Conformal subsystem symmetry

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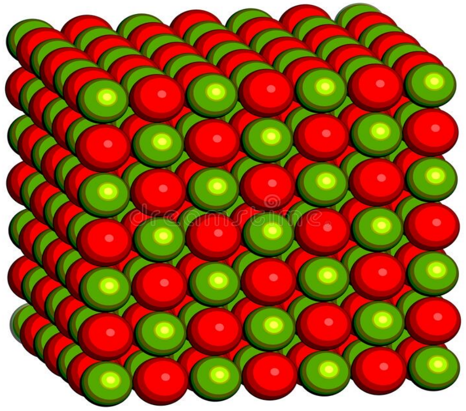
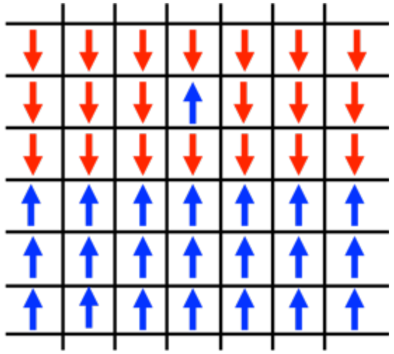
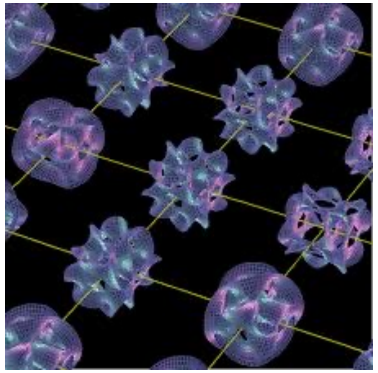
work done in collaboration with Amir Raz

QFT as the Universal Language

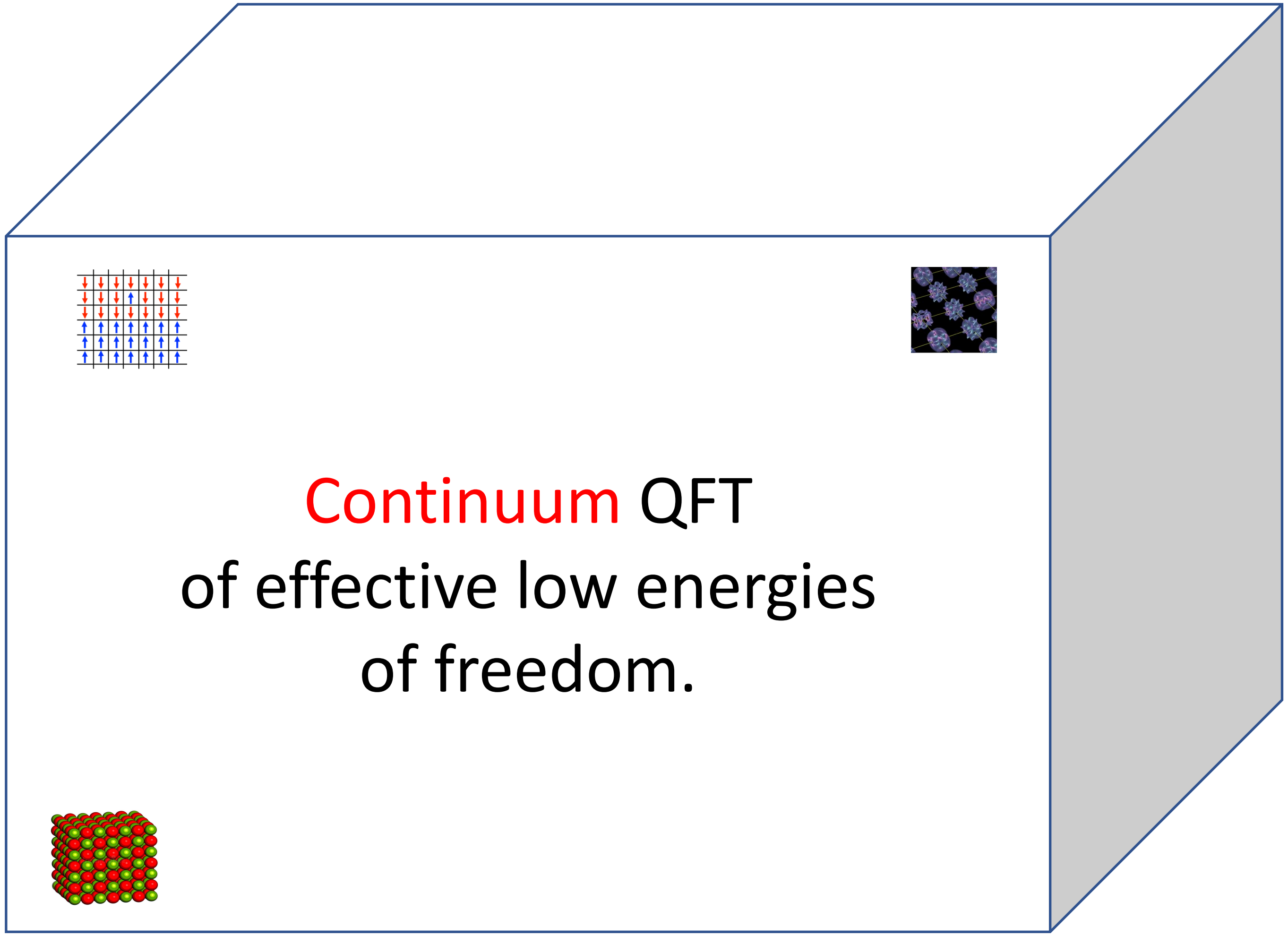




Zoom out

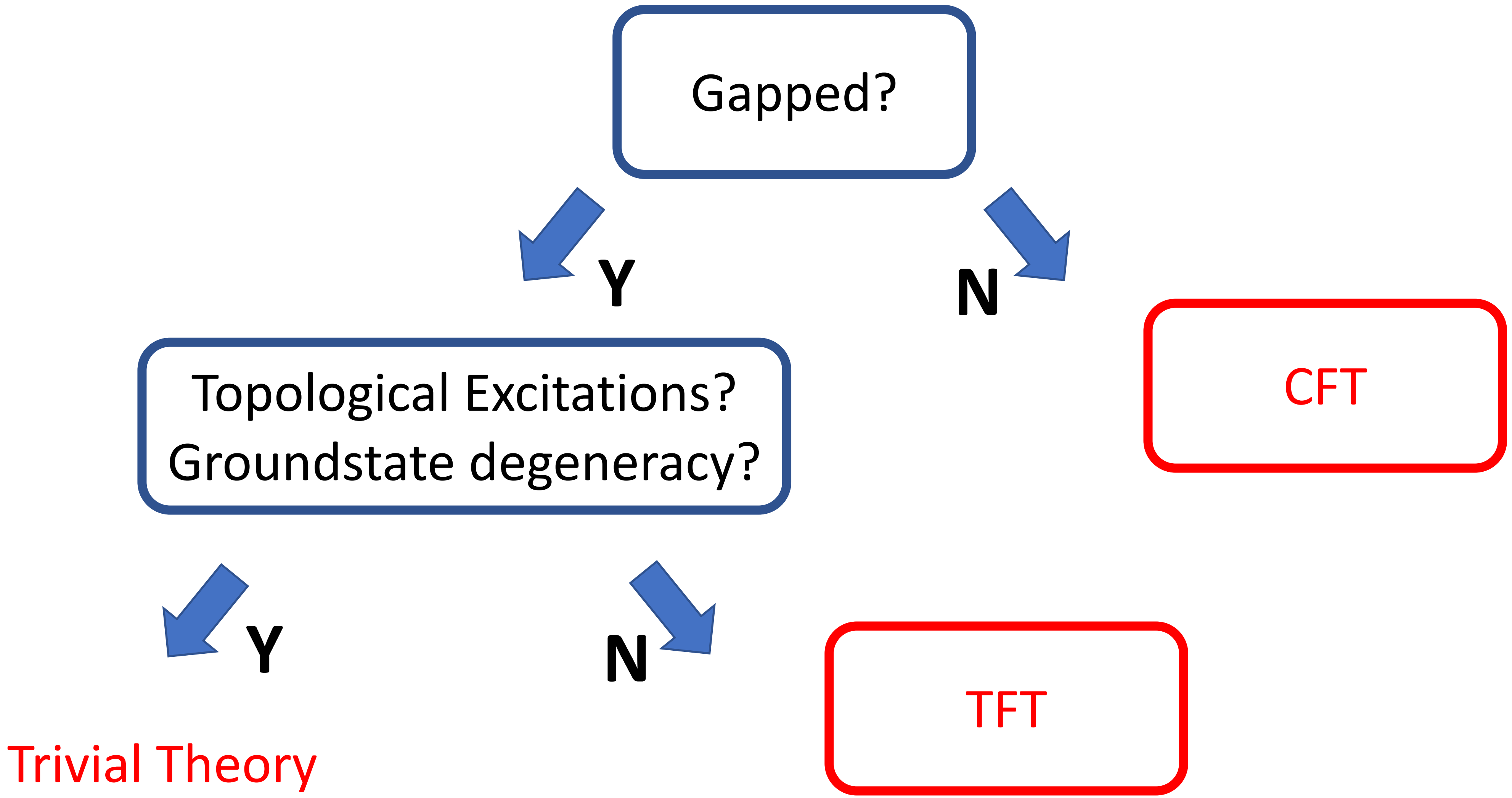


QFT as the Universal Language



Action uniquely
determined by just
a few low energy constants.

Very few possibilities



Wilsonian Paradigm

Classify all TFTs and CFTs

↓

Including non-relativistic CFTs,
and maybe also scale invariant but not conformally invariant theories, which seem to exist if relax requirements of Lorentz invariance

Classify all Phases of Matter

Fractons

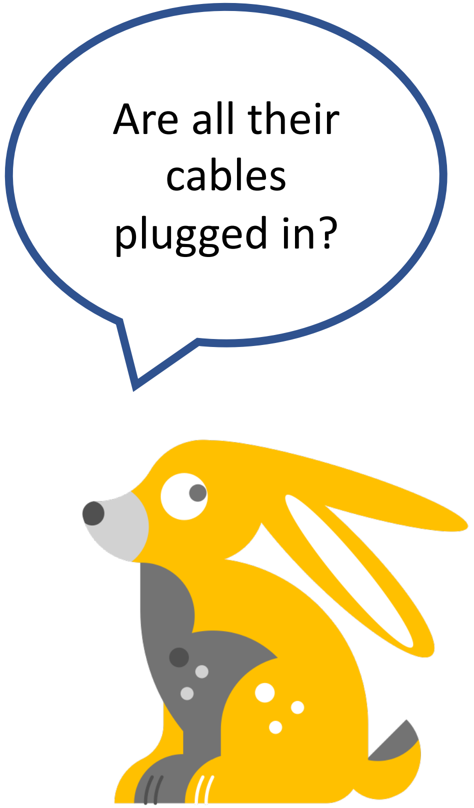
- Groundstate degeneracy but not topological

  Number of GS grows with volume

- Excitations with limited mobility (moving along lines and/or planes)

Surely an experimental error……

Whenever a dearly held theoretical believe is challenged, our first impulse is to blame it on the experimentalists……



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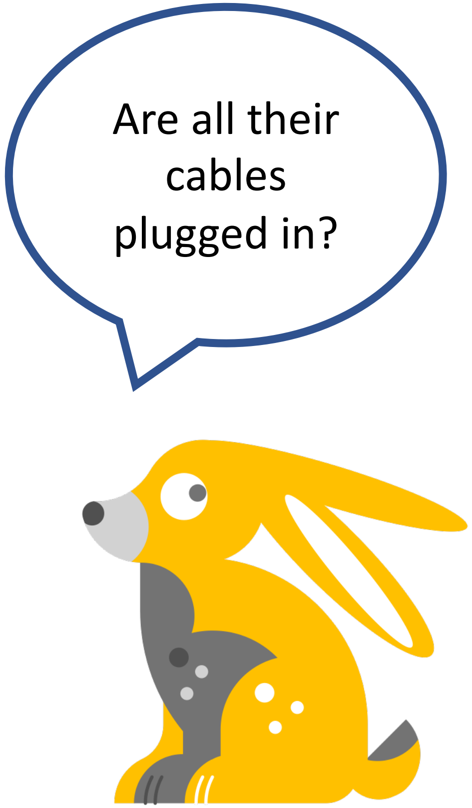


Problem: Fractons are a purely theoretical construct. In fact, at this stage we all can only wish they had some practical applications. Exactly solvable lattice models.

Simplest example: the X-cube model

(Vijay, Haah, Fu)

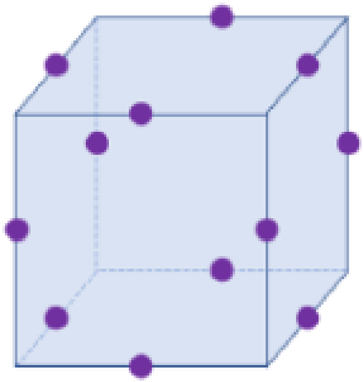
Lattice model with Ising spins on each lattice site.

Only interactions between near-by spins.

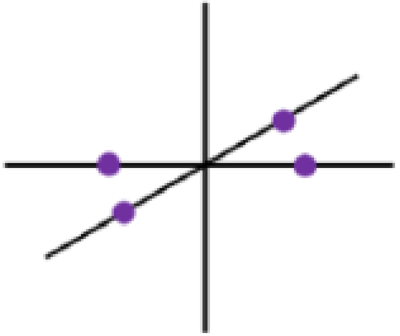
Generalization of Kitaev’s toric code
(“coupled layers of toric code”)

Simplest example: the X-cube model

(Vijay, Haah, Fu)



$$B_c = \prod_{\partial c} X$$



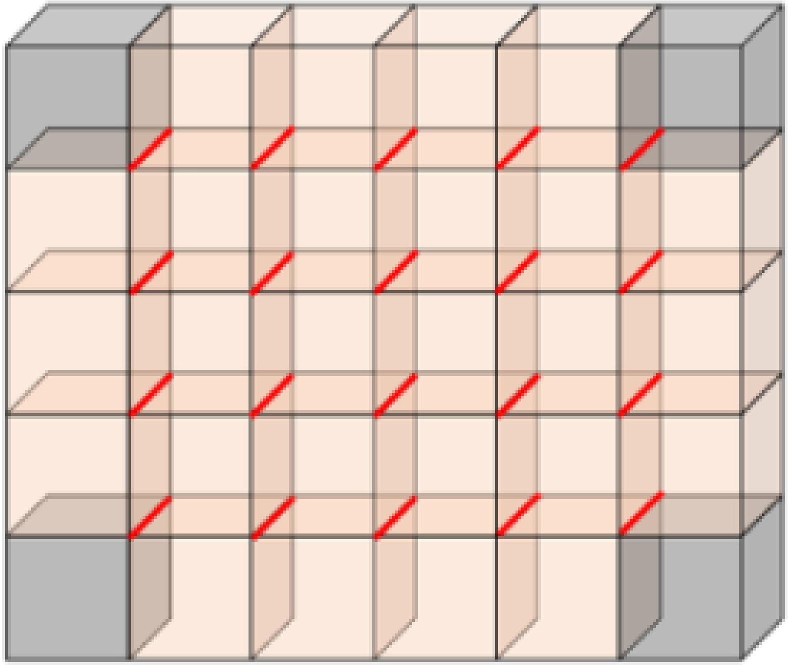
$$A_v^z = \prod_{+_{xy}} Z$$

Summed over all cubes, and over all crosses (in all orientations)

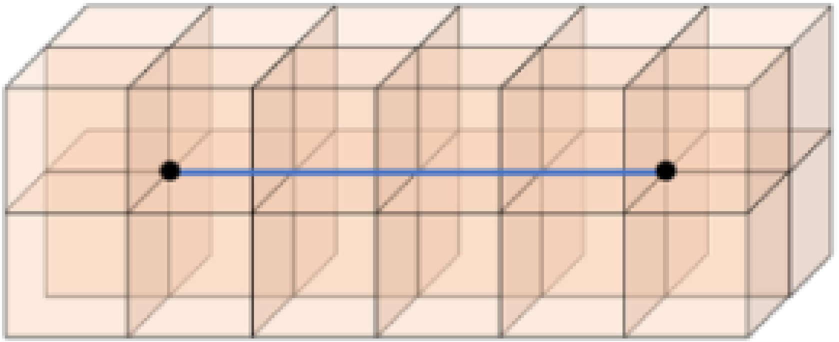
Commuting Projector Hamiltonian:

Groundstates simultaneous eigenstates of all A's and B's with eigenvalue -1.

Fractons in the X-cube model

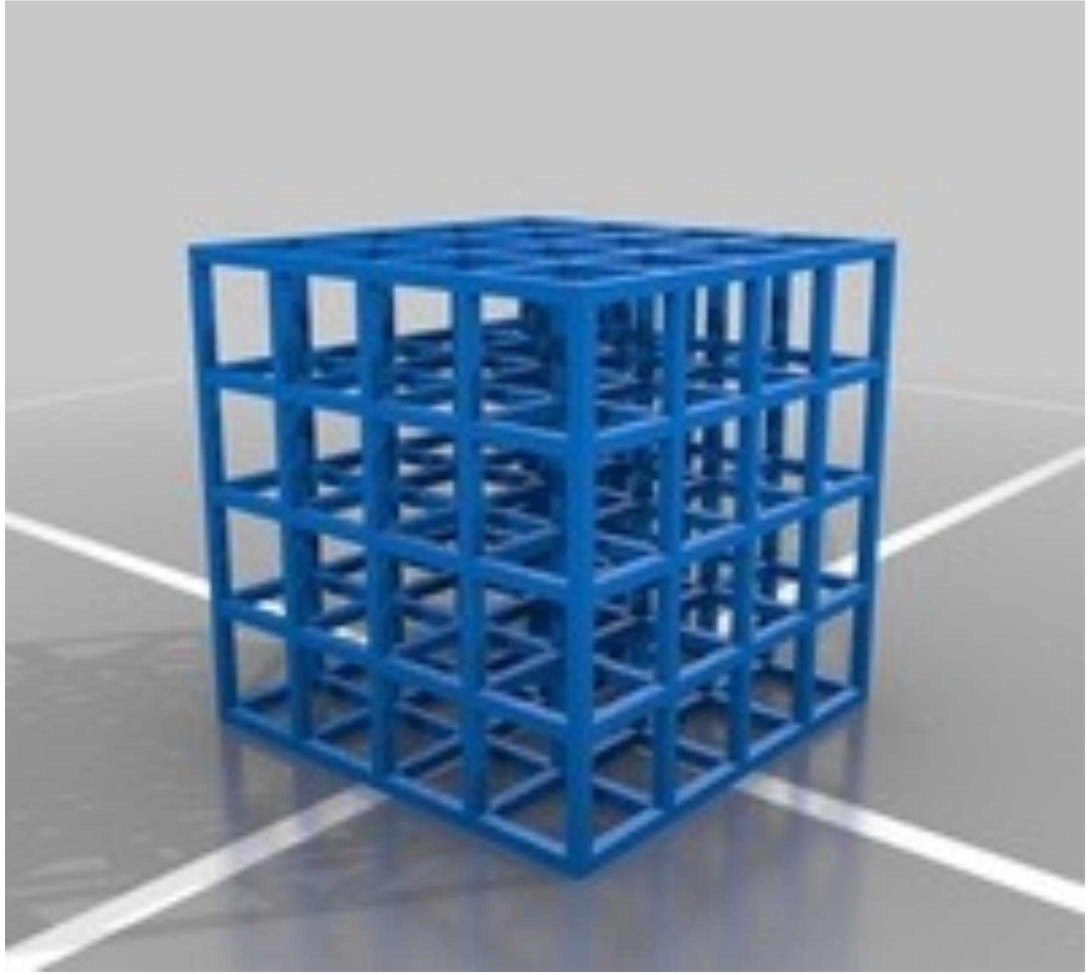


Z



X

Subsystem symmetry



Standard Ising spins:

Flipping all spins is a symmetry

X-cube:

Flipping all spins on
any given plane is a symmetry

Subsystem symmetry

In fact, every plane has two Z_2 symmetries.

These symmetries take ground states to ground states

$$\#_{GS} = 4^{\#_{planes}} = 2^{L_x+L_y+L_z-3}$$

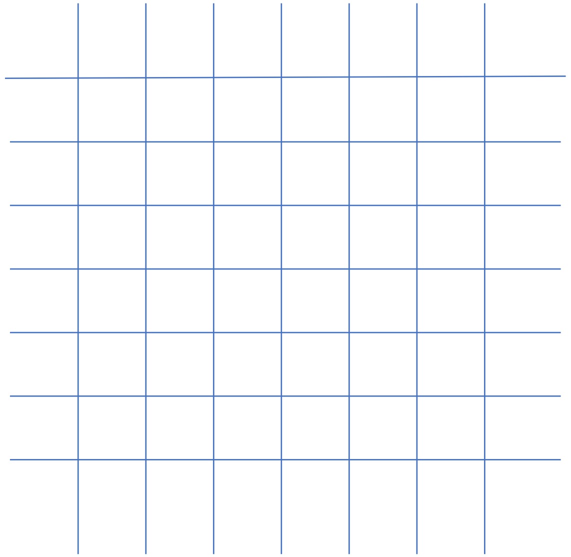
QFTs for Fractons?

Fractons do NOT allow a field theory description in terms of "standard" QFTs.

To describe their low-energy physics we need to extend our notion of what a QFT "is".

What does a "QFT for fractons" look like?

Two basic approaches (Shao, Slagle, ….)

Lattice →  → **Continuum QFT with discrete rotation symmetry**

(Seiberg-Shao)

Foliated Field theory

each layer: 2+1d QFT,
subsystem symmetry acts on layers

retain some sensitivity to lattice scale!

The simplest Seiberg-Shao theory

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_0\phi)^2 - \frac{1}{2\mu}(\partial_x\partial_y\phi)^2$$

- This is a 2+1 dimensional theory
- continuous translations
- 90 degree rotations: x→y, y→-x
- more bells and whistles needed for X-cube

The simplest Seiberg-Shao theory

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_0\phi)^2 - \frac{1}{2\mu}(\partial_x\partial_y\phi)^2$$

Subsystem symmetry:

$$\phi(t,x,y) \to \phi(t,x,y) + c^x(x) + c^y(y)$$

x labels lines. Shifting the field
at every point along a line of fixed x
is a symmetry.

The simplest Seiberg-Shao theory

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_0\phi)^2 - \frac{1}{2\mu}(\partial_x\partial_y\phi)^2$$

The theory is free!

- For a single scalar, all interactions consistent with symmetries are irrelevant!
- Interacting lattice fractons → free field theory
- Interacting theories possible if we couple SeSh scalar to other fields (Distler, AK, Raz – in progress)

Dispersion relation

$$\omega^2 = \frac{1}{\mu\mu_0} k_x^2 k_y^2 \, .$$

- $k_x=0$ gives $\omega=0$ for any k_y

UV/IR mixing!

- These would-be zero modes are lifted by quantum effects; mass set by lattice scale

A vast new realm of possibilities....

To understand what new possibilities arise once we allow QFTs with discrete rotations, we should first understand conformal Seiberg-Shao theories.

As always, we would expect the generic Seiberg-Shao theory to be thought of as a deformation of a conformal Seiberg-Shao theory.

Subsystem Conformal Symmetries

(Amir Raz, AK)

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_0\phi)^2 - \frac{1}{2\mu}(\partial_x\partial_y\phi)^2$$

Two independent scale symmetries:

$$D_x: \;\; t \to \lambda\, t, \qquad x \to \lambda\, x$$

$$D_y: \;\; t \to \lambda\, t, \qquad y \to \lambda\, y$$

Can they be extended to full conformal symmetries?

Yes! Subsystem Conformal Symmetries

Fourier transform in y:

$$\mathcal{L} = \left|\partial_t \phi_{k_y}\right|^2 + k_y^2 \left|\partial_x \phi_{k_y}\right|^2 .$$

standard 1+1 dimensional relativistic scalar, k_y^{-1} = "speed of light"

→ D_x part of standard relativistic 1+1 CFT in x and t

and similar for D_y

Full subsystem conformal algebra

- D_x and D_y do not commute
- Infinitely many additional generators needed to close algebra
- Are there simpler models where we can work out the full subsystem conformal symmetry?

A family of 2+1 dimensional theories with subsystem symmetries:

$$\mathcal{L} = |\partial_t\phi|^2 + |\partial_x\partial_y\phi|^2 .$$

$$\mathcal{L} = |\partial_t\phi|^2 + \phi^*\partial_x\partial_y\phi.$$

$$\mathcal{L} = i\phi^*\partial_t\phi + |\partial_x\partial_y\phi|^2 .$$

$$\mathcal{L} = i\phi^*\partial_t\phi + \phi^*\partial_x\partial_y\phi.$$

A family of 2+1 dimensional theories with subsystem symmetries:

relativistic kinetic term

$$\mathcal{L} = |\partial_t\phi|^2 + |\partial_x\partial_y\phi|^2 .$$

$$\mathcal{L} = |\partial_t\phi|^2 + \phi^*\partial_x\partial_y\phi.$$

$$\mathcal{L} = i\phi^*\partial_t\phi + |\partial_x\partial_y\phi|^2 .$$

$$\mathcal{L} = i\phi^*\partial_t\phi + \phi^*\partial_x\partial_y\phi.$$

non-relativistic kinetic term

A family of 2+1 dimensional theories with subsystem symmetries:

symmetries of cubic lattice

$$\mathcal{L} = |\partial_t \phi|^2 + |\partial_x \partial_y \phi|^2 .$$

$$\mathcal{L} = i\phi^* \partial_t \phi + |\partial_x \partial_y \phi|^2 .$$

x→y, y→-x

symmetries of rectangular lattice

$$\mathcal{L} = |\partial_t \phi|^2 + \phi^* \partial_x \partial_y \phi.$$

$$\mathcal{L} = i\phi^* \partial_t \phi + \phi^* \partial_x \partial_y \phi.$$

x→y, y→x

x→-x, y→-y

A family of 2+1 dimensional theories with subsystem symmetries:

$$\mathcal{L} = |\partial_t \phi|^2 + |\partial_x \partial_y \phi|^2 .$$

(2,4) model

(complex SeSh)

$$\mathcal{L} = |\partial_t \phi|^2 + \phi^* \partial_x \partial_y \phi.$$

(2,2) model

$$\mathcal{L} = i\phi^* \partial_t \phi + |\partial_x \partial_y \phi|^2 .$$

(1,4) model

$$\mathcal{L} = i\phi^* \partial_t \phi + \phi^* \partial_x \partial_y \phi.$$

(1,2) model

Zoo of conformal subsystem CFTs:

Model	Lagrangian	Sub-dimensional Conformal symmetry	Extended symmetry
$(2,4)$	$\lvert\partial_t\phi\rvert^2 + \lvert\partial_x\partial_y\phi\rvert^2$	Full relativistic $2d$	Infinite Dimensional, unknown
$(2,2)$	$\lvert\partial_t\phi\rvert^2 + \phi^*\partial_x\partial_y\phi$	Schrödinger $1+1$	$SO(3,2)$
$(1,4)$	$i\phi^*\partial_t\phi + \lvert\partial_x\partial_y\phi\rvert^2$	Schrödinger $1+1$	Large, unknown
$(1,2)$	$i\phi^*\partial_t\phi + \phi^*\partial_x\partial_y\phi$	Holomorphic $2d$	Schrödinger $1+(1,1)$

in two cases we can find the full extended subsystem conformal algebra!

The (1,2) model --- NR rectangular lattice

$$\mathcal{L} = i\phi^* \partial_t \phi + \phi^* \partial_x \partial_y \phi.$$

Finite dimensional sub-system conformal algebra:

$$H = \partial_t, \qquad P_x = \partial_x, \qquad P_y = \partial_y,$$

$$D_x = t\partial_t + x\partial_x + \frac{\Delta}{2}, \qquad D_y = t\partial_t + y\partial_y + \frac{\Delta}{2}, \qquad K_x = t\partial_x + Ny$$

$$K_y = t\partial_y + Nx, \qquad C = t^2\partial_t + tx\partial_x + ty\partial_y + t\Delta + Nxy.$$

The (1,4) model --- NR rectangular lattice

$$\mathcal{L} = i\phi^*\partial_t\phi + \phi^*\partial_x\partial_y\phi.$$

Finite dimensional sub-system conformal algebra:

standard translations

$$H = \partial_t, \qquad P_x = \partial_x, \qquad P_y = \partial_y,$$

$$D_x = t\partial_t + x\partial_x + \frac{\Delta}{2}, \qquad D_y = t\partial_t + y\partial_y + \frac{\Delta}{2}, \qquad K_x = t\partial_x + Ny$$

$$K_y = t\partial_y + Nx, \qquad C = t^2\partial_t + tx\partial_x + ty\partial_y + t\Delta + Nxy.$$

The (1,2) model --- NR rectangular lattice

$$\mathcal{L} = i\phi^* \partial_t \phi + \phi^* \partial_x \partial_y \phi.$$

Finite dimensional sub-system conformal algebra:

independent z=1 rescalings in (x,t) and (y,t)

$$H = \partial_t, \qquad P_x = \partial_x, \qquad P_y = \partial_y,$$

$$D_x = t\partial_t + x\partial_x + \frac{\Delta}{2}, \qquad D_y = t\partial_t + y\partial_y + \frac{\Delta}{2}, \qquad K_x = t\partial_x + Ny$$

$$K_y = t\partial_y + Nx, \qquad C = t^2\partial_t + tx\partial_x + ty\partial_y + t\Delta + Nxy.$$

The (1,2) model --- NR rectangular lattice

$$\mathcal{L} = i\phi^* \partial_t \phi + \phi^* \partial_x \partial_y \phi.$$

Finite dimensional sub-system conformal algebra:

the boost generators mix x and y

$$H = \partial_t, \qquad P_x = \partial_x, \qquad P_y = \partial_y,$$

$$D_x = t\partial_t + x\partial_x + \frac{\Delta}{2}, \qquad D_y = t\partial_t + y\partial_y + \frac{\Delta}{2}, \qquad \boxed{K_x = t\partial_x + Ny}$$

$$\boxed{K_y = t\partial_y + Nx,} \qquad C = t^2\partial_t + tx\partial_x + ty\partial_y + t\Delta + Nxy.$$

The (1,2) model --- NR rectangular lattice

$$\mathcal{L} = i\phi^* \partial_t \phi + \phi^* \partial_x \partial_y \phi.$$

Finite dimensional sub-system conformal algebra:

one special conformal generator needed to close the algebra

$$H = \partial_t, \qquad P_x = \partial_x, \qquad P_y = \partial_y,$$

$$D_x = t\partial_t + x\partial_x + \frac{\Delta}{2}, \qquad D_y = t\partial_t + y\partial_y + \frac{\Delta}{2}, \qquad K_x = t\partial_x + Ny$$

$$K_y = t\partial_y + Nx, \qquad C = t^2\partial_t + tx\partial_x + ty\partial_y + t\Delta + Nxy.$$

For completeness: the transformation laws

Boosts:

$$x' = x + ct, \qquad \phi' = e^{icy}\phi, \qquad \text{and} \qquad y' = y + ct, \qquad \phi' = e^{icx}\phi.$$

Conformal:

$$t' = \frac{at+b}{ct+d}, \qquad x' = \frac{x}{ct+d}, \qquad y' = \frac{y}{ct+d}, \qquad \phi' = (ct+d)e^{i\frac{cxy}{ct+d}}\phi, \qquad ad - bc = 1.$$

Scaling and Translations take “obvious” form

Why was this case so simple?

An old friend in disguise:

$$\mathcal{L} = i\phi^*\partial_t\phi + \phi^*\partial_x\partial_y\phi. \quad \longleftarrow \quad \mathcal{L} = i\phi^*\partial_t\phi + \frac{1}{2}\left(\partial_v\phi^*\partial_v\phi - \partial_u\phi^*\partial_u\phi\right),$$

Lightcone coordinates

Just a standard Schrodinger scalar in 2+1 dimensions
(with 2 time, on space dimension).

Not a useful way to study the theory, but it explains the algebra.

Conclusion and Future Directions

QFTs with subsystem symmetries and discrete rotation furnish novel and intricate conformal symmetries.

Many of these structures already apparent at the level of free theories in 2+1 dimensions.

Higher Dimensions?

Clearly very similar structures can also be found in other D:

E.g.: $$\mathcal{L} = \ldots + (\partial_{x^1} \ldots \partial_{x^D} \phi)^2$$

(You,Bi.Pretko;Gorantla, Lam,Seiberg, Shao)

$$D_i : \qquad x^i \to \lambda x^i, \quad t \to \lambda^z t$$

Interacting CFTs?

Natural candidate: CFT at second order phase transition between subsymmetry broken and unbroken phase.

Batista/Nussinov:

A subsystem symmetry acting on d+1 dimensional defects obeys the same theorems as a QFT in d+1 dimensions.

In particular, Mermin/Wagner tells us we need at least 2+1 dimensional subsystem symmetries to allow 2^{nd} order transitions

Examples?

$$\mathcal{L} = |\partial_t \phi|^2 + |\partial_x \partial_y \phi|^2 .$$

(2+1 Seiberg Shao)

Subsystem symmetry acting on lines.
Can not be broken.

$$\mathcal{L}_{3'} = \frac{\mu_0}{2}(\partial_0\theta)^2 - \frac{1}{2\mu}(\partial_x\partial_y\partial_z\theta)^2.$$

(3+1 from last slide)

Subsystem symmetry acting on lines.
Can not be broken.

$$\theta \to \theta + c_x(y,z) + c_y(z,x) + c_z(x,y)$$

Examples?

$$\mathcal{L}_3 = \frac{\mu_0}{2}(\partial_0\theta)^2 - \frac{1}{2\mu}\left[(\partial_x\partial_y\theta)^2 + (\partial_z\partial_x\theta)^2 + (\partial_y\partial_z\theta)^2\right]$$

$$\theta \to \theta + c_x(x) + c_y(y) + c_z(z)$$

(Alternate 3+1 theory)

Subsystem symmetry acting on planes.
Can be broken.

Distler, AK, Raz: Couple this system to 2+1 scalars living on planes to drive Wilson-Fisher like transition.

Stay Tuned!