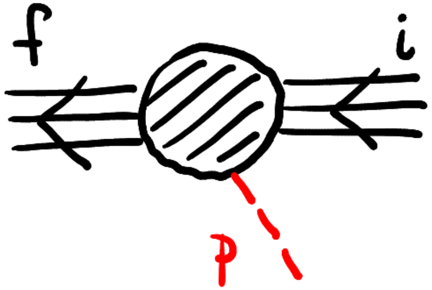
EXCEPTIONAL NONRELATIVISTIC EFFECTIVE FIELD THEORIES

PROLOGUE

ADLER ZERO

NG bosons interact as a rule weakly:

$$\mathcal{A}_{fi}(p) =$$



$$\lim_{p \to 0} \mathcal{A}_{fi}(p) = 0$$

typically $\mathcal{A}_{fi}(p) = \# p + \mathcal{O}(p^2)$

Exceptions to the rule:

- $\lim_{p \to 0} \mathcal{A}_{fi}(p) \neq 0$ Kampf et al. (2020)
- $\mathcal{A}_{fi}(p) = \# p^{\sigma} + \mathcal{O}(p^{\sigma+1})$, $\sigma > 1$: exceptional EFTs

ENHANCED SCATTERING AMPLITUDES

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\theta)^2 + c_4[(\partial_\mu\theta)^2]^2 + c_6[(\partial_\mu\theta)^2]^3 + c_8[(\partial_\mu\theta)^2]^4 + \ldots$$

arbitrary scale

fixed in terms of c_4 by requiring $\mathcal{A}_{fi}(p) \sim p^2$ order by order

$$S = \int d^D x \, \frac{1}{v}\sqrt{1 + v(\partial_\mu\theta)^2}$$

Cheung et al. (2015)

Enhanced (hidden) symmetry:

$$\theta \longrightarrow \theta + \beta_\mu x^\mu + \ldots$$

$$x^\mu \longrightarrow x^\mu - v\beta^\mu\theta + \ldots$$

EXCEPTIONAL THEORIES

(RELATIVISTIC)

with Mark Bogers

PRL 121 (2018) 171602

JHEP 05 (2018) 076

WHAT EXCEPTIONAL THEORIES ARE THERE?

Two possible approaches:

- bottom-up : scattering amplitudes ⟶ effective theories
- top-down : search for allowed hidden symmetries instead!

Classify allowed Lie algebras ⟶ Derive corresponding effective Lagrangians ⟶ Calculate scattering amplitudes

Bogers, TB (2018)
Roest et al. (2019)

THEORIES WITH ONE NG BOSON

Classification of Lie algebras :

$$\left.\begin{aligned}
[J_{\mu\nu}, J_{\kappa\lambda}] &= i\left(g_{\mu\lambda}J_{\nu\kappa} + g_{\nu\kappa}J_{\mu\lambda} - g_{\mu\kappa}J_{\nu\lambda} - g_{\nu\lambda}J_{\mu\kappa}\right)\\
[J_{\mu\nu}, P_\lambda] &= i\left(g_{\nu\lambda}P_\mu - g_{\mu\lambda}P_\nu\right)\\
[J_{\mu\nu}, K_\lambda] &= i\left(g_{\nu\lambda}K_\mu - g_{\mu\lambda}K_\nu\right)
\end{aligned}\right\}\ \text{fixed by Lorentz invariance}$$

$$\left.\begin{aligned}
[P_\mu, K_\nu] &= i\left(a\, g_{\mu\nu} Q + b\, J_{\mu\nu} + c\, \tilde{J}_{\mu\nu}\right)\\
[P_\mu, Q] &= i\left(d\, P_\mu + e\, K_\mu\right)\\
[K_\mu, K_\nu] &= i\left(f\, J_{\mu\nu} + g\, \tilde{J}_{\mu\nu}\right)\\
[K_\mu, Q] &= i\left(h\, P_\mu + i\, K_\mu\right)
\end{aligned}\right\}\ \text{constrained by Jacobi identities}$$

scalar generator responsible for the NG boson

vector generator of hidden symmetry

THEORIES WITH ONE NG BOSON

DBI scalar theory

$$S = \int d^Dx \sqrt{1 + v\,\partial_\mu\theta\,\partial^\mu\theta} + \ldots$$

Describes fluctuations of a D-brane in an extra dimension.

Symmetries :

- $\theta \to \theta + \alpha$
- $\theta \to \theta + \beta_\mu x^\mu + \ldots$

  $x^\mu \to x^\mu - v\beta^\mu\theta + \ldots$

$v > 0$: $SO(d,2) \ltimes \mathbb{R}^{D+1}$

$v < 0$: $SO(d+1,1) \ltimes \mathbb{R}^{D+1}$

# of spatial dimensions

$D = d+1$

THEORIES WITH ONE NG BOSON

Galileon theory

$$S = \sum_{k=0}^{D} \int d^D x \, \frac{c_k}{(D-k)!} \, \theta G_k + \dots$$

Symmetries :

- $\theta \to \theta + \alpha$
- $\theta \to \theta + \beta_\mu x^\mu$

$$G_k = \epsilon^{\mu_1 \dots \mu_k \lambda_{k+1} \dots \lambda_D} \epsilon^{\nu_1 \dots \nu_k}{}_{\lambda_{k+1} \dots \lambda_D} (\partial_{\mu_1} \partial_{\nu_1} \theta) \dots (\partial_{\mu_k} \partial_{\nu_k} \theta) = (D-k)! \det \{ \partial^{\mu_i} \partial_{\mu_j} \theta \}_{i,j=1}^{k}$$

Altogether D+1 different Lagrangians, all invariant only up to a surface term: Wess-Zumino-Witten terms.

Goon et al. (2012)

MULTI-FLAVOR THEORIES

Symmetry content :

- Poincaré algebra generated by $J_{\mu\nu}$, P_μ
- Q_i : scalar generators that may but need not be spontaneously broken
- $K_{\mu A}$: vector generators of hidden symmetries

Assumptions :

- $[P_\mu, Q_i] = 0$ ... to ensure Adler zero property
- $[P_\mu, K_{\nu A}] \supset Q_A$ ... to make $K_{\mu A}$ "redundant"

subset of Q_i,
all spontaneously broken

LIE ALGEBRA

$$[P_\mu, K_{\nu A}] = i g_{\mu\nu} Q_A$$

$$[K_{\mu A}, K_{\nu B}] = i\left(g_{AB} J_{\mu\nu} + g_{\mu\nu} Q_{AB}\right)$$

$$[K_{\mu A}, Q_B] = -i g_{AB} P_\mu$$

$$[K_{\mu C}, Q_{AB}] = i\left(g_{AC} K_{\mu B} - g_{BC} K_{\mu A}\right)$$

$$[Q_A, Q_B] = 0$$

$$[Q_{AB}, Q_C] = i\left(g_{BC} Q_A - g_{AC} Q_B\right)$$

$$[Q_{AB}, Q_{CD}] = i\left(g_{AD} Q_{BC} + g_{BC} Q_{AD} - g_{AC} Q_{BD} - g_{BD} Q_{AC}\right)$$

Pseudo-Euclidean algebra with:

- metric $g_{\mu\nu} \oplus g_{AB}$
- extra translations Q_A
- extra rotations $K_{\mu A}$, Q_{AB}

MULTI-FLAVOR THEORIES

DBI-like theories :

- theories with non-singular g_{AB}
- Q_i other than Q_A may also be spontaneously broken

$$S = \int d^D x \sqrt{-|g_{\mu\nu} - g_{AB}\,\partial_\mu \theta^A \partial^\nu \theta^B|} + \ldots$$

Describe fluctuations of a flat D-dimensional brane in a higher-dimensional pseudo-Euclidean spacetime.

MULTI-FLAVOR THEORIES

Galileon-like theories :

- theories with vanishing g_{AB} (and some additional assumptions)
- Q_i other than Q_A may also be spontaneously broken

$$S = \sum_{k=0}^{D} \int d^D x \, \frac{1}{(D-k)!} \, c_{A_1 \cdots A_{k+1}} \, \theta^{A_1} G_k^{A_2 \cdots A_{k+1}} + \ldots$$

fully symmetric invariant tensor

$$G_k^{A_1 \cdots A_k} = \epsilon^{\mu_1 \cdots \mu_k \lambda_{k+1} \cdots \lambda_D} \epsilon^{\nu_1 \cdots \nu_k}{}_{\lambda_{k+1} \cdots \lambda_D} (\partial_{\mu_1} \partial_{\nu_1} \theta^{A_1}) \cdots (\partial_{\mu_k} \partial_{\nu_k} \theta^{A_k})$$

EXCEPTIONAL THEORIES (NONRELATIVISTIC)

JHEP 02 (2021) 218

NONRELATIVISTIC EFFECTIVE THEORIES

Motivation :

- condensed-matter physics
- nonrelativistic gravity
- nonrelativistic string theory

LIE ALGEBRA

Reuse the previous results by interpreting :

- $g_{\mu\nu}$ as Euclidean metric
- $J_{\mu\nu}$ as spatial rotations
- P_{μ} as spatial translations

Include the Hamiltonian H among the scalar generators Q_i.

Include the boost vector K_{μ} among the vector generators $K_{\mu A}$.

DBI-LIKE THEORIES

Minimal action containing kinetic terms for all NG fields θ^A :

$$S = \int dt\, d^dx \, \sqrt{G} \left(1 + g_{AB} \nabla_0 \theta^A \nabla_0 \theta^B \right)$$

$$G_{\mu\nu} = g_{\mu\nu} - g_{AB}\, \partial_\mu \theta^A \partial_\nu \theta^B$$

$$(\tilde{G}^{-1/2})_C{}^B \, \partial_0 \theta^C$$

$$\tilde{G}^{AB} = g^{AB} - \partial_\mu \theta^A \partial^\mu \theta^B$$

WESS-ZUMINO-WITTEN TERM

Interesting possibility for two flavors of DBI scalars :

$$S_{WZW} = \int dt\, d^dx\, \epsilon_{AB}\, \theta^A \partial_0 \theta^B$$

- manifestly symmetric under ISO(2)
- secretly symmetric under two-flavor DBI-like symmetry

This describes a single complex (Schrödinger) scalar in disguise.

MINIMAL SCHRÖDINGER-DBI THEORY

$$S = \int dt\, d^2x \left[i\psi^\dagger \partial_0 \psi + \underbrace{\sqrt{1 - 2\vec{\nabla}\psi^\dagger \cdot \vec{\nabla}\psi + (\vec{\nabla}\psi^\dagger \cdot \vec{\nabla}\psi)^2 - |\vec{\nabla}\psi \cdot \vec{\nabla}\psi|^2}}_{\sqrt{G} \text{ in terms of } \psi,\, \psi^\dagger} \right]$$

Interesting features:

- single type-B NG boson in the spectrum
- nonzero commutator $[Q_1, Q_2]$ due to central charge generated by the WZW term

GALILEON - LIKE THEORIES

Scalar sector : $\{Q_i\} \longrightarrow \{\tilde{Q}_i\} \ltimes \{Q_A, Q_{AB}\}$

Abelian ideal of the scalar algebra

$$[P_\mu, K_{\nu A}] = i g_{\mu\nu} Q_A$$

$$[K_{\mu A}, K_{\nu B}] = i g_{\mu\nu} Q_{AB}$$

$$[\tilde{Q}_i, K_{\mu A}] = (t_i)^B{}_A K_{\mu B}$$

$$[\tilde{Q}_i, Q_A] = (t_i)^B{}_A Q_B$$

$$[\tilde{Q}_i, Q_{AB}] = (t_i)^C{}_A Q_{CB} + (t_i)^C{}_B Q_{AC}$$

$$[\tilde{Q}_i, \tilde{Q}_j] = i f_{ij}^k \tilde{Q}_k$$

The complete symmetry algebra is fixed by:

- the Lie algebra of Q_i
- its Abelian ideal generated by Q_A, Q_{AB}

COSET CONSTRUCTION

Coset space parametrization :

$$U(t,x,\theta,\xi) = e^{itH} e^{ix^\mu P_\mu} e^{i\theta^A Q_A} e^{\frac{i}{2}\theta^{AB} Q_{AB}} e^{i\xi^{\mu A} K_{\mu A}} e^{i\theta^a \tilde{Q}_a}$$

only include those generators that actually are spontaneously broken

Maurer-Cartan form :

$$\omega = -i U^{-1} dU = \omega_H H + \omega_P^\mu P_\mu + \omega_Q^A Q_A + \frac{1}{2}\omega_Q^{AB} Q_{AB} + \omega_K^{\mu A} K_{\mu A} + \Omega^i \tilde{Q}_i$$

vielbein

IHC $\xi_\mu^A = \partial_\mu \theta^A$

covariant derivative $\nabla_0 \theta^A$

covariant derivative of θ^a

WESS-ZUMINO-WITTEN TERMS

Cohomology problem: find all invariant closed $(D+1)$-forms that are not derivatives of an invariant D-form

Galileon forms: $G_k^{A_1\cdots A_k} = \epsilon_{\mu_1\cdots\mu_d}\,\omega_K^{\mu_1 A_1}\wedge\ldots\wedge\omega_K^{\mu_k A_k}\wedge\omega_P^{\mu_{k+1}}\wedge\ldots\wedge\omega_P^{\mu_d}$, $0\leqslant k\leqslant d$

Candidate WZW $(D+1)$-forms:

- $\boxed{\omega_Q^{A_1}\wedge\omega_H\wedge G_k^{A_2\cdots A_{k+1}}}$
- $\Omega^a\wedge\omega_H\wedge G_k^{A_1\cdots A_k}$
- $\boxed{\omega_Q^{A_1}\wedge\omega_Q^{A_2}\wedge G_k^{A_3\cdots A_{k+2}}}$
- $\Omega^a\wedge\omega_Q^{A_1}\wedge G_k^{A_2\cdots A_{k+1}}$
- $\Omega^a\wedge\Omega^b\wedge G_k^{A_1\cdots A_k}$

each of these classes leads to nontrivial WZW terms!

these have no equivalent in relativistic field theory!

these depend only on the Galileon fields θ^A

SAMPLE WZW TERMS

- $\mathcal{L} = c_{AB}\theta^A \partial_0 \theta^B$: Schrödinger kinetic term, leads to type-B NG bosons

- $\mathcal{L}_k = c_{aA_1\cdots A_{k+1}} \theta^a \partial_0 \theta^{A_1} G_k^{A_2\cdots A_{k+1}}$, $0 \leqslant k \leqslant d$

requires that $\rightarrow$ $c_{aA_1\cdots A_{k+1}}$ is fully symmetric in $A_1,\dots,A_{k+1}$ and invariant under the action of t_i on these

$\rightarrow$ $f^a_{ij} c_{aA_1\cdots A_{k+1}} = 0$

examples : $k=0$ $\rightsquigarrow$ $\mathcal{L}_0 = c_{aA}\theta^a \partial_0 \theta^A$

$k=1$ $\rightsquigarrow$ $\mathcal{L}_1 = c_{aAB}\theta^a \partial_0 \theta^A \partial_\mu \partial^\mu \theta^B$

SCATTERING AMPLITUDES

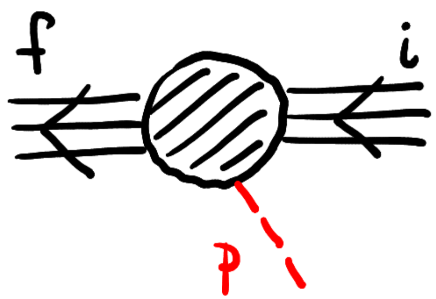
with Martin Mojahed

(preliminary)

PROOF OF ADLER ZERO: RELATIVISTIC

Extract the amplitude from a matrix element of the broken current:

$$\mathcal{A}_{fi}(p) =$$



$$\langle f | J^\mu(0) | i \rangle = \frac{i}{p^2} \underbrace{\langle 0 | J^\mu(0) | \theta(\vec{p}) \rangle}_{= ip^\mu F} \langle f + \theta(\vec{p}) | i \rangle + R^\mu(p)$$

Current conservation: $p_\mu \langle f | J^\mu(0) | i \rangle = 0 \rightsquigarrow \mathcal{A}_{fi}(p) = \frac{1}{F} p_\mu R^\mu(p)$

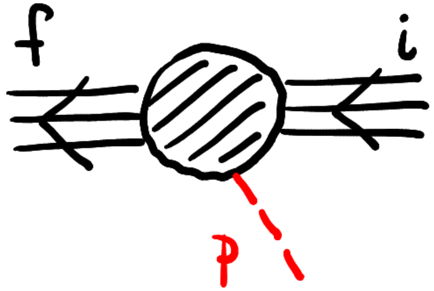
Hidden symmetry → Relations among Noether currents → Constraints on $R^\mu(p)$ → $\mathcal{A}_{fi}(p) \sim p^\sigma$, $\sigma > 1$

TB, Watanabe (2014)
TB (2020)

Cheung et al. (2017)

PROOF OF ADLER ZERO: NONRELATIVISTIC

$$\mathcal{A}_{fi}(p) =$$



$$\mathcal{A}_{fi}(p) = \frac{1}{F} p_\mu R^\mu(p)$$

still holds, but a spatial hidden symmetry only constrains the spatial part of $R^\mu(p)$!

Two roads towards enhanced scattering amplitudes in NR theories:

- hidden spatial and temporal symmetry
- hidden spatial symmetry and nonlinear dispersion relation

BOOTSTRAPPING REAL SCALAR EFTS

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \theta)^2 + \mathcal{L}_{int}(\partial_0 \theta, \vec{\nabla}\theta)$$

rotationally invariant, only first derivatives

impose $\mathcal{A}_{fi}(p) \sim p^2$

unique solution $\mathcal{L} = \frac{1}{v}\sqrt{1 + v(\partial_\mu \theta)^2}$ ... relativistic DBI

Lorentz invariance as emergent symmetry from soft limits ?!

BOOTSTRAPPING COMPLEX SCALAR EFTS

$$\mathcal{L} = \psi^\dagger(i\partial_0 + \vec{\nabla}^2)\psi + \mathcal{L}_{int}(\partial_0\psi, \partial_0\psi^\dagger, \vec{\nabla}\psi, \vec{\nabla}\psi^\dagger)$$

rotationally invariant, only first derivatives

impose $\mathcal{A}_{fi}(p) \sim p^2$

$$\mathcal{L} = \underbrace{\epsilon_{AB}\theta^A\partial_0\theta^B + \sqrt{G}\,[1}_{\text{minimal Schrödinger-DBI theory}} + \underbrace{c_4(\delta_{AB}\nabla_0\theta^A\nabla_0\theta^B)^2 + c_6(\delta_{AB}\nabla_0\theta^A\nabla_0\theta^B)^3}_{\text{higher-order corrections}} + \dots]$$

EPILOGUE

CONCLUSIONS

Dynamics of massless particles is strongly constrained by symmetry!

Adler zero (soft limit) $\longrightarrow$ coset effective theories with constraints on G/H

Kampf et al. (2020)

Enhanced soft limit $\longrightarrow$ Hidden symmetries $\longrightarrow$ Effective theories with special geometry

Cheung et al. (2015)

Hinterbichler, Joyce (2015)

Bogers, TB (2018)

Still a lot to understand, in both relativistic and nonrelativistic context!