

GRAVITATIONAL COUPLING TO COMPOSITE SYSTEMS AND THEIR QUANTUM INTERFERENCE

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INTERPLAY BETWEEN QUANTUM SYSTEMS AND GRAVITY



e.g. Kopp, Fragkos, IP [arXiv:2105.1345]

OVERVIEW

- Time dilation in interferometry: the "quantum twin paradox"
- Time dilation induced entanglement in composite quantum systems
- Decoherence due to time dilation
- Coupling of composite systems to gravity revisited
- Seeming violation of equivalence principle in passive gravitational mass
- Resolution: correct passive gravitational mass from first principles

INFLUENCE OF NEWTONIAN GRAVITY IN QM

Newtonian gravitational potential in matter waves:



What do we learn:

- Gravitational potential affects quantum wave function
- Coherent phase induced by Newtonian gravity
 What don't we learn:
- Beyond Newtonian limit: gravity is metric theory!
- Quantization of gravitational degrees of freedom: quantum gravity

Neutron interferometry

e.g. R. Colella, A.W. Overhauser, S.A. Werner, PRL 34, 1472-1474 (1975)



Atomic fountains



e.g.T. Kovachy et. al. Nature 528, 530–533 (2015)

BEYOND NEWTON: GRAVITATIONAL TIME DILATION



Two initially synchronized clocks placed at different gravitational potentials. Clock closer to a massive body ticks slower than the clock further away from the mass.



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QUANTUM COMPLEMENTARITY



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OVERLAP OF QM AND GR: "QUANTUM TWIN" SITUATION



 \Rightarrow no path coherence

MEASUREMENT OF GRAVITATIONAL TIME DILATION

Proper time in the presence of gravity:

 $\tau_2 \approx \tau_1 \left(1 + \frac{\Phi(x_2)}{c^2} + \cdots \right)$ Gravitational time dilation Two clocks: $|c(\tau_2)\rangle_2 = e^{-iH_0\tau_2/\hbar}|c(0)\rangle_2$ Clock mechanism is quantum mechanical, but the position remains classical. $|c(\tau_1)\rangle_1 = e^{-iH_0\tau_1/\hbar}|c(0)\rangle_1$

Demonstrated with Cs-clocks (classic test of general relativity):

J. Hafele, R. Keating, Science 177, 166–168 (1972)

Trapped ion clocks (30cm height difference): C. Chou, D. Hume, T. Rosenband, D. Wineland Science 329, 1630–1633 (2010)



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POST-NEWTONIAN HAMILTONIAN

Probe particle on a background metric:

$$S = mc^2 \int d\tau$$
$$L = -mc^2 \frac{d\tau}{dt}$$

The Schwarzschild metric

 $ds^2 - s^2 d\sigma^2 - \sigma x \mu x V$

$$g_{00} \simeq -\left(1 + 2\frac{\phi(x)}{c^2} + 2\frac{\phi(x)^2}{c^4}\right), \ g_{ij} \simeq \frac{1}{c^2}\delta_{ij}\left(1 - 2\frac{\phi(x)}{c^2}\right)$$

$$\dot{\tau} \simeq \sqrt{1 + 2\frac{\phi(x)}{c^2} + 2\frac{\phi(x)^2}{c^4} - \left(\frac{\dot{x}}{c}\right)^2 \left(1 - 2\frac{\phi(x)}{c^2}\right)}$$



$$p = mv + \frac{mv^3}{2c^2} - \frac{3mv\Phi(x)}{c^2}$$

Can also be obtained starting from the KG equation:

C. Lämmerzahl. "A Hamilton operator for quantum optics in gravitational fields." Physics Letters A 203, 12-17 (1995)

POST-NEWTONIAN HAMILTONIAN: INTERFEROMETRY

$$H = mc^{2} + \frac{p^{2}}{2m} + m\Phi(x) + \frac{m\Phi^{2}(x)}{2c^{2}} - \frac{p^{4}}{8m^{3}c^{2}} + \frac{3p\Phi(x)p}{2mc^{2}}$$

Can do quantum optics in Hamiltonian forumaltion Simpler approach:

Quantum phases depend on world-line:

$$\phi_j = -\frac{S_j}{\hbar} = mc^2 \int d\tau_j$$



S. Dimopoulos, P.W. Graham, J. M. Hogan, M.A. Kasevich. "General relativistic effects in atom interferometry." Physical Review D 78, 042003 (2008).

Phases depend on proper time. Can they be seen as ",clocks"? No! Global phase unobservable Proper time difference \neq measurement of time

HAMILTONIAN FOR TIME DILATION

Add degrees of freedom that measure time (clocks)

Total Hamiltonian?

Total mass of a system:



Generated by internal Hamiltonian H_0

Simple derivation to lowest order in 1/c²: GR time dilation as a result of mass-energy equivalence

Interaction with gravitational potential $\Phi(x)$:

$$m_{tot} = m + \frac{H_0}{c^2} \rightarrow \text{internal} \\ \text{dynamics} \\ \text{remaining static part} \quad \text{internal} \\ \text{dynamics} \\ \text{e.g:} \quad H_0 = \hbar\omega \rightarrow \text{redshifted to } \hbar\omega \left(1 + \frac{\Phi(x)}{c^2}\right) \quad \text{Gravitational} \\ \text{redshift} \\ \text{e.g:} \quad H_0 = \hbar\omega \rightarrow \text{redshifted to } \hbar\omega \left(1 + \frac{\Phi(x)}{c^2}\right) \quad \text{dynamics} \\ \text{dynamics} \\ \text{for a static part} \quad \text{for a static part} \quad \text{for a static part} \\ \text{for a static part} \quad \text{fo a static par$$

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \left(H_0 + mc^2 + \frac{p^2}{2m} + m\Phi(x) + \frac{m\Phi^2(x)}{2c^2} - \frac{p^4}{8m^3c^2} + \frac{3p\Phi(x)p}{2mc^2} + \left[\frac{\Phi(x)}{c^2} - \frac{p^2}{2m^2c^2}\right]H_0 + \cdots\right)|\psi\rangle$$

Gravitational part of interaction with $\Phi(\mathbf{x}) = g\mathbf{x}$: $H_{int} = \frac{g\mathbf{x}}{c^2}H_0$

Coupling between *internal* and *external* d.o.f. (revisit mass-energy equivalence later in talk)

CLOCKS IN SUPERPOSITION & TIME DILATION



Quantum mechanically: $\hat{H} \approx \hat{H}_{cm} + \hat{H}_0 + mg\hat{x} + \left|\frac{gx}{c^2}\hat{H}_0\right|$

 $|\psi\rangle =$

General relativity entangles any clock to the path due to time dilation.

Takes place only if both GR and QM present.

$$r_{m} + \hat{H}_{0} + mg\hat{x} + \frac{gx}{c^{2}}\hat{H}_{0}$$

$$\downarrow g$$

$$\downarrow g$$

$$\downarrow g$$

$$\frac{1}{\sqrt{2}}(|\psi_{down}\rangle|c_{down}\rangle + e^{-i\Delta\phi}|\psi_{up}\rangle|c_{up}\rangle)$$

Due to entanglement:

$$V = |\langle c_{down} | c_{up} \rangle| < 1$$

Change in Visibility

Test with matter-wave interferometry with additional internal clock-states $|c\rangle$ (e.g. $|c\rangle \propto |g\rangle + |e\rangle$)

M. Zych, F. Costa, I. Pikovski, Č. Brukner. Nature commun. 2, 505 (2011)

PHASE SHIFT VS. ENTANGLEMENT

Explainable by:

- Newtonian potential with absolute time
- analogues to a charged particle in EM
 Flat space-time: no notion of redshift
 necessary, even in post-Newton

Experimentally observed in Newtonian limit

Requires:

- proper time τ flows at different rates time dilation
- space-time geometry entangles clock to the path
- iff a particle is an operationally well defined ,,clock"– dynamical evolution of a degree-of-freedom

Experiment challanging

UNIVERSALITY OF TIME DILATION: EFFECT PRESENT IN ALL SYSTEMS



UNIVERSAL DECOHERENCE DUE TO GRAVITATIONAL TIME DILATION

Universality: any system affected equally by time dilation (fundamental in general relativity)

- Large System (N >> I internal degrees of freedom)
- No control of internal states
- No internal coherence (e.g. thermal state of each constitutent)

Internal states serve as "bath" for the center-of-mass

Gravitational degrees of freedom not the bath, time dilation mediates coupling between CoM + other modes

$$H_{int} = \left(\Phi(\hat{x}) - \frac{\hat{p}^2}{2m^2}\right)\hat{H}_0$$

GR time SR time

dilation dilation



I. Pikovski, M. Zych, F. Costa, Č. Brukner. Nature Physics 11, 668-672 (2015)

UNIVERSAL DECOHERENCE DUE TO GRAVITATIONAL TIME DILATION

 $|\psi_{cm}\rangle = \frac{1}{\sqrt{2}}(|x_1\rangle + |x_2\rangle)$

Any system affected by time dilation.

Simple model as an example: Composite system has N internal harmonic oscillators: Each constituent in equilibrium at $\widehat{H}_0 = \sum_{i=1}^N \widehat{n}_i \hbar \omega_i$

Each constituent in equilibrium at temperature T, center-of-mass in superposition:

Quantum coherence of center-of-mass reduces due to time-dilation: $V(t) \approx \left(1 + \left(\frac{k_B T g \Delta x t}{\hbar c^2}\right)\right)^{-N/2} \approx e^{-\left(\frac{t}{\tau_{dec}}\right)^2}$ $\tau_{dec} = \sqrt{2/N} \frac{\hbar c^2}{k_B T g \Delta x}$

- Universal for all composite systems
- Gaussian decay of quantum coherence in position
- Decoherence mediated by time dilation, depends on internal composition

- Relativistic, thermodynamic and quantum mechanical effect
- Regular quantum theory and general relativity, no new assumptions

I. Pikovski, M. Zych, F. Costa, Č. Brukner. Nature Physics 11, 668-672 (2015)



DECOHERENCE FOR ARBITRARY WORLDLINES



STRENGTH OF DECOHERENCE

$$\tau_{dec} = \frac{\sqrt{2}\hbar c^2}{\Delta H_0 g \Delta x} \approx \sqrt{\frac{2}{N} \frac{\hbar c^2}{k_B T g \Delta x}}$$

μm-scale object on Earth at room temperature, $\Delta x \sim \mu m$: $\tau_{dec} \sim m$ s

2-lvl clock at frequency ω , held for time T at superposition of size Δh : V =

$$\left|\cos\left(\frac{H_0\ \Delta\tau}{2}\right)\right| = \left|\cos\left(\frac{\omega T\ \Delta h}{2c^2}\right)\right|$$



Experimental requirements to see time dilation in quantum interference:

system	clock	$\omega [{\rm Hz}]$	$\Delta h^+ T \; [ms]$	$\Delta h^+ T \; [ms]$
			achieved	required
atoms	Optical states	10^{15}	10-1	10
electrons	spin precession	10^{13}	10^{-6}	10^{3}
molecules	vibrational modes	10^{12}	10^{-8}	10^{4}
neutrons	spin precession	10^{10}	10^{-7}	10^{6}
Photon	"Shapiro Delay"	Interferometer with ca. 10km arm length		



Analogue BEC experiment: Y. Margalit et al., Science 349, 1205-1208 (2015)

RELEVANT EXPERIMENTS

Effect simulated with magnetic fields (2015):

Sciencexpress

A self-interfering clock as a "which path" witness

Yair Margalit, Zhifan Zhou, Shimon Machluf,* Daniel Rohrlich, Yonathan Japha, Ron Folman†

Department of Physics, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel. *Present address: Van der Waals-Zeeman Institute, University of Amsterdam, Science Park 904, 1090 GL Amsterdam, Netherlands.

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In Einstein's general theory of relativity, time depends locally on gravity; in standard quantum theory, time is global—all clocks "tick" uniformly. We demonstrate a new tool for investigating time in the overlap of these two theories: a self-interfering clock, comprising two atomic spin states. We prepare the clock in a spatial superposition of quantum wave packets,

Also proposed space-based and ground-based photon experiments

Pallister et al., EPJ Quantum Technology 4 (2017) Hilweg et al., NJP 19, 033028 (2017)

Atoms across IOm in superposition: Planned MAGIS-100 detector at Fermilab



Holding atoms for 20s in superposition:



REVISTING THE GRAVITATIONAL COUPLING

$$H = \sqrt{-g_{00}(x)} H_0 \approx \left(1 + \frac{\phi(x)}{c^2}\right) H_0$$

Universal coupling to energy:

- Dictated by the equivalence principle
- Valid for any form of energy, i.e. arbitrary local Hamiltonian H_0
- Manifestation of time dilation
- Leads to entanglement between internal and external degrees of freedom
- Can lead to decoherence of COM of composite systems



CONSIDERING SPECIFIC COMPOSITE SYSTEMS

Problem: all concrete N-particle calculations in GR seem to show different result! E.g.:

A. Eddington, G. Clark, Proc. R. Soc. Lond. A166, 465 (1938)
K. Nordtvedt, Int. J. Theor. Phys. 3, 133-139 (1970)
E. Fishbach et al., Phys. Rev. D. 23, 2157-2180 (1981)
S. Carlip, Am. J. Phys. 66, 409-413 (1998)
A. G. Lebed, Cent. Eur. J. Phys. 11, 969–976 (2013)

$$H_{sys} = \sum_{i=1}^{N} \left(m_i c^2 + \frac{1}{2} m_i v_i^2 + k \sum_{j=1}^{N} \frac{q_i q_j}{2r_{ij}} \right) = R + T + U$$

On post-Newtonian metric, Results in gravitational coupling to:

 $m_G = \frac{1}{c^2}(R + 3T + 2U),$

$$H = H_{sys} + \frac{\phi}{c^2} \left(H_{sys} + 2T + U \right)$$

Problems:

 Coupling to energy not universal: different for T and U, different systems will fall differently

•
$$m_G \neq \frac{E}{c^2}$$

 Need correct Hamiltonian to do quantum physics

the retardation terms must be removed from $(4 \cdot 1)$. The mass is then

$$M = \Sigma_i m_i + \frac{3}{2} \Sigma_i m_i v_i^2 - \Sigma_i \Sigma_j \frac{m_i m_j}{\Delta_{ij}}, \qquad (4.5)$$

and the formal difficulty, caused by the divergence of (4.3) as $r \to \infty$, is avoided.

Let T, V be the kinetic and potential energies of the system. We have

$$T = \tfrac{1}{2} \varSigma_i m_i v_i^2, \quad V = - \tfrac{1}{2} \varSigma_i \varSigma_j \frac{m_i m_j}{\varDelta_{ij}}.$$

Hence (4.5) becomes

 $M = \Sigma m_i + 3T + 2V.$

(4.6)

A. Eddington, G. Clark, Proc. R. Soc. Lond. A166, 465 (1938)



DIFFERENT GRAVITATIONAL COUPLING

$$H = H_{sys} + \frac{\phi}{c^2} \left(H_{sys} + 2T + U \right)$$

Specific model systems

"Miraculous" resolution: Virial theorem Classically: $2\langle T \rangle_t = -\langle U \rangle_t$ QM: $2\langle T \rangle_{ensemble} = -\langle U \rangle_{ensemble}$

- No violation for current experimental tests
- Equivalence principle holds on timescales longer than internal dynamics

$$H = \left(1 + \frac{\phi}{c^2}\right) H_0$$

General considerations

$$- H = \left(1 + \frac{\phi}{c^2}\right) \langle H_{sys} \rangle$$

Vs.

- Inconsistent with exact universality of GR
- Deviations from equivalence principle in QM for composite systems beyond the mean

 $Q^{\mu}(t)$ world-line

RESOLUTION: DERIVATION OF MASS-ENERGY EQUIVALENCE

Generic action of N-particle system on a single world-line:

$$S = -\sum_{i=1}^{N} \left(m_i c^2 \int d\tau_i + q_i \int A_{\mu} (x_i) dx_i^{\mu} \right)$$

$$Co-moving coordinates:$$

$$Q_{local}^{\mu} = \frac{\partial x_{local}^{\mu}}{\partial x^{\nu}} Q^{\nu}(t)$$

$$Q_{local}^{0} = \tau$$

$$\frac{dQ_{local}^{i}}{\partial x^{\nu}} = 0$$

Can be re-written as:

Gives rise to:

$$S = -\int L_{local} \frac{d\tau}{dt} dt$$

Defined only wrt World-line in external internal, local quantities coordinates

$$L_{local} = L_{local}(x_i^{local}, p_i^{local}, \tau)$$

$$H = \sqrt{-g_{00}(c^2 p_k p^k + H_{local}^2)}$$

(if $g_{\mu\nu}$ same \forall particles)

dt

τ

- Completely general for arbitrary internal Hamiltonian (when tidal forces negligible)
- **Reproduces mass-energy equivalence**
- Energy expressed in terms of local physical quantities M. Zych, L. Rudnicki, I. Pikovski, PRD, 99, 104029 (2019)

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RESOLUTION: CORRECT GRAVITATIONAL MASS

Locally (sitting on the particle COM): $H_{local} = T_{local} + U_{local} + \sum m_i c^2$

From outside (observer sitting far away):

$$x_{i} = \left(1 + \frac{\phi(r)}{c^{2}}\right) x_{i}^{local} \qquad t = \left(1 - \frac{\phi(r)}{c^{2}}\right) \tau$$

$$U_{local} = \sum_{i,j=1}^{N} \frac{q_i q_j}{r_{ij}^{local}} = \sum_{i,j=1}^{N} \frac{q_i q_j}{r_{ij}} \left(1 + \frac{\phi(r)}{c^2}\right) = U\left(1 + \frac{\phi(r)}{c^2}\right)$$

$$p_i^{local} = m_i \frac{dx_i^{local}}{d\tau} = m_i \frac{dx_i}{dt} \frac{1 - \frac{\phi(r)}{c^2}}{1 + \frac{\phi(r)}{c^2}} \approx m_i v_i \left(1 - 2\frac{\phi(r)}{c^2}\right) = p_i \left(1 + \frac{\phi(r)}{c^2}\right)$$

$$T_{local} = \sum_{i} \frac{1}{2} m_{i} v_{i}^{local 2} = \sum_{i} \frac{p_{i}}{2m_{i}} \left(1 + 2\frac{\phi(r)}{c^{2}} \right) = T \left(1 + 2\frac{\phi(r)}{c^{2}} \right)$$



or Pikovski

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RESOLUTION: CORRECT GRAVITATIONAL MASS

Locally (sitting on the particle COM): $H_{local} = T_{local} + U_{local} + \sum m_i c^2$

From outside (observer sitting far away):

$$U_{local} = U\left(1 + \frac{\phi(r)}{c^2}\right) \qquad T_{local} = T\left(1 + 2\frac{\phi(r)}{c^2}\right)$$

$$H = H_{sys} + \frac{\phi}{c^2} \left(H_{sys} + 2T + U \right) = \left(1 + \frac{\phi}{c^2} \right) H_{local}$$

$$m_{G} = \frac{H_{local}}{c^{2}} = \sum_{i=1}^{N} m_{i} + \frac{1}{c^{2}} (T_{local} + U_{local})$$

Total passive mass is total, local energy $H_{local} = H_0 \neq H_{sys}$ No anomalous terms.

LOCAL PHYSICAL QUANTITIES COUPLING TO GRAVITY

$$U_{local} = U\left(1 + \frac{\phi(r)}{c^2}\right) \qquad T_{local} = T\left(1 + 2\frac{\phi(r)}{c^2}\right)$$

 $H = H_{sys} + \frac{\phi}{c^2} (H_{sys} + 2T + U) = \left(1 + \frac{\phi}{c^2}\right) H_{local} \qquad m_G = \sum_{i=1}^N m_i + \frac{1}{c^2} (T_{local} + U_{local})$

- Same expression in different coordinates
- T and U include the redshifted quantities, are not the physical kinetic and potential energies of the particles inside the system
- Becomes apparent when using 2 sets of coordinates: (r, t) for describing gravity and COM, and (r_{loc}, τ) for describing the internal DOF

SUMMARY

- "Quantum twin paradox": a single clock in superpositions with different proper times
- Leads to time dilation induced entanglement & decoherence, can be probed experimentally
- Due to composite systems coupling to gravity: $\sqrt{-g_{00}(x)} H_{local}$



- Apparent discrepancy with 80-year-long results in general relativity: $m_G = 3T + 2U$?
- Resolution: Passive gravitational mass is exactly m_G = R+T+U when defined in rest frame
- Derivation of GR mass-energy equivalence from first principles: $H = \sqrt{-g_{00}(c^2p_kp^k + H_{local}^2)}$

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COLLABORATORS & FUNDING





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Thank you for your attention