rijksuniversiteit groningen

Carroll versus Galilei from a Brane Perspective

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- Eric Bergshoeff, José Manuel Izquierdo, Tomás Ortín, and LR (2019). "Lie Algebra Expansions and Actions for Non-Relativistic Gravity". In: *JHEP* 08, p. 048. DOI: 10.1007/JHEP08(2019)048. arXiv: 1904.08304 [hep-th]

- Eric Bergshoeff, José Manuel Izquierdo, and LR (Mar. 2020). "Carroll versus Galilei from a Brane Perspective". In: arXiv: 2003.03062 [hep-th]

- Carroll is kinematical group [Bacry and Levy-Leblond 1968]
- Carroll Symmetry in different physical context (near horizon geometry, gravitational waves, strong coupling limit of GR and ...) [Donnay and Marteau 2019; Penna 2018; Duval, Gibbons, Horvathy, and P. .-M. Zhang 2017; Marc Henneaux, Pilati, and Teitelboim 1982; Marc Henneaux 1979; Damour, M. Henneaux, and Nicolai 2003]
- Carroll dual to Galilei [Barducci, Casalbuoni, and Joaquim Gomis 2018; Duval, Gibbons, Horvathy, and P. Zhang 2014; E. Bergshoeff, Joaquim Gomis, Rollier, Rosseel, and Veldhuis 2017; Hartong 2015]
- Ultra-relativistic limit as a test

Galilei vs Carroll

Poincaré Algebra

$$[J_{ab}, J_{cd}] = 4\delta_{[a[c}J_{d]b]} \qquad [J_{ab}, G_d] = 2\delta_{d[b}G_{a]}$$
$$[G_a, G_b] = J_{ab} \qquad [J_{ab}, P_d] = 2\delta_{d[b}P_{a]}$$
$$[G_a, H] = P_a \qquad [G_a, P_b] = \delta_{ab}H$$

Inönü-Wigner Contraction $V_0 = \{J_{ab}, H\}$

Inönü-Wigner Contraction $V_0 = \{J_{ab}, P_a\}$

Galilei Algebra

$$[J_{ab}, J_{cd}] = 4\delta_{[a[c}J_{d]b]} \qquad [J_{ab}, G_d] = 2\delta_{d[b}G_{a]}$$
$$[G_a, G_b] = 0 \qquad [J_{ab}, P_d] = 2\delta_{d[b}P_{a]}$$
$$[G_a, H] = P_a \qquad [G_a, P_b] = 0$$

Carroll Algebra

$$[J_{ab}, J_{cd}] = 4\delta_{[a[c}J_{d]b]} \qquad [J_{ab}, G_d] = 2\delta_{d[b}G_{a]}$$
$$[G_a, G_b] = 0 \qquad [J_{ab}, P_d] = 2\delta_{d[b}P_{a]}$$
$$[G_a, H] = 0 \qquad [G_a, P_b] = \delta_{ab}H$$

$$H \xleftrightarrow{1 : D-1} P_a$$

p-Brane Decompostion of Poincaré Algebra

Poincaré Algebra

$$[\hat{J}_{\hat{A}\hat{B}}, \hat{J}_{\hat{C}\hat{D}}] = 4\eta_{[\hat{A}[\hat{C}}\hat{J}_{\hat{D}]\hat{B}]}$$
$$[\hat{J}_{\hat{A}\hat{B}}, \hat{P}_{\hat{C}}] = 2\eta_{\hat{C}[\hat{B}}\hat{P}_{\hat{A}]}$$

Decomposition

$$\hat{A} = \begin{cases} A = 0, ..., p & \text{Longitudinal} \\ a = p+1, ..., D-1 & \text{Transverse} \end{cases}$$

$$\eta_{\hat{A}\hat{B}} = \begin{cases} \eta_{AB} & = \text{diag}(-1, +1, ..., +1) \\ \eta_{ab} & = \text{diag}(+1, +1, ..., +1) \end{cases}$$

Generators

$J_{AB} = \hat{J}_{AB}$ long. rotations

$J_{ab} = \hat{J}_{ab}$ tr. rotations

$G_{Ab} = \hat{J}_{Ab}$ boost

$H_A = \hat{P}_A$ long. translations

$P_a = \hat{P}_a$ tr. translations

Poincaré Algebra

$$[J_{AB}, J_{CD}] = 4\eta_{[A[C}J_{D]B]} \qquad [J_{ab}, J_{cd}] = 4\delta_{[a[c}J_{d]b]}$$
$$[J_{AB}, G_{Cd}] = 2\eta_{C[B}G_{A]d} \qquad [J_{ab}, G_{Cd}] = 2\delta_{d[b|}G_{C|a]}$$
$$[J_{ab}, P_d] = 2\delta_{d[b}P_{a]} \qquad [J_{AB}, H_C] = 2\eta_{C[B}H_{A]}$$
$$[G_{Aa}, P_b] = \delta_{ab}H_A \qquad [G_{Aa}, H_B] = -\eta_{AB}P_a$$
$$[G_{Aa}, G_{Bb}] = -\eta_{AB}J_{ab} - \delta_{ab}J_{AB}$$

Poincaré Algebra

$$[G_{Aa}, P_b] = \delta_{ab} H_A \qquad [G_{Aa}, H_B] = -\eta_{AB} P_a$$
$$[G_{Aa}, G_{Bb}] = -\eta_{AB} J_{ab} - \delta_{ab} J_{AB}$$

$$[J_{AB}, J_{CD}] = 4\eta_{[A[C} J_{D]B]} \qquad [J_{ab}, J_{cd}] = 4\delta_{[a[c} J_{d]b]}$$
$$[J_{AB}, G_{Cd}] = 2\eta_{C[B} G_{A]d} \qquad [J_{ab}, G_{Cd}] = 2\delta_{d[b|} G_{C|a]}$$
$$[J_{ab}, P_d] = 2\delta_{d[b} P_{a]} \qquad [J_{AB}, H_C] = 2\eta_{C[B} H_{A]}$$

Inönü-Wigner Contraction $V_0 = \{J_{ab}, J_{AB}, H_A\}$

Inönü-Wigner Contraction $V_0 = \{J_{ab}, J_{AB}, P_a\}$

p-Brane Galilei Algebra

$$[G_{Aa}, G_{Bb}] = 0$$
$$[G_{Aa}, P_b] = 0$$
$$[G_{Aa}, H_B] = -\eta_{AB} P_a$$

q-Brane Carroll Algebra

$$[G_{Aa}, G_{Bb}] = 0$$
$$[G_{Aa}, P_b] = \delta_{ab} H_A$$
$$[G_{Aa}, H_B] = 0$$

Duality

$$H_A \overset{1:1}{\longleftrightarrow} P_a$$

if $q = D - p - 2$

Galilie-Carroll Duality

Duality

p-Brane Galilei				**$(D-p-2)$-Brane Carroll**
Longitudinal	A	$\longleftrightarrow$	a	Transverse
Transverse	a	$\longleftrightarrow$	A	Longitudinal

[Barducci, Casalbuoni, and Joaquim Gomis 2018]

$D = 3$ Non-Zero Commutation Relations

Galilei		**Carroll**	
$p=0$	$p=1$	$p=0$	$p=1$
$[J, G_a] = \epsilon_a{}^b G_b$	$[M, G_A] = \epsilon_A{}^B G_B$	$[J, G_a] = \epsilon_a{}^b G_b$	$[M, G_A] = \epsilon_A{}^B G_B$
$[J, P_a] = \epsilon_a{}^b P_b$	$[M, H_A] = \epsilon_A{}^B H_B$	$[J, P_a] = \epsilon_a{}^b P_b$	$[M, H_A] = \epsilon_A{}^B H_B$
$[G_a, H] = -P_a$	$[G_A, H_B] = -\eta_{AB} P$	$[G_a, P_b] = \eta_{ab} H$	$[G_A, P] = H_A$

Lie Algebra Expansion

Lie Algebra Expansion

The Lie algebra expansion is a technique to generate a new algebra, usually bigger, from a given one $\mathfrak{g}$ [de Azcarraga, Jose M. Izquierdo, Moises Picon, and Oscar Varela 2003; Hatsuda and Sakaguchi 2002; Hatsuda and Sakaguchi 2003]

Initial Algebra

Algebra $\mathfrak{g}$ with generators T_A and commutation relations

$$[T_A, T_B] = f_{AB}{}^C T_C$$

Decomposition

$$\mathfrak{g} = V_0 \oplus V_1$$
$$T_{A_0} \in V_0, \; T_{A_1} \in V_1$$

symmetric space structure

$$[V_0, V_0] \subseteq V_0$$
$$[V_0, V_1] \subseteq V_1$$
$$[V_1, V_1] \subseteq V_0$$

Infinite Expansion

$$T_{A_0} = \overset{(0)}{T}_{A_0} + \lambda^2 \overset{(2)}{T}_{A_0} + \ldots$$
$$T_{A_1} = \lambda \overset{(1)}{T}_{A_1} + \lambda^3 \overset{(3)}{T}_{A_1} + \ldots$$

New Generators

Each term in the expansion could be seen as an independent generator with commutation relations $[\overset{(m)}{T}_A, \overset{(n)}{T}_B] = f_{AB}{}^C \overset{(n+m)}{T}_C$

TRUNCATED EXPANSION

$$T_{A_0} = \overset{(0)}{T}_{A_0} + \lambda^2 \overset{(2)}{T}_{A_0} + \ldots + \lambda^{N_0} \overset{(N_0)}{T}_{A_0} \qquad N_0 \in 2\mathbb{Z}$$

$$T_{A_1} = \lambda \overset{(1)}{T}_{A_1} + \lambda^3 \overset{(3)}{T}_{A_1} + \ldots + \lambda^{N_1} \overset{(N_1)}{T}_{A_1} \qquad N_1 \in 2\mathbb{Z} + 1$$

TRUNCATION RULES

Consistency (Jacobi) requires $N_1 = N_0 \pm 1$

FINAL ALGEBRA

We get a new algebra denoted with $\mathfrak{g}(N_0, N_1)$ with commutation relations

$$[\overset{(m)}{T}_A, \overset{(n)}{T}_B] = \begin{cases} f_{AB}{}^C \overset{(n+m)}{T}_C & if\ m+n \leqslant \max\{N_0, N_1\} \\ 0 & otherwise \end{cases}$$

INÖNÜ-WIGNER CONTRACTION

The lowest order algebra $\mathfrak{g}(0,1)$ is the Inönü-Wigner contraction of $\mathfrak{g}$ withe respect to the subalgebra V_0

Lie Algebra Expansion and the Duality Map

Properties

- The expansion is uniquely defined by the decompostion
- Duality map holds order by order
- Truncation could spoil the duality

Duality and Expansion

p-Brane Galilei (N_0, N_1)				**$(D-p-2)$-Brane Carroll (N_0, N_1)**
Longitudinal	A	$\Longleftrightarrow$	a	Transverse
Transverse	a	$\Longleftrightarrow$	A	Longitudinal

GALILEI GAUGING EXPANSION

	V_0			V_1	
Generator	J_{ab}	J_{AB}	H_A	G_{Ab}	P_a
Field	Ω_μ^{ab}	Ω_μ^{AB}	τ_μ^A	Ω_μ^{Ab}	E_μ^a
Parameter	λ^{ab}	λ^{AB}	η^Z	λ^{Ab}	η^a

[Azcarraga, J. Izquierdo, M. Picon, and O. Varela 2007; Hansen, Hartong, and Obers 2019b]

$$H_A = \overset{(0)}{H}_A + \lambda^2 \overset{(2)}{H}_A + \dots$$
$$\tau_\mu^A = \overset{(0)}{\tau}{}_\mu^A + \lambda^2 \overset{(2)}{\tau}{}_\mu^A + \dots$$
$$\eta^A = \overset{(0)}{\eta}{}^A + \lambda^2 \overset{(2)}{\eta}{}^A + \dots$$
$$P_a = \lambda \overset{(1)}{P}_a + \lambda^3 \overset{(3)}{P}_a + \dots$$
$$E_\mu^a = \lambda \overset{(1)}{E}{}_\mu^a + \lambda^3 \overset{(3)}{E}{}_\mu^a + \dots$$
$$\eta^a = \lambda \overset{(1)}{\eta}{}^a + \lambda^3 \overset{(3)}{\eta}{}^a + \dots$$

CARROLL GAUGING EXPANSION

	V_0			V_1	
Generator	J_{ab}	J_{AB}	P_a	G_{Ab}	H_A
Field	Ω_μ^{ab}	Ω_μ^{AB}	E_μ^a	Ω_μ^{Ab}	τ_μ^A
Parameter	λ^{ab}	λ^{AB}	η^a	λ^{Ab}	η^A

[Hartong 2015]

$$P_a = \overset{(0)}{P}_a + \lambda^2 \overset{(2)}{P}_a + \dots$$
$$E_\mu^a = \overset{(0)}{E}{}_\mu^a + \lambda^2 \overset{(2)}{E}{}_\mu^a + \dots$$
$$\eta^a = \overset{(0)}{\eta}{}^a + \lambda^2 \overset{(2)}{\eta}{}^a + \dots$$
$$H_A = \lambda \overset{(1)}{H}_A + \lambda^3 \overset{(3)}{H}_A + \dots$$
$$\tau_\mu^A = \lambda \overset{(1)}{\tau}{}_\mu^A + \lambda^3 \overset{(3)}{\tau}{}_\mu^A + \dots$$
$$\eta^A = \lambda \overset{(1)}{\eta}{}^A + \lambda^3 \overset{(3)}{\eta}{}^A + \dots$$

$D = 4$ Einstein-Hilbert Action

$$
\begin{aligned}
\mathcal{L}_{p=0} &= \epsilon_{abc} \left[-R^{ab}(J) \wedge E^c \wedge \tau + R^a(G) \wedge E^b \wedge E^c \right] , \\
\mathcal{L}_{p=1} &= \epsilon_{ABab} \left[R^{ab}(J) \wedge \tau^A \wedge \tau^B + R^{AB}(J) \wedge E^a \wedge E^b - 4R^{Aa}(G) \wedge \tau^B \wedge E^b \right] , \\
\mathcal{L}_{p=2} &= -\, \epsilon_{ABC} \left[R^{AB}(J) \wedge \tau^C \wedge E + R^A(G) \wedge \tau^B \wedge \tau^C \right] .
\end{aligned}
$$

Leading Order Action Term (Algebra $\mathfrak{g}(2,3)$)

$\mathcal{L}_p$	**Galilei**	**Carroll**
$p = 0$	$-\epsilon_{abc} \overset{(0)}{R}{}^{ab}(J) \wedge \overset{(1)}{E}{}^c \wedge \overset{(0)}{\tau}$	$\epsilon_{abc} \left[-\overset{(0)}{R}{}^{ab}(J) \wedge \overset{(0)}{E}{}^c \wedge \overset{(1)}{\tau} + \overset{(1)}{R}{}^a(G) \wedge \overset{(0)}{E}{}^b \wedge \overset{(0)}{E}{}^c \right]$
$p = 1$	$\epsilon_{ABab} \overset{(0)}{R}{}^{ab}(J) \wedge \overset{(0)}{\tau}{}^A \wedge \overset{(0)}{\tau}{}^B$	$\epsilon_{ABab} \overset{(0)}{R}{}^{AB}(J) \wedge \overset{(0)}{E}{}^a \wedge \overset{(0)}{E}{}^b$
$p = 2$	$-\epsilon_{ABC} \left[\overset{(0)}{R}{}^{AB}(J) \wedge \overset{(0)}{\tau}{}^C \wedge \overset{(1)}{E} + \overset{(1)}{R}{}^A(G) \wedge \overset{(0)}{\tau}{}^B \wedge \overset{(0)}{\tau}{}^C \right]$	$-\epsilon_{ABC} \overset{(0)}{R}{}^{AB}(J) \wedge \overset{(1)}{\tau}{}^C \wedge \overset{(0)}{E}$

Next-to-Leading Order Action Term (Algebra $\mathfrak{g}(2,3)$)

$\mathcal{L}_p$	Galilei	Carroll
$p=0$	$\epsilon_{abc}\Big[-\overset{(0)}{R}{}^{ab}(J)\wedge\overset{(3)}{E}{}^{c}\wedge\overset{(0)}{\tau}-\overset{(2)}{R}{}^{ab}(J)\wedge\overset{(1)}{E}{}^{c}\wedge\overset{(0)}{\tau}+ -\overset{(0)}{R}{}^{ab}(J)\wedge\overset{(1)}{E}{}^{c}\wedge\overset{(2)}{\tau}+\overset{(1)}{R}{}^{a}(G)\wedge\overset{(1)}{E}{}^{b}\wedge\overset{(1)}{E}{}^{c}\Big]$	$\epsilon_{abc}\Big[-\overset{(0)}{R}{}^{ab}(J)\wedge\overset{(0)}{E}{}^{c}\wedge\overset{(3)}{\tau}-\overset{(2)}{R}{}^{ab}(J)\wedge\overset{(0)}{E}{}^{c}\wedge\overset{(1)}{\tau}+ -\overset{(0)}{R}{}^{ab}(J)\wedge\overset{(2)}{E}{}^{c}\wedge\overset{(1)}{\tau}+2\overset{(1)}{R}{}^{a}(G)\wedge\overset{(2)}{E}{}^{b}\wedge\overset{(0)}{E}{}^{c}+ +\overset{(3)}{R}{}^{a}(G)\wedge\overset{(0)}{E}{}^{b}\wedge\overset{(0)}{E}{}^{c}\Big]$
$p=1$	$\epsilon_{ABab}\Big[\overset{(2)}{R}{}^{ab}(J)\wedge\overset{(0)}{\tau}{}^{A}\wedge\overset{(0)}{\tau}{}^{B}+2\overset{(0)}{R}{}^{ab}(J)\wedge\overset{(2)}{\tau}{}^{A}\wedge\overset{(0)}{\tau}{}^{B}+ +\overset{(0)}{R}{}^{AB}(J)\wedge\overset{(1)}{E}{}^{a}\wedge\overset{(1)}{E}{}^{b}-4\overset{(1)}{R}{}^{Aa}(G)\wedge\overset{(0)}{\tau}{}^{B}\wedge\overset{(1)}{E}{}^{b}\Big]$	$\epsilon_{ABab}\Big[\overset{(0)}{R}{}^{ab}(J)\wedge\overset{(1)}{\tau}{}^{A}\wedge\overset{(1)}{\tau}{}^{B}+\overset{(2)}{R}{}^{AB}(J)\wedge\overset{(0)}{E}{}^{a}\wedge\overset{(0)}{E}{}^{b}+ +2\overset{(0)}{R}{}^{AB}(J)\wedge\overset{(2)}{E}{}^{a}\wedge\overset{(0)}{E}{}^{b}-4\overset{(1)}{R}{}^{Aa}(G)\wedge\overset{(1)}{\tau}{}^{B}\wedge\overset{(0)}{E}{}^{b}\Big]$
$p=2$	$-\epsilon_{ABC}\Big[\overset{(0)}{R}{}^{AB}(J)\wedge\overset{(0)}{\tau}{}^{C}\wedge\overset{(3)}{E}+\overset{(2)}{R}{}^{AB}(J)\wedge\overset{(0)}{\tau}{}^{C}\wedge\overset{(1)}{E}+ +\overset{(0)}{R}{}^{AB}(J)\wedge\overset{(2)}{\tau}{}^{C}\wedge\overset{(1)}{E}+2\overset{(1)}{R}{}^{A}(G)\wedge\overset{(2)}{\tau}{}^{B}\wedge\overset{(0)}{\tau}{}^{C}+ +\overset{(3)}{R}{}^{A}(G)\wedge\overset{(0)}{\tau}{}^{B}\wedge\overset{(0)}{\tau}{}^{C}\Big]$	$-\epsilon_{ABC}\Big[\overset{(2)}{R}{}^{AB}(J)\wedge\overset{(1)}{\tau}{}^{C}\wedge\overset{(0)}{E}+\overset{(0)}{R}{}^{AB}(J)\wedge\overset{(3)}{\tau}{}^{C}\wedge\overset{(0)}{E}+ \overset{(0)}{R}{}^{AB}(J)\wedge\overset{(1)}{\tau}{}^{C}\wedge\overset{(2)}{E}+\overset{(1)}{R}{}^{A}(G)\wedge\overset{(1)}{\tau}{}^{B}\wedge\overset{(1)}{\tau}{}^{C}\Big]$

[Hansen, Hartong, and Obers 2019a; Ozdemir, Ozkan, Tunca, and Zorba 2019; E. A. Bergshoeff, Grosvenor, Simsek, and Yan 2019]

Invariance and Truncation

Truncation	Order 3 Action Term
$\mathfrak{g}$	$\epsilon_{abc}\Big[-\overset{(0)}{R}{}^{ab}(J)\wedge\overset{(3)}{E}{}^{c}\wedge\overset{(0)}{\tau}-\overset{(2)}{R}{}^{ab}(J)\wedge\overset{(1)}{E}{}^{c}\wedge\overset{(0)}{\tau}-\overset{(0)}{R}{}^{ab}(J)\wedge\overset{(1)}{E}{}^{c}\wedge\overset{(2)}{\tau}+\overset{(1)}{R}{}^{a}(G)\wedge\overset{(1)}{E}{}^{b}\wedge\overset{(1)}{E}{}^{c}\Big]$
$\mathfrak{g}(2,1)$	$\epsilon_{abc}\Big[-\overset{(2)}{R}{}^{ab}(J)\wedge\overset{(1)}{E}{}^{c}\wedge\overset{(0)}{\tau}-\overset{(0)}{R}{}^{ab}(J)\wedge\overset{(1)}{E}{}^{c}\wedge\overset{(2)}{\tau}+\overset{(1)}{R}{}^{a}(G)\wedge\overset{(1)}{E}{}^{b}\wedge\overset{(1)}{E}{}^{c}\Big]$

$$\delta_J(\overset{(2)}{\lambda})(\overset{(0)}{R}{}^{ab}(J)\wedge\overset{(3)}{E}{}^{c}\wedge\overset{(0)}{\tau})=-\overset{(2)}{\lambda}{}^{c}{}_{d}\overset{(0)}{R}{}^{ab}(J)\wedge\overset{(1)}{E}{}^{d}\wedge\overset{(0)}{\tau}$$

$$\delta_J(\overset{(2)}{\lambda})\overset{(2)}{R}{}^{ab}(J)\wedge\overset{(1)}{E}{}^{c}\wedge\overset{(0)}{\tau})=\Big(\overset{(2)}{\lambda}{}^{a}{}_{d}\overset{(0)}{R}{}^{bd}(J)-\overset{(2)}{\lambda}{}^{b}{}_{d}\overset{(0)}{R}{}^{ad}(J)\Big)\wedge\overset{(1)}{E}{}^{c}\wedge\overset{(0)}{\tau}$$

Invariance Condition

Invariance under $\mathfrak{g}(N_0,N_1)$: no missing term in the n-th order action, after the truncation, with respect to the infinite expansion

Invariance Conditions

Galilei		Carroll	
$p = 0$	$\begin{cases} D = 3 & n \leqslant N_0 \\ D \neq 3 & n \leqslant N_1 + D - 4 \end{cases}$	$p = D - 1$	$n \leqslant N_1 + D - 3$
$p = D - 3$	$n \leqslant N_0$	$p = D - 2$	$\begin{cases} D = 3 & n \leqslant N_0 \\ D \neq 3 & n \leqslant N_1 + D - 4 \end{cases}$
$1 \leqslant p < D - 3$	$n \leqslant N_1 + D - p - 4$	$1 < p \leqslant D - 3$	$n \leqslant N_1 + p - 2$
$p = D - 2$	$n \leqslant N_1$	$p = 1$	$n \leqslant N_0$
$p = D - 1$	$n \leqslant N_0$	$p = 0$	$n \leqslant N_1$.

The Starting Point: 4D Enhanced Galilei String

The Algebra

$$[J, P_a] = -\epsilon_a{}^b P_b \qquad [J', H_A] = -\epsilon_A{}^B H_B$$
$$[J', G_{Aa}] = -\epsilon_A{}^B G_{Ba} \qquad [J, G_{Aa}] = -\epsilon_a{}^b G_{Ab}$$
$$[G_{Aa}, H_B] = -\eta_{AB} P_a \qquad [G_{Aa}, G_{Bb}] = \delta_{ab} Z_{[AB]}$$
$$[J', M_A] = -\epsilon_A{}^B M_B \qquad [J', Z_{AB}] = -\epsilon_A{}^C Z_{CB} - \epsilon_B{}^C Z_{AC}$$
$$[G_{Aa}, P_b] = \delta_{ab} M_A \qquad [H_A, Z_{BC}] = 2\eta_{AC} M_B - \eta_{BC} M_A$$

The Sigma Model

$$\mathcal{L} = T\sqrt{h} h^{\alpha\beta} \partial_\alpha X^{\hat\mu} \partial_\beta X^{\hat\nu} [E_{\hat\mu}{}^{\hat a} E_{\hat\nu}{}^{\hat b} \delta_{\hat a \hat b} + 2 m_{(\hat\mu}{}^{\hat A} \tau_{\hat\nu)}{}^{\hat B} \eta_{AB}]$$
$$h_{\alpha\beta} = \partial_\alpha X^{\hat\mu} \partial_\beta X^{\hat\nu} \tau_{\hat\mu}{}^{\hat A} \tau_{\hat\nu}{}^{\hat B} \eta_{\hat A \hat B}$$
$$D_{[\hat\mu} \tau_{\hat\nu]}{}^{\hat A} = 0 \qquad (zero\ torsion)$$

Symmetries

$$\delta E_{\hat\mu}{}^{\hat a} = \lambda^{\hat a}{}_{\hat A} \tau_{\hat\mu}{}^{\hat A}$$
$$\delta m_{\hat\mu}{}^{\hat A} = D_{\hat\mu} \sigma^{\hat A} + \lambda^{\hat A}{}_{\hat a} E_{\hat\mu}{}^{\hat a} + \\ + \sigma^{\hat A}{}_{\hat B} \tau_{\hat\mu}{}^{\hat B}$$

[E. Bergshoeff, Jaume Gomis, and Yan 2018]

4D Enhanced Galilei/Carroll String Algebras

Commutation Rules 4D $p = 1$

Galilei	Carroll
$[J, P_a] = -\epsilon_a{}^b P_b$	$[J, P_a] = -\epsilon_a{}^b P_b$
$[J', H_A] = -\epsilon_A{}^B H_B$	$[J', H_A] = -\epsilon_A{}^B H_B$
$[J', G_{Aa}] = -\epsilon_A{}^B G_{Ba}$	$[J', G_{Aa}] = -\epsilon_A{}^B G_{Ba}$
$[J, G_{Aa}] = -\epsilon_a{}^b G_{Ab}$	$[J, G_{Aa}] = -\epsilon_a{}^b G_{Ab}$
$[G_{Aa}, H_B] = -\eta_{AB} P_a$	$[G_{Aa}, P_b] = -\delta_{ab} H_A$
$[G_{Aa}, G_{Bb}] = \delta_{ab} Z_{[AB]}$	$[G_{Aa}, G_{Bb}] = \eta_{AB} Z_{[ab]}$
$[J', M_A] = -\epsilon_A{}^B M_B$	$[J, M_a] = -\epsilon_a{}^b M_b$
$[J', Z_{AB}] = -\epsilon_A{}^C Z_{CB} - \epsilon_B{}^C Z_{AC}$	$[J, Z_{ab}] = -\epsilon_a{}^c Z_{cb} - \epsilon_b{}^c Z_{ac}$
$[G_{Aa}, P_b] = \delta_{ab} M_A$	$[G_{Aa}, H_B] = \eta_{AB} M_a$
$[H_A, Z_{BC}] = 2\eta_{AC} M_B - \eta_{BC} M_A$	$[P_a, Z_{bc}] = 2\delta_{ac} M_b - \delta_{bc} M_a$

4D String Sigma Models

Polyakov Action

$$\mathcal{L}_{Pol} = \sqrt{h} h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \hat{E}_\mu{}^{\hat{A}} \hat{E}_\nu{}^{\hat{B}} \eta_{\hat{A}\hat{B}} + \\ - (p-1) T \sqrt{h}$$

Expansion

Galilei

$$\hat{E}_\mu{}^A = c\tau_\mu{}^A + \frac{1}{c} m_\mu{}^A$$

$$\hat{E}_\mu{}^a = E_\mu{}^a$$

Carroll

$$\hat{E}_\mu{}^A = \tau_\mu{}^A$$

$$\hat{E}_\mu{}^a = cE_\mu{}^a + \frac{1}{c} n_\mu{}^a$$

4D Enhanced Galilei String

$$\mathcal{L} = T\sqrt{h} h^{\alpha\beta} \partial_\alpha X^{\hat{\mu}} \partial_\beta X^{\hat{\nu}} [E_{\hat{\mu}}{}^{\hat{a}} E_{\hat{\nu}}{}^{\hat{b}} \delta_{\hat{a}\hat{b}} + 2 m_{(\hat{\mu}}{}^{\hat{A}} \tau_{\hat{\nu})}{}^{\hat{B}} \eta_{AB}]$$

$$h_{\alpha\beta} = \partial_\alpha X^{\hat{\mu}} \partial_\beta X^{\hat{\nu}} \tau_{\hat{\mu}}{}^{\hat{A}} \tau_{\hat{\nu}}{}^{\hat{B}} \eta_{\hat{A}\hat{B}}$$

$$D_{[\hat{\mu}} \tau_{\hat{\nu}]}{}^{\hat{A}} = 0 \qquad (\textit{zero torsion})$$

Symmetries

$$\delta E_{\hat{\mu}}{}^{\hat{a}} = \lambda^{\hat{a}}{}_{\hat{A}} \tau_{\hat{\mu}}{}^{\hat{A}}$$

$$\delta m_{\hat{\mu}}{}^{\hat{A}} = D_{\hat{\mu}} \sigma^{\hat{A}} + \lambda^{\hat{A}}{}_{\hat{a}} E_{\hat{\mu}}{}^{\hat{a}} + \\ + \sigma^{\hat{A}}{}_{\hat{B}} \tau_{\hat{\mu}}{}^{\hat{B}}$$

4D Enhanced Carroll String

$$\mathcal{L} = T\sqrt{h} h^{\alpha\beta} \partial_\alpha X^{\hat{\mu}} \partial_\beta X^{\hat{\nu}} [\tau_{\hat{\mu}}{}^{\hat{A}} \tau_{\hat{\nu}}{}^{\hat{B}} \eta_{\hat{A}\hat{B}} + 2 n_{(\hat{\mu}}{}^{\hat{a}} E_{\hat{\nu})}{}^{\hat{b}} \delta_{\hat{a}\hat{b}}]$$

$$h_{\alpha\beta} = \partial_\alpha X^{\hat{\mu}} \partial_\beta X^{\hat{\nu}} E_{\hat{\mu}}{}^{\hat{a}} E_{\hat{\nu}}{}^{\hat{b}} \delta_{\hat{a}\hat{b}}$$

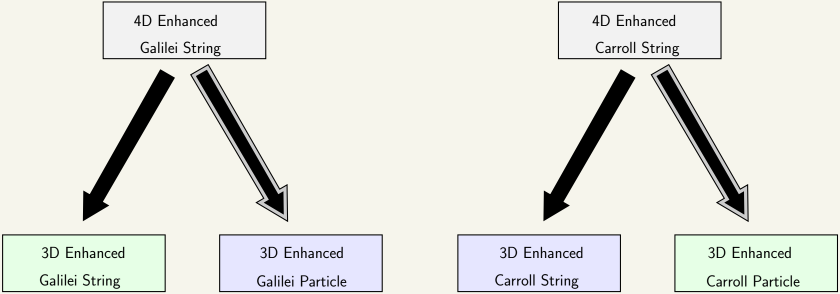
$$D_{[\hat{\mu}} E_{\hat{\nu}]}{}^{\hat{a}} = 0 \qquad (\textit{zero torsion})$$

Symmetries

$$\delta \tau_{\hat{\mu}}{}^{\hat{A}} = \lambda^{\hat{A}}{}_{\hat{a}} E_{\hat{\mu}}{}^{\hat{a}}$$

$$\delta n_{\hat{\mu}}{}^{\hat{a}} = D_{\hat{\mu}} \sigma^{\hat{a}} + \lambda^{\hat{a}}{}_{\hat{A}} \tau_{\hat{\mu}}{}^{\hat{A}} + \\ + \sigma^{\hat{a}}{}_{\hat{b}} E_{\hat{\mu}}{}^{\hat{b}}$$

DIMENSIONAL REDUCTION



Commutation Rules		Galilei	Carroll
	$D=3\ p=0$	$[J,P_a]=-\epsilon_a{}^b P_b$ $[J,G_a]=-\epsilon_a{}^b G_b$ $[G_a,H]=P_a$ $[G_a,P_b]=\delta_{ab}M$	$[J,P_a]=-\epsilon_a{}^b P_b$ $[J,G_a]=-\epsilon_a{}^b G_b$ $[J,M_a]=-\epsilon_a{}^b M_b$ $[J,Z_{ab}]=-\epsilon_a{}^c Z_{cb}-\epsilon_b{}^c Z_{ac}$ $[G_a,G_b]=Z_{[ab]}$ $[G_a,P_b]=-\delta_{ab}H$ $[G_a,H]=M_a$ $[P_a,Z_{bc}]=2\delta_{ac}M_b-\delta_{bc}M_a$
	$D=3\ p=1$	$[J',H_A]=-\epsilon_A{}^B H_B$ $[J',G_A]=-\epsilon_A{}^B G_B$ $[J',M_A]=-\epsilon_A{}^B M_B$ $[J',Z_{AB}]=-\epsilon_A{}^C Z_{CB}-\epsilon_B{}^C Z_{AC}$ $[G_A,G_B]=Z_{[AB]}$ $[G_A,H_B]=-\eta_{AB}P$ $[G_A,P]=M_A$ $[H_A,Z_{BC}]=2\eta_{AC}M_B-\eta_{BC}M_A$	$[J',H_A]=-\epsilon_A{}^B H_B$ $[J',G_A]=-\epsilon_A{}^B G_B$ $[G_A,H_B]=-\eta_{AB}M$ $[G_A,P]=H_A$

3D Enhanced Galilei String

$$\mathcal{L} = T\sqrt{h}h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu [E_\mu E_\nu + 2m_{(\mu}{}^A \tau_{\nu)}{}^B \eta_{AB}]$$
$$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \tau_\mu{}^A \tau_\nu{}^B \eta_{AB}$$
$$D_{[\mu}\tau_{\nu]}{}^A = 0$$

Symmetries

$$\delta E_\mu = \lambda^A \tau_\mu{}^B \eta_{AB}$$
$$\delta m_\mu{}^A = D_\mu \sigma^A - \lambda^A E_\mu + + \sigma^A{}_B \tau_\mu{}^B$$

3D Enhanced Galilei Particle

$$\mathcal{L} = me^{-1}\dot{X}^\mu \dot{X}^\nu [E_\mu{}^a E_\nu{}^b \delta_{ab} - 2m_{(\mu}\tau_{\nu)}]$$
$$e = \dot{X}^\mu \tau_\mu$$
$$\partial_{[\mu}\tau_{\nu]} = 0$$

Symmetries

$$\delta E_\mu{}^a = \lambda^a \tau_\mu$$
$$\delta m_\mu = \partial_\mu \sigma + \lambda^a E_\mu{}^b \delta_{ab}$$

3D Enhanced Carroll String

$$\mathcal{L} = T\sqrt{h}h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu [\tau_\mu{}^A \tau_\nu{}^B \eta_{AB} + 2n_{(\mu}E_{\nu)}]$$
$$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu E_\mu E_\nu + \partial_\alpha X^y \partial_\beta X^y$$
$$\partial_{[\mu}E_{\nu]}{}^a = 0$$

Symmetries

$$\delta \tau_\mu{}^A = \lambda^A E_\mu$$
$$\delta n_\mu{}^a = \partial_\mu \sigma - \lambda^A \tau_\mu{}^B \eta_{AB}$$

3D Enhanced Carroll Particle

$$\mathcal{L} = -Te^{-1}\dot{X}^\mu (E_y{}^a \tau_\mu - E_\mu{}^a \tau_y)(E_{ya}\tau - E_a \tau_y) +$$
$$+ 2T\dot{X}^\mu \epsilon_{ab}(E_\mu{}^a n_y{}^B - E_y{}^a n_\mu{}^b)$$
$$e = \epsilon_{ab}\dot{X}^\mu E_\mu{}^a E_y{}^b$$
$$D_\mu E_y{}^a = 0$$

Symmetries

$$\delta \tau_\mu = \lambda_a E_\mu{}^a$$
$$\delta \tau_y = \lambda_a E_y{}^a$$
$$\delta n_\mu{}^a = D_\mu \sigma^a + \lambda^a \tau_\mu + \sigma^a{}_b E_\mu{}^b$$
$$\delta n_y{}^a = \lambda^a \tau_y + \sigma^a{}_b E_y{}^b$$

3D Enhanced Carroll String

$$\mathcal{L} = T\sqrt{h}h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu [\tau_\mu{}^A \tau_\nu{}^B \eta_{AB} + 2n_{(\mu}E_{\nu)}]$$
$$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu E_\mu E_\nu + \partial_\alpha X^y \partial_\beta X^y$$
$$\partial_{[\mu}E_{\nu]}{}^a = 0$$

Symmetries

$$\delta\tau_\mu{}^A = \lambda^A E_\mu$$
$$\delta n_\mu{}^a = \partial_\mu \sigma - \lambda^A \tau_\mu{}^B \eta_{AB}$$

3D Enhanced Carroll Particle

$$\mathcal{L} = -Te^{-1}\dot{X}^\mu (E_y{}^a \tau_\mu - E_\mu{}^a \tau_y)(E_{ya}\tau - E_a \tau_y) +$$
$$+ 2T\dot{X}^\mu \epsilon_{ab}(E_\mu{}^a n_y{}^B - E_y{}^a n_\mu{}^b)$$
$$e = \epsilon_{ab}\dot{X}^\mu E_\mu{}^a E_y{}^b$$
$$D_\mu E_y{}^a = 0$$

Symmetries

$$\delta\tau_\mu = \lambda_a E_\mu{}^a$$
$$\delta\tau_y = \lambda_a E_y{}^a$$
$$\delta n_\mu{}^a = D_\mu \sigma^a + \lambda^a \tau_\mu + \sigma^a{}_b E_\mu{}^b$$
$$\delta n_y{}^a = \lambda^a \tau_y + \sigma^a{}_b E_y{}^b$$

Degeneracy

Condition	Extra Fields	
range$\{a\}$ < range$\{A\}$	X_{y_i}	Transverse Scalars
range$\{A\}$ < range$\{a\}$	$E_{y_i}{}^a$	Matter

- Extension to Supersymmetric Cases
- Analysis of the duality in different physical contexts (near horizon-asymptotics)
- Holography, Conformal Extension and BMS symmetries...

Thank You!