Non-relativistic limit of Jackiw-Teitelboim gravity

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Zoom Seminars on Non-Lorentzian Theories

February 11, 2021

Based on 2011.15053, 21......
in collaboration with Diego Hidalgo and Patricio Salgado-Rebolledo

Overlap with Grumiller, Hartong, Prohazka, Salzer 2011.13870

Plan of the talk

- Motivation
- JT gravity
- NR JT gravity, first order
- Boundary theory. Non-linear realizations and Schwarzians
- Newton-Cartan structures
- Comments on 2d Carroll symmetries
- Conclusions

Motivation

- (NR) Non-relativistic gravity and NR holography
- Soluble sectors of the SYK model
- Non-relativistic Schwarzian

2d Jackiw-Teitelboim gravity

- The JT gravity action is Jackiw 1985, Teitelboim 1983

$$S_{\rm JT} = \tilde{\kappa} \int d^2x \sqrt{-g}\, \Phi(\mathcal{R} - 2\tilde{\Lambda})\,,$$

where $\mathcal{R}$ is the Ricci scalar, $\tilde{\Lambda}$ is the cosmological constant, Φ a scalar field, $\tilde{\kappa}$ a two-dimensional coupling constant.

JT gravity

- Equations of motion

$$\mathcal{R} - 2\tilde{\Lambda} = 0$$

constant curvature,
scalar field

$$\left(\nabla_\mu \nabla_\nu - \frac{1}{2} g_{\mu\nu} \nabla^2\right)\Phi = 0$$

$$(\nabla^2 - 2\tilde{\Lambda})\Phi = 0$$

2d Jackiw-Teitelboim gravity

- The action including the boundary term is

Almheiri,Grumiller, Myers, Polchinski

$$S_{\rm JT} = \tilde{\kappa} \int_{\mathcal{M}} d^2x \sqrt{-g}\, \Phi(\mathcal{R} - 2\tilde{\Lambda}) + 2\tilde{\kappa} \int_{\partial\mathcal{M}} dy \sqrt{-g}\, \Phi(\mathcal{K} - 2\mathcal{K}_0)$$

where K is the extrinsic curvature and h is the induced metric on the boundary

JT gravity

- A first order formulation can be defined by gauging $(\mathrm{A})\mathrm{dS}_2$ symmetry and constructing a BF theory Fukuyama, Kamimura 1985; Isler, Trugenberger 1989; Chamseddine, Wyler 1989;Jackiw 1993

$$\left[\tilde{J}, \tilde{P}_a\right] = \epsilon_a{}^b \tilde{P}_b \,, \qquad \left[\tilde{P}_a, \tilde{P}_b\right] = -\tilde{\Lambda}\epsilon_{ab}\tilde{J} \,,$$

The scalar field B takes value on the Lie algebra

$$B = \Phi^a \, \tilde{P}_a + \Phi \, \tilde{J}$$

JT gravity

- The gauge field and the field strength are

$$A = E^a \tilde{P}_a + \Omega \tilde{J} ,$$

$$F = R^a(\tilde{P})\tilde{P}_a + R(\tilde{J})\tilde{J} , \qquad R^a(\tilde{P}) = dE^a - \epsilon^a{}_b \, \Omega E^b , \qquad R(\tilde{J}) = d\Omega - \frac{\tilde{\Lambda}}{2}\epsilon_{ab}E^a E^b$$

The BF action is given by

$$S[B, A] = \int \langle B, F \rangle = \tilde{\mu} \int \left[\Phi R(\tilde{J}) - \tilde{\Lambda} \Phi_a R^a(\tilde{P}) \right]$$

JT gravity

- the invariant metric is given by

$$\langle \tilde{J}, \tilde{J} \rangle = \tilde{\mu}, \qquad \langle \tilde{P}_a, \tilde{P}_b \rangle = -\tilde{\mu}\,\tilde{\Lambda}\,\eta_{ab}\,,$$

The equations of motion are

$$\begin{aligned}
\delta\Phi^a : &\quad R^a(\tilde{P}) = 0\,,\\
\delta\Phi : &\quad R(\tilde{J}) = 0\,,\\
\delta E^a : &\quad d\Phi^a - \epsilon^a{}_b\left(\Phi E^b + \Omega\Phi^b\right) = 0\\
\delta\Omega : &\quad d\Phi - \tilde{\Lambda}\epsilon_{ab}E^a\Phi^b = 0\,.
\end{aligned}$$

Non-relativistic JT gravity

- The NR limit of JT gravity in the first-order formulation can be obtained from NR limit ofBF theory with gauge algebra $(\mathrm{A})\mathrm{dS}_2 \times \mathbb{R}$

The NR limit is given by

$$\tilde{P}_0 = \frac{\varepsilon}{2} H + \frac{1}{\varepsilon} M\,,$$

$$\tilde{P}_1 = P\,,$$

$$\tilde{J} = \frac{1}{\varepsilon} G\,,$$

$$\tilde{Y} = \frac{\varepsilon}{2} H - \frac{1}{\varepsilon} M$$

$$\Lambda = \frac{1}{\varepsilon^2}\tilde{\Lambda}\,,$$

$$\varepsilon = 1/c \to 0$$

NR JT gravity

- The contracted algebra is the Newton-Hooke algebra Bacry, Levy-Leblond 1968

$$[G, H] = P, \qquad [G, P] = M, \qquad [H, P] = -\Lambda\, G.$$

The relativistic and NR gauge fields are related

Bergshoeff, Rosseel, Zojer 2015

$$A = E^a \tilde{P}_a + \Omega \tilde{J} + X\tilde{Y} = \tau H + eP + \omega G + mM$$

$$E^0 = \frac{1}{\varepsilon}\tau + \frac{\varepsilon}{2} m\,, \qquad E^1 = e\,,$$

$$\Omega = \varepsilon\,\omega\,, \qquad X = \frac{1}{\varepsilon}\tau - \frac{\varepsilon}{2} m$$

NR JT gravity

- For the scalar fields we impose

$$\Phi^0 = \frac{1}{\varepsilon}\eta + \frac{\varepsilon}{2}\zeta\,,$$

$$\Phi^1 = \rho\,,$$

$$\Phi = \varepsilon\,\phi\,,$$

$$\Psi = \frac{1}{\varepsilon}\eta - \frac{\varepsilon}{2}\zeta\,.$$

The action of NR gravity is

$$S = \mu \int \left(\phi R(G) + \Lambda\left(\eta R(M) + \zeta R(H) - \rho R(P)\right)\right)$$

$$\mu = \varepsilon^2\,\tilde{\mu}\,,$$

a quadratic divergent term appearing in Sl(2,R) term of the action has been cancelled by the U(1) term

NR JT gravity

$$R(H) = d\tau\,, \qquad R(G) = d\omega - \Lambda\tau e\,,$$

$$R(P) = de + \omega\tau \qquad R(M) = dm + \omega e\,.$$

The equations of motion are

$$\begin{aligned} \delta\eta:& \quad R(H) = 0\,, \\ \delta\rho:& \quad R(P) = 0\,, \\ \delta\phi:& \quad R(G) = 0\,, \\ \delta\zeta:& \quad R(M) = 0\,, \end{aligned} \qquad \begin{aligned} \delta\tau:& \quad d\eta = 0\,, \\ \delta e:& \quad d\rho + \omega\eta - \tau\phi = 0\,, \\ \delta\omega:& \quad d\phi - \Lambda(\tau\rho - e\eta) = 0\,, \\ \delta m:& \quad d\zeta + \omega\rho - e\phi = 0\,. \end{aligned}$$

There is no torsion $\quad d\tau = 0 \quad \Longrightarrow \quad \tau = d\lambda$

NR JT gravity

- Using the equation of motion R(P)=0 we can express the spin connection as

$$\omega_\mu = 2\,\tau^{[\alpha} e^{\beta]} \left(e_\mu \partial_\alpha e_\beta - \tau_\mu \partial_\alpha m_\beta\right).$$

The second order NR action is

$$S = \mu \int \phi\, R(G) \;=\; -\mu \int \det(\tau e)\phi \left(\mathcal{R}^{NR} - 2\Lambda\right)$$

$$\mathcal{R}^{NR} \equiv \overset{(-2)}{\mathcal{R}} = 8\tau^{[\mu} e^{\nu]} \partial_\mu \left(\tau^{[\alpha} e^{\beta]} (\partial_\alpha e_\beta e_\nu - \partial_\alpha m_\beta \tau_\nu)\right)$$

Boundary theory

- In order to have a well-defined variational problem. We need to pay attention to the boundary terms

$$S[B,A] = \int_{\mathcal{M}} \langle B, F \rangle + \int_{\partial\mathcal{M}} b\,,$$

We consider the bulk coordinates $x^\mu = (t, r)$

The boundary is defined by $r \to \infty$

$$A = A_t\, dt + A_r\, dr\,, \qquad\qquad b = b_t dt\,.$$

Boundary theory

- Imposing the boundary condition

$$B\Big|_{\partial\mathcal{M}} = k\, A_t \qquad \Longrightarrow \qquad b_t = -\frac{k}{2}\langle A_t, A_t\rangle$$

Saad, Shenker, Stanford, 2019

On the bulk the gauge field is pure gauge $A = g^{-1}dg$ Bañados 1996

If we choose the gauge fixing condition $\partial_t A_r = 0 \implies g(t,r) = U(t)\, b(r)$

$A = b^{-1}db + b^{-1}ab$ where $a = U^{-1}dU$ U only depends on t

We consider the boundary target coordinate t as a world-line parameter of a particle and

$$a = \Omega^\star$$

Boundary theory

The boundary action becomes

$$S_{\mathrm{bdy}}[U] = -\frac{k}{2}\int dt \left\langle U^{-1}U', U^{-1}U' \right\rangle = -\frac{k}{2}\int dt \, \langle \Omega, \Omega \rangle^{\star}$$

We have an action of a particle moving in a group manifold

Boundary theory

(A)dS$_2$ case

$\mathfrak{sl}(2,\mathbb{R})$ algebra

$$[\tilde{\mathcal{H}},\tilde{\mathcal{D}}]=\tilde{\mathcal{H}}\,,\qquad [\tilde{\mathcal{K}},\tilde{\mathcal{D}}]=-\tilde{\mathcal{K}}\,,\qquad [\tilde{\mathcal{H}},\tilde{\mathcal{K}}]=2\,\tilde{\mathcal{D}}\,.$$

The non-degenerate bilinear form is

$$\langle\tilde{\mathcal{D}},\tilde{\mathcal{D}}\rangle=\gamma_0\,,\qquad \langle\tilde{\mathcal{H}},\tilde{\mathcal{K}}\rangle=-2\gamma_0$$

We consider the local parametrization of the group element

$$\tilde{U}\;=\;e^{\tilde{\rho}\tilde{\mathcal{H}}}\,e^{\tilde{y}\tilde{\mathcal{K}}}\,e^{\tilde{u}\tilde{\mathcal{D}}}$$

Boundary action

- The MC form Ω

$$\tilde{\Omega} = \tilde{\Omega}_{\tilde{\mathcal{H}}}\,\tilde{\mathcal{H}} + \tilde{\Omega}_{\tilde{\mathcal{D}}}\,\tilde{\mathcal{D}} + \tilde{\Omega}_{\tilde{\mathcal{K}}}\,\tilde{\mathcal{K}} = \begin{pmatrix} \frac{1}{2}\tilde{\Omega}_{\tilde{\mathcal{D}}} & -\tilde{\Omega}_{\tilde{\mathcal{K}}} \\ \tilde{\Omega}_{\tilde{\mathcal{H}}} & -\frac{1}{2}\tilde{\Omega}_{\tilde{\mathcal{D}}} \end{pmatrix}$$

$$\tilde{\mathcal{H}} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \qquad \tilde{\mathcal{D}} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \tilde{\mathcal{K}} = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$$

$$\tilde{\Omega}_{\tilde{\mathcal{H}}} = e^{\tilde{u}}\,d\tilde{\rho}\,, \qquad \tilde{\Omega}_{\tilde{\mathcal{K}}} = e^{-\tilde{u}}\left(\tilde{y}^2\,d\tilde{\rho} + d\tilde{y}\right), \qquad \tilde{\Omega}_{\tilde{\mathcal{D}}} = d\tilde{u} + 2\tilde{y}\,d\tilde{\rho}$$

Boundary theory

- The action is

$$S[U] = -\frac{k}{2}\int dt\,\langle\tilde{\Omega},\tilde{\Omega}\rangle^\star = -\frac{k\gamma_0}{2}\int dt\left(4\tilde{\rho}'\,(\tilde{y}\,\tilde{u}' - \tilde{y}') + \tilde{u}'^2\right)$$

depends on three variables, we can reduce to two of them by imposing the Inverse Higgs mechanism Ivanov, Oglevestsky 1975

$$\tilde{\Omega}_{\tilde{\mathcal{H}}} = \mu\,, \qquad \tilde{\Omega}_{\tilde{\mathcal{D}}} = 0$$

$$\tilde{u} = \log\left(\frac{\mu}{\tilde{\rho}'}\right), \qquad \tilde{y} = -\frac{\tilde{u}'}{2\tilde{\rho}'}$$

The gauge field on the boundary becomes

$$a \equiv \tilde{\Omega}^\star\Big|_{IHM} = \mu\,\tilde{\mathcal{H}} + L(\tilde{\rho})\,\tilde{\mathcal{K}} = \begin{pmatrix} 0 & -L(\tilde{\rho}) \\ \mu & 0 \end{pmatrix}$$

Drinfeld-Sokolov reduction

Boundary theory

$$L(\tilde{\rho}) := \frac{1}{2\,\mu}\mathrm{Sch}(\tilde{\rho}, t)$$

$$\mathrm{Sch}(\tilde{\rho}, t) = \frac{\tilde{\rho}'''}{\tilde{\rho}'} - \frac{3}{2}\left(\frac{\tilde{\rho}''}{\tilde{\rho}'}\right)^2$$

We have the action of the Schwarzian mechanical model invariant under global SL(2,R) transformations

$$S[\tilde{\rho}] = k\gamma_0 \int dt\, \mathrm{Sch}(\tilde{\rho}, t)$$

$$\tilde{\rho}_\epsilon = \frac{a\,\tilde{\rho} + b}{c\,\tilde{\rho} + d}, \qquad a\,b - c\,d = 1\,, \qquad \Longrightarrow \qquad S[\tilde{\rho}_\epsilon] = S[\tilde{\rho}]$$

Boundary theory

- Let us find the associated gauge field and metric on the bulk. We can introduce the radial dependence performing a gauge transformation

$$b = \begin{pmatrix} e^{r/2} & 0 \\ 0 & e^{-r/2} \end{pmatrix}$$

$$A = b^{-1}db + b^{-1}\Omega^{\star}b$$

$$A = \begin{pmatrix} \frac{dr}{2} & 0 \\ 0 & -\frac{dr}{2} \end{pmatrix} + \frac{dt}{2}\begin{pmatrix} 0 & -2\,\mathrm{Sch}(\tilde{\rho},t)\,e^{-r} \\ e^{r} & 0 \end{pmatrix}$$

From which we can read the zweibein

$$E^0 = \left(\frac{1}{2}e^{r} - \mathrm{Sch}(\tilde{\rho},t)e^{-r}\right)dt$$

$$E^1 = dr\,,$$

$$\Omega = \left(\frac{1}{2}e^{r} + \mathrm{Sch}(\tilde{\rho},t)e^{-r}\right)dt$$

Sl(2,R) is isomorphic to AdS_2

$$\tilde{\mathcal{D}} = \ell P_0\,, \qquad \tilde{\mathcal{H}} = \ell P_1 + J\,, \qquad \tilde{\mathcal{K}} = \ell P_1 - J$$

Boundary theory

- The associated AdS metric is

$$ds^2_{AdS} = -\left(\frac{1}{4}e^{2r} - \mathrm{Sch}(\tilde{\rho}, t) + e^{-2r}\,\mathrm{Sch}(\tilde{\rho}, t)^2\right)dt^2 + dr^2$$

it is a metric of constant curvature R=-2

The metric close to the boundary is

$$ds^2|_{\mathrm{boundary}} = -e^{2r}\frac{dt^2}{4} = -\frac{dt^2}{\epsilon^2}$$

Boundary theory

(A) $\mathbf{dS_2 \times \mathbb{R}}$ case

We consider the local parametrization of the group element

$$\tilde{U} = e^{\tilde{s}\tilde{\mathcal{Y}}}\, e^{\tilde{\rho}\tilde{\mathcal{H}}}\, e^{\tilde{y}\tilde{\mathcal{K}}}\, e^{\tilde{u}\tilde{\mathcal{D}}}$$

The MC form is

$$\tilde{\Omega} = \tilde{\Omega}_{\tilde{\mathcal{H}}}\,\tilde{\mathcal{H}} + \tilde{\Omega}_{\tilde{\mathcal{D}}}\,\tilde{\mathcal{D}} + \tilde{\Omega}_{\tilde{\mathcal{K}}}\,\tilde{\mathcal{K}} + \tilde{\Omega}_{\tilde{\mathcal{Y}}}\,\tilde{\mathcal{Y}}\,,$$

$$\tilde{\Omega}_{\tilde{\mathcal{H}}} = e^{\tilde{u}}\, d\tilde{\rho}\,,$$

$$\tilde{\Omega}_{\tilde{\mathcal{K}}} = e^{-\tilde{u}}\left(\tilde{y}^2\, d\tilde{\rho} + d\tilde{y}\right)$$

$$\tilde{\Omega}_{\tilde{\mathcal{D}}} = d\tilde{u} + 2\tilde{y}\, d\tilde{\rho}\,,$$

$$\tilde{\Omega}_{\tilde{\mathcal{Y}}} = d\tilde{s}\,.$$

Boundary action

- The action is

$$S[\tilde\rho,\tilde y,\tilde u,\tilde s] = -\frac{k}{2}\int dt\left(\langle\tilde\Omega,\tilde\Omega\rangle\right)^{\star} = -\frac{k}{2}\int dt\left(\gamma_0\left(4\tilde\rho'(\tilde y\,\tilde u' - \tilde y') + \tilde u'^2\right) + \gamma_1\,\tilde s'^2\right),$$

depends on four variables, we can reduce to two of them by imposing the Inverse Higgs mechanism Ivanov, Ogievestsky 1975 Galajinsky 2019

$$\tilde\Omega_{\tilde{\mathcal{H}}} = \mu\,\tilde\Omega_{\tilde{\mathcal{Y}}}\,,$$

$$\tilde\Omega_{\tilde{\mathcal{D}}} = -2\nu\,\tilde\Omega_{\tilde{\mathcal{Y}}}\,,$$

$$\rho' = e^{-u}s'\,,$$

$$\tilde y = -\frac{2\,\nu\,\tilde s' + \tilde u'}{2\tilde\rho'}\,,$$

The action becomes

$$S[\tilde u,\tilde s] = \int dt\left(\gamma_0\left(\tilde u'^2 + 2\tilde u'' - \frac{2\tilde s''\tilde u'}{\tilde s'}\right) + \gamma_1\,\tilde s'^2\right).$$

NR boundary action

Non-relativistic limit of the $SL(2,\mathbb{R})\times\mathbb{R}$ boundary action

A contraction of the $SL(2,\mathbb{R})\times\mathbb{R}$ algebra is

$$[\mathcal{H},\mathcal{D}]=\mathcal{H}\,,\qquad [\mathcal{K},\mathcal{D}]=-\mathcal{K}\,,\qquad [\mathcal{H},\mathcal{K}]=2\mathcal{Z}\,,$$

where we have used

$$\tilde{\mathcal{D}}=\frac{1}{2}\mathcal{D}+\frac{1}{\varepsilon^2}\mathcal{Z}\,,\qquad \tilde{\mathcal{Y}}=\frac{1}{2}\mathcal{D}-\frac{1}{\varepsilon^2}\mathcal{Z}\,,$$

$$\mathcal{H}=\varepsilon\,\tilde{\mathcal{H}}\,,\qquad \mathcal{K}=\varepsilon\,\tilde{\mathcal{K}}\,.$$

Extended Galilean Conformal algebra which is isomorphic NH^+

$$\mathcal{D}=\ell\,H\,,\qquad \mathcal{H}=\ell\,P+G\,,\qquad \mathcal{K}=\ell\,P-G\,,\qquad \mathcal{Z}=\ell\,M$$

NR boundary theory

- The relation among relativistic Goldstone and NR Goldstone fields is

$$\tilde{u} = u + \frac{\varepsilon^2}{2} s \,, \qquad \tilde{\rho} = \varepsilon\, \rho \,,$$

$$\tilde{y} = \varepsilon\, y \,, \qquad \tilde{s} = u - \frac{\varepsilon^2}{2} s \,.$$

If we use $\gamma_1 = -\gamma_0$ the action becomes

$$S[u,s] = 2\,\varepsilon^2\,\gamma_0 \int dt \left(s'' + s' \left(u' - \frac{u''}{u'} \right) \right) + \mathcal{O}\left(\varepsilon^4\right) \quad \rightarrow$$

$$2\,\alpha_0 \int dt \left(s'' + s' \left(u' - \frac{u''}{u'} \right) \right) .$$

NR Schwarzian

where $\gamma_0 := \alpha_0/\varepsilon^2$

NR boundary theory

- Another contraction that leads to NH^- is

$$\tilde{\mathcal{H}} = \frac{1}{2}\mathcal{D} + \frac{1}{2\varepsilon}(\mathcal{H} - \mathcal{K}) + \frac{1}{\varepsilon^2}\mathcal{Z}\,, \qquad (6.9a) \qquad \tilde{\mathcal{D}} = \frac{1}{2\varepsilon}(\mathcal{H} + \mathcal{K})$$

$$\tilde{\mathcal{K}} = \frac{1}{2}\mathcal{D} - \frac{1}{2\varepsilon}(\mathcal{H} - \mathcal{K}) + \frac{1}{\varepsilon^2}\mathcal{Z}\,, \qquad (6.9b) \qquad \tilde{\mathcal{Y}} = \frac{1}{2}\mathcal{D} - \frac{1}{\varepsilon^2}\mathcal{Z}$$

$$[\hat{\mathcal{H}}, \hat{\mathcal{D}}] = \hat{\mathcal{K}}\,, \qquad [\hat{\mathcal{K}}, \hat{\mathcal{D}}] = -\hat{\mathcal{H}}\,, \qquad [\hat{\mathcal{H}}, \hat{\mathcal{K}}] = 2\hat{\mathcal{Z}}$$

Twisted conformal algebra that is isomorphic to NH^-

$$S[\hat{u}, \hat{s}] \;=\; 2i\,\alpha_0 \int dt \left(\hat{s}'' - \hat{s}'\left(i\,\hat{u}' + \frac{\hat{u}''}{\hat{u}'}\right)\right)$$

Afshar, Gonzalez, Grumiller, Vassilevich 2019

Newton-Cartan structure

Extended Newton-Hooke$_2^+$ case

Group element $U = e^{s\mathcal{Z}}\, e^{\rho\mathcal{H}}\, e^{y\mathcal{K}}\, e^{u\mathcal{D}}$

MC form $\Omega = U^{-1}dU = \Omega_{\mathcal{Z}}\,\mathcal{Z} + \Omega_{\mathcal{H}}\,\mathcal{H} + \Omega_{\mathcal{K}}\,\mathcal{K} + \Omega_{\mathcal{D}}\,\mathcal{D}$

$$\Omega_{\mathcal{Z}} = ds + 2\,y\,d\rho\,, \qquad \Omega_{\mathcal{H}} = e^{u}\,d\rho\,, \qquad \Omega_{\mathcal{K}} = e^{-u}\,dy\,, \qquad \Omega_{\mathcal{D}} = du$$

The action after IHM and change of variables gives the NR schwarzian

$$S[v,s] = 2\,c_1 \int dt\left(s'' + s'\left(v' - \frac{v''}{v'}\right)\right)$$

Newton-Cartan structure

- The boundary gauge field takes the form

$$a = \Omega^*\Big|_{IHM} = \Big(\alpha\mathcal{H} + \mathcal{T}(t)\,\mathcal{D} + \mathcal{L}(t)\mathcal{K}\Big)dt = \begin{pmatrix} 0 & \alpha & 0 \\ 0 & \mathcal{T}(t) & \mathcal{L}(t) \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{T}(t) \equiv v' - \frac{v''}{v'}, \qquad \mathcal{L}(t) \equiv \frac{1}{2\alpha v'}\Big(s'\Big(v'' - v'^2\Big) - s''v'\Big)$$

We introduce the radial dependence by a gauge transformation

$$b = e^{-r\mathcal{H}} = \begin{pmatrix} 1 & -r & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Newton-Cartan structure

- The gauge field becomes

$$A = (-dr + (r\mathcal{T}(t) + \alpha)dt)\mathcal{H} + (\mathcal{L}(t)\,\mathcal{K} + \mathcal{T}(t)\,\mathcal{D} + 2r\mathcal{L}(t)\,\mathcal{Z})dt$$

using $\mathcal{D} = \ell\, H\,, \qquad \mathcal{H} = \ell\, P + G\,, \qquad \mathcal{K} = \ell\, P - G\,, \qquad \mathcal{Z} = \ell\, M$

we get the Newton-Cartan structure

$$h_{\mu\nu} = \ell^2 \Big[(r\mathcal{T} + \alpha + \mathcal{L})^2 dt^2 - 2(r\mathcal{T} + \alpha + \mathcal{L}) dr dt + dr^2 \Big]$$

$$\tau^\mu = \frac{1}{\ell\mathcal{T}}(1\,, \mathcal{L} + \alpha + r\mathcal{T}) \qquad h_{\mu\nu}\tau^\nu = 0$$

Carroll symmetries in 2d

Levy-Leblond1965

- The Carroll $(\mathrm{A})\mathrm{dS}_2$ has a central extension

$$[G, P] = H, \qquad [G, H] = M, \qquad [H, P] = -\Lambda\, G$$

Barducci, Casalbuoni,Gomis 2018

If we interchange H by P and change thesign of the cosmological constant we have the following dualities

$$\text{Extended } \mathrm{NH}_2^+ \leftrightarrow \text{Extended Carroll } \mathrm{AdS}_2$$
$$\text{Extended } \mathrm{NH}_2^- \leftrightarrow \text{Extended Carroll } \mathrm{dS}_2\,.$$

Conclusions

- We have constructed the action of NR JT gravity as a NR limit.
- The NR Boundary action is constructed by means of non-linear realizations.
- These analysis can be extended to the ultra-relativistic limit, Carroll limit.

Extra material

- NR JT gravity in second order formalism
- Carroll JT gravity