EFFECTIVE FIELD THEORY FOR NON-LORENTZIAN HYDRODYNAMICS

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[2008.03994] AJ; [2010.15782] J Armas, AJ
[2009.01356] AJ, P Kovtun

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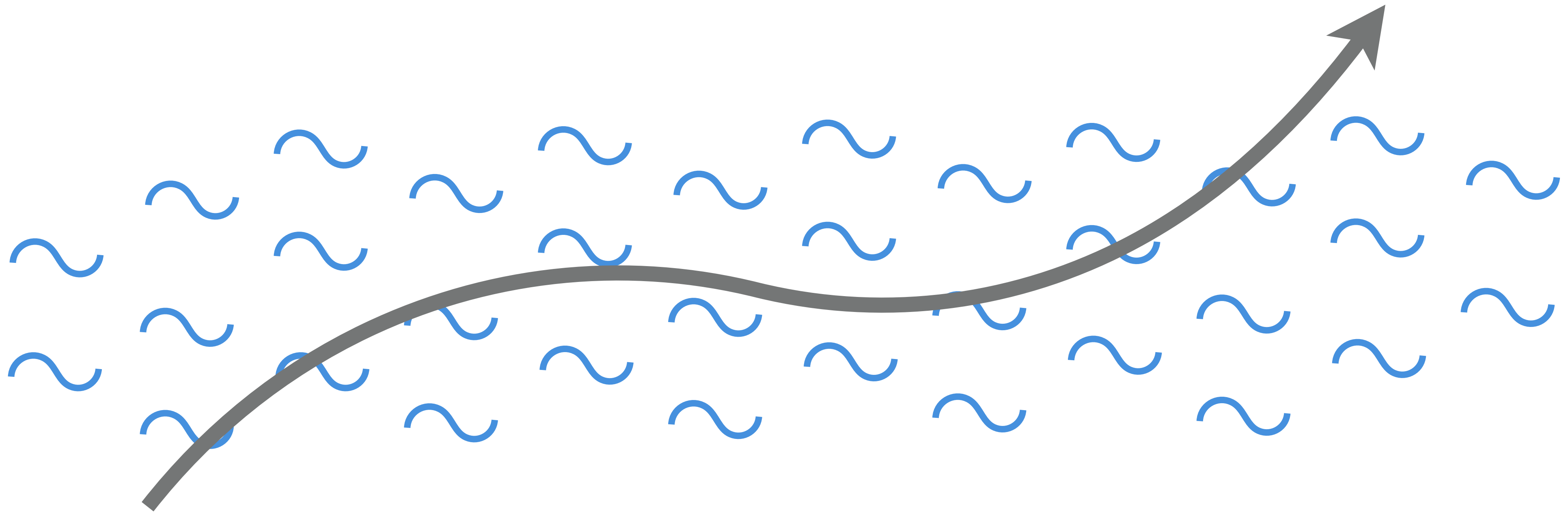
CLASSICAL HYDRODYNAMICS

- Hydrodynamics is an effective description for low-energy long-wavelength fluctuations in a macroscopic system around the equilibrium thermal state.
- At macroscopic scales, all the microscopic excitations have effectively dissipated and the dynamics is dominated by the macroscopic conserved charges (e.g. energy, momentum, particle number) that cannot dissipate locally.
- Hydrodynamics is described by transport coefficients (e.g. viscosities, conductivity) that characterise how fluxes (e.g. energy current, stress tensor, particle flux) respond to variations in the conserved densities.
- Dynamic evolution of the conserved charges is governed by the respective conservation equations.

CLASSICAL HYDRODYNAMICS

- The classical formulation of hydrodynamics is incomplete.
- The non-linear hydrodynamic equations account for mutual interactions between conserved charge modes, but not for possible interactions with the background thermal noise arising due to the ignored microscopic excitations.



- There isn't a first principle derivation of the hydrodynamic equations in the classical framework. It isn't even clear what are the correct degrees of freedom out of equilibrium.
- One would like a better understanding of how the second law of thermodynamics emerges from the underlying effective degrees of freedom and symmetries.

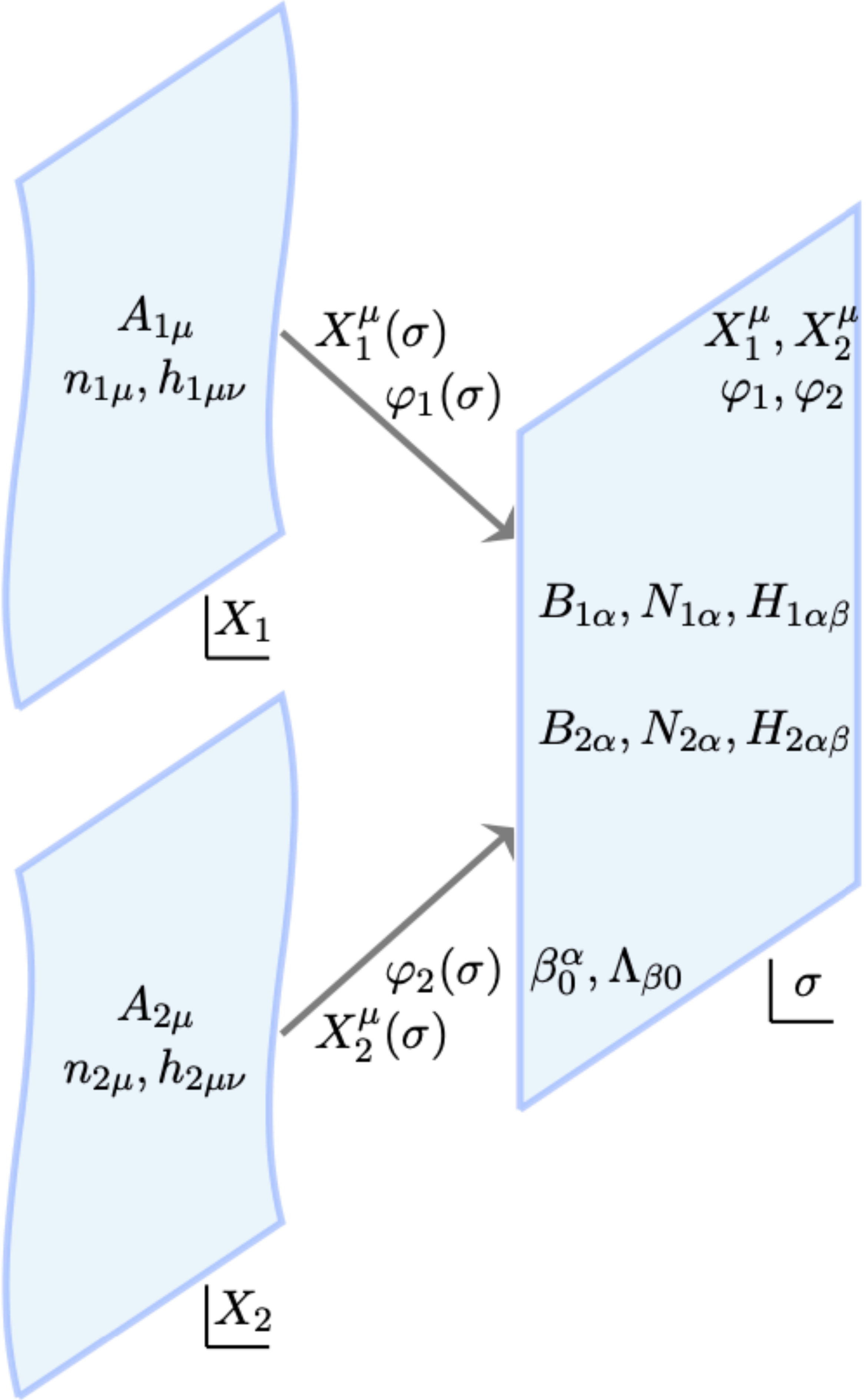
SCHWINGER-KELDYSH FRAMEWORK

- Schwinger-Keldysh (SK) framework provides a consistent path-integral based formulation of hydrodynamics. [1]
- SK framework allows us to classify the most generic (non-Gaussian) stochastic noise into the hydrodynamic setup. [2]
- The framework proposes a set of effective degrees of freedom and symmetries that can be used to construct an effective action for hydrodynamics from scratch.
- The SK framework provides a derivation of the second law of thermodynamics within the context of the hydrodynamic effective field theory. [3]

[1] Crossley, Glorioso, Liu [1511.03646]; Haehl, Loganayagam, Rangamani [1511.07809]; Jensen, Pinzani-Fokeeva, Yarom [1701.07436]
[2] AJ, Kovtun [2009.01356]
[3] Glorioso, Liu [1612.07705]

SCHWINGER-KELDYSH FRAMEWORK

- SK framework of hydrodynamics is setup as a sigma model with two copies of target spacetime fields living on the fluid worldvolume.
- The system respects various worldvolume and target spacetime symmetries. In particular, there is a discrete KMS symmetry that implements fluctuation-dissipation theorem.
- The original models were adapted specifically to relativistic fluids. The aim of the present talk is to revisit these ideas in the absence of Lorentz boost symmetries. [1]



[1] AJ [2008.03994]; J Armas, AJ [2010.15782]

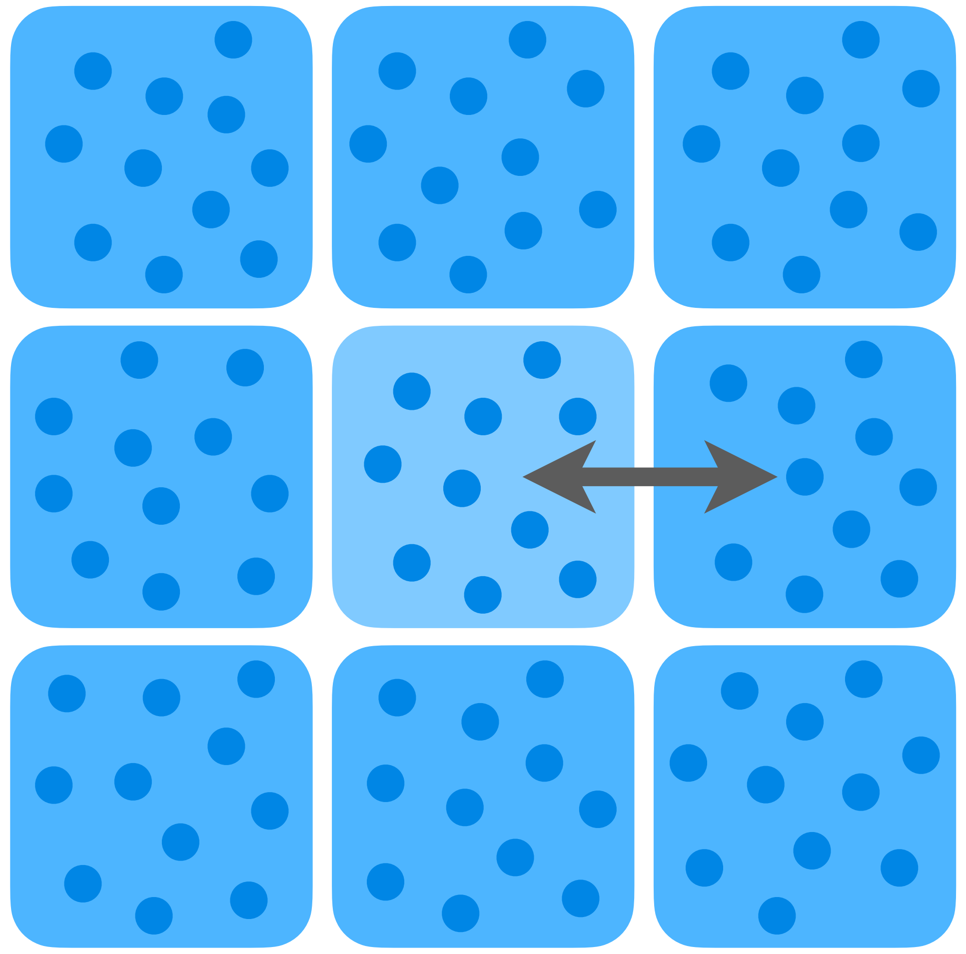
CONSERVATION EQUATIONS

- The dynamical equations of classical hydrodynamics are a set of **conservation equations** associated with the global symmetries that the system enjoys.

Particle number conservation: $\partial_t n + \partial_i j^i = 0$

Energy conservation: $\partial_t \epsilon + \partial_i \epsilon^i = 0$

Momentum conservation: $\partial_t \pi^i + \partial_j \tau^{ij} = 0$



- τ^{ij} is symmetric due to rotational invariance

Angular momentum conservation: $\partial_t \left(\pi^{[i} x^{j]}\right) + \partial_k \left(\tau^{k[i} x^{j]}\right) = \tau^{[ji]}$

- For a relativistic theory with **Lorentz boost symmetry**

Center-of-energy conservation: $\partial_t \left(c^{-2} \epsilon\, x^i - \pi^i t\right) + \partial_k \left(c^{-2} \epsilon^k x^i - \tau^{ki} t\right) = c^{-2} \epsilon^i - \pi^i$

- For Galilean hydrodynamics, we have **Milne boost symmetry**

Center-of-mass conservation: $\partial_t \left(m\, n\, x^i - \pi^i t\right) + \partial_k \left(m\, j^k x^i - \tau^{ki} t\right) = m\, j^i - \pi^i$

COUPLING TO SOURCES

- We can introduce the background sources [1]

$$\begin{pmatrix} n \\ j^i \end{pmatrix} \to A_\mu, \qquad \begin{pmatrix} \epsilon \\ \epsilon^i \end{pmatrix} \to n_\mu, \qquad \begin{pmatrix} -n_k\pi^k/n_t \\ \pi^i \end{pmatrix}, \begin{pmatrix} n_k n_l \tau^{kl}/n_t^2 & -n_k\tau^{kj}/n_t \\ -n_k\tau^{ki}/n_t & \tau^{ij} \end{pmatrix} \to h_{\mu\nu} \text{ with } \det h = 0$$

$$j^\mu \qquad \epsilon^\mu \qquad \pi^\mu \qquad \tau^{\mu\nu}$$

- The coupling structure is given as

$$\delta W = \int dt\, d^3x \sqrt{\gamma} \left[j^\mu \delta A_\mu - \epsilon^\mu \delta n_\mu + \left(v^\mu \pi^\nu + \frac{1}{2}\tau^{\mu\nu} \right) \delta h_{\mu\nu} \right]$$

$$\gamma = \det n_\mu n_\mu + h_{\mu\nu}$$
$$v^\mu h_{\mu\nu} = 0$$
$$v^\mu n_\nu + h^{\mu\lambda}h_{\lambda\nu} = \delta^\mu_\nu$$

We can use the symmetry-invariance of W to obtain the covariant conservation equations.

- One derivative Galilean fluids

$$j^\mu = n\, u^\mu, \qquad \epsilon^\mu = (\epsilon + p)u^\mu - p\, v^\mu + T^2\kappa h^{\mu\nu} L_\beta n_\nu - \frac{1}{2} T\eta^{\mu\nu\rho\sigma}\bar{u}_\nu L_\beta h_{\rho\sigma}$$

$$\pi^\mu = \rho\, u^\mu, \qquad \tau^{\mu\nu} = \rho\, u^\mu u^\nu + p\, h^{\mu\nu} - \frac{1}{2} T\eta^{\mu\nu\rho\sigma}\left(L_\beta h_{\rho\sigma} - 2\bar{u}_{(\rho} L_\beta n_{\sigma)} \right)$$

$$u^\mu n_\mu = 1$$
$$\bar{u}_\mu = h_{\mu\nu}u^\nu, \quad \bar{u}^\mu = h^{\mu\nu}\bar{u}_\nu$$
$$\beta^\mu = \frac{u^\mu}{T}$$
$$\eta^{\alpha\beta\gamma\delta} = 2\eta\, h^{\alpha(\gamma}h^{\delta)\beta} + \left(\zeta - \tfrac{2}{d}\eta\right) h^{\alpha\beta}h^{\gamma\delta}$$

[1] de Boer, Hartong, Have, Obers, Sybesma [2004.10759]

SCHWINGER-KELDYSH SIGMA MODEL

- Schwinger-Keldysh (SK) field theory for hydrodynamics is naturally formulated on the fluid worldvolume characterised by coordinates σ^α.

- On this worldvolume, lives the double copy dynamical fields

$$X_1^\mu(\sigma), \quad X_2^\mu(\sigma), \qquad \varphi_1(\sigma), \quad \varphi_2(\sigma)$$

- We also introduce two copies of background sources

$$A_{1\mu}, \quad A_{2\mu}, \qquad n_{1\mu}, \quad n_{2\mu}, \qquad h_{1\mu\nu}, \quad h_{2\mu\nu}$$

- The theory is invariant under background diffeomorphisms and gauge transformations on the two SK spacetimes. All the dependence must arise through the invariants

$$B_{1,2\alpha} = \partial_\alpha X^\mu_{1,2} A_{1,2\mu} + \partial_\alpha \varphi_{1,2}, \qquad N_{1,2\alpha} = \partial_\alpha X^\mu_{1,2} n_{1,2\mu}, \qquad H_{1,2\alpha\beta} = \partial_\alpha X^\mu_{1,2} \partial_\beta X^\nu_{1,2} h_{1,2\mu\nu}$$

[1] Kamenev, 2011. [2] Crossley, Glorioso, Liu [1511.03646]; Haehl, Loganayagam, Rangamani [1511.07809]; Jensen, Pinzani-Fokeeva, Yarom [1701.07436]

WORLDVOLUME SYMMETRIES

- The worldvolume features a reference thermal vector $\beta_0^\alpha = 1/T_0 \delta_\tau^\alpha$ and reference chemical shift $\Lambda_{\beta 0} = \mu_0/T_0$, characterising the global thermal state.

- Theory is invariant under $(\beta_0^\alpha, \Lambda_{\beta 0})$ preserving local diffeomorphisms and gauge transformations on the worldvolume

$$\sigma^\alpha \to \sigma^\alpha + f^\alpha \quad \textit{such that} \quad \partial_\tau f^\alpha = 0, \qquad \varphi_{1,2} \to \varphi_{1,2} + \lambda \quad \textit{such that} \quad \partial_\tau \lambda = 0$$

- Finally, we have a discrete **KMS symmetry** which acts on the invariants as

$$B_{1\alpha}(\sigma) \to B_{1\alpha}(-\sigma), \qquad B_{2\alpha}(\sigma) \to B_{2\alpha}(-\tau - i\hbar/T_0, -\vec{\sigma})$$

$$N_{1\alpha}(\sigma) \to N_{1\alpha}(-\sigma), \qquad N_{2\alpha}(\sigma) \to N_{2\alpha}(-\tau - i\hbar/T_0, -\vec{\sigma})$$

$$H_{1\alpha\beta}(\sigma) \to H_{1\alpha\beta}(-\sigma), \qquad H_{2\alpha\beta}(\sigma) \to H_{2\alpha\beta}(-\tau - i\hbar/T_0, -\vec{\sigma})$$

This non-local symmetry is quite challenging to implement at the full quantum level, but can be made manifest in the statistical limit $(\hbar \to 0)$.

SCHWINGER-KELDYSH EFFECTIVE ACTION

- The effective action for hydrodynamics is written as

$$
\begin{aligned}
S[X^{\mu}_{1,2}, \varphi_{1,2}; A_{1,2\mu}, n_{1,2\mu}, h_{1,2\mu\nu}] &= \int dt\, d^3x \sqrt{\gamma_r}\, \mathscr{L}[B_{1,2\alpha}, N_{1,2\alpha}, H_{1,2\alpha\beta}] \\
&= \int dt\, d^3x \sqrt{\gamma_r}\, \mathscr{L}[B_{r\alpha}, N_{r\alpha}, H_{r\alpha\beta}; B_{a\alpha}, N_{a\alpha}, H_{a\alpha\beta}]
\end{aligned}
$$

  Here “r” and “a” are average and difference combinations of invariants respectively.

- The action is required to satisfy the SK constraints

$$
\mathscr{L}^*[B_{r\alpha}, N_{r\alpha}, H_{r\alpha\beta}; B_{a\alpha}, N_{a\alpha}, H_{a\alpha\beta}] = -\mathscr{L}[B_{r\alpha}, N_{r\alpha}, H_{r\alpha\beta}; -B_{a\alpha}, -N_{a\alpha}, -H_{a\alpha\beta}]
$$

$$
\mathscr{L}[B_{r\alpha}, N_{r\alpha}, H_{r\alpha\beta}; B_{a\alpha} = 0, N_{a\alpha} = 0, H_{a\alpha\beta} = 0] = 0
$$

$$
\mathrm{Im}\, \mathscr{L}[B_{r\alpha}, N_{r\alpha}, H_{r\alpha\beta}; B_{a\alpha}, N_{a\alpha}, H_{a\alpha\beta}] \geq 0
$$

RESTORING BOOST SYMMETRY

- To restore Lorentz boost symmetry, one requires that the dependence on invariants only enters via the combinations

$$G_{1,2\alpha\beta} = -\,c^2 N_{1,2\alpha} N_{1,2\beta} + H_{1,2\alpha\beta}, \qquad B_{1,2\alpha}$$

  These are pullbacks of the relativistic metric on the SK spacetimes.

- To restore Milne boost symmetry, one requires that the dependence on invariants enters only via the higher-dimensional combinations

$$G_{1,2AB} = \begin{pmatrix} 0 & -N_{1,2\beta} \\ -N_{1,2\alpha} & \frac{1}{m} B_{1,2\alpha} N_{1,2\beta} + \frac{1}{m} N_{1,2\alpha} B_{1,2\beta} + H_{1,2\alpha\beta} \end{pmatrix}$$

  The worldvolume diffeomorphisms and gauge-shifts combine to become higher-dimensional diffeomorphisms.

- This theory can also be obtained directly as an uncharged higher-dimensional relativistic fluid in the presence of a null Killing vector — **null fluid**.

EXPLICIT EFFECTIVE ACTION

- The explicit effective action for dissipative hydrodynamics at one-derivative order is given as

$$\mathscr{L} = n\, u^\alpha B_{a\alpha} - \epsilon\, u^\alpha N_{a\alpha} + p\left(\frac{1}{2} H_r^{\alpha\beta} H_{a\alpha\beta} - u^\alpha H_{r\alpha}^{\ \ \beta} N_{a\beta}\right) + \frac{1}{2}\rho\, u^\alpha u^\beta H_{a\alpha\beta}$$

$$+iT \begin{pmatrix} B_{a\alpha} \\ -N_{a\alpha} \\ \frac{1}{2} H_{a\alpha\beta} \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & T\kappa\, H_r^{\alpha\gamma} & \eta^{\alpha\beta\gamma\delta}\bar{u}_\beta \\ 0 & \eta^{\alpha\beta\gamma\delta}\bar{u}_\delta & \eta^{\alpha\beta\gamma\delta} \end{pmatrix} \begin{pmatrix} B_{a\gamma} + \frac{i}{T_0}\partial_\tau B_{r\gamma} \\ -N_{a\gamma} - \frac{i}{T_0}\partial_\tau N_{r\gamma} \\ \frac{1}{2} H_{a\gamma\delta} + \frac{1}{2}\frac{i}{T_0}\partial_\tau H_{r\gamma\delta} \end{pmatrix} + \mathcal{O}(\hbar)$$

$$u^\alpha = \frac{\delta_\tau^\alpha}{N_{r\tau}}, \quad T = \frac{T_0}{N_{r\tau}}, \quad \mu = \frac{\mu_0 + B_{r\tau}}{N_{rt}} \qquad \bar{u}_\alpha = H_{r\alpha\beta} u^\beta, \quad \bar{u}^\alpha = H_r^{\alpha\beta}\, \bar{u}_\beta \qquad \eta^{\alpha\beta\gamma\delta} = 2\eta\, H_r^{\alpha(\gamma} H_r^{\delta)\beta} + \left(\zeta - \frac{2}{d}\eta\right) H_r^{\alpha\beta} H_r^{\gamma\delta}$$

All coefficients are functions of T, μ, $\bar{u}^2 = \bar{u}^\alpha \bar{u}_\alpha$. We have constraints

$$\epsilon + p = T\frac{\partial p}{\partial T} + \mu\frac{\partial p}{\partial \mu} + 2\bar{u}^2\frac{\partial p}{\partial \bar{u}^2}, \qquad n = \frac{\partial p}{\partial \mu}, \qquad \rho = 2\frac{\partial p}{\partial \bar{u}^2}, \qquad \eta, \zeta, \kappa \geq 0$$

We have only included dissipative corrections for Galilean hydrodynamics for simplicity.

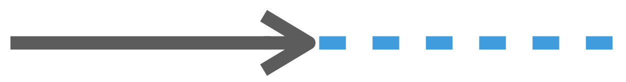
WHY SHOULD YOU CARE?

- Schwinger-Keldysh framework provides a first principle derivation of classical hydrodynamic equations via a variational principle.
- It provides a symmetry-based understanding of the second law of thermodynamics.
- The formalism can be used to consistently introduce various patters of spontaneous and explicit symmetry breaking, leading to effective theories of a host of real world systems e.g. superfluids, crystals [1], boost-non-invariant fluids etc.
- Upon consistently introducing stochastic noise corrections within the EFT, one finds that classical transport coefficients do not universally characterise the long-distance late-time behaviour of near-equilibrium thermal systems. [2]

[1] Landry [1912.12301]
[2] AJ, Kovtun [2009.01356]

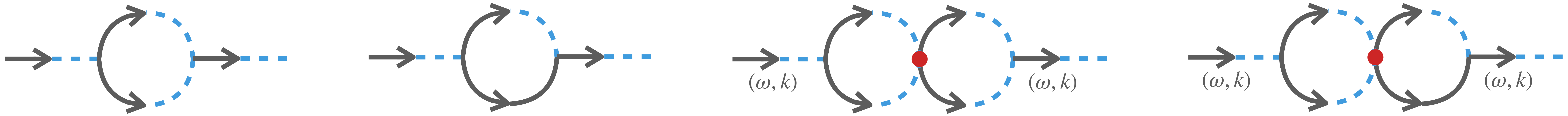
NON-UNIVERSAL STOCHASTIC CORRECTIONS

- Tree-level retarded two-point Green's function can be recovered using the mixed propagator



$$G^{R,cl}_{nn}(\omega, k) = \frac{ik^2\,\sigma}{\omega + iDk^2} + \dots$$

- One loop correction to this comes from the diagrams [1,2]



$$G^{R}_{nn}(\omega, k) = \frac{ik^2}{\omega + iDk^2}\left[\chi D + \frac{\omega k^2}{\omega + iDk^2}\frac{\chi^2 T}{32\pi D}\left(\frac{\partial D}{\partial n}\right)^2\sqrt{k^2 - 2i\omega/D} + \dots\right.$$
$$\left. - \frac{\omega k^2 T}{1024\pi^2 D^2}\left(\frac{\partial D}{\partial n}\right)^2 (k^2 - 2i\omega/D)\left(\frac{1/6\,\vartheta_1 k^4}{\omega + iDk^2} + \frac{2/3\,\vartheta_1 + \vartheta_2}{D^2}(\omega + iDk^2)\right) + \dots\right]$$

Hydrodynamic correlation functions get **non-universal** corrections that are not fixed by classical transport coefficients. [2]

[1] Chen-Lin, Delacretaz, Hartnoll [1811.12540]
[2] AJ, Kovtun [2009.01356]

OUTLOOK

- Classical hydrodynamics is an incomplete description of long-wavelength low-energy description of near-equilibrium thermal systems.
- A better and more complete description is offered by the Schwinger-Keldysh effective field theory framework.
- While the original construction of SK EFTs assumes Lorentz boost symmetry, the framework can be generalised to Galilean hydrodynamics or the absence of any boost symmetry altogether.
- This is helpful for a plethora of applications to real world scenarios, where the setting is typically non-relativistic (Galilean). The formalism also applies to systems with a preferred frame of reference leading to no boost invariance in the effective theory such as active matter.

[2009.01356] Non-universality

arxiv.org/abs/2009.01356

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High Energy Physics – Theory

[Submitted on 2 Sep 2020]

Non-universality of hydrodynamics

Akash Jain, Pavel Kovtun

mySpires. Diffusion | Effective Action | Schwinger-Keldysh | Stochastic Hydrodynamics

We investigate the effects of stochastic interactions on hydrodynamic correlation functions using the Schwinger-Keldysh effective field theory. We identify new "stochastic transport coefficients" that are invisible in the classical constitutive relations, but nonetheless affect the late-time behaviour of hydrodynamic correlation functions through loop corrections. These results indicate that classical transport coefficients do not provide a universal characterisation of long-distance, late-time correlations even within the framework of fluctuating hydrodynamics.

Comments: 5 pages + Supplementary Material

Subjects: **High Energy Physics – Theory (hep-th)**; Statistical Mechanics (cond-mat.stat-mech); High Energy Physics – Phenomenology (hep-ph); Mathematical Physics (math-ph); Fluid Dynamics (physics.flu-dyn)

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(or arXiv:2009.01356v1 **[hep-th]** for this version)

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