NON-RIEMANNIAN GRAVITY FROM DFT

BASED ON 2012.07765 WITH U. GURSOY, N. ZINNATO AND S. VERMA

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NON-RIEMANNIAN GEOMETRY

- **Covariantization of non-Lorentzian symmetries**

- **We will focus on three particular examples of non-riemannian geometries**

1. **Torsional Newton Cartan (Non-relativistic with Galilean symmetry)**
2. **Carrol (Ultra relativistic with Carroll symmetry)**
3. **String Newton Cartan (Non-relativistic with String-Galilean symmetry)**

NON-RIEMANNIAN GRAVITY

- **NR-geometries have notions of distance (degenerate metric),curvature, etc**

For SNC there are two temporal directions

$h^{\mu\nu}$
spatial degenerate Inverse metric

τ_μ
Clock one form (temporal vielbein)

$h_{\mu\nu}$
spatial degenerate metric

υ^μ
Inverse temporal vielbein

$$h^{\mu\nu}\tau_\mu = 0$$

$$h_{\mu\nu}\upsilon^\nu = 0$$

$$\tau_\mu \upsilon^\mu = -1$$

$$h_{\mu\rho}h^{\rho\nu} - \tau_\mu \upsilon^\nu = \delta^\mu_\nu$$

$$\{\Gamma^\lambda_{\mu\nu}, R^\lambda{}_{\rho\mu\nu}, \Gamma^\lambda_{[\mu\nu]}\}$$

Parallel transport, curvature And torsion

- **We want to look at the dynamics of these NR geometric objects: equations of motion and possibly action principles. Many possible applications.**

PATHS TO NON-RIEMANNIAN GRAVITY

- **Using symmetries** 1505.05011 Hartong, 1807.04765 Hansen et al.

- **NR limits of Lorentz action/equations of motion**
  Ex 1. $c \to \infty$ or $c \to 0$ limits 1810.09387 Bergshoeff et al. , 1604.08054 Hartong et al.
  Ex 2. 1/c expansion 1703.03459 Van den Bleeken, 2001.10277 Hansen et al. , 2102.06974 Bergshoeff et al.

- **Embedding in known models**
  Ex 1. Null reduction
  Ex2. DFT (Doubled gravity) 2012.07765 Gallegos et al.

- **Other approaches**
  Beta functions 1905.07315 Gomis et al. , 1906.01607 Gallegos et al.

WHY DFT?

- **There exist a relation between T-duality, null isometries and non-riemannian theories**

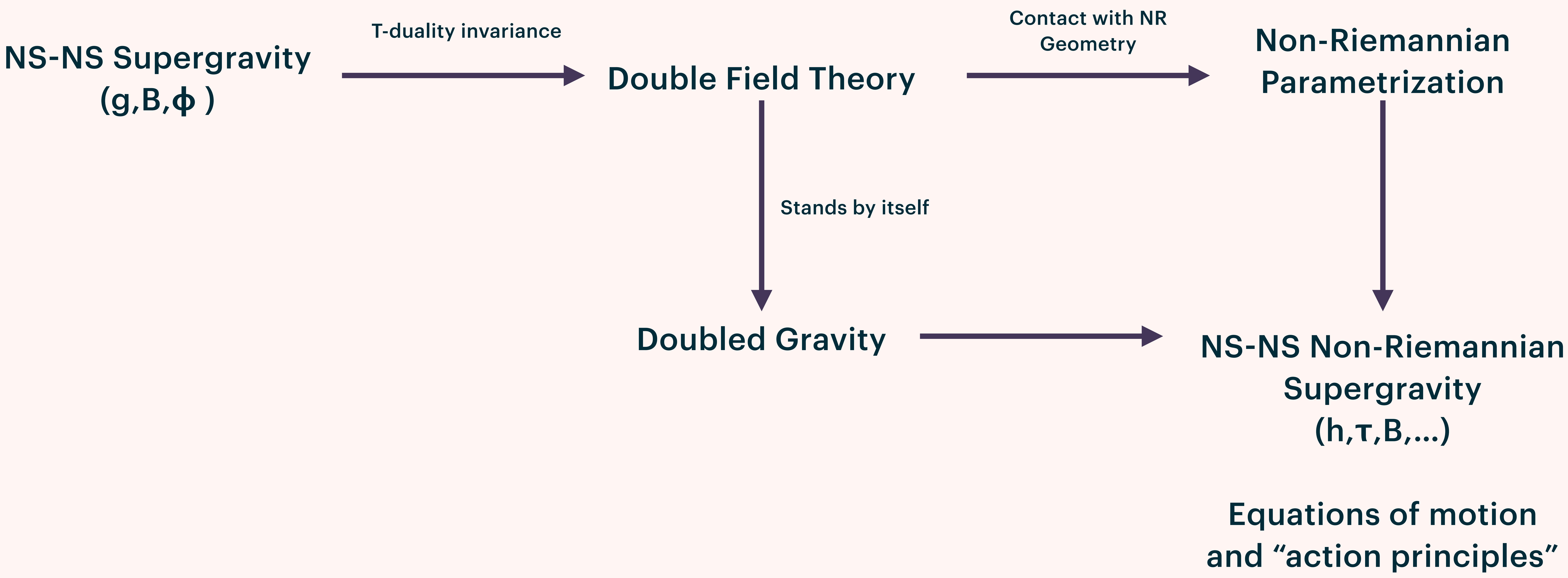
  1806.06071 Bergshoeff et al.

- **DFT provides a T-duality invariant formulation of supergravity**

  0904.4664 Hull & Zwiebach, 1109.1782 Zwiebach, many more

- **DFT is a natural playground to study these theories in a unified way**

  1508.01121 Moon Ko et al. , 1707.03713 Morand and Park


NS-NS Supergravity
(g,B,ϕ)
T-duality invariance
Double Field Theory
Contact with NR
Geometry
Non-Riemannian
Parametrization
Stands by itself
Doubled Gravity
NS-NS Non-Riemannian
Supergravity
(h,τ,B,...)
Equations of motion
and “action principles”

DOUBLED GRAVITY

NS-NS SUPERGRAVITY

- **The basic fields are (g,B,ϕ) and are subject to the equations of motion**

$$R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \phi - \frac{1}{4} H_\mu{}^{\rho\sigma} H_{\nu\rho\sigma} = 0$$

$$\nabla_\rho H^{\rho\mu\nu} - 2\nabla_\rho \phi H^{\rho\mu\nu} = 0$$

- **The equations of motion can be derived from the action**

$$S = \int d^d x e \left[R + 4\nabla_\mu \phi \nabla^\mu \phi - \frac{1}{12} H^{\mu\nu\rho} H_{\mu\nu\rho} \right]$$

NS-NS SUPERGRAVITY AND T-DUALITY

- **The D-dimensional theory exhibits a T-duality symmetry (large compact dimensions are dual to small compact dimensions)**
- **This symmetry is geometrized into an O(D,D) symmetry.**
- **The elements of O(D,D) can be defined as the set of 2Dx2D matrices $\mathscr{H}$ that preserve the O(D,D) invariant metric η_{MN}**

$$\mathcal{H}_M{}^P \eta_{PQ} \mathcal{H}^Q{}_N = \eta_{MN}$$

$$\eta_{MN} = \begin{pmatrix} 0 & \delta^\mu_\nu \\ \delta^\nu_\mu & 0 \end{pmatrix}$$

NS-NS SUPERGRAVITY AND T-DUALITY

- **The theory should be rewritten in terms of O(D,D) covariant objects**
- **We should double the coordinates** $X^M = (X^\mu, \tilde{X}_\nu)$
- **To recover NS-NS gravity we need to impose independence of dual coordinates**
- **(g,B) are captured by a single O(D,D) tensor and ϕ is captured by a tensor density** d

$$\mathcal{H} = \begin{pmatrix} g_{\mu\nu} - B_{\mu\rho} g^{\rho\sigma} B_{\sigma\nu} & -g^{\nu\rho} B_{\rho\mu} \\ -g^{\mu\rho} B_{\rho\nu} & g^{\mu\nu} \end{pmatrix}$$

Generalized Metric

$$e^{-2d} = e^{-2\phi}\sqrt{-g}$$

Generalized Dilaton

A DFT FORMULATION OF GRAVITY

1003.5027. Hohm et al. 1006.4823 Hohm et al.

- **We can rewrite the action in terms of O(D,D) objects**

$$S_{\rm DFT} = \int d^D X d^D \tilde{X} e^{-2d} \mathcal{R}$$

Reduces to NS-NS action

DFT Ricci Scalar

$$\mathcal{R} = 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL}$$

- **From the variation of the generalized metric we find the equations of motion**

DFT Ricci Tensor $\mathcal{R}_{MN} = 0$

$$\mathcal{R} = 0$$

A DFT FORMULATION OF GRAVITY

- The same action can be constructed in the same way the Einstein-Hilbert action is constructed by using O(D,D) symmetry instead of Lorentz symmetry.

  1105.6294 Jeon et al.

- A notion of generalized diffeomorphisms needs to be introduced and from there parallel transport, curvature, torsion can be constructed.

- DFT action stands on its own without referencing its NS-NS origins.

$$\mathcal{H} = \begin{pmatrix} g_{\mu\nu} - B_{\mu\rho} g^{\rho\sigma} B_{\sigma\nu} & -g^{\nu\rho} B_{\rho\mu} \\ -g^{\mu\rho} B_{\rho\nu} & g^{\mu\nu} \end{pmatrix}$$

NS-NS Supergravity (g,B,ϕ) → **Double Field Theory** → **Doubled Gravity**

$$S_{\mathrm{DFT}} = \int d^D X d^D \tilde{X} e^{-2d} \mathcal{R}$$

$$\mathcal{R} = 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL}$$

DFT AND NON-RIEMANNIAN GEOMETRY

GENERAL PARAMETRIZATION OF THE GENERALIZED METRIC

1707.03713 Morand and Park

- **The Riemann parametrization of the generalized metric is not the most general one**

$$\mathcal{H} = \begin{pmatrix} g_{\mu\nu} - B_{\mu\rho} g^{\rho\sigma} B_{\sigma\nu} & -g^{\nu\rho} B_{\rho\mu} \\ -g^{\mu\rho} B_{\rho\nu} & g^{\mu\nu} \end{pmatrix}$$

- **The most general parametrization has been worked out to be of the form**

$$\mathcal{H}_{MN} = \begin{pmatrix} K_{\mu\nu} - \mathcal{B}_{\mu\rho} H^{\rho\sigma} \mathcal{B}_{\sigma\nu} + 2 \left(Z^T\right)^{\rho}{}_{(\mu} \mathcal{B}_{\nu)\rho} & -H^{\nu\rho}\mathcal{B}_{\rho\mu} + Z^{\nu}_{\mu} \\ -H^{\mu\rho}\mathcal{B}_{\rho\nu} + Z^{\nu}_{\mu} & H^{\mu\nu} \end{pmatrix} \qquad Z^{\nu}_{\mu} = y^{\nu}_{i} x^{i}_{\mu} - \bar{y}^{\nu}_{\bar{I}} \bar{x}^{\bar{i}}_{\mu}$$

$$\{0 \le i \le n, 0 \le \bar{i} \le \bar{n}\} \quad \text{s.t.} \quad n + \bar{n} < D$$

- **There are two symmetric tensors ($K_{\mu\nu}$,$H^{\mu\nu}$), an antisymmetric two form $B_{\mu\nu}$, and four sets of vectors {x,$\overline{\text{x}}$,y,$\overline{\text{y}}$}.**

GENERAL PARAMETRIZATION OF THE GENERALIZED METRIC

1707.03713 Morand and Park

- **The parametrization is characterized by the pair of integers $(n, \bar{n})$, their sum is equal to the number of null eigenvectors {x,$\bar{\text{x}}$} and {y,$\bar{\text{y}}$} of the symmetric tensors**

$$H^{\mu\nu} x^i_\nu = H^{\mu\nu} \bar{x}^{\bar{i}}_\nu = 0 \qquad K_{\mu\nu} y^\nu_j = K_{\mu\nu} \bar{y}^\nu_{\bar{j}} = 0$$

- **The difference of these integers is an O(D,D) invariant**

$$\mathcal{H}^M_M = 2(n - \bar{n})$$

- **The fields are subject to the completeness relation**

$$H^{\mu\rho} K_{\rho\nu} + y^\mu_i x^i_\nu + \bar{y}^\mu_{\bar{i}} \bar{x}^{\bar{i}}_\nu = \delta^\mu_\nu$$

$$\{y^\mu_i x^j_\mu = \delta^j_i, \bar{y}^\mu_{\bar{i}} \bar{x}^{\bar{j}}_\mu = \delta^{\bar{j}}_{\bar{i}}, y^\mu_i \bar{x}^{\bar{j}}_\mu = \bar{y}^\mu_{\bar{i}} x^j_\mu = 0\}$$

- **These fields are reminiscent of the usual TNC or SNC metrics**

SYMMETRIES OF THE PARAMETRIZATION

1707.03713 Morand and Park

- **The parametrization has a GL(n)xGL(bar n) local symmetry and a generalize Milne shift symmetry**

$$
\begin{aligned}
(y_i^{\mu})' &= y_i^{\mu} + V_i^{\mu} \\
(\bar{y}_{\bar{\imath}}^{\mu})' &= \bar{y}_{\bar{\imath}}^{\mu} + \bar{V}_{\bar{\imath}}^{\mu} \\
(K_{\mu\nu})' &= K_{\mu\nu} - 2x^i_{(\mu}K_{\nu)\rho}V_i^{\rho} - 2\bar{x}^{\bar{\imath}}_{(\mu}K_{\nu)\rho}\bar{V}_{\bar{\imath}}^{\rho} + \left(x^i_{\mu}V_{\rho i} + \bar{x}^{\bar{\imath}}_{\mu}\bar{V}_{\rho\bar{\imath}}\right)\left(x^i_{\nu}V_i^{\rho} + \bar{x}^{\bar{\imath}}_{\nu}\bar{V}_{\bar{\imath}}^{\rho}\right) \\
(\mathcal{B}_{\mu\nu})' &= \mathcal{B}_{\mu\nu} - 2x^i_{[\mu}V_{\nu]i} + 2\bar{x}^{\bar{\imath}}_{[\mu}\bar{V}_{\nu]\bar{\imath}} + 2x^i_{[\mu}\bar{x}^{\bar{\jmath}}_{\nu]}\left(y_i^{\rho}\bar{V}_{\rho\bar{\jmath}} + \bar{y}_{\bar{\jmath}}^{\rho}V_{\rho i} + V_{\rho i}\bar{V}_{\bar{\jmath}}^{\rho}\right)
\end{aligned}
$$

- **This shift symmetry will reduce to the known Galilean and Carroll boosts**

SOME EXAMPLES

- **The (0,1) or (1,0) theory can be identified with TNC (without the U(1) central extension)**

$$H^{\mu\nu} = h^{\mu\nu} \qquad \{x^1_\mu = \tau_\mu, \bar{x}^{\bar{i}}_\mu = 0\}$$

$$K_{\mu\nu} = h_{\mu\nu} \qquad \{y^\mu_1 = -\upsilon_\mu, \bar{y}^\mu_{\bar{i}} = 0\}$$

- **The (D-1,0) or (0,D-1) theory can be identified with Carroll**

$$H^{\mu\nu} = \upsilon^\mu \upsilon^\nu \qquad \{x^{i=1,..,D}_\mu = e^a_\mu, \bar{x}^{\bar{i}}_\mu = 0\} \qquad e^a_\mu e^b_\nu \delta_{ab} = h_{\mu\nu}$$

$$K_{\mu\nu} = \tau_\mu \tau_\nu \qquad \{y^\mu_{i=1,..,D} = e^\mu_a, \bar{y}^\mu_{\bar{i}} = 0\} \qquad e^\mu_a e^\nu_b \delta^{ab} = h^{\mu\nu}$$

- **SNC can be identified with the (1,1) parametrization**
- **Riemann geometry is identified with (0,0) parametrization**

(SMALL DETOUR) STRING COUPLED TO DFT BACKGROUND

- **For reference it will be convenient to write down the coupling of a string with a DFT background parametrized this way** 1707.03713 Morand and Park, 1908.00074 Blair

$$S = \int d^2\sigma \left[-\frac{1}{2}\sqrt{-\gamma}\gamma^{\alpha\beta}K_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu - \frac{1}{2}\epsilon^{\alpha\beta}B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu \right.$$
$$\left. + \beta_{\alpha i} x^i_\mu \left(\sqrt{-\gamma}\gamma^{\alpha\beta} - \epsilon^{\alpha\beta}\right)\partial_\beta X^\mu - \bar{\beta}_{\alpha\bar{i}} \bar{x}^{\bar{i}}_\mu \left(\sqrt{-\gamma}\gamma^{\alpha\beta} + \epsilon^{\alpha\beta}\right)\partial_\beta X^\mu \right]$$

Lagrange multiplier   Lagrange multiplier

There will be n+bar n Lagrange multipliers enforcing Hamilltonian constraints

- **This action will correspond to the Polyakov action of a string. On the appropriate parametrization it reduces to the known NR-Polyakov actions for TNC and SNC**

Double Field Theory → (Contact with NR Geometry) → General Parametrization

Double Field Theory → (Stands by itself) → Doubled Gravity

General Parametrization → NS-NS Non-Riemannian Supergravity (h,τ,B,...)

Doubled Gravity → NS-NS Non-Riemannian Supergravity (h,τ,B,...)

$$\mathcal{H}_{MN} = \begin{pmatrix} K_{\mu\nu} - \mathcal{B}_{\mu\rho}H^{\rho\sigma}\mathcal{B}_{\sigma\nu} + 2\left(Z^T\right)^{\rho}_{\ (\mu}\mathcal{B}_{\nu)\rho} & -H^{\nu\rho}\mathcal{B}_{\rho\mu} + Z^{\nu}_{\mu} \\ -H^{\mu\rho}\mathcal{B}_{\rho\nu} + Z^{\nu}_{\mu} & H^{\mu\nu} \end{pmatrix}$$

$$S_{\mathrm{DFT}} = \int d^D X d^D \tilde{X} e^{-2d}\mathcal{R}$$

$$\begin{aligned}\mathcal{R} = {} & 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d \\ & + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL}\end{aligned}$$

TORSIONAL NEWTON CARTAN GRAVITY

GENERAL CONSIDERATIONS

- **Recall that there is a transverse metric $h_{\mu\nu}$ and a longitudinal clock one form τ_μ**

$$h^{\mu\rho}h_{\rho\nu} - \upsilon^\mu\tau_\nu = \delta^\mu_\nu \qquad \upsilon^\mu\tau_\mu = -1 \qquad h^{\mu\nu}\tau_\nu = h_{\mu\nu}\upsilon^\nu = 0$$

$$h_{\mu\nu} = e^a_\mu e^b_\nu \delta_{ab}$$

- **The theory has intrinsic torsion defined via** 2009.01948 Figueroa-O'Farrill

$$F = d\tau$$

- **The theory is locally invariant under Galilean Boosts**

$$\delta e^a_\mu = \lambda^a \tau_\mu$$

- **We can add a U(1) central extension by adding the one form U(1) connection m**

$$\delta m_\mu = \lambda_a e^a_\mu + \partial_\mu \sigma$$

GENERAL CONSIDERATIONS

- **It us useful to introduce Galilean boost invariant objects**

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \tau_\mu m_\nu - \tau_\nu m_\mu$$
$$\hat{\upsilon}^\mu \equiv \upsilon^\mu - h^{\mu\nu} m_\nu$$

Newton's potential $\Phi \equiv -\upsilon^\rho m_\rho + \frac{1}{2} h^{\rho\sigma} m_\rho m_\sigma$

- **We will assume there is non-vanishing torsion and fix the connection to be compatible with (τ_μ,$h^{\mu\nu}$)**

$$D_\rho h^{\mu\nu} = 0$$
$$D_\rho \tau_\mu = 0$$

$$\tau_\rho \Gamma^\rho_{[\mu\nu]} = \partial_{[\mu} \tau_{\nu]} \equiv F_{\mu\nu}$$

Acceleration $a_\mu \equiv \upsilon^\mu F_{\mu\nu}$

EMBEDDING INTO DFT

- **TNC with U(1) extension can be embedded into a relativistic metric with a null isometry**

$$\mathcal{H}_{MN} = \begin{pmatrix} \bar{h}_{\mu\nu} & \tau_\mu & 0 & 0 \\ \tau_\mu & 0 & 0 & 0 \\ 0 & 0 & h^{\mu\nu} & -\hat{v}^\mu \\ 0 & 0 & -\hat{v}^\mu & 2\Phi \end{pmatrix} \xrightarrow{\text{O(D,D) rotation}} \mathcal{H}_{MN} = \begin{pmatrix} \bar{h}_{\mu\nu} & 0 & 0 & \tau_\mu \\ 0 & 2\Phi & -\hat{v}^\nu & 0 \\ 0 & -\hat{v}^\mu & h^{\mu\nu} & 0 \\ \tau_\nu & 0 & 0 & 0 \end{pmatrix}$$

(0,0) parametrization

From here we can read the following

$$K_{mn} = \begin{pmatrix} h_{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix} \qquad H^{mn} = \begin{pmatrix} h^{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix} \qquad \mathcal{B}_{mn} = \begin{pmatrix} 0 & -m_\mu \\ m_\nu & 0 \end{pmatrix}$$

Both K and H have two null eigenvectors, so we are dealing with a **(1,1) parametrization**

The U(1) central extension is inherited from the 1-form symmetry of the B-field

EMBEDDING INTO DFT (ADDING MATTER)

- **We can add a two form field $\bar{B}_{\mu\nu}$ and a one form field $\mathcal{N}_\mu$, they have the interpretation of the KK-components of the d+1-dimensional relativistic B-field.**

$$K_{mn} = \begin{pmatrix} h_{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix} \qquad H^{mn} = \begin{pmatrix} h^{\mu\nu} & h^{\mu\rho}\aleph_\rho \\ h^{\nu\rho}\aleph_\rho & h^{\rho\sigma}\aleph_\rho\aleph_\sigma \end{pmatrix} \qquad \mathcal{B}_{mn} = \begin{pmatrix} \bar{B}_{\mu\nu} & -m_\mu \\ m_\nu & 0 \end{pmatrix}$$

$$x_m = \frac{1}{\sqrt{2}}\begin{pmatrix} \tau_\mu - \aleph_\mu \\ 1 \end{pmatrix}, \qquad \bar{x}_m = \frac{1}{\sqrt{2}}\begin{pmatrix} \tau_\mu + \aleph_\mu \\ -1 \end{pmatrix},$$

$$y^m = \frac{1}{\sqrt{2}}\begin{pmatrix} -\upsilon^\mu \\ 1 - \upsilon^\mu\aleph_\mu \end{pmatrix}, \qquad \bar{y}^m = \frac{1}{\sqrt{2}}\begin{pmatrix} -\upsilon^\mu \\ -1 - \upsilon^\mu\aleph_\mu, \end{pmatrix}.$$

- **There is an additional U(1) symmetry associated to $\mathcal{N}$, we have chosen $\bar{B}_{\mu\nu}$ such that it is invariant under Galilean boosts but not under this additional U(1) transformation**

EMBEDDING INTO DFT (ADDING MATTER)

- **It is convenient to define the field strengths and the electric field $\mathfrak{e}$**

$$H \equiv d\bar{B} \qquad b \equiv d\aleph \qquad \mathfrak{e} = \hat{v}^{\rho} b_{\rho\mu}$$

- **The "standard" dilation can also be added via the identification**

$$e^{-2d} = \sqrt{\frac{det\bar{h}}{2\Phi}}e^{-2\phi}$$

ACTION

- **Using the DFT generalized metric we find the action**

$$S=\int d^dx\, e\left[\mathcal{R}+\frac{1}{2}a^\mu a_\mu+\frac{1}{2}\mathfrak{e}^\mu\mathfrak{e}_\mu-4a^\mu D_\mu\phi+4D^\mu\phi D_\mu\phi-\frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho}\right.$$
$$\left.-\frac{1}{2}\hat{\upsilon}^\rho H_{\rho\mu\nu}b^{\mu\nu}-\frac{1}{2}\left(F^{\mu\nu}F_{\mu\nu}+b^{\mu\nu}b_{\mu\nu}\right)\Phi\right]$$

- **We will obtain the same action from null reducing the NS-NS action**

- **Due to the identity $\tau_\mu\tau_\nu\delta h^{\mu\nu}=0$ the action will be missing one equation (Newton's law)**

EQUATIONS OF MOTION AND COMPARISON WITH TNC BETA FUNCTIONS

- **The TNC beta function computation was performed under some assumptions/restrictions**

The twistless constraint was imposed from the onset

$$F_{\mu\nu}F^{\mu\nu} = 0 \leftrightarrow F \wedge \tau = 0$$

U(1) invariance was imposed, rather than derived

- **Using DFT (or directly the null reduction of the NS-NS sector) we can generalize to arbitrary torsion**

EQUATIONS OF MOTION AND COMPARISON WITH TNC BETA FUNCTIONS

- From the $\delta\Phi$ variation the generalization of the twistless constraint shows up

"Raised indices" is short hand notation for contraction with the inverse metric h

$$F_{\mu\nu}F^{\mu\nu} = b_{\mu\nu}b^{\mu\nu}$$

Only the "magnetic piece" of b sources twist

The equivalent of this expressions follows from the beta function associated to the Lagrange multipliers

$$\delta\langle\beta\bar{\beta}\rangle = b_{\mu\nu}b^{\mu\nu} = 0$$

Obtained TNC beta function - twistless torsion was imposed

$$F_{\mu\nu}F^{\mu\nu} = 0 \leftrightarrow F \wedge \tau = 0$$

EQUATIONS OF MOTION AND COMPARISON WITH TNC BETA FUNCTIONS

- **From the variation of the matter fields {δϕ,δB,δ$\mathcal{N}$} we find the equations**

$$D_\mu D^\mu \phi + a^\mu D_\mu \phi - 2 D_\mu \phi D^\mu \phi = \frac{1}{2}\mathfrak{e}^2 - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{1}{2}\hat{\upsilon}^\lambda H_{\lambda\mu\nu} b^{\mu\nu} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \Phi$$

Dilaton equation - Beta function computation involves two loops and has not been done

$$D^\rho H_{\rho\mu\nu} + a^\rho H_{\rho\mu\nu} - 2 D^\rho \phi \, H_{\rho\mu\nu} = 2 D_{[\mu} \mathfrak{e}_{\nu]} - 2\hat{\upsilon}^\lambda D_\lambda b_{\mu\nu} + 2 a_{[\mu} \mathfrak{e}_{\nu]} + 4 b^\rho{}_{[\mu} F_{\nu]\rho} \Phi$$
$$+ \left(2\hat{\upsilon}^\rho D_\rho \phi - D_\rho \hat{\upsilon}^\rho\right) b_{\mu\nu}$$

Maxwell Equation - Same as the computed TNC beta function for B (when torsion is twistless)

$$D^\rho b_{\rho\mu} - 2 D^\rho \phi \, b_{\rho\mu} = \frac{1}{2} F^{\rho\sigma} H_{\rho\sigma\mu}$$

Equation for $\mathcal{N}$ - Same as the computed TNC beta function for $\mathcal{N}$ (when torsion is twistless)

EQUATIONS OF MOTION AND COMPARISON WITH TNC BETA FUNCTIONS

- **From the variation $\delta\tau$ we find the vector equation**

$$D^\rho F_{\rho\mu} + a^\rho F_{\rho\mu} - 2D^\rho \phi F_{\rho\mu} = \frac{1}{2} b^{\rho\sigma} H_{\rho\sigma\mu} + 2\, \mathfrak{e}^\rho b_{\rho\mu}$$

Equation for τ, when the electric field is zero it matches the TNC beta functions.
Otherwise there seems to be a factor of 2 mismatch

- **When torsion has twist we need the factor appearing in the DFT computation to preserve the U(1) mass invariance**
- **During the TNC beta function computation we are forced to make torsion twistless to keep the computation covariant.**
- **U(1) invariance was imposed, rather than derived, on the TNC beta function computation. There is the possibility that we missed some terms that are U(1) invariant when torsion is twistless.**

THE MISSING EQUATION

- **The variation of the TNC metrics {δυ,δh} and the missing equation (Newton's law) can be combined into the Einstein equation**

$$\mathcal{R}_{(\mu\nu)} + 2D_{(\mu}D_{\nu)}\phi - \frac{1}{4}h^{\rho\sigma}h^{\lambda\kappa}H_{\mu\rho\lambda}H_{\nu\sigma\kappa} = \frac{a_\mu a_\nu - \mathfrak{e}_\mu \mathfrak{e}_\nu}{2} - \hat{\upsilon}^\rho D_{(\mu}F_{\nu)\rho} + \hat{\upsilon}^\lambda h^{\rho\sigma} b_{\rho(\mu}H_{\nu)\lambda\sigma} - \left(F_{\mu\rho}F_{\nu\sigma}h^{\rho\sigma} - b_{\mu\rho}b_{\nu\sigma}h^{\rho\sigma}\right)\Phi$$

Einstein equation - It matches the bar h TNC beta function (twistless limit)

- **The missing equation (Newton's law) can be read directly from the DFT equations of motion but not from a variation of the action**

Function of extrinsic curvature

$$D^\mu D_\mu \Phi + 3a^\mu D_\mu \Phi + m_\Phi^2 \Phi - 2F_{\mu\nu}F^{\mu\nu}\Phi^2 = \rho_{\mathcal{K}} + \rho_m$$

Time-time projection of Einstein equation

Function of H and dilaton

Function of torsion and matter

WHY IS IT MISSING?

- **Relativistic perspective: The null reduction sets g_{uu}=0 off-shell so its equation of motion is missing.**
- **Symmetry perspective: Possible emergence of a dilation symmetry (inherited from its SNC embedding).**
- **DFT perspective: This is a known DFT phenomenon due to the fixing of (n, bar n).**

  1909.10711 K. Cho and J-H Park

  Analysis of the DFT fluctuations show that $n\bar{n}$ equations change the type of non-Riemannianity. Fixing (n,bar n) removes this equations.

- **We should think of these sectors of DFT as particular solutions rather than independent theories.**

CARROLL

GENERAL CONSIDERATIONS

- **The metric structure is the same as in TNC (same notation as well)**

$$h^{\mu\rho}h_{\rho\nu} - \upsilon^\mu\tau_\nu = \delta^\mu_\nu \qquad \upsilon^\mu\tau_\mu = -1 \qquad h^{\mu\nu}\tau_\nu = h_{\mu\nu}\upsilon^\nu = 0$$

$$h_{\mu\nu} = e^a_\mu e^b_\nu \delta_{ab}$$

- **The theory has intrinsic torsion defined through** 2009.01948 Figueroa-O'Farrill

$$\mathcal{K}_{\mu\nu} = -\frac{1}{2}\mathcal{L}_\upsilon h_{\mu\nu}$$

- **The theory is locally invariant under Carroll Boosts**

$$\delta_{\mathcal{C}}\tau_\mu = \lambda_\mu, \qquad \delta_{\mathcal{C}}h^{\mu\nu} = 2h^{\rho(\mu}\upsilon^{\nu)}\lambda_\rho$$

- **A contravariant vector M can be added (in a similar fashion as m was for TNC)**

$$\delta M^\mu = h^{\mu\nu}\lambda_\nu$$

From the nature of the embedding there is no additional symmetry associated to M

GENERAL CONSIDERATIONS

- **It us useful to introduced Boost invariant objects**

$$\hat{\tau}_\mu = \tau_\mu - h_{\mu\nu} M^\nu ,$$
$$\hat{h}^{\mu\nu} = h^{\mu\nu} - 2M^{(\mu}\upsilon^{\nu)} + 2\Phi\upsilon^\mu\upsilon^\nu \equiv \bar{h}^{\mu\nu} + 2\Phi\upsilon^\mu\upsilon^\nu ,$$
$$\Phi = -M^\mu\tau_\mu + \frac{1}{2}h_{\mu\nu}M^\mu M^\nu = -M^\mu\hat{\tau}_\mu - \frac{1}{2}h_{\mu\nu}M^\mu M^\nu ,$$

- **We will consider a connection satisfying**

$$\bar{D}_\mu \hat{h}^{\rho\sigma} = -4\mathcal{K}_{\mu\lambda}\bar{h}^{\lambda(\rho}\upsilon^{\sigma)}\Phi ,$$
$$\bar{D}_\mu \hat{\tau}_\nu = -2\mathcal{K}_{\mu\nu}\Phi$$

- **The intrinsic torsion of Carroll is associated with the extrinsic curvature instead of F**

EMBEDDING INTO DFT (COMPARISON WITH TNC)

- **Carrol with U(1) extension can be embedded into a relativistic metric with a null isometry**

$$(\mathcal{H}_{\text{Carroll}})_{MN} = \begin{pmatrix} h_{\mu\nu} & -\hat{\tau}_\mu & 0 & 0 \\ -\hat{\tau}_\mu & 2\Phi & 0 & 0 \\ 0 & 0 & \bar{h}^{\mu\nu} & \upsilon^\mu \\ 0 & 0 & \upsilon^\mu & 0 \end{pmatrix}$$

O(D,D) rotation →

We swapped all directions

$$(\mathcal{H}_{\text{Carroll}})_{MN} \to \begin{pmatrix} \bar{h}^{\mu\nu} & 0 & 0 & \upsilon^\mu \\ 0 & 2\Phi & -\hat{\tau}_\nu & 0 \\ 0 & -\hat{\tau}_\mu & h_{\mu\nu} & 0 \\ \upsilon^\nu & 0 & 0 & 0 \end{pmatrix}$$

Comparison with TNC ↕ They are the same when m=0 and M=0

Some identifications can be made between TNC and Caroll ↕

$$(\mathcal{H}_{\text{TNC}})_{MN} = \begin{pmatrix} \bar{h}_{\mu\nu} & \tau_\mu & 0 & 0 \\ \tau_\mu & 0 & 0 & 0 \\ 0 & 0 & h^{\mu\nu} & -\hat{\upsilon}^\mu \\ 0 & 0 & -\hat{\upsilon}^\mu & 2\Phi \end{pmatrix}$$

O(D,D) rotation →

We only swapped null directions

$$(\mathcal{H}_{\text{TNC}})_{MN} = \begin{pmatrix} \bar{h}_{\mu\nu} & 0 & 0 & \tau_\mu \\ 0 & 2\Phi & -\hat{\upsilon}^\nu & 0 \\ 0 & -\hat{\upsilon}^\mu & h^{\mu\nu} & 0 \\ \tau_\nu & 0 & 0 & 0 \end{pmatrix}$$

TNC AND CARROLLL IDENTIFICATION

- By comparing TNC and Carroll generalized metric we can map one to the other after the identification

$$\bar{h}^{\mu\nu}_{(\mathcal{C})} \leftrightarrow \bar{h}^{(TNC)}_{\mu\nu}, \qquad \hat{\tau}^{(\mathcal{C})}_{\mu} \leftrightarrow \hat{\upsilon}^{\mu}_{(TNC)}, \qquad \Phi^{(\mathcal{C})} \leftrightarrow \Phi^{(TNC)},$$
$$h^{(\mathcal{C})}_{\mu\nu} \leftrightarrow h^{\mu\nu}_{(TNC)}, \qquad \upsilon^{\mu}_{(\mathcal{C})} \leftrightarrow \tau^{(TNC)}_{\mu}, \qquad M^{\mu}_{(\mathcal{C})} \leftrightarrow m^{(TNC)}_{\mu}$$

This identification has been made before in 1505.05011

- DFT makes this evident and seems to imply Carroll and TNC might be related through a T-duality transformation, equivalent to a c -> 1/c rule.

- We have not explored this further

CAROLL ACTION AND EQUATIONS OF MOTION

- **The action for Carroll is given by**

$$S = \int d^dx\, e\, \Big[\bar{\mathcal{R}} + \frac{1}{2}a^\mu a_\mu + \frac{1}{2}\mathfrak{e}^\mu \mathfrak{e}_\mu - 4a^\mu \bar{D}_\mu\phi + 4\bar{D}^\mu\phi\bar{D}_\mu\phi + 4\upsilon^\mu\upsilon^\nu\bar{D}_\mu\phi\,\bar{D}_\nu\Phi$$
$$-8\Phi\mathcal{K}\upsilon^\rho\bar{D}_\rho\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + \frac{1}{2}b^{\mu\nu}\upsilon^\rho H_{\mu\nu\rho}\Big]$$

- **There is also a missing equation, the corresponding Newton's law is**

$$\bar{D}^\mu\bar{D}_\nu\Phi - a^\mu\bar{D}_\mu\Phi - 2\bar{D}^\mu\phi\,\bar{D}_\mu\Phi = \frac{1}{4}\left(F^{\mu\nu}F_{\mu\nu} - b^{\mu\nu}b_{\mu\nu}\right)$$

- **There are no constraints on F, the equivalent Carroll torsion constraints will be constraints on the extrinsic curvature that follow from the $\delta\Phi$ variation**

QUICK GLANCE AT THE EQUATIONS OF MOTION

Torsion constraints

$$\bar{\mathcal{R}}_{\mu\nu}\upsilon^\mu\upsilon^\nu = \frac{1}{4}\upsilon^\rho\upsilon^\sigma H^{\mu\nu}{}_\rho H_{\mu\nu\sigma} - 2\upsilon^\mu\upsilon^\nu\bar{D}_\mu\bar{D}_\nu\phi$$

Equations for τ , and Einstein equation

$$\begin{aligned}\bar{D}^\rho F_{\rho\mu} + a^\rho F_{\rho\mu} - 2\bar{D}^\rho\phi\, F_{\rho\mu} &= \frac{1}{2}b^{\rho\sigma}H_{\rho\sigma\mu} + \mathfrak{e}^\rho b_{\mu\rho} + 2\bar{D}^\rho\mathcal{K}_{\rho\mu} - 2\mathcal{K}\bar{D}_\mu\Phi \\ &\quad + 2\upsilon^\rho\bar{D}_\mu\bar{D}_\rho\Phi - 4\upsilon^\rho\bar{D}_\rho\phi\, D_\mu\Phi \\ \bar{\mathcal{R}}_{(\mu\nu)} + 2\bar{D}_{(\mu}\bar{D}_{\nu)}\phi - \frac{1}{4}H_\mu{}^{\rho\sigma}H_{\nu\rho\sigma} &= \frac{a_\mu a_\nu - \mathfrak{e}_\mu\mathfrak{e}_\nu}{2} + D_{(\mu}a_{\nu)} - F_{(\mu}{}^\rho\mathcal{K}_{\nu)\rho} + \upsilon^\rho b_{(\mu}{}^\sigma H_{\nu)\rho\sigma} \\ &\quad + 4\Phi\upsilon^\rho\left(\bar{D}_\rho\mathcal{K}_{\mu\nu} - \mathcal{K}\,\mathcal{K}_{\mu\nu} + \frac{\bar{D}_\rho\Phi}{\Phi}\mathcal{K}_{\mu\nu}\right)\end{aligned}$$

QUICK GLANCE AT THE EQUATIONS OF MOTION

- **Equations for the matter fields**

$$\bar{D}_\mu \bar{D}^\mu \phi + a^\mu \bar{D}_\mu \phi - 2\bar{D}_\mu \phi \bar{D}^\mu \phi = 2\upsilon^\mu \upsilon^\nu D_\mu \Phi D_\nu \phi - \frac{1}{2} H^{\rho\mu\nu} H_{\rho\mu\nu} + \frac{1}{2} \mathfrak{e}^\mu \mathfrak{e}_\mu + \frac{1}{2} \upsilon^\rho b^{\mu\nu} H_{\rho\mu\nu}$$

$$\bar{D}^\rho b_{\rho\mu} - 2\bar{D}^\rho \phi \, b_{\rho\mu} = \frac{1}{2} F^{\rho\sigma} H_{\rho\sigma\mu} + 2\upsilon^\rho \bar{D}^\sigma \Phi \, H_{\rho\sigma\mu}$$

$$\bar{D}^\rho H_{\rho\mu\nu} + a^\rho H_{\rho\mu\nu} - 2\bar{D}^\rho \phi \, H_{\rho\mu\nu} = 2b_{[\mu}{}^\rho \mathcal{K}_{\nu]\rho} - \mathcal{K} b_{\mu\nu} + \upsilon^\rho \left(\bar{D}_\rho b_{\mu\nu} - 2\bar{D}_\rho \phi \, b_{\mu\nu} + 2\upsilon^\sigma \bar{D}_\sigma \Phi \, H_{\rho\mu\nu} \right)$$

STRING NEWTON CARTAN

GENERAL CONSIDERATIONS

- **Recall that there are two longitudinal directions with a Lorentz structure**

$$h^{\mu\rho}h^{\perp}_{\rho\nu} - \upsilon^{\mu}_{A}\tau_{\nu}^{A} = \delta^{\mu}_{\nu}, \qquad \upsilon^{\mu}_{A}\tau_{\mu}^{B} = -\delta^{B}_{A}, \qquad \tau_{\mu}^{A}h^{\mu\rho} = \upsilon^{\mu}_{A}h^{\perp}_{\mu\rho} = 0$$

$$h_{\mu\nu} = E_{\mu}^{A'}E_{\nu}^{B'}\delta_{A'B'}$$

Indices are raised with Minkowski metric

- **We can define torsion as**

$$F^{A}_{\mu\nu} \equiv 2\partial_{[\mu}\tau_{\nu]}{}^{A} + 2\epsilon^{A}{}_{B}\omega_{[\mu}\tau_{\nu]}{}^{B} = 2\nabla_{[\mu}\tau^{A}_{\nu]}$$

- **Galilean boosts become String Galilean boosts**

$$\delta_{\Sigma}\upsilon^{\mu}_{A} = -E^{\mu}_{A'}\Sigma_{A}^{A'}, \qquad \delta_{\Sigma}E_{\mu}^{A'} = -\tau_{\mu}^{A}\Sigma_{A}^{A'}, \qquad \delta E^{\mu}_{A'} = \delta_{\Sigma}\tau_{\mu}^{A} = 0$$

- **A U(1) mass symmetry, called the Z_A symmetry, is usually included by the addition of a gauge field m**

$$\delta_{Z}m_{\mu}^{A} = D_{\mu}\sigma^{A} \qquad \delta_{\Sigma}m_{\mu}^{A} = E_{\mu}^{A'}\Sigma_{A'}^{A}$$

LET'S TALK ABOUT Z_A

- For a classical SNC theory (string in SNC background) to have arbitrary torsion and preserve Z_A invariance we need a B-field that transforms as 1907.01663 Harmark et al.

$$\delta B_{\mu\nu} = 2\epsilon_{AB}\tau^A_{[\mu}D_{\nu]}\sigma^B$$

- We can use m to write down Boost invariant (but no Z_A invariant) fields

$$\begin{aligned}\bar{h}_{\mu\nu} &= h^{\perp}_{\mu\nu} + 2\eta_{AB}m^A_{(\mu}\tau^B_{\nu)},\\ u^\mu_A &= v^\mu_A + h^{\mu\rho}m_{\rho A},\\ \Phi^{AB} &= 2u^{\rho(A}m_\rho{}^{B)} - h^{\mu\nu}m^A_\mu m^B_\nu = -u^{\mu A}u^{\nu B}\bar{h}_{\mu\nu},\end{aligned}$$

- The Z_A symmetry is part of a larger Stueckelberg symmetry

This shift symmetry is related to the DFT generalized shift symmetry

$$\delta\bar{h}_{\mu\nu} = 2\tau^A_{(\mu}D_{\nu)}\sigma_A, \qquad \delta B_{\mu\nu} = 2\epsilon_{AB}\tau^A_{[\mu}D_{\nu]}\sigma^B,$$

- m can be gauge away but B will not longer be Boost invariant This choice has been used in 2102.06974 were they derived SNC equations from a 1/c expansion

DFT EMBEDDING

- **We got the DFT embedding by matching the SNC Polyakov and DFT Polyakov actions**

$$\mathcal{H}_{AB} = \begin{pmatrix} 1 & \mathcal{B} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} K & Z \\ Z^T & H \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\mathcal{B} & 1 \end{pmatrix}$$

$$K_{\mu\nu} = \bar{h}_{\mu\nu} + \Phi_{AB}\tau_\mu^A\tau_\nu^B, \qquad Z^\mu_\nu = -\epsilon_{AB}u^{\mu A}\tau_\nu^B,$$

$$H^{\mu\nu} = h^{\mu\nu}, \qquad \mathcal{B}_{\mu\nu} = B_{\mu\nu} - \frac{1}{2}\Phi\epsilon_{AB}\tau_\mu^A\tau_\nu^B,$$

- **We can reduce SNC to TNC by comparing the generalized metrics and assuming an isometry along a longitudinal compact direction** 1907.01663 Harmark et al.

$$\upsilon_1^\mu = 0, \qquad \upsilon_1^u = 1,$$

$$\upsilon_0^\mu = \upsilon^\mu, \qquad \upsilon_0^u = \upsilon^\mu \aleph_\mu,$$

$$h^{\mu\nu}_{(SNC)} = h^{\mu\nu}_{(TNC)}, \qquad h^{\mu u}_{(SNC)} = h^{\mu\rho}\aleph_\rho,$$

$$h^{uu}_{(SNC)} = h^{\rho\sigma}\aleph_\rho\aleph_\sigma, \qquad B^{(SNC)}_{\mu\nu} = B^{(TNC)}_{\mu\nu}$$

$$B^{(SNC)}_{\mu u} = -m_\mu.$$

$$m_M^A = 0, \qquad \tau_\nu^0 = 0, \qquad \tau_u^1 = 1, \qquad E_u^{A'} = 0$$

ACTION AND EQUATIONS OF MOTION

- **The SNC action is found to be**

Acceleration, defined by the contraction of F and u

$$S = \int d^D x\, e \Big[\mathcal{R} - a^{\mu AB}\big(a_{\mu(AB)} - \frac{1}{2}\eta_{AB}a_\mu\big) + (a^\mu - 2D^\mu\phi)(a_\mu - 2D_\mu\phi)$$

$$-\frac{1}{2}F^{\mu\nu A}F^{B}_{\mu\nu}\big(\Phi_{AB} - \frac{1}{2}\eta_{AB}\Phi\big) + \frac{1}{2}\epsilon_{AB}u^{\rho A}F^{\mu\nu B}H_{\rho\mu\nu} - \frac{1}{12}H^{\rho\mu\nu}H_{\rho\mu\nu}\Big]$$

- **There is also a missing equation**

Spatial piece of acceleration

$$u^{\mu A}u^{\nu}{}_{A}\mathcal{R}_{\mu\nu} + 2u^{\mu A}u^{\nu}{}_{A}D_\mu D_\nu\phi - \frac{1}{4}u^{\mu A}u^{\nu}{}_{A}H_{\mu\rho\sigma}H_{\nu}{}^{\rho\sigma} = \frac{3}{2}\mathcal{S}^{\rho AB}D_\rho\Phi_{AB} - \frac{3}{4}a^\mu D_\mu\Phi$$

$$-\frac{3}{4}a^\mu a_\mu\Phi + \mathcal{S}^{\mu AB}\mathcal{S}_{\mu AC}\Phi_B^{\ C} - 2f^A f_A + \epsilon^{AB}\mathcal{Q}_{AB}$$

Totally antisymmetric longitudinal piece of torsion

- **It agrees with the SNC beta functions computed at zero torsion (foliation constraint)**

QUICK GLANCE AT THE EQUATIONS OF MOTION

- **Torsion constraint**

$$F^{\mu\nu A}F_{\mu\nu A} = 0$$

- **We could not combine the u equation into a single expression, we only found its projections**

$$D^\rho F^\mu{}_{\rho A} + a_{\rho A}{}^B F^{\mu\rho}{}_B - 2F^\mu{}_{\rho A}D^\rho\phi = \frac{1}{2}\epsilon_{AB}F^{\rho\sigma B}H^\mu{}_{\rho\sigma}$$

$$D^\rho \mathcal{S}_\rho{}^{AB} + a^\rho \mathcal{S}_\rho{}^{AB} - 2\mathcal{S}^{\rho AB}D_\rho\phi = -\frac{1}{4}F^{\mu\nu A}F^B_{\mu\nu}\Phi + 2\epsilon^{(A}{}_C\mathcal{F}^{B)C}$$

QUICK GLANCE AT THE EQUATIONS OF MOTION

- **The same happened for Einstein equations**

$$\mathcal{R}^{(\mu\nu)} + 2D^{(\mu}D^{\nu)}\phi - \frac{1}{4}H^{\mu}{}_{\rho\sigma}H^{\nu\rho\sigma} = u^{\rho}_{A}D^{(\mu}F_{\rho}{}^{\nu)A} + \mathcal{S}^{\mu AB}\mathcal{S}^{\nu}{}_{AB} - \epsilon^{AB}u^{\sigma}{}_{A}F^{\rho(\mu}{}_{B}H^{\nu)}{}_{\rho\sigma} - \frac{1}{2}F^{\mu\rho A}F^{\nu}{}_{\rho A}\Phi$$

$$\begin{aligned} h^{\mu\rho}u^{\sigma A}\mathcal{R}_{(\rho\sigma)} + 2u^{\rho A}D^{\mu}D_{\rho}\phi + a^{\mu}u^{\rho A}D_{\rho}\phi - \frac{1}{4}H^{\mu}{}_{\rho\sigma}H_{\lambda}{}^{\rho\sigma}u^{\lambda A} &= \frac{1}{2}u^{\rho A}u^{\sigma}{}_{B}D_{\rho}F_{\sigma}{}^{\mu B} - u^{\rho}{}_{B}D^{\mu}a_{\rho}{}^{(AB)} + u^{\rho A}D^{\mu}a_{\rho} \\ &- \frac{1}{2}u^{\rho B}D_{\rho}a^{\mu}_{BA} + \frac{1}{2}u^{\rho A}D_{\rho}a^{\mu} - 2a^{\mu[AB]}u^{\rho}{}_{B}D_{\rho}\phi - a^{\mu}u^{\rho A}D_{\rho}\phi - \frac{1}{2}a^{\mu}\mathcal{K}^{A} + \frac{1}{2}a^{\mu BA}\mathcal{K}_{B} \\ &- F^{\mu}{}_{\rho C}a^{\rho B(A}\Phi_{B}{}^{C)} + a^{\rho}F^{\mu}{}_{\rho C}\Phi^{AC} + \frac{1}{2}a^{\rho}F_{\rho}{}^{\mu A}\Phi + \frac{1}{2}F^{\mu\rho B}D_{\rho}\Phi^{A}_{B} - \frac{1}{2}F^{\mu\rho A}D_{\rho}\Phi \\ &+ \epsilon^{AB}\mathcal{P}^{\mu}{}_{B} \end{aligned}$$

QUICK GLANCE AT THE EQUATIONS OF MOTION

- **The matter equations look slightly nicer**

$$D^\rho H_\rho{}^{\mu\nu} + a^\rho H_\rho{}^{\mu\nu} - 2D^\rho\phi\, H_\rho{}^{\mu\nu} = 2\Omega^{\mu\nu} + \epsilon_{AB}\mathcal{H}^{\mu\nu AB}$$

$$D^\mu D_\mu\phi - \frac{1}{2}D^\mu a_\mu + 2a_\mu D^\mu\phi - 2D^\mu\phi\, D_\mu\phi - \frac{1}{2}a_\mu a^\mu = \frac{1}{4}\epsilon_{AB}u^{\rho A}F^{\mu\nu B}H_{\rho\mu\nu} - \frac{1}{12}H^{\rho\mu\nu}H_{\rho\mu\nu}$$

COMPARISON WITH 1/C EXPANSION OF NS-NS GRAVITY

2102.06974 Bergshoeff et al.

- **The actions match after imposing the m=0 gauge**

- **The missing equation is explained by an emergent dilatation symmetry**

$$\delta\tau_\mu^A = \lambda_D \tau_\mu^A \qquad \delta\phi = \lambda_D \qquad \delta\Phi^{AB} = -2\lambda_D \Phi^{AB}$$

- **This symmetry is, of course, present in the action we computed**

CONCLUSIONS/SUMMARY

- **DFT unifies many Non-Riemannian geometries into a single framework**
- **We used this framework to write down gravitational equations for various non-Riemannian geometries**
- **Some interesting relations between non-Riemannian geometries can be infer (Carroll and TNC/ TNC and SNC)**
- **We can try to push this further into understand non-Riemannian notions of "Entropy" or thermal states**

1608.04734 A. Arvanitakis and C. Blair

We only have to understand it once

FUTURE DIRECTIONS

- Repeat an analogous computation to the Exceptional Field Theory case
- ExFT unifies not only 10d supergravity but also 11d M-theory, and has been shown to admit Non-Riemannian parametrizations 1902.01867 Berman et. al.
- We are pushing to write 11d Non-Riemannian actions as well as writing other supergravity sectors Work in progress with Natale, Chris, and Umut