Spacetime Symmetry Breaking:

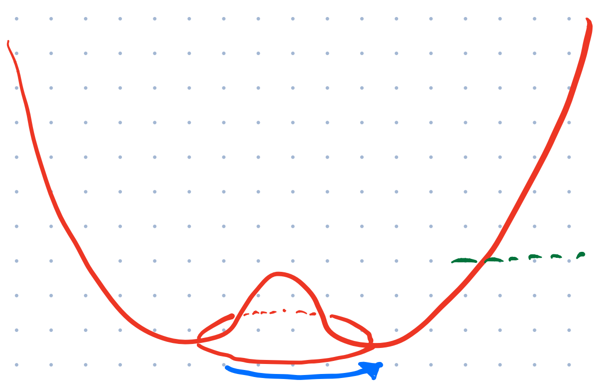
Goldstone Modes, Soft Limits & Double copies

- Introduction
- Non-Abelian
- Space-time symmetries
- Exceptional case
- Double copies.

Introduction

Symmetries key to Nature:

- linearly realised : multiplets (vectors, metric tensor)
- non-linearly ..... : Goldstone modes (pions, inflaton, phonons).



linear (UV)

spontaneous symmetry breaking scale

non-linear (IR)

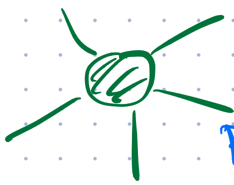
non-linear / shift symmetry

Goldstone mode : $\phi \to \phi + c$

derivatively coupled : $\mathcal{L} = c_2 (\partial\phi)^2 + c_4 (\partial\phi)^4 + \ldots\ldots$

(plus Wess-Zumino terms)

special amplitudes :



$p_\mu \to \epsilon \cdot p_\mu$

vanish in soft limit $(\epsilon \to 0)$

Adler zero

$$A_n = 0 + (\ldots)\epsilon + (\ldots)\epsilon^2$$

Shift-symmetric scalar is simplest case $=$ non-linear $U(1)$.

Generalisation to

* non-Abelian ?
* non-trivial spacetime ?

$\rightsquigarrow$ Goldstone modes, soft limits ? (double copy) relations ?

Non-Abelian case

Generalisation to any "coset" of internal symmetry breaking:

[Coleman, Callan, Wess, Zumino, 1969]

non-linear

$\frac{G}{H}$

linear

(e.g. $\frac{SU(N) \times SU(N)}{SU(N)}$ for pions.)

Coset universality: fixed by symmetry breaking pattern.

Goldstone mode for every broken generator: ϕ in adjoint of G/H.

Non-linear symmetry: $\phi \to \phi + c + \phi \cdot c \cdot \phi$

fixed by non-Abelian symmetry

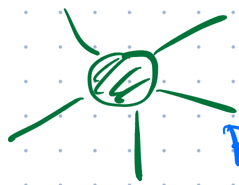
Interacting field theory:

$$\mathcal{L} = c_2 \cdot \left((\partial\phi)^2 + \phi^2 (\partial\phi)^2 + \ldots \right) + \text{higher-derivatives.}$$

non-linear sigma model (NLSM)

note: similarity to gravity - all two-derivative interactions fixed by symmetry.

Soft limit:



$p_\mu \to \epsilon \cdot p_\mu$

$$A_n = 0 + \epsilon \cdot (\ldots) + \epsilon^2 \cdot (\ldots)$$

$\Rightarrow$ Adler zero for all Goldstones.

Internal symmetries:

	Abelian	vs	non-Abelian
Goldstone modes?	1-on-1		1-on-1
soft limits?	$A_n \sim \epsilon$		$A_n \sim \epsilon$
interactions?	free		fixed.

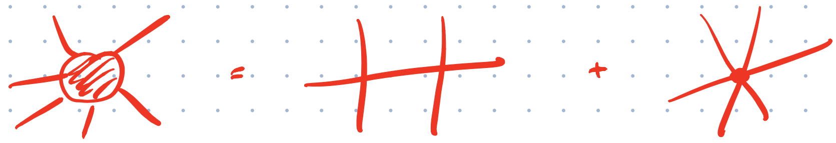
Space-time Symmetries.

Back to shift-symm. scalar, with

$$\mathcal{L} = c_2 \cdot (\partial\phi)^2 + c_4 \cdot (\partial\phi)^4 + \ldots$$

Q: tune coefficients to get symmetry enhancement?

Approach 1: soft limits, eg. for 6-point:



$$\mathcal{O}(\epsilon^2) \Leftarrow \mathcal{O}(\epsilon) + \mathcal{O}(\epsilon)$$

Requires $\mathcal{L} = \sqrt{1+(\partial\phi)^2} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{4}(\partial\phi)^4 + \ldots$

Dirac-Born-Infeld (DBI).

[Cheung, Kampf, Novotny, Trnka 2014]

Approach 2: classify possible algebras

shift symmetry + vector transformation = Poincaré in 5D

$$\delta\phi = c \quad + \quad c_\mu (x^\mu + \phi \partial^\mu \phi)$$

space-time symmetry

Requires same Lagrangian

Algebraic constraints only allow for a handful of such "exceptional" field theories.

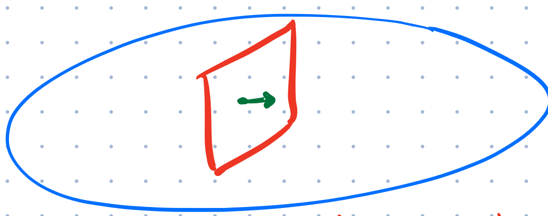
[Bogers, Brauner 2018, DR, Stefanyszyn, Wertman, 2019].

Note: more broken symmetries than Goldstones. "inverse Higgs constraints" [Ivanov, Ogievetsky 1975].

Natural multi-field generalisation

$$\frac{G}{H} = \frac{\text{Poincare in } D=4+N}{\text{Poincare in } D=4} \quad \Rightarrow \quad SO(N) \text{ interacting scalars.}$$

Geometric interpretation:



Mink_{4+N} bulk.

Mink_4 brane

Exceptional case

More space-time symmetries / even softer limits? Answered with dynamical & algebraic approaches.

Only ONE more case:

amplitudes

symmetry $\qquad \delta\phi = c + c_\mu \cdot x^\mu + c_{\mu\nu}\left(x^\mu x^\nu + \partial^\mu\phi\,\partial^\nu\phi\right)$

Lagrangian $\qquad \mathcal{L} = (\partial\phi)^2 + (\partial\phi)^2 \cdot (\partial\partial\phi)^2$

[Hinterbichler, Joyce 2015].

"special Galileon" (SG)

One Goldstone mode,
two inverse Higgs constraints,
three Adler zeroes.

$\rightsquigarrow$ most symmetric scalar?

Similarity to gravity :

* symm. group : linear coord. transformations $\quad x^\mu \to \Lambda^\mu{}_\nu \cdot x^\nu$

* covariant metric $\quad g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \partial_\nu \phi.$

* dynamics in terms of invariants : $\quad \det(g) = 1$ [DR, 2020].

* no multi-field generalisation

* highest soft limit.

	NLSM	DBI	SG
inverse Higgs	0	1	2
soft limit	$\mathcal{O}(\epsilon)$	$\mathcal{O}(\epsilon^2)$	$\mathcal{O}(\epsilon^3)$
SO(N) irrep	adjoint	fund.	singlet.

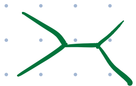
Double Copy

$g_{\mu\nu}$ $\quad$ A_μ^a

Intriguing relations between GR and YM:
at amplitude level [Bern, Carrasco, Johansson, 2008]

Yang-Mills amplitude factorise:

$$A_4 = \sum_{s,t,u} \frac{\text{colour} * \text{kinematics}}{\text{propagator}}$$



colour: $\quad c_s = f_{ab}{}^e \cdot f_{cde} \qquad c_s + c_t + c_u = 0.$
(Jacobi id.)

kinematics: $\quad n_s = n_s(p_-, e_-) \qquad n_s + n_t + n_u = 0$
(kinematic Jacobi)

"colour-kinematics duality"

General Relativity closely related:

$$A_4 = \sum_{s,t,u} \frac{\text{kinematics} * \text{kinematics}}{\text{propagator}}$$

"double copy"

bi-adjoint cubic scalar	-	adjoint YM	-	singlet GR
colour * colour		colour * kinematics		kinematics * kinematics

Similar double copy relations for
three exceptional scalar field theories:

adjoint		fundamental		singlet
NLSM	-	DBI	-	SG
flavour * flavour		flavour * kinematics		kinematics * kinematics

"flavour - kinematics duality".

e.g. 4-point numerators:

kinematics : $n_s = s \cdot (t-u)$ $\qquad n_s + n_t + n_u = 0.$

flavour : $f_s = \delta_{ab} \cdot \delta_{cd} \cdot (t-u) + \delta_{a[c} \delta_{d]b} \cdot s$ $\qquad f_s + f_t + f_u = 0.$

[DR, Wang, in progress]

Q: new perspective on soft limits / exceptional interactions of these theories?

Q: similar classification / relation in non-relativistic case?

Thanks