Quantum Field Theory for Gravity at All Scales

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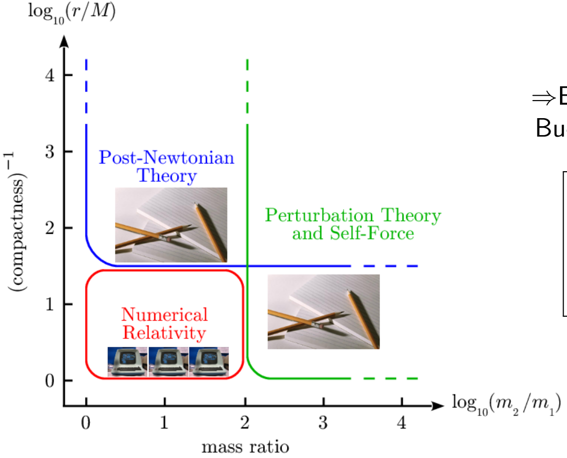
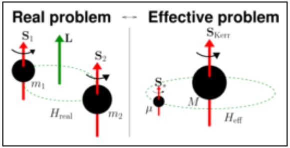
Theory of GW templates



⇒Effective One-Body (EOB), Buonanno and Damour 1999

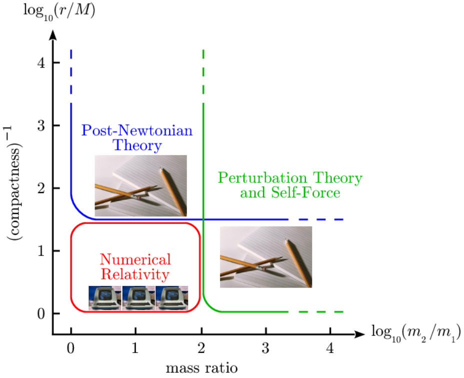


⇒Numerical Relativity breakthrough, Pretorius 2005

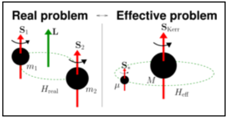
Detection by matched filtering

- High demand on accurate theoretical waveform templates
- Modeling of waveforms currently uses PN parameters of up to 6th PN order! Up to 10% possible discrepancy w simulations at 5PN [nPN≡ v^{2n}]

Theory of GW templates



⇒Effective One-Body (EOB)
Buonanno & Damour 1999



- Increasing influx of real-world GW data
  ⇒ PN gravity is key for theoretical GW data → EFTs of PN Gravity
- Underlying Science: Informs on strong gravity, QFT ↔ Gravity
- Can we get insight on the graviton Compton amplitude with $s \geq 5/2$ from PN gravity? [Arkani-Hamed, Huang, Huang; 2017]

Precision gravity – marry 2 pillars of theoretical physics!

Quantum Field Theory + Gravity

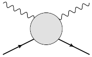
State of the Art in PN Gravity

State of the Art for Generic Compact Binary Dynamics

l \ n	(N^0)LO	$N^{(1)}$LO	N^2LO	N^3LO	N^4LO	N^5LO
S^0	1	0	3	0	25	0
S^1	2	7	32	**174**		
S^2	2	2	**18**	**52**		
S^3	4	**24**				
S^4	3	**5**				

- (n, l) entry at $n + l + \text{Parity}(l)/2$ PN order
- A measure for loop computational scale:
  n = highest n-loop graphs at N^nLO, l = highest multipole moment S^l
- Gray area corresponds to gravitational Compton scattering

  with $s \geq 3/2$ since classical $S^l \leftrightarrow$ quantum $s = l/2$



- All (but top right ones) are derived in the public `EFTofPNG` code:
  `https://github.com/miche-levi/pncbc-eftofpng`

State of the Art

State of the Art for Generic Compact Binary Dynamics

l \ n	$(N^0)LO$	$N^{(1)}LO$	N^2LO	N^3LO	N^4LO	N^5LO
S^0	++	++	++	++	++	+
S^1	++	++	++	+		
S^2	++	++	+	+		
S^3	++	+				
S^4	++	+				

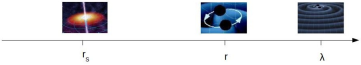
- ++ = fully done/verified; + = partial/not verified
- Even l easier than odd l; Also in particular at $l = 0 \rightarrow n$ odd easier
- As of 2PN – UV dependence needed to complete accuracy
- At 4PN all sectors fully verified except (n,l)=(2,2) [Levi+ 2016]
- At 4.5PN & 5PN – NO sector is currently fully done/verified!

EFTs are Universal

Levi, Rept. Prog. Phys. 2020

There is a Hierarchy of Scales

1. r_s, scale of internal structure, $r_s \sim m$
2. r, orbital separation scale, $\frac{r_s}{r} \sim v^2$
3. λ, radiation wavelength scale, $\frac{r}{\lambda} \sim v$



$v \ll 1 \rightarrow nPN \equiv v^{2n}$ correction in classical gravity to Newtonian gravity

Multistage strategy for EFTs of inspiraling binaries

[Goldberger & Rothstein 2007]

1. One-Particle EFT
2. EFT of a Composite Particle
3. Effective Theory of Dynamical Multipoles

It's a multiscale!

Setup of EFTs is Universal

Bottom-Up and/or Top-Down Strategies

Two generic procedures to construct Effective Field Theories:

1. Top-Down, perturbative theory in high resolution/close-up is known, so we reduce resolution by systematically removing extra pixels. Feynman formalism/technology enables that.
   – *"Wilsonian approach", Wilson 1971-1974*
2. Bottom-Up, no perturbative theory for the system in close-up, so we look from afar, and gradually zoom in. This is done by identifying the Degrees Of Freedom and Symmetries.
   – *"Decoupling theorem", Appelquist & Carazzone 1975*

One-Particle EFT

1st Stage Remove scale r_S of isolated compact object

In the full theory we only have a vacuum gravitational field:

$$S\left[g_{\mu\nu}\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{g}R\left[g_{\mu\nu}\right]$$

"Integrate out" strong field modes $g^s_{\mu\nu}$, $g_{\mu\nu} \equiv g^s_{\mu\nu} + \bar{g}_{\mu\nu}$ via bottom-up approach:

$$S_{\text{eff}}\left[\bar{g}_{\mu\nu}, y^\mu(\sigma), e^\mu_A(\sigma)\right] = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}R\left[\bar{g}_{\mu\nu}(x)\right] + \underbrace{\sum_{i=1}^{\infty} C_i(r_s)\int d\sigma \mathcal{O}_i(\sigma)}_{\equiv S_{pp}(\sigma) \text{ with Wilson coefficients}}$$

The operators $\mathcal{O}_i(\sigma)$ must respect the symmetries that pertain at low energies.

$$\begin{aligned} S_{\text{eff}}[\bar{g}_{\mu\nu}, y^\mu] = &-\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}R\left[\bar{g}_{\mu\nu}(x)\right] \\ &-\int m d\sigma + \underbrace{c_{5\text{PN}}\int d\sigma\left(R_{\mu\alpha\nu\beta}\dot{y}^\alpha\dot{y}^\beta\right)^2 + \cdots}_{\text{finite size effects}} \end{aligned}$$

EFT of Composite Particle

2nd Stage Remove orbital scale r of binary, first via the top-down approach:

$$\bar{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \underbrace{H_{\mu\nu}}_{\text{orbital}} + \underbrace{\widetilde{h}_{\mu\nu}}_{\text{radiation}}$$

$$\partial_t H_{\mu\nu} \sim \frac{v}{r} H_{\mu\nu}, \quad \partial_i H_{\mu\nu} \sim \frac{1}{r} H_{\mu\nu}, \quad \partial_\rho \widetilde{h}_{\mu\nu} \sim \frac{v}{r} \widetilde{h}_{\mu\nu}$$

$$S_{\text{eff}} \left[\bar{g}_{\mu\nu}, y_1^\mu, y_2^\mu, e_{(1)}{}_A^\mu, e_{(2)}{}_A^\mu\right] = -\frac{1}{16\pi G} \int d^4x \sqrt{\bar{g}} R\left[\bar{g}_{\mu\nu}\right] + S_{\text{pp}}(\sigma_1) + S_{\text{pp}}(\sigma_2)$$

Integrate out orbital field modes - in this classical context - only tree level

$$\Rightarrow e^{iS_{\text{eff(composite)}}\left[\widetilde{h}_{\mu\nu}, y^\mu, e^\mu_{(Comp)A}\right]} \equiv \int \mathcal{D}H_{\mu\nu} \; e^{iS_{\text{eff}}\left[\bar{g}_{\mu\nu}, y_1^\mu, y_2^\mu, e_{(1)}{}_A^\mu, e_{(2)}{}_A^\mu\right]}$$

Stop here for effective action strictly in conservative sector, that is **WITHOUT** any remaining (orbital scale) field modes

EFTs of Extended Gravitating Objects

$$S_{\text{eff}} = S_{\text{g}}[g_{\mu\nu}] + \sum_{a=1}^{2} S_{\text{pp}}(\lambda_a); \quad S_{pp}(\lambda_a) = \sum_{i=1}^{\infty} C_i(r_s) \int d\lambda_a \mathcal{O}_i(\lambda_a)$$

$$S_{\text{g}}[g_{\mu\nu}] = -\frac{1}{16\pi G_d} \int d^{d+1}x \sqrt{g}\, R + \frac{1}{32\pi G_d} \int d^{d+1}x \sqrt{g}\, g_{\mu\nu} \Gamma^{\mu} \Gamma^{\nu},$$

$$G_d \equiv G_N \left(\sqrt{4\pi e^{\gamma}}\, R_0\right)^{d-3},$$

To facilitate computations in PN: [Kol & Smolkin 2008]

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \equiv e^{2\phi} \left(dt - A_i dx^i\right)^2 - e^{-\frac{2}{d-2}\phi} \gamma_{ij} dx^i dx^j,$$

$$\langle\, \phi(x_1) \;\; \phi(x_2)\, \rangle = \text{———} \quad = \quad \frac{16\pi\, G_d}{c_d} \cdot \delta(t_1 - t_2) \int_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{x}_1 - \vec{x}_2)}}{\vec{k}^2},$$

$$\langle A_i(x_1)\; A_j(x_2) \rangle = \text{- - - - - - - -} \quad = -16\pi\, G_d \cdot \delta(t_1 - t_2) \int_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{x}_1 - \vec{x}_2)}}{\vec{k}^2}\, \delta_{ij}.$$

Spinning Particle: DOFs

[**ML**, Rept. Prog. Phys. 2020]

Assume isolated object has no intrinsic permanent multipoles beyond mass (monopole) and spin (dipole)

1. *Gravitational field*
   - Metric $g_{\mu\nu}(x)$
   - Tetrad field $\eta^{ab}\tilde{e}_a{}^{\mu}(x)\tilde{e}_b{}^{\nu}(x) = g^{\mu\nu}(x)$

2. *Particle Coordinate*

   $y^{\mu}(\sigma)$ function of arbitrary affine parameter σ
   Particle worldline position does not in general coincide with object's 'center'

3. *Particle rotating DOFs*

   Worldline tetrad, $\eta^{AB}e_A{}^{\mu}(\sigma)e_B{}^{\nu}(\sigma) = g^{\mu\nu}$

   $\Rightarrow$ Angular velocity $\Omega^{\mu\nu}(\sigma) \equiv e^{\mu}_A \frac{De^{A\nu}}{D\sigma}$ + conjugate spin $S_{\mu\nu}(\sigma)$

   $\Rightarrow$ Lorentz matrices $\eta^{AB}\Lambda_A{}^{a}(\sigma)\Lambda_B{}^{b}(\sigma) = \eta^{ab}$ + conjugate local spin $S_{ab}(\sigma)$

Spinning Particle: Symmetries

[**ML**, Rept. Prog. Phys. 2020]

1. General coordinate invariance, and parity invariance
2. Worldline reparametrization invariance
3. Internal Lorentz invariance of local frame field
4. SO(3) invariance of “body-fixed” spatial triad
5. Spin gauge invariance, that is invariance under choice of completion of “body-fixed” spatial triad through timelike vector

Spin as Extra Particle DOF

Effective Action of Spinning Particle

- $u^{\mu} \equiv dy^{\mu}/d\sigma$, $\Omega^{\mu\nu} \equiv e^{\mu}_{A}\frac{De^{A\nu}}{D\sigma} \Rightarrow L_{\text{pp}}\left[\bar{g}_{\mu\nu}, u_{\mu}, \Omega^{\mu\nu}\right]$
  [Hanson & Regge 1974, Bailey & Israel 1975]
- $S_{\mu\nu} \equiv -2\frac{\partial L}{\partial \Omega^{\mu\nu}}$ spin as further particle DOF – classical source
  [...Levi+ JHEP 2015]

$$\Rightarrow S_{\text{pp}}(\sigma) = \int d\sigma \left[-p_{\mu}u^{\mu} - \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + L_{\text{NMC}}\left[\bar{g}_{\mu\nu}\left(y^{\mu}\right), u^{\mu}, S_{\mu\nu}\right]\right]$$

For EFT of spin – gauge of both rotational DOFs
should be fixed at level of one-particle action

- This form implicitly assumes initial "covariant gauge":
  $$e^{\mu}_{[0]} = \frac{p^{\mu}}{\sqrt{p^2}}, \quad S_{\mu\nu}p^{\nu} = 0$$
  [Tulczyjew 1959]
- Linear momentum $p_{\mu} \equiv -\frac{\partial L}{\partial u^{\mu}} = m\frac{u^{\mu}}{\sqrt{u^2}} + \mathcal{O}(RS^2)$

EFT of Spinning Particle

Effective Action of Spinning Particle

- $u^\mu \equiv dy^\mu/d\sigma$, $\Omega^{\mu\nu} \equiv e^\mu_A \frac{De^{A\nu}}{D\sigma} \Rightarrow L_{\rm pp}\left[\bar{g}_{\mu\nu}, u_\mu, \Omega^{\mu\nu}\right]$
  [Hanson & Regge 1974, Bailey & Israel 1975]
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$$\Rightarrow S_{\rm pp}(\sigma) = \int d\sigma \left[-p_\mu u^\mu - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{\rm NMC}\left[\bar{g}_{\mu\nu}\left(y^\mu\right), u^\mu, S_{\mu\nu}\right]\right]$$

Theory challenges tackled [...Levi+ JHEP 2015, Levi Rept. Prog. Phys. 2020]

1. Relativistic spin has a minimal finite measure S/M
   $\rightarrow$ Clashes with the EFT/point-particle viewpoint
   $\Rightarrow$ Introduce “gauge freedom” in choice of rotational variables
2. Fix non-minimal coupling part of the action, $L_{\rm NMC}$

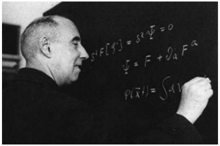
Introduce Gauge Freedom in Tetrad & Spin

[**ML** & Steinhoff, JHEP 2015]

Introduce gauge freedom into tetrad

Transform from a gauge condition

$$e_{A\mu}q^{\mu} = \eta_{[0]A} \Leftrightarrow e_{[0]\mu} = q_{\mu}$$



Ernst Stueckelberg

to

$$\hat{e}_{A\mu}w^{\mu} = \eta_{[0]A} \Leftrightarrow \hat{e}_{[0]\mu} = w_{\mu}$$

with a boost-like transformation in covariant form

$$\hat{e}^{A\mu} = L^{\mu}{}_{\nu}(w,q)e^{A\nu}$$

with q_{μ}, w_{μ} timelike unit 4-vectors

Generic gauge for the tetrad entails the generic "SSC"

$$\hat{e}_{[0]\mu} = w_{\mu} \quad \Rightarrow \quad \hat{S}^{\mu\nu}\left(p_{\nu} + \sqrt{p^2}w_{\nu}\right) = 0$$

Extra term in Minimal Coupling

[**ML** & Steinhoff, JHEP 2015]

$$\Rightarrow \hat{S}^{\mu\nu} = S^{\mu\nu} - \delta z^{\mu} p^{\nu} + \delta z^{\nu} p^{\mu}, \qquad \delta z^{\mu} p_{\mu} = 0$$

$\Rightarrow$ Extra term in action appears!

- From minimal coupling

$$\frac{1}{2} S_{\mu\nu} \Omega^{\mu\nu} = \frac{1}{2} \hat{S}_{\mu\nu} \hat{\Omega}^{\mu\nu} + \frac{\hat{S}^{\mu\rho} p_{\rho}}{p^2} \frac{D p_{\mu}}{D\sigma}$$

- Extra term with covariant derivative of momentum, contributes to finite size effects, yet carries no Wilson coefficient
- As of LO with spin, to all orders in spin!
- Essentially Thomas precession (later recovered as "Hilbert space matching")

- We transform between spin variables by projecting onto the hypersurface orthogonal to p_{μ}

$$S_{\mu\nu} = \hat{S}_{\mu\nu} - \frac{\hat{S}_{\mu\rho} p^{\rho} p_{\nu}}{p^2} + \frac{\hat{S}_{\nu\rho} p^{\rho} p_{\mu}}{p^2}$$

Why Generalized Canonical Gauge?

Here are some of the obvious reasons to use it:

1. Allows to disentangle DOFs in EFT and land on well-defined effective action.
2. Standard procedure to land on Hamiltonian, similar to non-spinning sectors.
3. Essential for Effective One-Body framework – needed to generate waveforms.
4. Direct and simple derivation of physical EOMs for position + spin.
5. Enables most stringent consistency check of Poincaré algebra of invariants.
6. Natural classical treatment to be promoted/confronted with QFT.

Non-minimal coupling: Under construction

[**Levi**+, JHEP 2014, JHEP 2015]

Spin-induced higher multipoles

- Consider the spin vector similar to the Pauli-Lubanski pseudovector

$$S^{\mu} \equiv *S^{\mu\nu} \frac{p_{\nu}}{\sqrt{p^2}} \simeq *S^{\mu\nu} \frac{u_{\nu}}{\sqrt{u^2}}, \qquad *S_{\alpha\beta} \equiv \frac{1}{2} \epsilon_{\alpha\beta\mu\nu} S^{\mu\nu}$$

$$\Rightarrow \; S_{\mu} S^{\mu} = -\frac{1}{2} S_{\mu\nu} S^{\mu\nu} \equiv -S^2$$

- Consider dependence of higher powers of spin:

$$S^{\alpha}{}_{\mu} S^{\mu}{}_{\beta} = -S^{\alpha} S_{\beta} - S^2 \left(\delta^{\alpha}{}_{\beta} - \frac{u^{\alpha} u_{\beta}}{u^2} \right)$$

$$S^{\alpha}{}_{\mu} S^{\mu}{}_{\nu} S^{\nu}{}_{\beta} = -S^2 S^{\alpha}{}_{\beta}$$

$$\Rightarrow X(X + iS)(X - iS) = 0$$

$\Rightarrow$ Independent combinations: S^{α}, $S^{\alpha}{}_{\mu} S^{\mu\beta} \sim S^{\alpha} S^{\beta} \dots$

Non-minimal coupling: Under construction

[**Levi**+, JHEP 2014, JHEP 2015]

Spin-induced higher multipoles

- Considering body-fixed frame: Spin multipoles are SO(3) irreps tensors
- Recall we start from "covariant gauge": $e_{[0]}{}^{\mu} = u^{\mu}/\sqrt{u^2}$, $e_{[i]}{}^{\mu} u_{\mu} = 0$

$\Rightarrow$ Spin-induced higher multipoles are symmetric, traceless, and spatial, constant tensors in body-fixed frame

Curvature

- Electric component $E_{\mu\nu} \equiv R_{\mu\alpha\nu\beta} u^{\alpha} u^{\beta}$
- Magnetic component $B_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\alpha\beta\gamma\mu} R^{\alpha\beta}{}_{\delta\nu} u^{\gamma} u^{\delta}$

Field is vacuum solution at LO, properties of Riemann, Bianchi identities
$\Rightarrow$ E/B are symmetric, traceless, and orthogonal to u^{μ}, also when projected to body-fixed frame, where they are spatial

Building blocks $\sim$ Riemann components • Spin-induced multipoles

Even/odd spin-induced multipoles couple to even/odd parity electric/magnetic curvature components, and their covariant derivatives

Leading Non-Minimal Couplings to All Orders in Spin

[**Levi**+, JHEP 2014, JHEP 2015]

Key: Consider classical spin vector similar to Pauli-Lubanski vector

→ Massive spinor-helicity, Arkani-Hamed+ 2017 – resonates with this form

New Wilson coefficients of linear-in-curvature couplings → "Love numbers":

$$L_{\text{NMC}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}$$

Leading - linear in curvature - spin couplings up to 5PN order

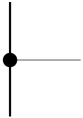
- $L_{ES^2} = -\frac{C_{ES^2}}{2m} \frac{E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu,$ Quadrupole @2PN
- $L_{BS^3} = -\frac{C_{BS^3}}{6m^2} \frac{D_\lambda B_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda,$ Octupole @3.5PN
- $L_{ES^4} = \frac{C_{ES^4}}{24m^3} \frac{D_\lambda D_\kappa E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda S^\kappa,$ Hexadecapole @4PN

Kaluza-Klein decomposition of field

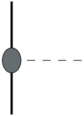
Reduction over time dimension à la Kaluza-Klein

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \equiv e^{2\phi}(dt - A_i dx^i)^2 - e^{-2\phi}\gamma_{ij} dx^i dx^j$$

- $\phi, A_i, \gamma_{ij} \equiv \delta_{ij} + \sigma_{ij}$, KK fields



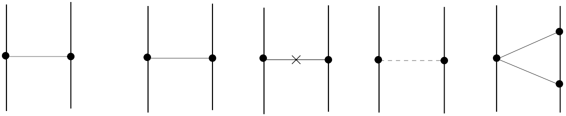
- Newtonian potential scalar ϕ



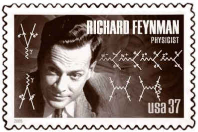
- Gravitomagnetic vector A_i
- Hierarchy in coupling to mass and to spin
- Advantageous, preferable e.g. over Lorentz covariant, ADM decompositions...

LO sectors beyond Newtonian

Feynman graphs of non-spinning sector to 1PN order



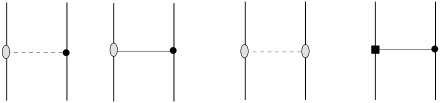
(a0) (a) (b) (c) (d)



Newton
.

One-loop diagram – absent from 1PN
with KK parametrization of field

LO Feynman diagrams with spin – to quadratic-in-spin



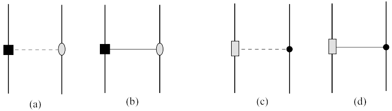
(a) (b) (a1) (a2)

Spin-Orbit Spin-Spin

LO cubic & quartic in spin

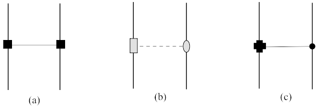
[**Levi+**, JHEP 2014]

Feynman diagrams of LO **cubic** in spin sector



- On left pair – quadrupole-dipole, on right – octupole-monopole
- Note analogy of each pair with LO spin-orbit

Feynman diagrams of LO **quartic** in spin sector

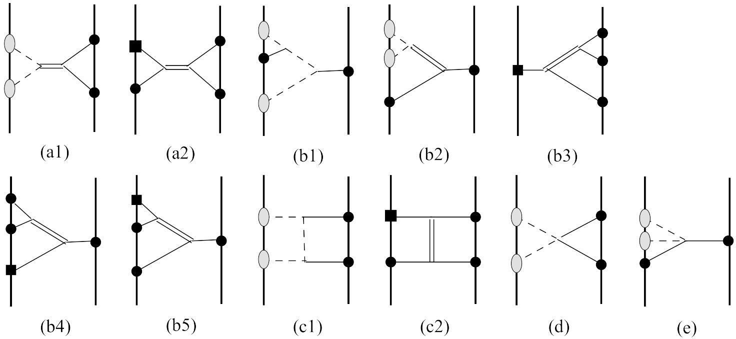


- On left and right – quadrupole-quadrupole and hexadecapole-monopole
  Each is analogous to LO spin-squared
- In middle – octupole-dipole analogous to LO spin1-spin2

NNLO spin-squared sector

[**ML** & Steinhoff, 2016]

Feynman diagrams of order G^3 with 2 loops

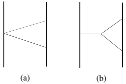


- Recall at N^{n}LO – n-loop graphs are realized, in particular with spin!
- Five 2-loop topologies actually fall into 3 kinds
- I (or H rotated) topology (c1,c2) – is the leading nasty one, i.e. IBP needed!

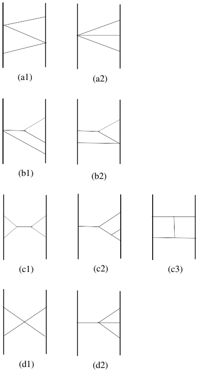
Graph Topologies up to 2-Loop

[Rept. Prog. Phys. 2020, **Levi+** + x2 2020]

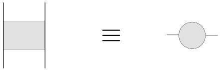
Single topology at $O(G)$:
One-graviton exchange.



Topologies at $O(G^2)$:
(a) Two-graviton exchange;
(b) Cubic self-interaction
$\equiv$ One-loop topology.



Topologies at $O(G^3)$



Standard QFT multi-loops: n-loop master integrals and IBPs (Integration By Parts) EFTofPNG code [**Levi+** 2017, 2020,...]

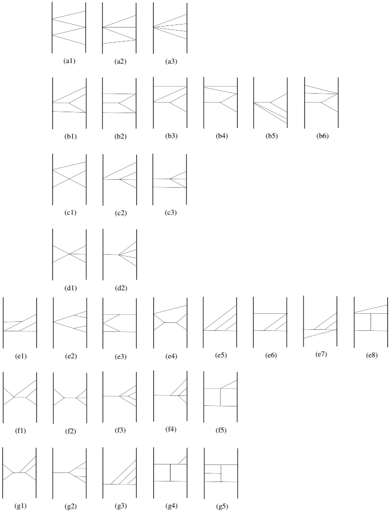
A topology at G^{n+1} is rank r, when r basic n-loop integral types form its n-loop integral

$$\int_{\vec{p}_1} \frac{e^{i\vec{p}_1\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{p}_1^2} \int_{\vec{p}_2} \frac{e^{i\vec{p}_2\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{p}_2^2},$$

$$p_1 + p_2 \to p, \quad p_2 \to k_1, \quad \to \int_{\vec{p}} e^{i\vec{p}\cdot(\vec{x}_1-\vec{x}_2)} \int_{\vec{k}_1} \frac{1}{\vec{k}_1^2(\vec{p}-\vec{k}_1)^2}$$

Graph Topologies at G^4=up to 3-Loop

[**Levi**+ 2020]



Topologies at $O(G^4)$

At G^n the loop order n_L

$$n_L \equiv 2n - \sum_{i=1}^{n+1} m_i$$

with m_i gravitons on insertion i

A topology at G^{n+1} is rank r, when r basic n-loop integral types form its n-loop integral

Complete N^3LO Quadratic-in-Spin

Considering first quadratic-in-curvature couplings with spin

[**Levi**+ 2020; Kim, **ML**, Yin, in prep.]

Graph distribution in this sector with a total(?) of 1121 – from linear in R

Topology Order in G	1	2	3	4
No. of graphs	19	251	688	163

Do we have more contributions beyond linear in curvature? [Yes, at R^2!]

Integration and Scalability

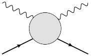
- Building on publicly-available `EFTofPNG` code [**ML** & Steinhoff 2017] `https://github.com/miche-levi/pncbc-eftofpng`
- Higher-rank graphs reduced using IBP method, e.g. 87 at G^3, 31 at G^4
- Upgrade using projection method for integrand numerators as high as rank-8
- Upgrade from IBP "by hand" to algorithmic IBP – our variation of Laporta

Nonlinear Higher-in-Spin

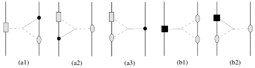
What is the nature of massive particles of $s > 2$?

- Gravitational interaction with spins $\leftrightarrow$ Scattering of graviton and massive spin
  Classical $S^l \leftrightarrow$ Quantum $s = l/2$
- Insight on Compton scattering of graviton and massive higher-spin $s \geq 5/2$

  [Arkani-Hamed+ 2017]



NLO cubic-, quartic-in-spin [**Levi+**, Teng, JHEP 2021 x 2, + Morales in prep.]



- Graphs with "elementary" worldline-graviton couplings up to 1-loop
- Some worldline-graviton couplings become quite intricate and subtle, new "composite" multipoles in terms of "elementary" spin multipoles
- Operators quadratic-in-curvature at NLO S^4

Extending Non-Minimal Action with Spin

Extending effective action beyond linear-in-curvature

[**Levi+** 2020, JHEP 2021]

$$
\begin{aligned}
L_{\mathrm{NMC(R^2)}} = {} & C_{E^2}\frac{E_{\alpha\beta}E^{\alpha\beta}}{\sqrt{u^2}^{\,3}} + C_{B^2}\frac{B_{\alpha\beta}B^{\alpha\beta}}{\sqrt{u^2}^{\,3}} + \dots \\
& + C_{E^2S^2}S^\mu S^\nu\frac{E_{\mu\alpha}E_\nu^\alpha}{\sqrt{u^2}^{\,3}} + C_{B^2S^2}S^\mu S^\nu\frac{B_{\mu\alpha}B_\nu^\alpha}{\sqrt{u^2}^{\,3}} \\
& + C_{E^2S^4}S^\mu S^\nu S^\kappa S^\rho\frac{E_{\mu\nu}E_{\kappa\rho}}{\sqrt{u^2}^{\,3}} + C_{B^2S^4}S^\mu S^\nu S^\kappa S^\rho\frac{B_{\mu\nu}B_{\kappa\rho}}{\sqrt{u^2}^{\,3}} \\
& + C_{\nabla EBS}S^\mu\frac{D_\mu E_{\alpha\beta}B^{\alpha\beta}}{\sqrt{u^2}^{\,3}} + C_{E\nabla BS}S^\mu\frac{E_{\alpha\beta}D_\mu B^{\alpha\beta}}{\sqrt{u^2}^{\,3}} \\
& + C_{\nabla EBS^3}S^\mu S^\nu S^\kappa\frac{D_\kappa E_{\mu\alpha}B_\nu^\alpha}{\sqrt{u^2}^{\,3}} + C_{E\nabla BS^3}S^\mu S^\nu S^\kappa\frac{E_{\mu\alpha}D_\kappa B_\nu^\alpha}{\sqrt{u^2}^{\,3}} \\
& + C_{(\nabla E)^2S^2}S^\mu S^\nu\frac{D_\mu E_{\alpha\beta}D_\nu E^{\alpha\beta}}{\sqrt{u^2}^{\,3}} + C_{(\nabla B)^2S^2}S^\mu S^\nu\frac{D_\mu B_{\alpha\beta}D_\nu B^{\alpha\beta}}{\sqrt{u^2}^{\,3}} \\
& + C_{(\nabla E)^2S^4}S^\mu S^\nu S^\kappa S^\rho\frac{D_\kappa E_{\mu\alpha}D_\rho E_\nu^\alpha}{\sqrt{u^2}^{\,3}} + C_{(\nabla B)^2S^4}S^\mu S^\nu S^\kappa S^\rho\frac{D_\kappa B_{\mu\alpha}D_\rho B_\nu^\alpha}{\sqrt{u^2}^{\,3}} + \dots,
\end{aligned}
$$

- New (unstudied) Wilson coefficients
- Are there any redundant terms (“on-shell operators”) here?

Curious Findings at NLO Cubic- & Quartic-in-Spin

Dependence in product of Wilson coefficients

Originating from lower-order sectors, e.g. at NLO cubic-in-spin we get $(C_{ES^2})^2$

"Composite" worldline couplings

$$p_\mu = -\frac{\partial L}{\partial u^\mu} = \tfrac{m}{u} u_\mu + \Delta p_\mu(RS^2)$$

Application of gauge at NLO as of cubic-in-spin → New type of worldine-graviton couplings to "composite" multipoles, in terms of "elementary" ones

Quadratic-in-curvature contributions

$$L_{\mathrm{S}^4(\mathrm{R}^2)} = \frac{C_{E^2S^4}}{24m^3} S^\mu S^\nu S^\kappa S^\rho \frac{E_{\mu\nu}E_{\kappa\rho}}{\sqrt{u^2}^{\,3}} + \frac{C_{B^2S^4}}{24m^3} S^\mu S^\nu S^\kappa S^\rho \frac{B_{\mu\nu}B_{\kappa\rho}}{\sqrt{u^2}^{\,3}}$$

Turns out only electric operator enters at S^4 - only single 2-graviton exchange

QFT for Gravity at All Scales

Levi Rept. Prog. Phys. 2020
Levi+ 2x 2020, 2x JHEP 2021, x in prep. + Kim, Morales, Yin
My Public Webpage: DEFYING GRAVITY

- Real-world scalability:
  - EFT of gravitating spinning objects - self-contained framework
    ⇒ Direct derivation of useful & physical quantities
    ⇒ Self-consistency checks
  - Precision frontier with spins being pushed to 5PN order!
  - Continuous development of public computational tools
    → EFTofPNG code [CQG Highlights 2017, upgrades...]

- Fundamental lessons:
  - PN gravity informs us about gravity in general
  - New features in NLO higher-spin sectors resonate with picture of composite (rather than elementary) particles at higher quantum spins
  - Possible insights for graviton Compton amplitude with higher spins?
  - Can field theory advances further simplify computations, or even enable analytical predictions to capture the strong gravity regime of the GW signal?