An exact Galilean supersymmetric model

Silvia Penati, University of Milano-Bicocca and INFN

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R. Auzzi, S. Baiguera, G. Nardelli, SP, arXiv:1904.08404

Motivations

We have studied at quantum level a non-relativistic version of the Wess-Zumino model in (2+1)D with $\mathcal{N} = 2$ SUSY

- In models describing CM systems SUSY has been observed to be an emergent symmetry, that is it appears in the effective theory describing the low-energy modes. On the other hand, at these scales the system is typically in a non-relativistic regime.

  Therefore, it is physically relevant to construct NR SUSY models

- Non-relativistic holography: Non-relativistic generalisation of the AdS/CFT is of interest for the holographic description of CM systems. Which is the role of supersymmetry in NR gauge/gravity correspondence?

  D.T. Son, PRD78 (2008);
  K. Balasubramanian, J. McGreevy, PRL101 (2008);
  W.D. Goldberger, JHEP03 (2009);
  S. Kachru, X. Liu, M. Mulligan, PRD78 (2008);
  M. Taylor, 0812.0530;
  S. Janiszewski, A. Karch, JHEP02 (2013)
  M. H. Christensen, J. Hartong, N. A. Obers and B. Rollier, PRD89, JHEP01 (2014)

- Supersymmetry often helps in solving exactly the theory

State of the art

NR models **with NO SUSY** have been extensively studied, which have

- Galilean symmetry $(H, \vec{P}, \vec{J}, \vec{G})$ or Bargmann symmetry ($U(1)$ central extension) in (3+1)D **J-M. Lévy-Leblond, CMP6 286 (1967)**

- Schroedinger symmetry = NR version of conformal symmetry
  C. R. Hagen, PRD5 (1972) 377

- Lifshitz symmetry $(H, \vec{P}, \vec{J}, D)$ **E. Lifshitz, ZETF11 (1941) 255, 269**

- NR + gauge symmetries: CS-matter theories **G. Lozano, PLB283 (1992)**
  Galilean Electrodynamics
  G. Festuccia, D. Hansen, J. Hartong, N. Obers, 1607.01753
  S. Chapman, L. Di Pietro, K.T. Grosvenor, Z. Yan, 2007.03033

NR models **with SUSY** have been studied, in

- (3+1)D $\Rightarrow$ WZ model, Lifshitz models, NR corners of N=4 SYM

  R. Puzalowski, Acta Phys. Austriaca 50 (1978) 45; T. E. Clark and S. T. Love, NPB 231 (1984); J. A. de Azcarraga and D. Ginestar, JMP 32 (1991); P. Fendley, K. Schoutens, J. de Boer, 0210161; R. Dijkgraaf, D. Orlando, and S. Reffert, 0903.0732; S. Chapman, Y. Oz, A. Raviv-Moshe, 1508.03338; A. Meyer, Y. Oz, A. Raviv-Moshe, 1703.04740; S. Baiguera, T. Harmark, N. Wintergerst, 2009.03799

- (2+1)D $\Rightarrow$ $\mathcal{N}=2$ Chern-Simons-matter theory (enhanced Schroedinger symmetry)

  M. Leblanc, G. Lozano, H. Min, AP 219 (1992) 328; O. Bergman, C. B. Thorn, PRD 52 (1995) 5997

$\Rightarrow$ NR curved backgrounds (Newton-Cartan geometry)

D.T. Son, M. Wingate, AoP321 (2006)
R. Auzzi, S. Baiguera, G. Nardelli, JHEP02 (2016)
K. Jensen, SciPost Phys. 5 (2018)
R. Andringa, E. A. Bergshoeff, J. Rosseel and E. Sezgin, CQG30 (2013)

$\Rightarrow$ NR strings

J. Gomis, H. Ooguri, 0009181
T. Harmark, J. Hartong, N.A. Obers, 1705.03535
E. Bergshoeff, J. Gomis, Z. Yan, 1806.06071
J. Gomis, Z. Yan, M. Yu, 2007.01886

From a theoretical point of view there are still interesting open questions:

- Which are the renormalization properties of NR SUSY theories?
- Does SUSY conspire with the NR space-time symmetry to mild UV divergences?
- Do non-renormalization theorems still work ?

We focus on (2+1)D field theories

Plan of the talk

1) Construction of the NR $\mathcal{N} = 2$ Superspace

2) NR Wess-Zumino Model. Perturbative analysis, renormalization properties, one-loop exactness

3) Conclusions and future directions

SUSY Extended Galilean algebra

There are different ways to obtain the Bargmann algebra in (d+1)D

- Taking the Inönü-Wigner contraction of the (d+1)D Poincaré $\otimes\, U(1)$ algebra in the $c \to \infty$ limit
- By dimensionally reducing the ((d+1)+1)D Poincaré algebra along a **null direction**

Similarly, we can construct the Super-Bargmann algebra in (d+1)D

- Completing the Galilean algebra with a set of fermionic generators and impose constraints on the algebra
- Taking the Inönü-Wigner contraction of the (d+1)D super-Poincaré $\otimes\, U(1)$ algebra in the $c \to \infty$ limit
- By dimensionally reducing the ((d+1)+1)D relativistic SUSY algebra along a **null direction**

$\Rightarrow$ To construct a NR Superspace the most convenient approach is null reduction

$\mathcal{N} = 2$ SUSY Bargmann algebra in (2+1)D

$$[P_j, G_k] = i\delta_{jk} M \,, \qquad [H, G_j] = iP_j \,,$$
$$[P_j, J] = -i\epsilon_{jk} P_k \,, \qquad [G_j, J] = -i\epsilon_{jk} G_k \,, \qquad j,k = 1,2$$

$$[Q_1, J] = \tfrac{1}{2} Q_1 \,, \qquad \{Q_1, Q_1^\dagger\} = \sqrt{2} H \,,$$
$$[Q_2, J] = -\tfrac{1}{2} Q_2 \,, \quad [Q_2, G_1 - iG_2] = -iQ_1 \,, \quad \{Q_2, Q_2^\dagger\} = \sqrt{2} M \,,$$
$$\{Q_1, Q_2^\dagger\} = -(P_1 - iP_2) \,, \quad \{Q_2, Q_1^\dagger\} = -(P_1 + iP_2)$$

Null reduction: We start from the (3+1)D super-Poincarè algebra realized on the spacetime

$$(x^+, x^-, x^{i=1,2}) \qquad x^\pm = \frac{x^3 \pm x^0}{\sqrt{2}} \qquad \text{light} - \text{cone coords.}$$

⋆ compactify x^- on a tiny circle of radius R and rescale $x^+ \to x^+/R$, $x^- \to Rx^-$.

⋆ Define $P_- \equiv M$ and select only the generators which commute with M $\Rightarrow P_+ \equiv H,\ P_{i=1,2},\ M_{12} \equiv J,\ M_{i-} \equiv G_i$ bosonic subalgebra

⋆ In the fermionic sector, write the (3+1)D anticommutator $\{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot\beta}\} = i\sigma^\mu_{\alpha\dot\beta}\partial_\mu$ in terms of the light-cone coordinates

$$\{\mathcal{Q}, \bar{\mathcal{Q}}\} = i\begin{pmatrix} \sqrt{2}\partial_+ & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\sqrt{2}\partial_- \end{pmatrix}$$

Field theory realization: Reduce a generic field as

$$\Phi(x^\mu) = e^{imx^-}\tilde{\Phi}(x^+ \equiv t, x^i) \qquad m \to M - \text{eigenvalue } (\textbf{adimensional})$$

Identify $\partial_+ \to \partial_t\,,\ \partial_- \to im \qquad \mathcal{Q}_\alpha \to Q_\alpha, \quad \bar{\mathcal{Q}}_{\dot\alpha} \to Q^\dagger_\alpha$

Non-relativistic Superspace

Since the null reduction does not affect the fermionic coordinates

$$(3+1)\ \mathcal{N}=1 \text{ relativistic superspace} \Longrightarrow (2+1)\ \mathcal{N}=2 \text{ NR superspace}$$

$$(x^+, x^-, x^i, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}) \qquad \Longrightarrow \qquad (t, x^i, \theta^1, \theta^2, (\theta^1)^\dagger, (\theta^2)^\dagger)$$

$$[t]=-2,\ [x^i]=-1,\ [\theta^1]=-1,\ [\theta^2]=0$$

Reduction of a generic superfield

$$\Phi(x^M, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}) = e^{imx^-}\,\tilde{\Phi}(t, x^i, \theta^1, \theta^2, (\theta^1)^\dagger, (\theta^2)^\dagger)$$

⋆ Covariant derivatives

$$\begin{cases} \mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - \frac{i}{2}\bar{\theta}^{\dot{\beta}}\partial_{\alpha\dot{\beta}} \\ \bar{\mathcal{D}}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - \frac{i}{2}\theta^{\beta}\partial_{\beta\dot{\alpha}} \end{cases} \Longrightarrow \begin{cases} D_1 = \frac{\partial}{\partial\theta^1} - \frac{i}{2}\bar{\theta}^2(\partial_1 - i\partial_2) - \frac{i}{\sqrt{2}}\bar{\theta}^1\partial_t \\ \bar{D}_1 = \frac{\partial}{\partial\bar{\theta}^1} - \frac{i}{2}\theta^2(\partial_1 + i\partial_2) - \frac{i}{\sqrt{2}}\theta^1\partial_t \\ D_2 = \frac{\partial}{\partial\theta^2} - \frac{i}{2}\bar{\theta}^1(\partial_1 + i\partial_2) - \frac{1}{\sqrt{2}}\bar{\theta}^2 M \\ \bar{D}_2 = \frac{\partial}{\partial\bar{\theta}^2} - \frac{i}{2}\theta^1(\partial_1 - i\partial_2) - \frac{1}{\sqrt{2}}\theta^2 M \end{cases}$$

⋆ (Anti)chiral superfields $\bar{D}_\alpha \Sigma = 0, \qquad D_\alpha \bar{\Sigma} = 0$

$$\Sigma(x_L, \theta^\alpha) = \varphi(x_L) + \theta^\alpha \psi_\alpha(x_L) - \theta^2 F(x_L)$$

$$\bar{\Sigma}(x_R, \bar{\theta}^\beta) = \bar{\varphi}(x_R) + \bar{\theta}_\gamma \bar{\psi}^\gamma(x_R) - \bar{\theta}^2 \bar{F}(x_R) \qquad x^{\alpha\beta}_{L,R} = x^{\alpha\beta} \mp i\, \theta^\alpha \bar{\theta}^\beta$$

⋆ Berezin and spacetime integrations

$$\int d^4x d^4\theta\, \Phi = \int d^4x\, \mathcal{D}^2 \bar{\mathcal{D}}^2 \Phi \Big|_{\theta=\bar{\theta}=0} \qquad (\Phi = e^{imx^-}\, \tilde{\Phi})$$

$$\longrightarrow \underbrace{\int d^3x D^2 \bar{D}^2 \tilde{\Phi} \Big|_{\theta=\bar{\theta}=0}} \times \frac{1}{2\pi} \int_0^{2\pi} dx^-\, e^{imx^-}$$

$$\downarrow \qquad\qquad \Downarrow$$

$$\equiv \int d^3x\, d^4\theta\, \tilde{\Phi} \qquad \text{Non} - \text{vanishing result only if } \mathrm{M}(\Phi) = 0$$

Relativistic $\mathcal{N} = 1$ WZ model in (3+1)D

$$S = \int d^4x\, d^4\theta\, \bar{\Sigma}\Sigma + \int d^4x\, d^2\theta \left(\frac{\mu}{2}\Sigma^2 + \frac{\lambda}{3!}\Sigma^3 \right) + \text{h.c.} \qquad \mu = 0$$

- **The WZ model is renormalizable**
- Renormalization in Superspace: UV divergent contributions only to the Kähler potential (no chiral divergent terms). Therefore,

$$\mathcal{L} \to \mathcal{L}_{\text{ren}} = \int d^4\theta\, Z_\Sigma(\bar{\Sigma}\Sigma) + \int d^2\theta\, Z_\lambda Z_\Sigma^{3/2}(\frac{\lambda}{3!}\Sigma^3)$$

The absence of chiral divergences implies

$$Z_\lambda Z_\Sigma^{3/2} = 1 \implies Z_\lambda = Z_\Sigma^{-3/2}$$

- **Non-renormalization theorem**

$\rightarrow$ Perturbative — M.T. Grisaru, W. Siegel, M. Rocek, NPB 159 (1979) 429

$\rightarrow$ Non-perturbative — N. Seiberg, PLB 318 (1993) 469

Holomorphicity, SUSY and R-symmetry

Non-relativistic Wess-Zumino model in (2+1)D

Particle number conservation requires at least two superfields

$$S = \int d^3x d^4\theta \left(\bar{\Phi}_1 \Phi_1 + \bar{\Phi}_2 \Phi_2 \right) + g \int d^3x d^2\theta \, \Phi_1^2 \Phi_2 + \text{h.c.}$$

$$M(\Phi_1) = m, \; M(\Phi_2) = -2m$$

Manifestly invariant under NR $\mathcal{N} = 2$ SUSY

$$\Phi_1 = \varphi_1 + \theta^1 \xi_1 + \theta^2 \, 2^{\frac{1}{4}} \sqrt{m} \, \chi_1 - \frac{1}{2} \theta^\alpha \theta_\alpha F_1$$

$$\Phi_2 = \varphi_2 + \theta^1 \xi_2 + \theta^2 \, i 2^{\frac{1}{4}} \sqrt{2m} \, \chi_2 - \frac{1}{2} \theta^\alpha \theta_\alpha F_2$$

$(\xi_1, F_1) \; (\xi_2, F_2) \longrightarrow$ auxiliary (non-dynamical) fields

Technical subtely: $M(\varphi_2) < 0, M(\chi_2) < 0$

$$S = \int d^3x \left[2im\bar{\varphi}_1\partial_t\varphi_1 + \bar{\varphi}_1\nabla^2\varphi_1 \underbrace{-4im\bar{\varphi}_2\partial_t\varphi_2} + \bar{\varphi}_2\nabla^2\varphi_2 + \ldots \right]$$

Integrate by parts and exchange $\varphi_2 \leftrightarrow \bar{\varphi}_2$ (the same for fermions)

$$S = \int d^3x \left[\bar{\varphi}_1 \left(2im\partial_t + \nabla^2\right)\varphi_1 + \bar{\varphi}_2 \left(4im\partial_t + \nabla^2\right)\varphi_2 \right.$$
$$\left. + \bar{\chi}_1 \left(2im\partial_t + \nabla^2\right)\chi_1 + \bar{\chi}_2 \left(4im\partial_t + \nabla^2\right)\chi_2 \right] + S_{\text{int}}$$

$$S_{\text{int}} = \int d^3x \left[-4|g|^2|\varphi_1\varphi_2|^2 - |g|^2|\varphi_1|^4 \right.$$
$$- ig\left(\sqrt{2}\varphi_1\chi_1(\partial_1 - i\partial_2)\bar{\chi}_2 - 2\bar{\varphi}_2\chi_1(\partial_1 - i\partial_2)\chi_1 + 2\sqrt{2}\varphi_1((\partial_1 - i\partial_2)\chi_1)\bar{\chi}_2\right) + \text{h.c.}$$
$$\left. + 2|g|^2\left(-|\varphi_1|^2\bar{\chi}_1\chi_1 - 4|\varphi_1|^2\bar{\chi}_2\chi_2 + 2|\varphi_2|^2\bar{\chi}_1\chi_1 + 2\sqrt{2}\varphi_1\varphi_2\bar{\chi}_1\bar{\chi}_2 + 2\sqrt{2}\bar{\varphi}_1\bar{\varphi}_2\chi_2\chi_1\right) \right]$$

Renormalization in Superspace

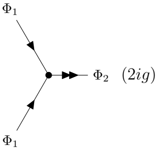
- Superfield propagators

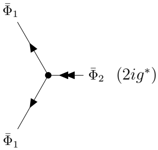
$$\langle \Phi_a(\omega, \vec{p}, \theta_1, \bar{\theta}_1) \bar{\Phi}_a(-\omega, -\vec{p}, \theta_2, \bar{\theta}_2) \rangle = \frac{i}{2m_a\omega - \vec{p}^2 + i\varepsilon} \, \delta^{(4)}(\theta_1 - \theta_2) \qquad a = 1, 2$$

In configuration space

$$D_a(\vec{x}, t) = \int \frac{d^2p \, d\omega}{(2\pi)^3} i \frac{\delta^{(4)}(\theta_1 - \theta_2)}{2m_a \, \omega - \vec{p}^2 + i\varepsilon} \, e^{-i(\omega t - \vec{p} \cdot \vec{x})} = -\frac{i \, \Theta(t)}{4\pi \, t} e^{i \frac{m_a \vec{x}^2}{2t}} \, \delta^{(4)}(\theta_1 - \theta_2)$$

- Supervertices



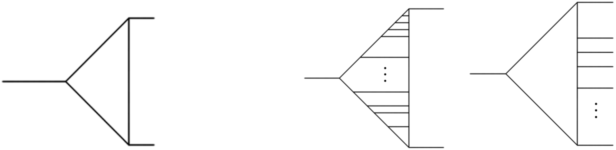


Number of incoming arrows = Number of outgoing arrows

Selection rules

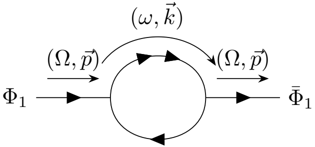
Loop diagrams are formally the same as in the relativistic 2-field WZ model, but....

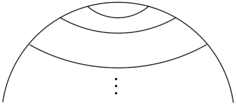
- Selection rule 1 - Particle number conservation at each vertex

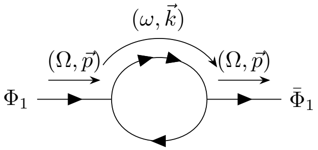


- Selection rule 2 - Arrows inside a Feynman diagram cannot form a closed loop.

O. Bergman, PRD 46 (1992) 5474







⋆ In momentum space

$$i\Gamma_1^{(2)}(\Phi_1, \bar{\Phi}_1) \to 4|g|^2 \int \frac{d\omega\, d^2k}{(2\pi)^3} \frac{1}{\left[4m\omega - \vec{k}^2 + i\varepsilon\right]\left[2m(\omega - \Omega) - (\vec{k} - \vec{p})^2 + i\varepsilon\right]} = 0$$

⋆ In configuration space it would be proportional to $\Theta(t)\Theta(-t) = 0$

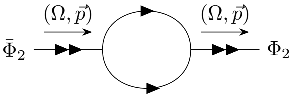
The model is renormalizable for power counting.

$\delta = 2 - E$ **and** ω**-integrations always convergent**

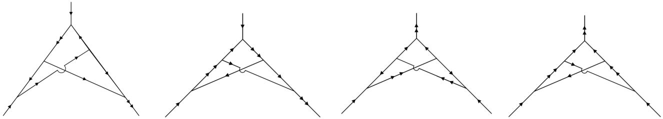
Results

Self-energy corrections - The only non-vanishing diagram at one loop

$$\Gamma_2^{(2)}(\Phi_2, \bar{\Phi}_2) \to \frac{|g|^2}{4\pi m}\frac{1}{\varepsilon}\int d^4\theta\, \Phi_2(\Omega, \vec{p}, \theta)\bar{\Phi}_2(\Omega, \vec{p}, \theta)$$



Vertex corrections - No one-loop. At two loops



They vanish due to circulating loops.

No non-vanishing diagrams arise at higher loops $\Rightarrow$ One-loop exactness

⋆ Non-relativistic non-renormalization theorem

We have proved the NR perturbative non-renormalization theorem (no vertex corrections allowed)
Seiberg's argument can be easily imported and a non-perturbative non-renormalization theorem holds

⋆ Exact beta-function

$$\mathcal{L} \to \mathcal{L}_{\rm ren} = \int d^4\theta \, \left(Z_1 \bar{\Phi}_1 \Phi_1 + Z_2 \bar{\Phi}_2 \Phi_2 \right) + g \int d^2\theta \, Z_g Z_1 Z_2^{1/2} \Phi_1^2 \Phi_2 + \text{h.c.}$$

with $Z_1 = 1, \quad Z_2 = 1 - \frac{|g|^2}{4\pi m} \frac{1}{\varepsilon}$

There are no UV divergent vertex corrections. Therefore,

$$Z_g Z_1 Z_2^{1/2} = 1 \implies Z_g = Z_2^{-1/2} \sim 1 + \frac{|g|^2}{8\pi m} \frac{1}{\varepsilon} \implies \boxed{\beta_g = \frac{g^3}{4\pi m}}$$

The model is classically scale invariant, but scale invariance is lost due to quantum corrections. It is restored at the IR fixed point $g = 0$.
Similar behavior in the bosonic case (O. Bergman, PRD46 (1992))

Conclusions

We have studied quantum properties of the simplest NR susy model in (2+1)D. Working in NR Superspace we have found that:

1. The model is one-loop exact. Scale invariance is broken by one-loop effects.
2. At quantum level the model cannot be obtained as the null reduction of the quantum (3+1)D relativistic model. In particular, the (2+1)D NR theory has much nicer properties compared to its (3+1) parent theory.

Future directions

1. Coupling to gauge fields
2. Coupling to supergravity. Models coupled to NC supergravity as null reduction of relativistic models coupled to Poincaré supergravity in Bergshoeff, Chatzistavrakidis, Lahnsteiner, Romano, Rosseel, 2005.09001
3. Theories with more SUSY (ex: NR ABJM, Y. Nakayama 0902.2267; K.-M. Lee, S. Lee, S. Lee 0902.3857)
4. Integrability in NR systems
5. Opposite limit ($c \to 0$) - super-Carroll models