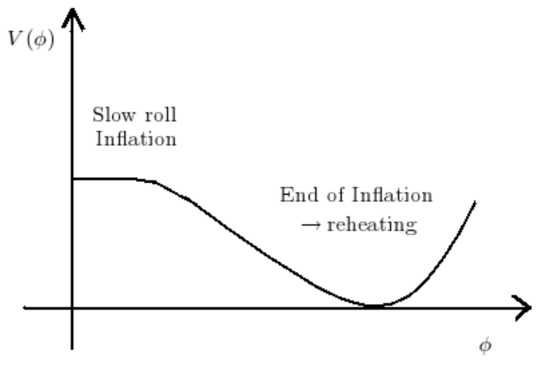
Carroll perfect fluid at rest: $-\mathcal{E} := T^0{}_0 = P$

Carroll as organizing principle

Single scalar inflation: $$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right]$$

Reading off EMT in homogeneous setting: $$\mathcal{E} = \frac{1}{2}c^2\pi_\phi^2 + V\,, \quad P = \frac{1}{2}c^2\pi_\phi^2 - V$$

Conjugate momentum: $$\pi_\phi := \frac{1}{c^2}\partial_t\phi$$



Slow roll inflation: $$w = \frac{\frac{1}{2}c^2\pi_\phi^2 - V(\phi)}{\frac{1}{2}c^2\pi_\phi^2 + V(\phi)} \approx -1$$

Carroll limit as organizing principle: $$w = \frac{\frac{1}{2}c^2\pi_\phi^2 - V(\phi)}{\frac{1}{2}c^2\pi_\phi^2 + V(\phi)} = -1 + \frac{\pi_\phi^2}{V}\,c^2 + \mathcal{O}(c^4)$$

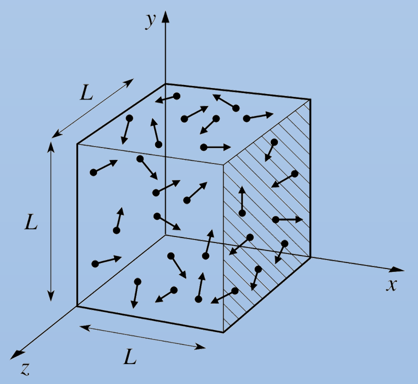
Observable universe within Hubble radius: $$r_H = c\frac{a}{\partial_t a}$$

Reflects ultralocal behavior

Opposite limit as organizing principle has been considered in: [Van den Bleeken'17][Hansen, Hartong, Obers'19+'20]

Carroll Microscopics

partition function of a Boltzmann gas of N relativistic particles: $H_1 = c|p|$ $\quad Z \sim \left[V\gamma(T/c)^3\right]^N$



Relation to EMT:

$$T^0{}_0 = \langle n H_1 \rangle \qquad T^i{}_0 = \left\langle n H_1 \frac{\partial H_1}{\partial p_j} \right\rangle$$

$$T^0{}_j = \langle n p_j \rangle \qquad T^i{}_j = \left\langle n p_i \frac{\partial H_1}{\partial p_j} \right\rangle$$

Carroll limits:

	relativistic	$\lvert v\rvert < c$	$\lvert v\rvert > c$
$\mathcal{E}$	$\frac{n}{\beta}\frac{dc^2+v^2}{c^2-v^2}$	$d\frac{n}{\beta}$	$-\frac{n}{\beta}$
$T^i{}_0$	$-\frac{n}{\beta}\frac{(d+1)c^2}{c^2-v^2}v^j$	0	0
$T^0{}_j$	$\frac{n}{\beta}\frac{d+1}{c^2-v^2}v^j$	0	$-n\frac{d+1}{\beta}\frac{v_j}{v^2}$
P	$\frac{n}{\beta}$	$\frac{n}{\beta}$	$\frac{n}{\beta}$

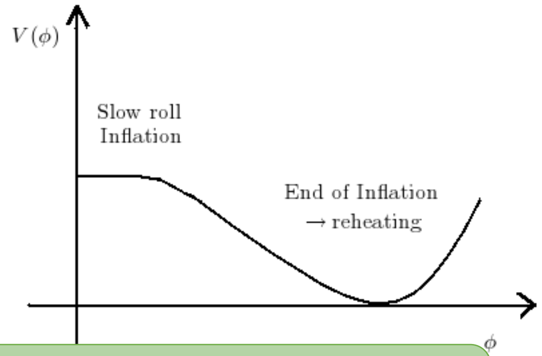
For v<c in order to have EMT with right covariance: $\mathcal{E} = -P$

partition function not finite :(

Reflects the fact that c->0 limit of microscopic systems is often tricky

Some works on specific Carroll systems: [Bergshoeff, Gomis, Longhi'14][Duval, Gibbons, Horvathy, Zhang'14][Bagchi, Mehra, Nandi'19x2]

Some closing statements



Assuming Carroll boost, there are two Carroll fluids which can’t be boosted into one another

Carroll fluids have a w=-1 equation of state. This might be used to explore e.g. inflation.

Can we understand Carroll microscopics better?

Thank you for tuning in!