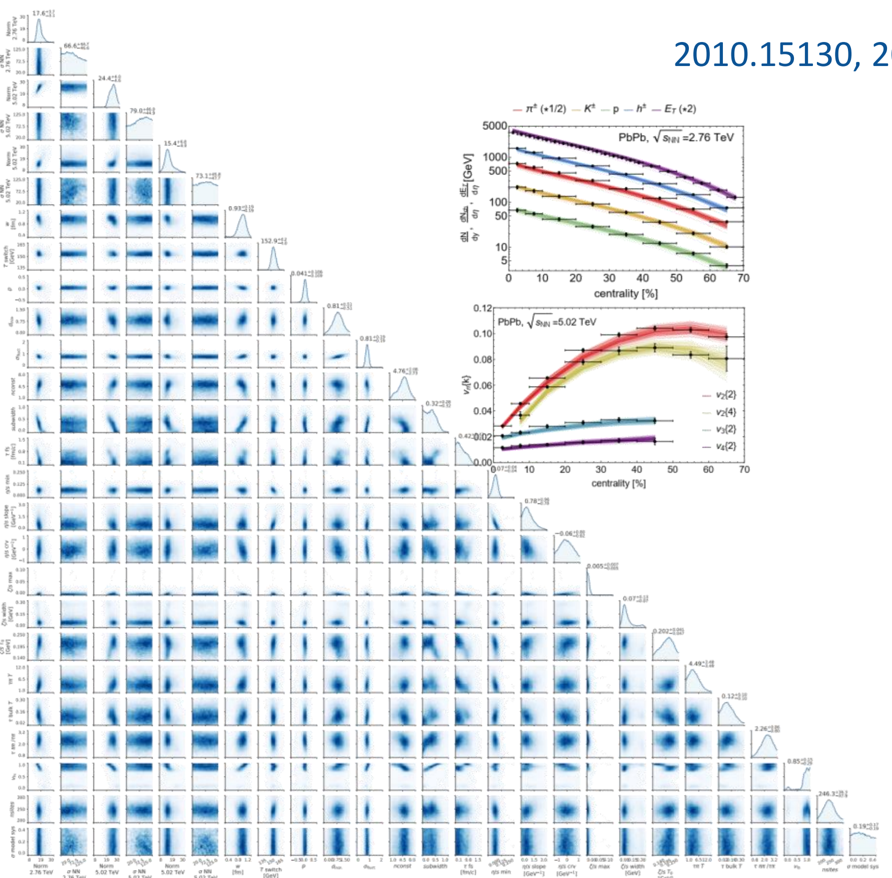
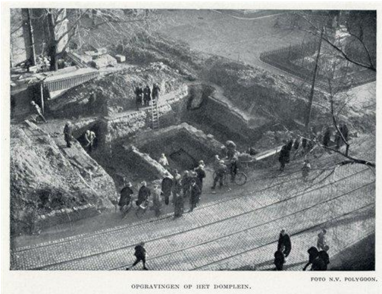
A global analysis of Heavy Ion Collisions with transverse momentum dependence

Towards a precision analysis of heavy ion collisions

Based on *Trajectum* with Govert Nijs
2010.15130, 2010.15134 with Govert Nijs, Umut Gursoy and Raimond Snellings





Roman excavations in Utrecht (from Trajectum, or bridge) in 1929

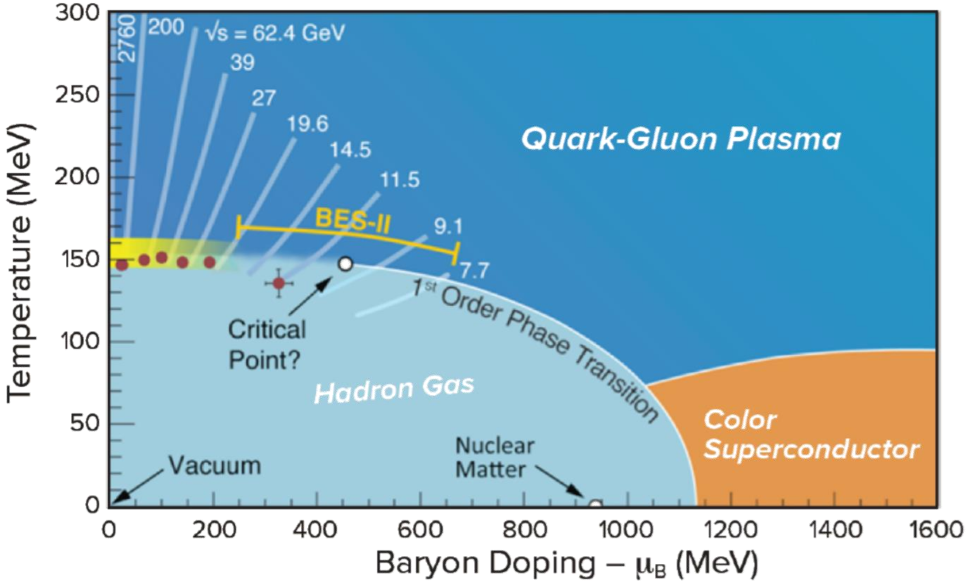
Wilke van der Schee

Heavy Ion seminar, Niels Bohr Institute
Kopenhagen, 26 November 2020

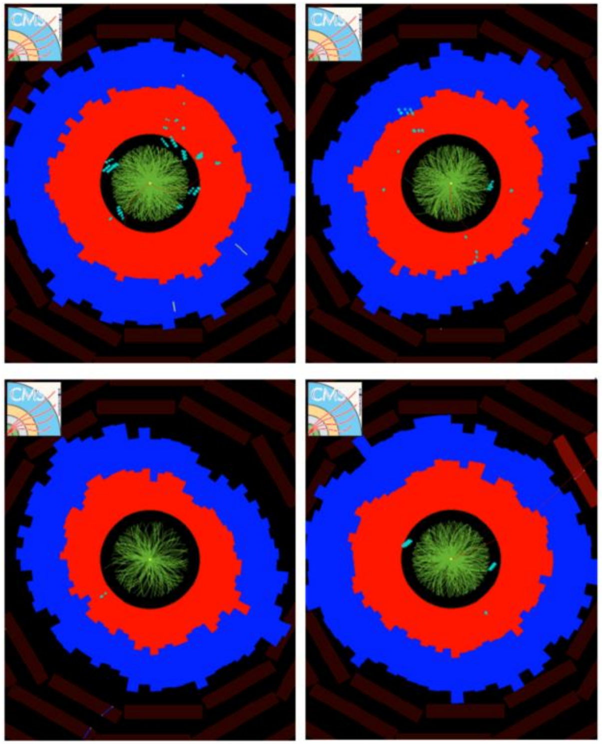
Why do we study heavy ion collisions?

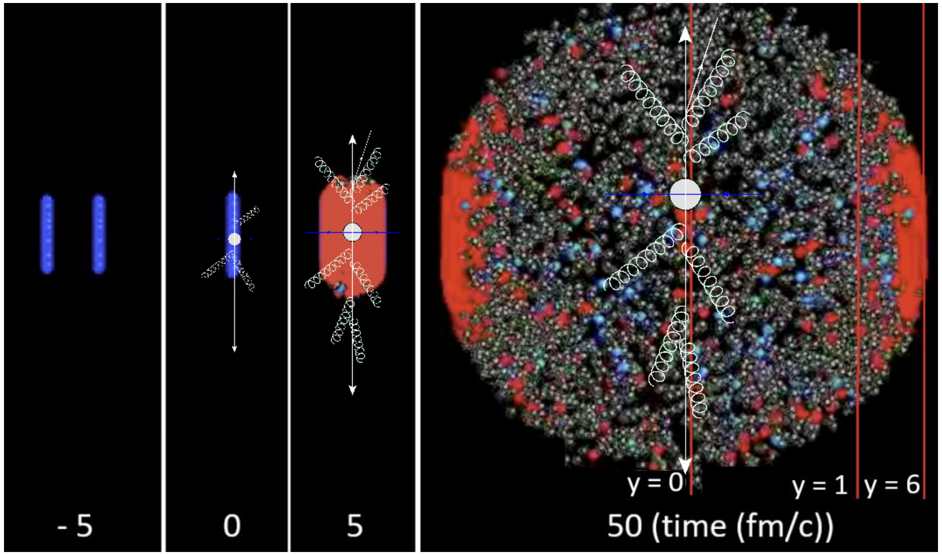
1. Fundamental force of nature

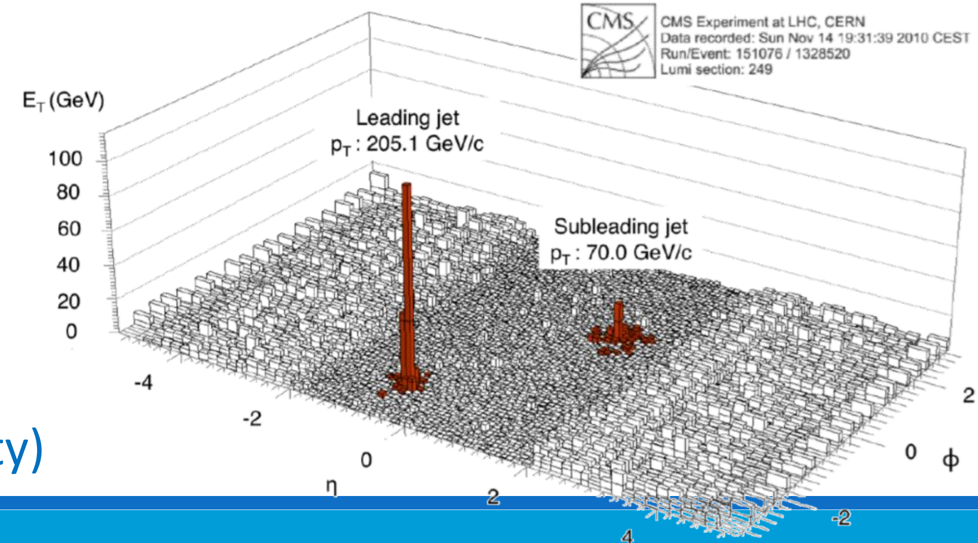
2. Perhaps the simplest form of complex matter
   - Confinement: hadron gas in IR
   - Cross-over to quark-gluon plasma
   - QGP: strongly coupled
   - A critical point?



Quark-gluon plasma is strongly interacting



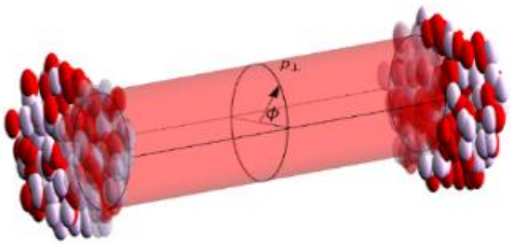




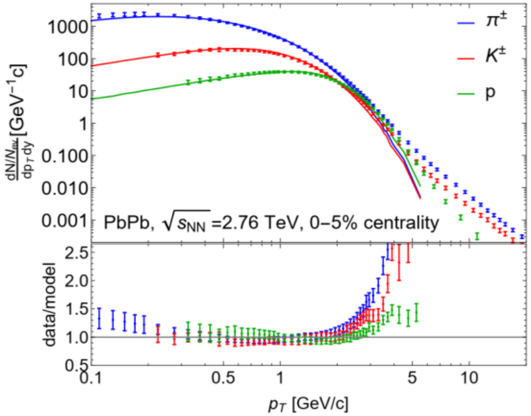
Studying the most perfect liquid

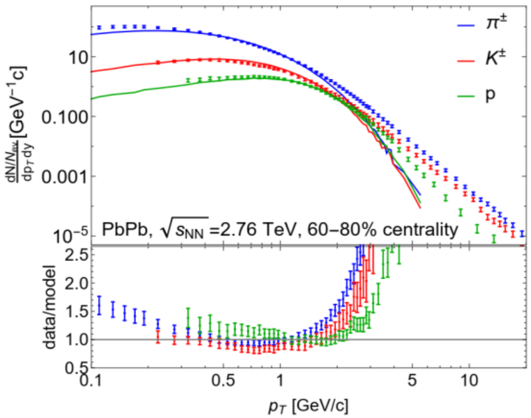
- Jet energy loss in dijet pair
- Anisotropic flow (small viscosity)

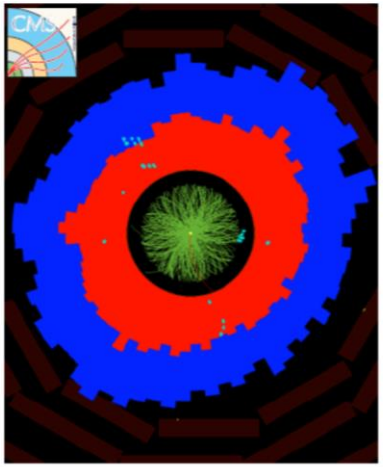
A wealth of experimental data



Spectra for pions, kaons and protons



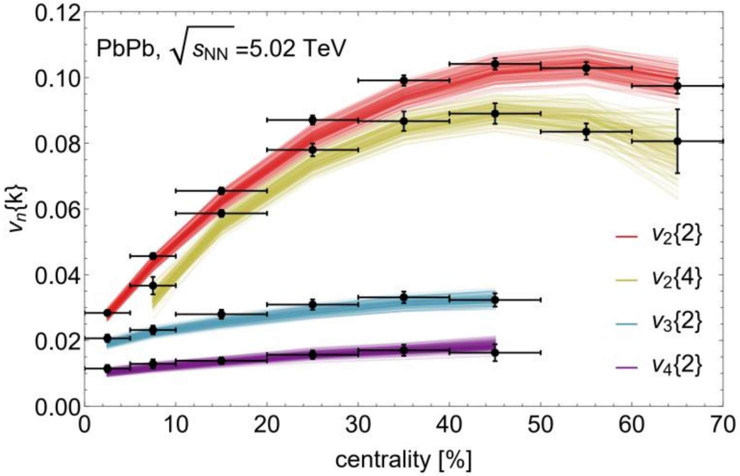




Quantify anisotropic flow:

Fourier coefficients:

- v_2 : elliptic flow
- v_3 : triangular flow, etc



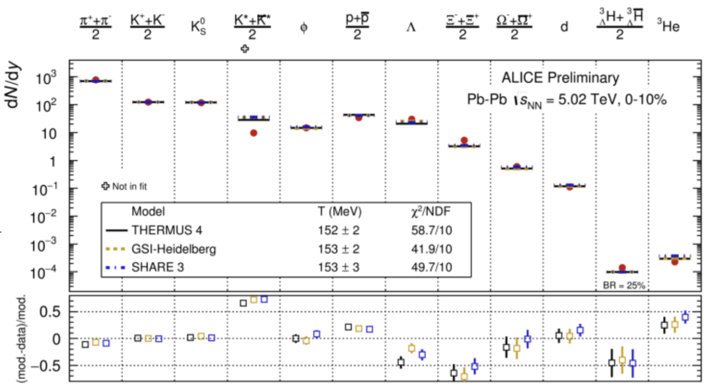
What do we study in heavy ion collisions?

1. Fundamental force of nature
   - Asymptotic freedom: from quasi-particles to strong coupling
   - Can we understand transport when it is close to equilibrium?
   - Can we understand the collision itself? Hydrodynamisation?

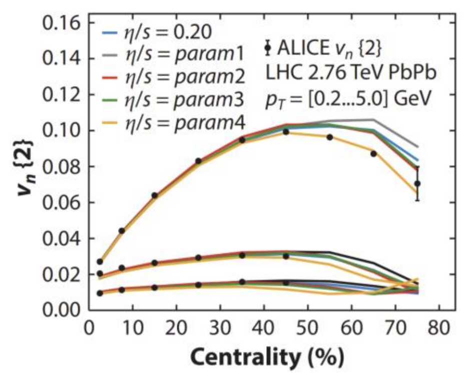
2. A lot of qualitative progress in (simple?) models
   - Ideally simple relations: elliptic flow → shear viscosity
   - Realistically all aspects of heavy ion collisions influence elliptic flow

What have we learnt?

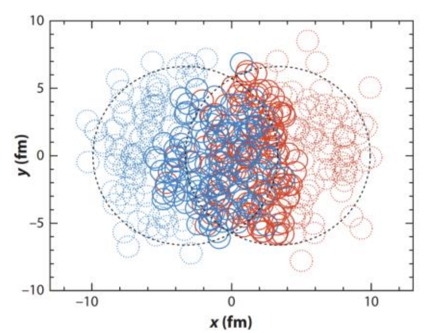
1. Particle ratios: approximately thermal



2. Hydrodynamics for PbPb up to 50% centrality
   - `fast' applicability: within 1 or 2 fm/c
   - Small specific shear viscosity



3. Initial shape: some variation of Monte Carlo Glauber



ALICE, Testing the system size dependence of hydrodynamical expansion and thermal particle production in Xe–Xe and Pb–Pb collisions (2018)
H. Niemi, K. J. Eskola, and R. Paatelainen, Event-by-event fluctuations in a EKRT: Determining QCD matter shear viscosity in heavy-ion collisions (2016)

What are the open questions?

1. Particle ratios: (sizeable) deviations from thermal equilibrium
   - Viscous corrections within hydrodynamics (later)

Hydro at large Reynolds

2. Hydrodynamics for very peripheral PbPb and *p*Pb?
   - How fast? 0.1 fm/c or 1.5 fm/c?
   - T-dependent shear viscosity, bulk viscosity
   - Second order transport relevant?

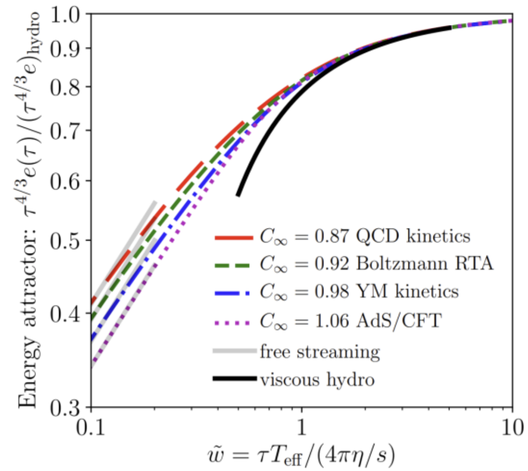
Is QGP strongly coupled?
At which energy scale?
Non-conformal?

3. Initial shape: how to convert colliding nucleons to energy density
   - Not even settled if binary collisions are ruled out (!)
   - More profound in *p*Pb: spherical proton unlikely

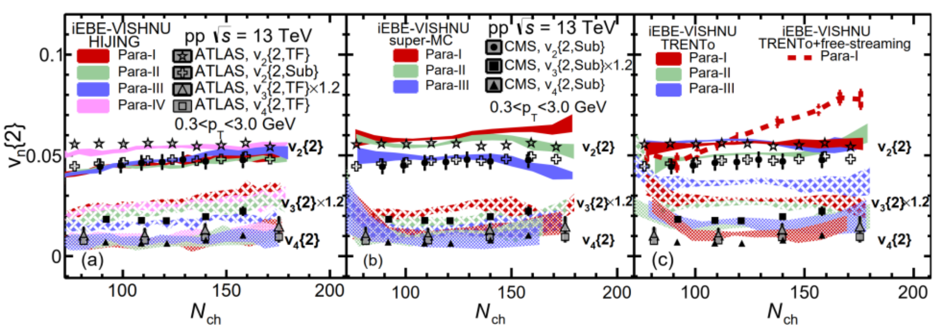
What are the d.o.f.?
Partons? Glasma?

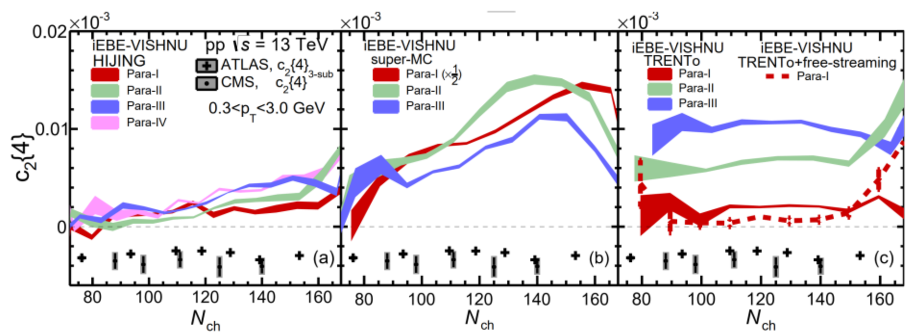
Questions are becoming more precise

1. Theoretically: are we sensitive to the first 2 fm/c?
   - A (perceived?) equivalence between weakly/strongly coupled approaches
   - But note dependence of η/s is scaled out
   - *Smaller systems may be more sensitive to microscopics*



2. Experimentally: can we understand soft observables in small systems?





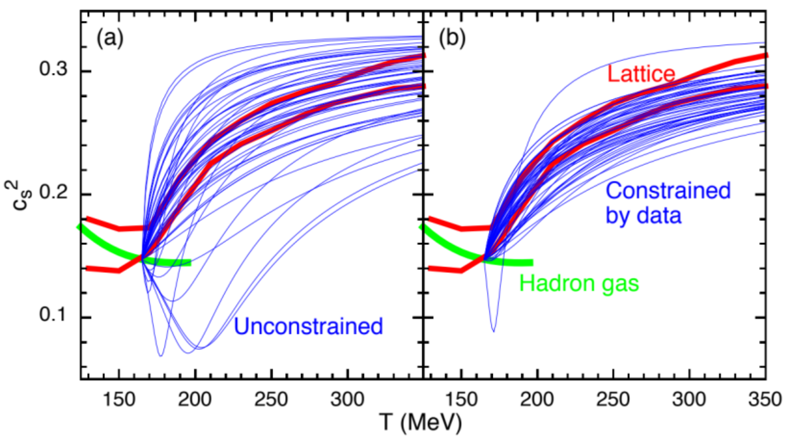
G. Giacalone, A. Mazeliauskas and S. Schlichting, Hydrodynamic attractors, initial state energy and particle production in relativistic nuclear collisions (2019)
Wenbin Zhao, You Zhou, Koichi Murase and Huichao Song, Searching for small droplets of hydrodynamic fluid in proton-proton collisions at the LHC (2020)

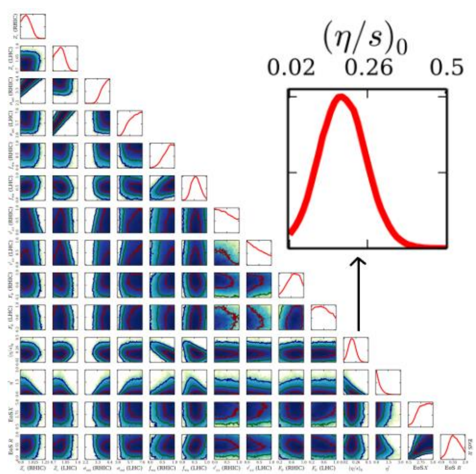
First global analyses

Precise questions require precise understanding of interplay of rich physics in heavy ion collisions

Constraining EOS (Jan 2015)

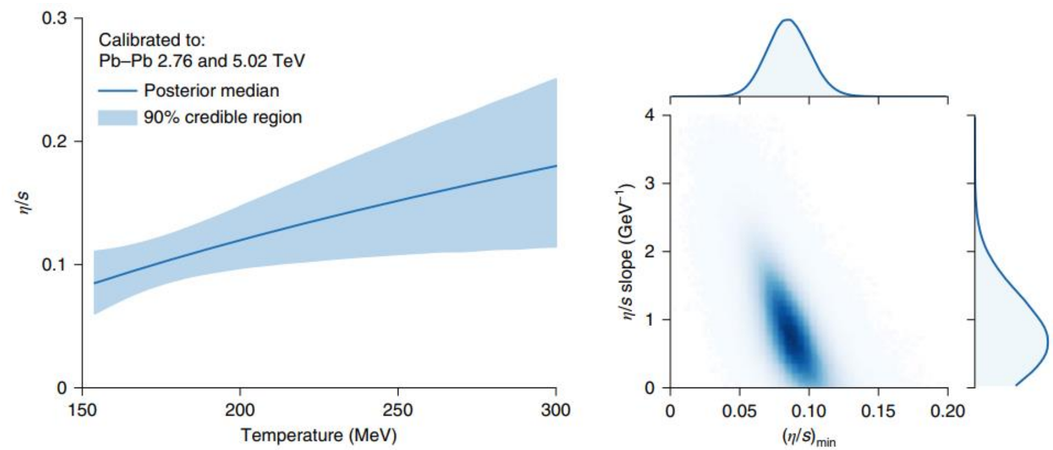
Prior Posterior (data)





Constraining η/s (2019, Nature Physics)

η/s versus temperature Posterior (**η/s+slope**)

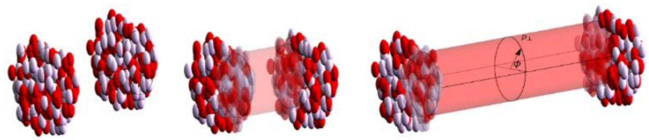


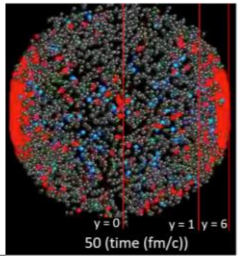
Jonah E. Bernhard, J. Scott Moreland and Steffen A. Bass
Bayesian estimation of the specific shear and bulk viscosity of quark–gluon plasma

Important: `average' viscosity better constrained than T-dependent viscosity

S. Pratt, E. Sangaline, P. Sorensen and H. Wang, Constraining the Eq. of State of Super-Hadronic Matter from Heavy-Ion Collisions (2015)
Evan Sangaline and Scott Pratt, Towards a Deeper Understanding of How Experiments Constrain the Underlying Physics of Heavy-Ion Collisions (2015)

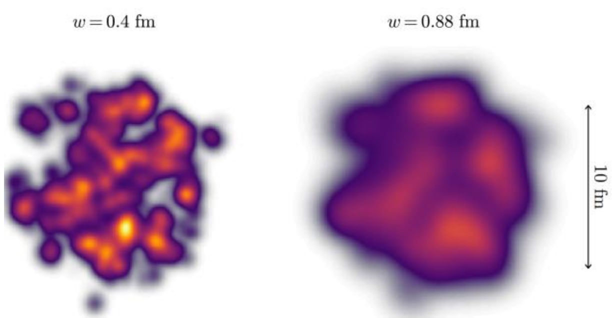
Standard model of heavy ion collisions





Initial stage (9)

Subnucleonic structure? (7)

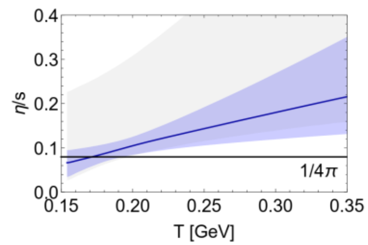


Non-thermal flow? (2)
Yes: for time τ with varying speed

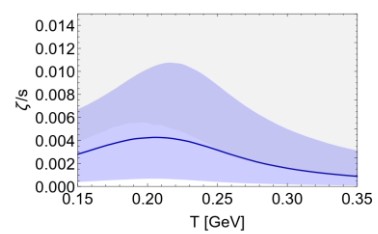
Fluctuations? (1)

Viscous hydrodynamics (9)

Shear viscosity (3)



Bulk viscosity (3)



Second order transports: 3

Cascade of hadrons (1)

Convert quark-gluon plasma at T_{switch} to particles following Boltzmann distribution (particlization, 1)

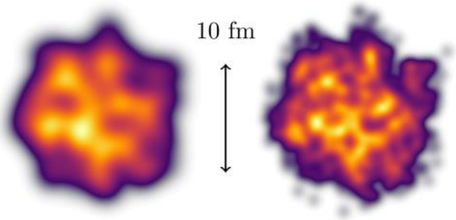
Subtle: viscous corrections

Evolve particles with hadronic code: SMASH

Jonah Bernhard, Scott Moreland and Steffen Bass, Bayesian estimation of the specific shear and bulk viscosity of quark–gluon plasma (2019)
Govert Nijs, WS, Umut Gursoy and Raimond Snellings, A Bayesian analysis of Heavy Ion Collisions with Trajectum (2020)

Initial geometry: two (three?) uncertainties

1. The structure of nucleons
   - n_c constituents of Gaussian subwidth *v* within a nucleon of width *w*
   - Nucleons placed according to MC Glauber



2. How do colliding (sub)nucleons deposit their energy? $\mathcal{T} = \left(\frac{\mathcal{T}_A^p + \mathcal{T}_B^p}{2} \right)^{1/p}$
   - For $p = 0$ we get $\mathcal{T} = \sqrt{\mathcal{T}_A \mathcal{T}_B}$: close to EKRT or Holography ($\mathcal{T} \approx (\mathcal{T}_A \mathcal{T}_B)^{4/9}$)
   - Does not quite allow binary scaling?

3. (Quantum) fluctuations in the above: Gamma-distribution:
   - Goes beyond MC Glauber fluctuations $p(T) = \frac{1}{\Gamma(1/\sigma)\sigma^{1/\sigma}} T^{(1-\sigma)/\sigma} e^{-T/\sigma}$

Scott Moreland, Jonah Bernhard and Steffen Bass, Estimating nucleon substructure properties in a unified model of p-Pb and Pb-Pb collisions (2018)
WS and Bjoern Schenke, Rapidity dependence in holographic heavy ion collisions (2015)

Pre-flow and initial conditions

1. Free streaming with free streaming velocity
   - → Match stress energy tensor to hydro

$$\hat{p}^\mu = \begin{pmatrix} 1 \\ v_{fs} \cos\phi \\ v_{fs} \sin\phi \end{pmatrix}$$

$$T^{\mu\nu}(x,y) = \frac{1}{2\pi\tau_{fs}} \int_0^{2\pi} d\phi \hat{p}^\mu \hat{p}^\nu \; \mathcal{T}(x - v_{fs}\tau_{fs}\cos\phi, y - v_{fs}\tau_{fs}\sin(\phi))$$

2. Important: velocity also determines pressure
   - Allows pressureless fluid (P=0) all the way till conformal EOS (P=e/3)
   - In reality in equilibrium around 400 MeV we have P = 0.85 e/3

Hydrodynamics: first and second order

1. Constitutive relations for the stress tensor, with p(ρ) EOS from HotQCD

$$T^{\mu\nu} = \rho u^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}, \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

With shear and bulk tensors:

$$D\Pi = -\frac{1}{\tau_\Pi}\left[\Pi + \zeta \nabla \cdot u + \delta_{\Pi\Pi} \nabla \cdot u \Pi - \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu}\right],$$

$$\Delta^\mu_\alpha \Delta^\nu_\beta D\pi^{\alpha\beta} = -\frac{1}{\tau_\pi}\Big[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu} + \delta_{\pi\pi}\pi^{\mu\nu}\nabla \cdot u - \phi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} + \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} - \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}\Big].$$

2. We vary the green coefficients, η and ζ as a function of temperature, 2nd order according to $\frac{\tau_\Pi s T \delta^2}{\zeta}$, $\frac{\tau_\pi s T}{\eta}$ and $\frac{\tau_{\pi\pi}}{\tau_\pi}$

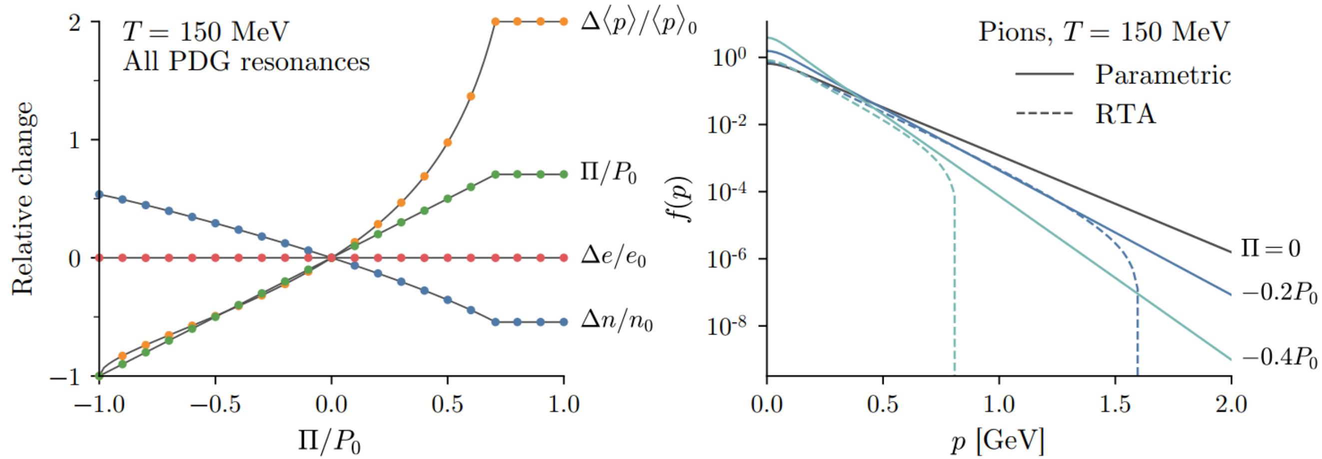
Particlization: viscous corrections

$$T^{\mu\nu} = \sum_{\text{sp}} g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E} f(p),$$

1. Particles in fluid restframe cannot be in thermal equilibrium
2. Several methods that (only) agree for small deviations

$f(p) \to z_{\text{bulk}} f(p + \lambda_{\text{bulk}} p)$ parametric, rescale p: fix z and λ such that e and P match

$\delta f = f_0(1 \pm f_0) \frac{\tau}{ET} \left[\frac{1}{2\eta} p^i p^j \pi_{ij} + \frac{1}{\zeta} \left(\frac{p^2}{3} - c_s^2 E^2 \right) \Pi \right]$ change *f(p)* directly, motivated by RTA



`Parametric' clearly better at high p_T, but somewhat ad-hoc and species independent

Performing a global analysis

Bayes theorem:

$$\mathcal{P}(\boldsymbol{x}|\mathbf{y}_{\text{exp}}) = \frac{e^{-\Delta^2/2}}{\sqrt{(2\pi)^n \det\left(\Sigma(\boldsymbol{x})\right)}}\mathcal{P}(\boldsymbol{x})$$

$$\text{with } \Delta^2 = (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\text{exp}}) \cdot \Sigma(\boldsymbol{x})^{-1} \cdot (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\text{exp}})$$

We have a 20-dimensional parameter space and 514 datapoints

- Run model on 1000 `design’ points, spaced on a latin hypercube (lxplus ☺)
- `Interpolate’ results by training a Gaussian Process Emulator

Markov Chain Monte Carlo (emcee2.2)

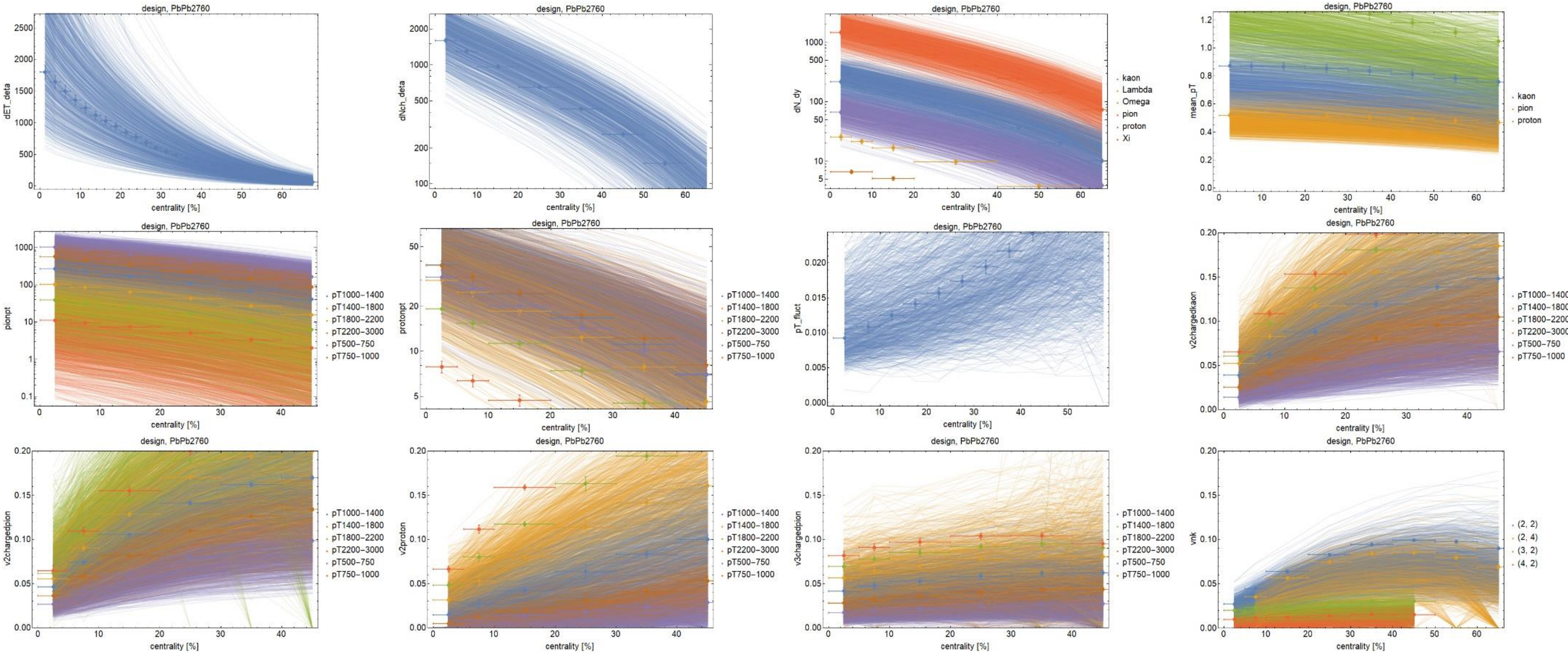
- Obtain sample of 10^6 likely values

Compare posterior with data

- From emulator (emulator has its own uncertainty estimate)
- A high statistics run at the optimal value (MAP, maximum a posteriori)

Wilke van der Schee, CERN

Overview of selected model output



1000 design points for PbPb, 2000 for pPb. 6k hydro events for PbPb, 40k for pPb

Experimental observables: *a wealth of data*

1. Yields, spectra, identified $v_n\{2\}$ versus p_T, *p*Pb and PbPb (514 datapoints)

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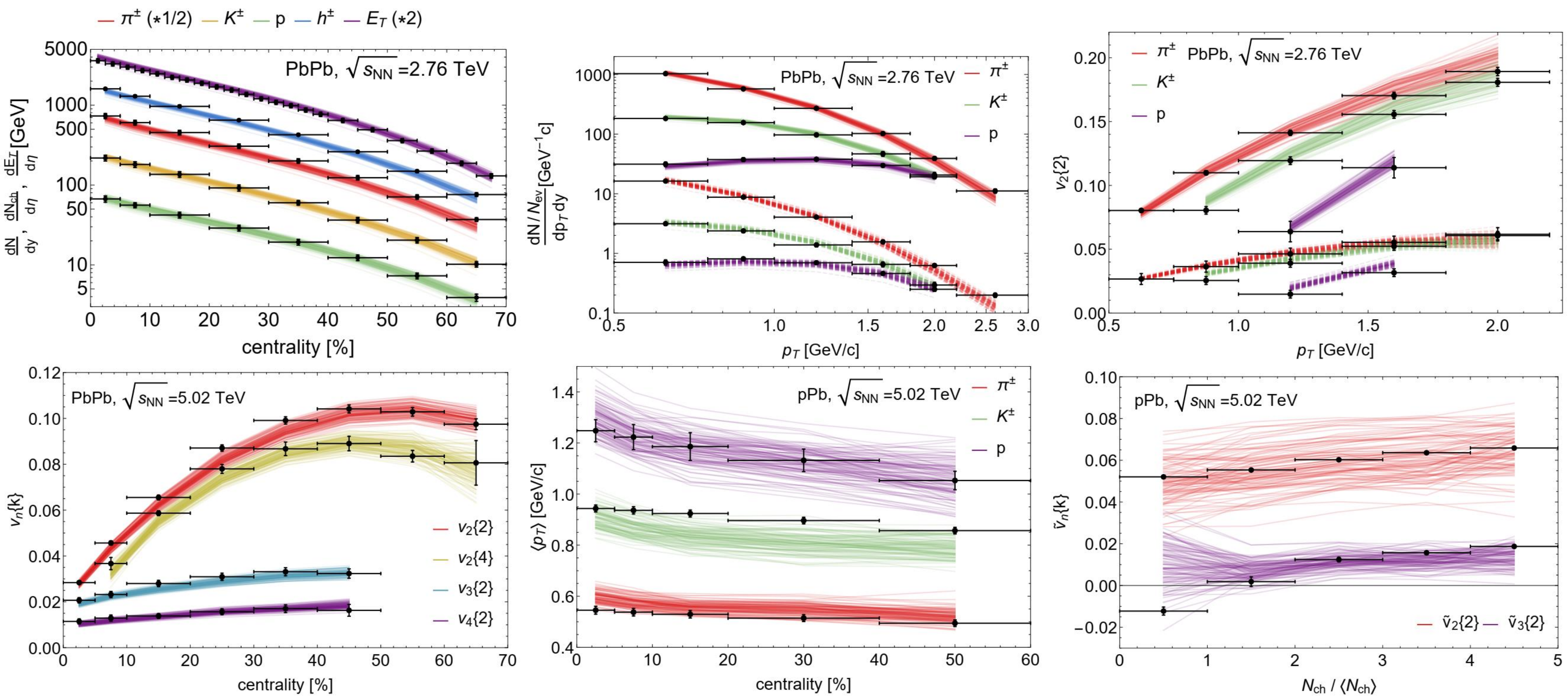
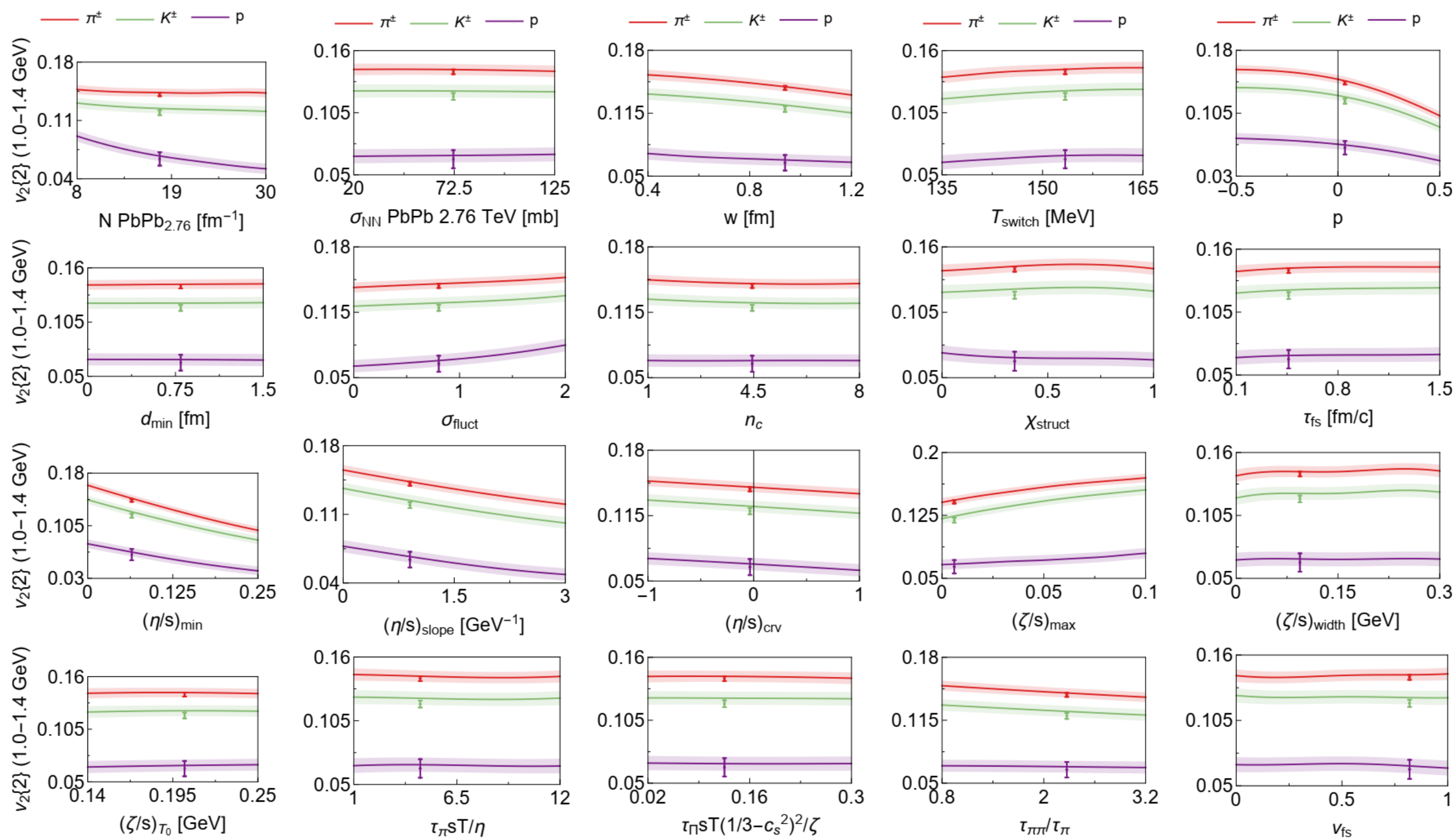
2. Note: points are highly correlated, without p_T dependence effectively only 6 to 8 principal components (PCs), for 20 parameters. With p_T-dependence roughly 12 PCs.



Selected results: the emulator

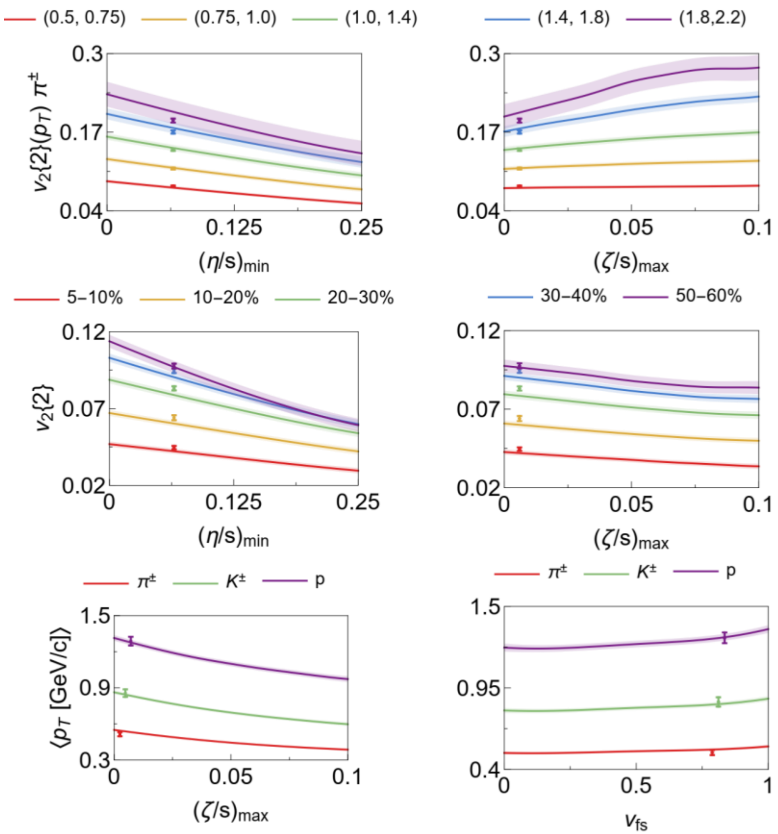
1. v_2 in 1.0-1.4 GeV/c p_T-bin, for optimal values, varying our parameters



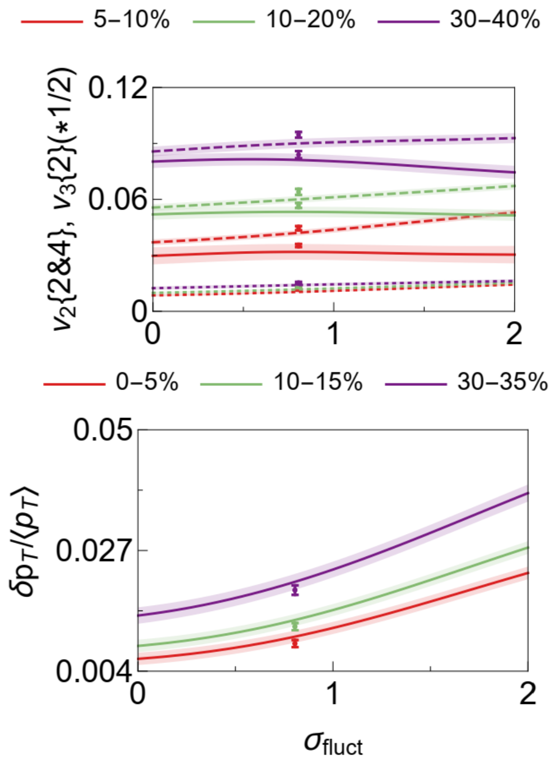
The emulator: Viscosities and fluctuations

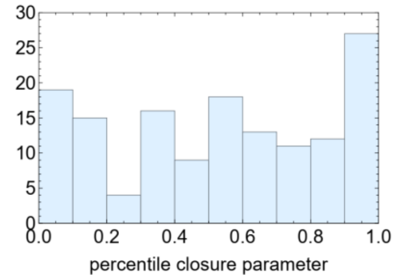
also note: emulator uncertainty (50-60%, or $v_2\{4\}$)

v_2 increases in every p_T bin, but decreases on average for ζ.



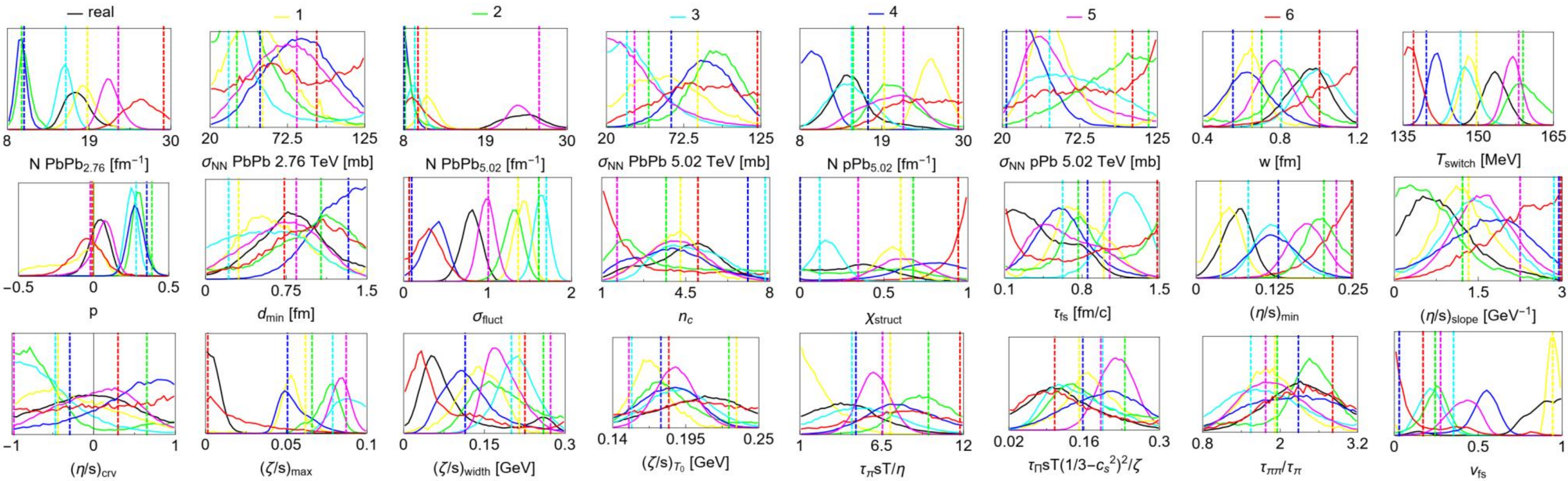
$v_2\{2\}$ - $v_2\{4\}$ increases when increasing fluctuations





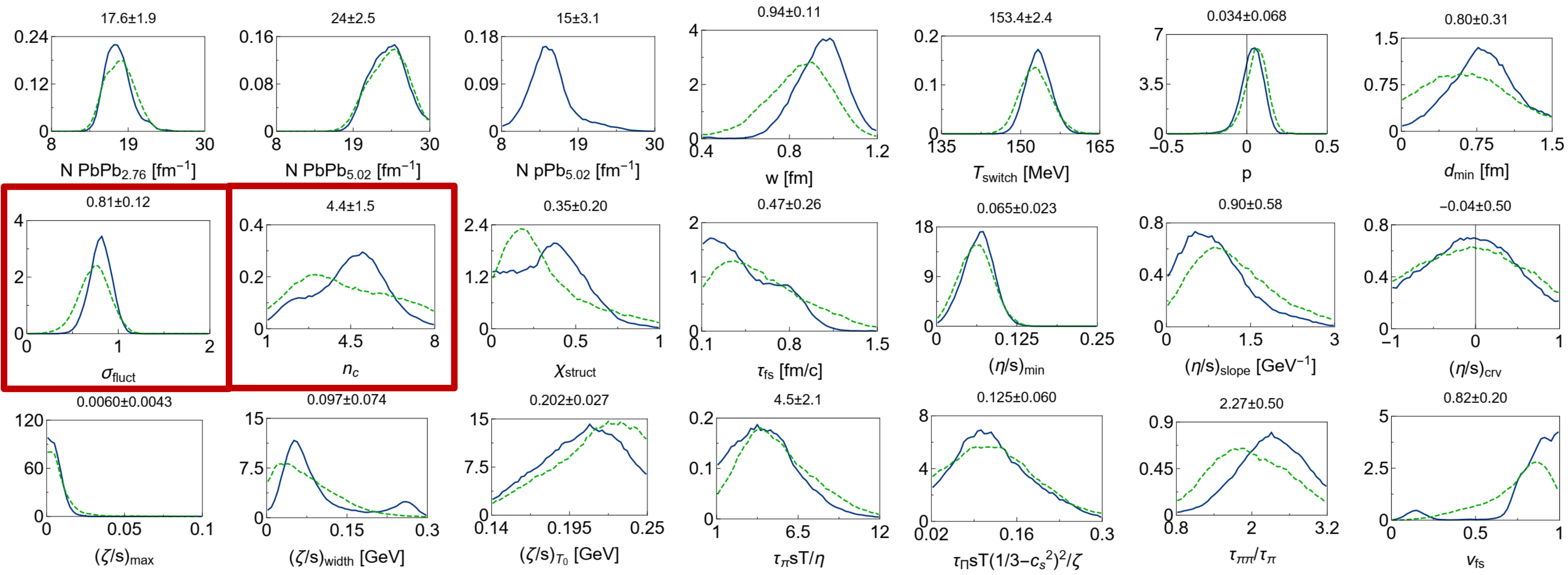
Selected results: closure test

1. We chose six random parameter points (sometimes at edge of prior)
   - Try to extract parameters from model-generated `experimental' data
2. Verifies model + shows sensitivity data on parameter
   - Output indeed consistent with input
   - Sensitive to viscosities, less so for second order



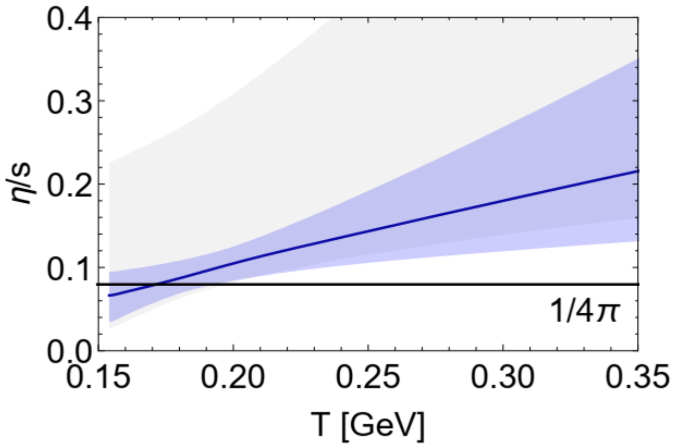
Posterior distributions

1. Dashed: without *p*Pb: indeed much flatter for e.g. n_c
2. Somewhat surprising constraint on fluctuations? (first found at Duke)

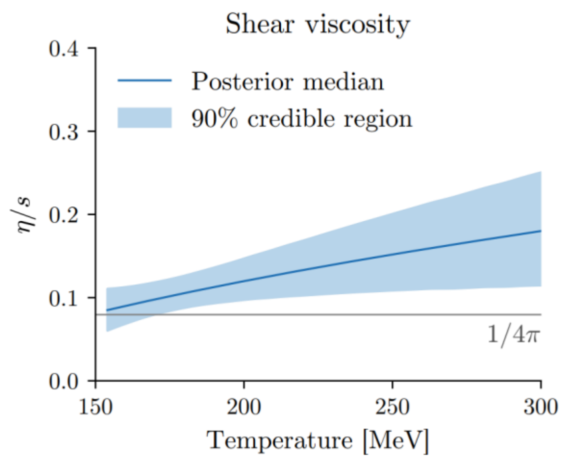


Posterior distributions – shear viscosity

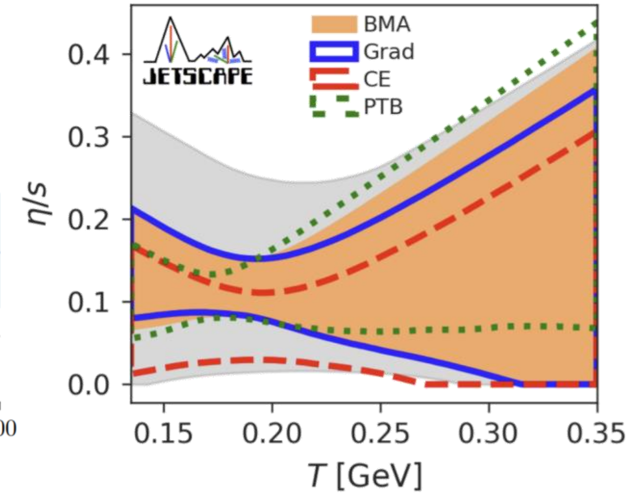
1. Shear viscosity consistent with previous work
   - More data, but also enlarged model → similar constraint on η/s
   - New JETSCAPE slightly broader band (larger priors, single PbPb energy but with RHIC)



Current work (2020)



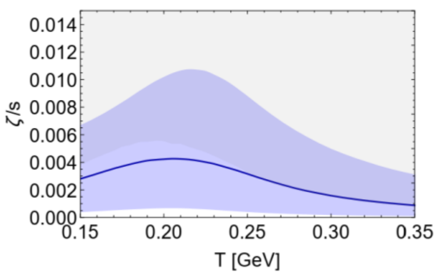
J. Bernhard, S. Moreland and S. Bass, Nature Physics (2019)

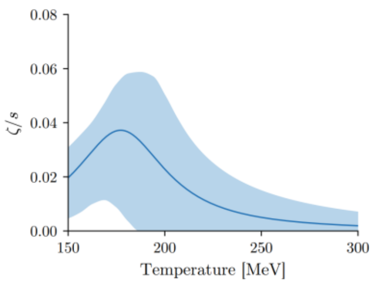


JETSCAPE (2020)

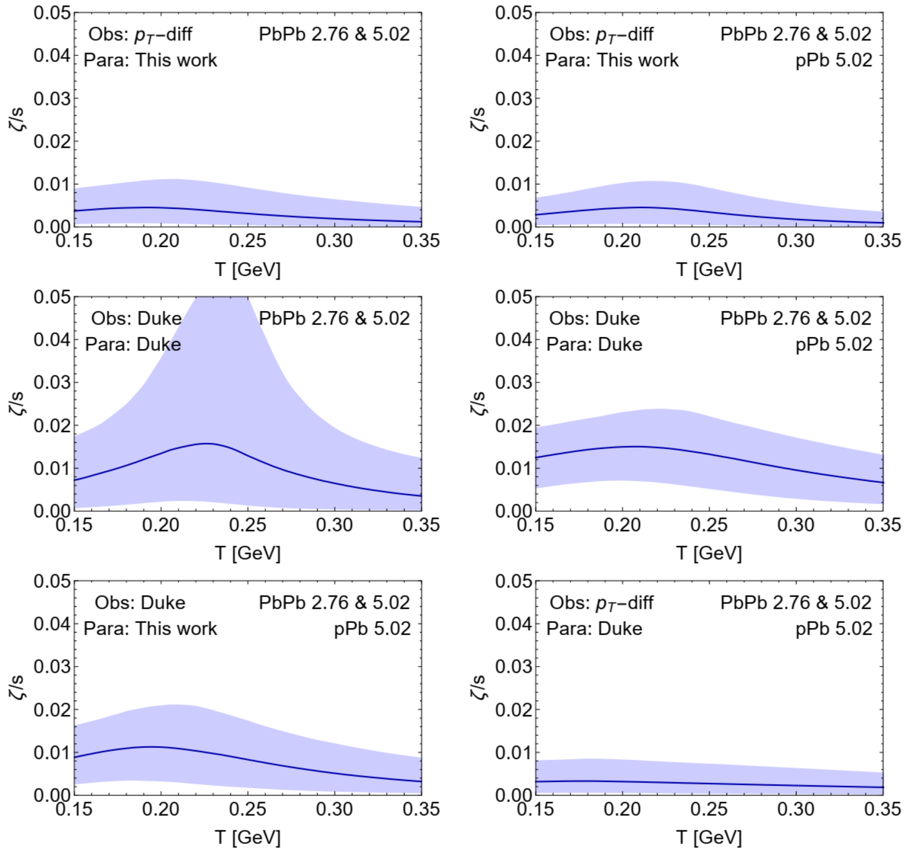
Posterior distributions – bulk viscosity:

Much smaller, even consistent with zero





J. Bernhard, S. Moreland and S. Bass, Nature Physics (2019)

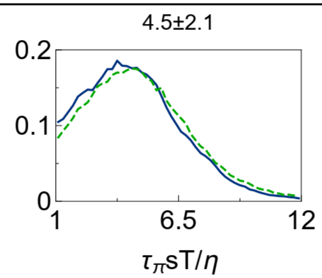
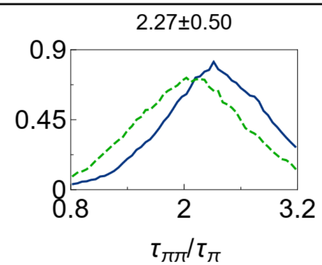


Bulk viscosity, varied several aspects:

- More limited parameter set
  - All versus only `Duke’
- Include or not include p-Pb collisions
- **Include p_T-differential observables**

Posterior distributions – 2nd order transport

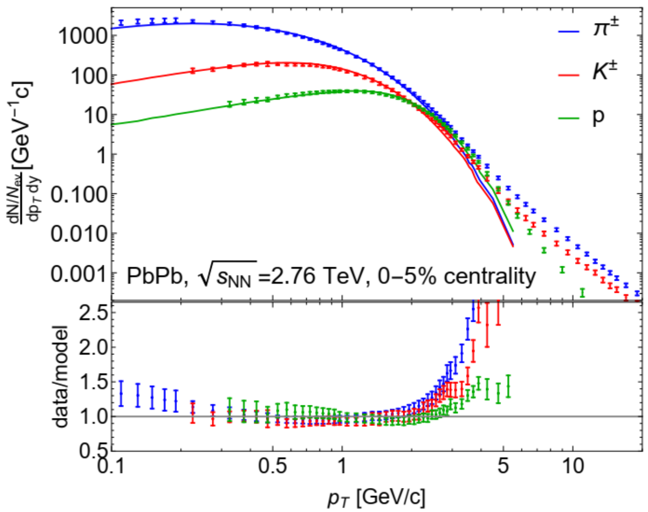
1. τ_π and $\tau_{\pi\pi}$ can be compared to strong and weak coupling values
   - Both consistent, AdS/CFT slightly favoured for $\tau_{\pi\pi}$

	this work	AdS/CFT	kinetic theory
$\frac{\tau_\pi s T}{\eta}$		$4 - \log(4) \approx 2.61$	5
$\frac{\tau_{\pi\pi}}{\tau_\pi}$		$\frac{88}{35(2-\log 2)} \approx 1.92$	$\frac{10}{7} \approx 1.43$

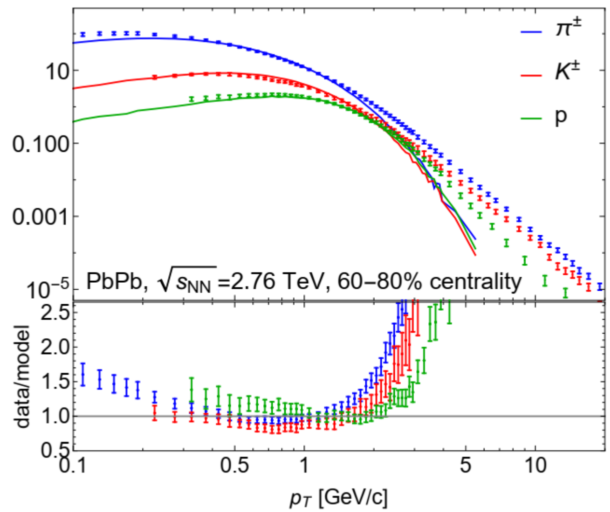
MAP: maximum a posteriori: spectra

1. High statistics run at optimal parameters, compared with ALICE data

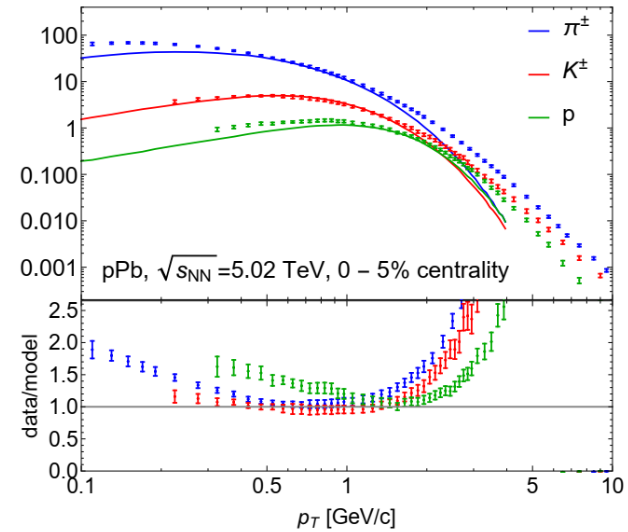
Central PbPb



Very peripheral PbPb

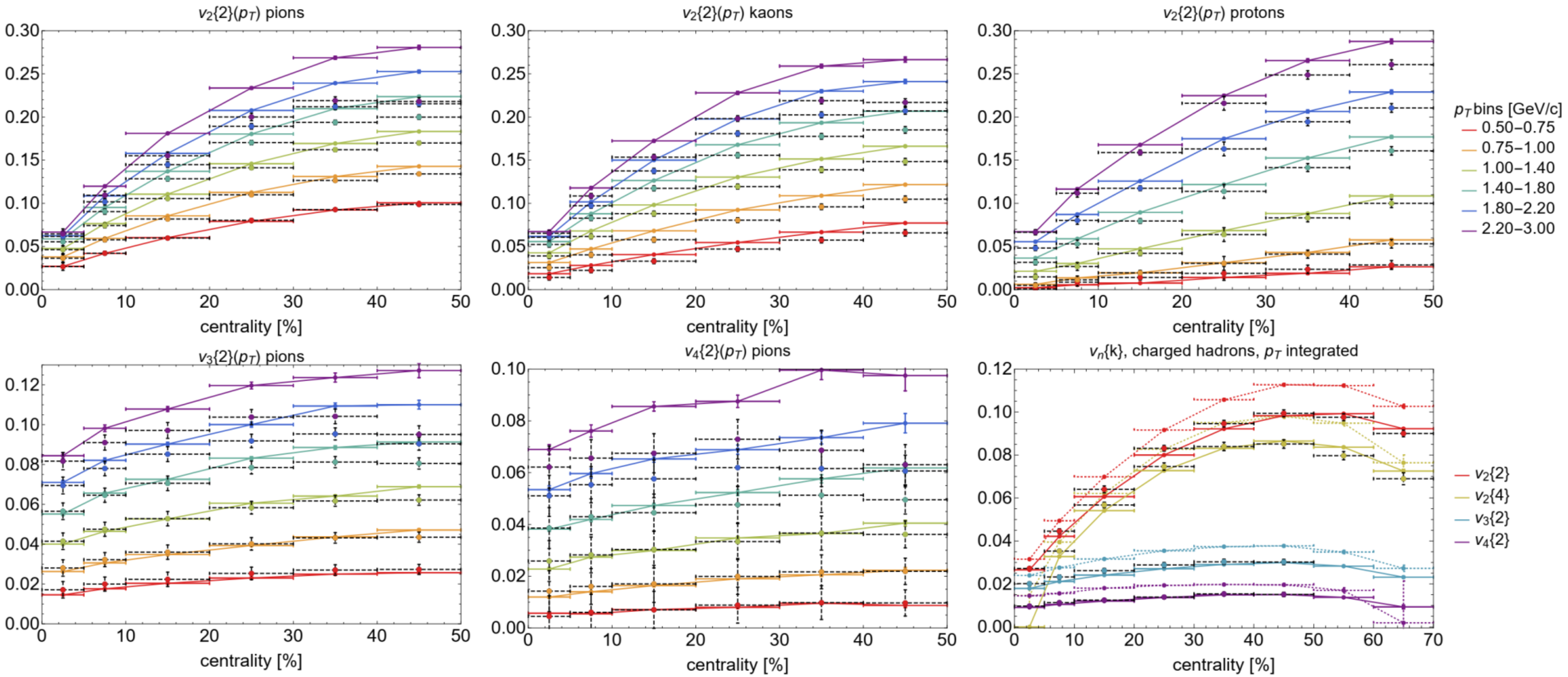


Central *p*Pb



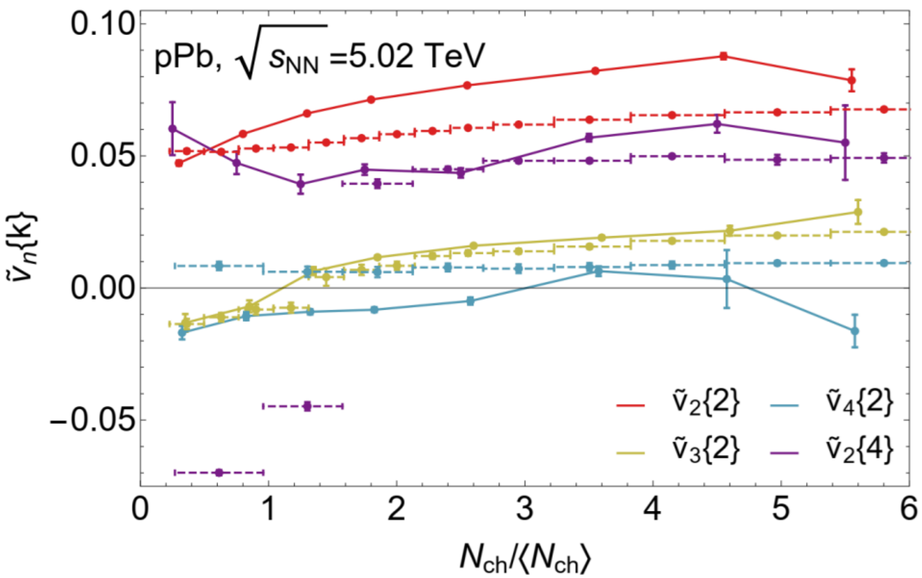
MAP: PbPb anisotropic flow

1. Anisotropic flow matches well, except for a few high p_T bins



MAP: *p*Pb anisotropic flow

1. Emulator and MCMC are less precise for *p*Pb: theory errors is statistical only
2. Shows potential to obtain imaginary $v_n\{2\}$ (= negative $\tilde{v}_n\{k\}$), in agreement with ATLAS low multiplicity result
3. Sheds new light on discussion of hydro versus sign of $\langle\langle 2\rangle\rangle_n$



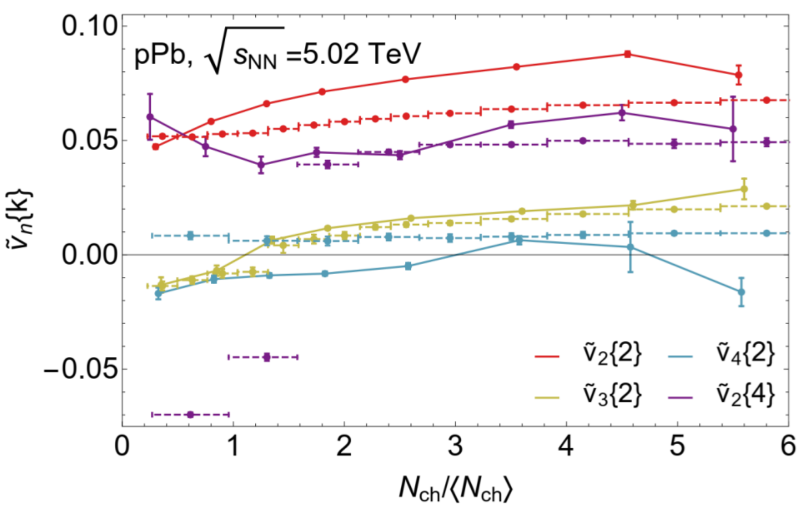
$$Q_n = \sum_{i=1}^{M} e^{in\phi_i}$$

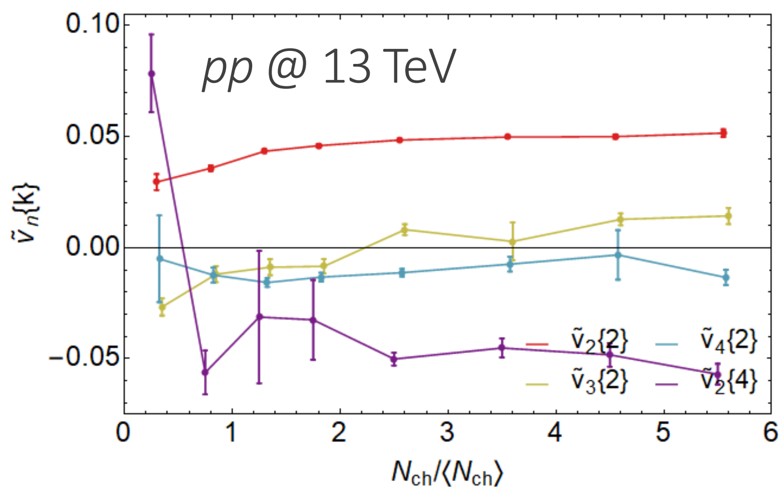
$$\langle 2\rangle_n = \frac{|Q_n|^2 - M}{M(M-1)}$$

$$v_n\{2\} = \sqrt{\langle\langle 2\rangle\rangle_n}$$

MAP: *pp* anisotropic flow

1. Preliminary results for You; different sign for $v_2\{4\}^2$ (?)





Discussion

1. A road to precision analysis of the quark-gluon plasma
   - Measuring transport and initial stage `beyond η/s’, revisiting bulk viscosity
   - Hints on constraints for second order transport

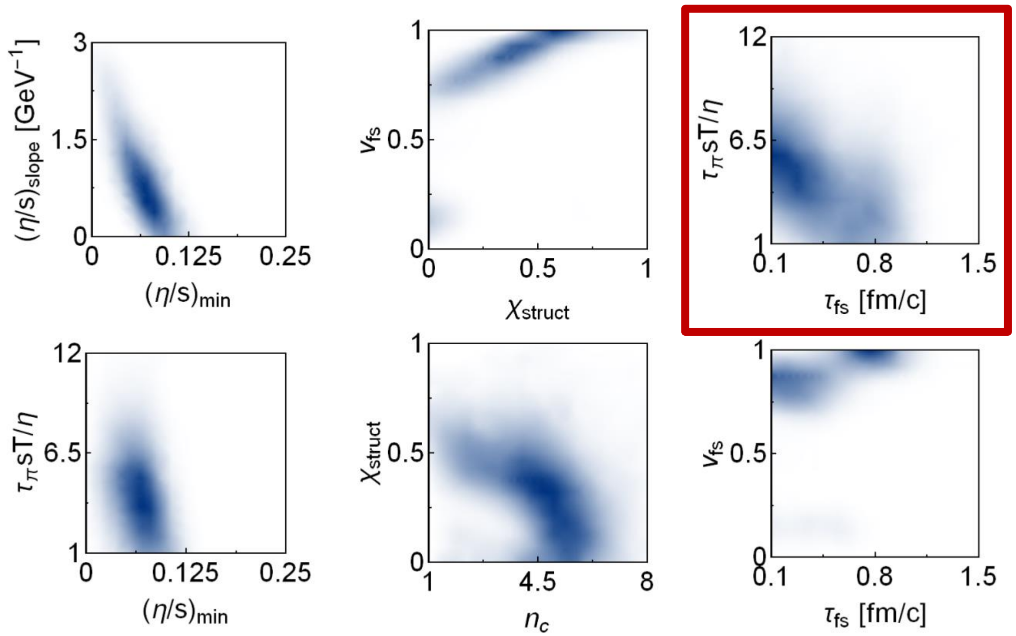
2. Encouraging results
   - p_{T}-differential anisotropic flow sheds new light on global analyses
   - Interesting MAP results on flow in *p*Pb

3. Study is still limited:
   - Still significant uncertainties in initial stage and particlisation
   - No variation of uncertainties in SMASH
   - Data set still fairly small, should still include RHIC results

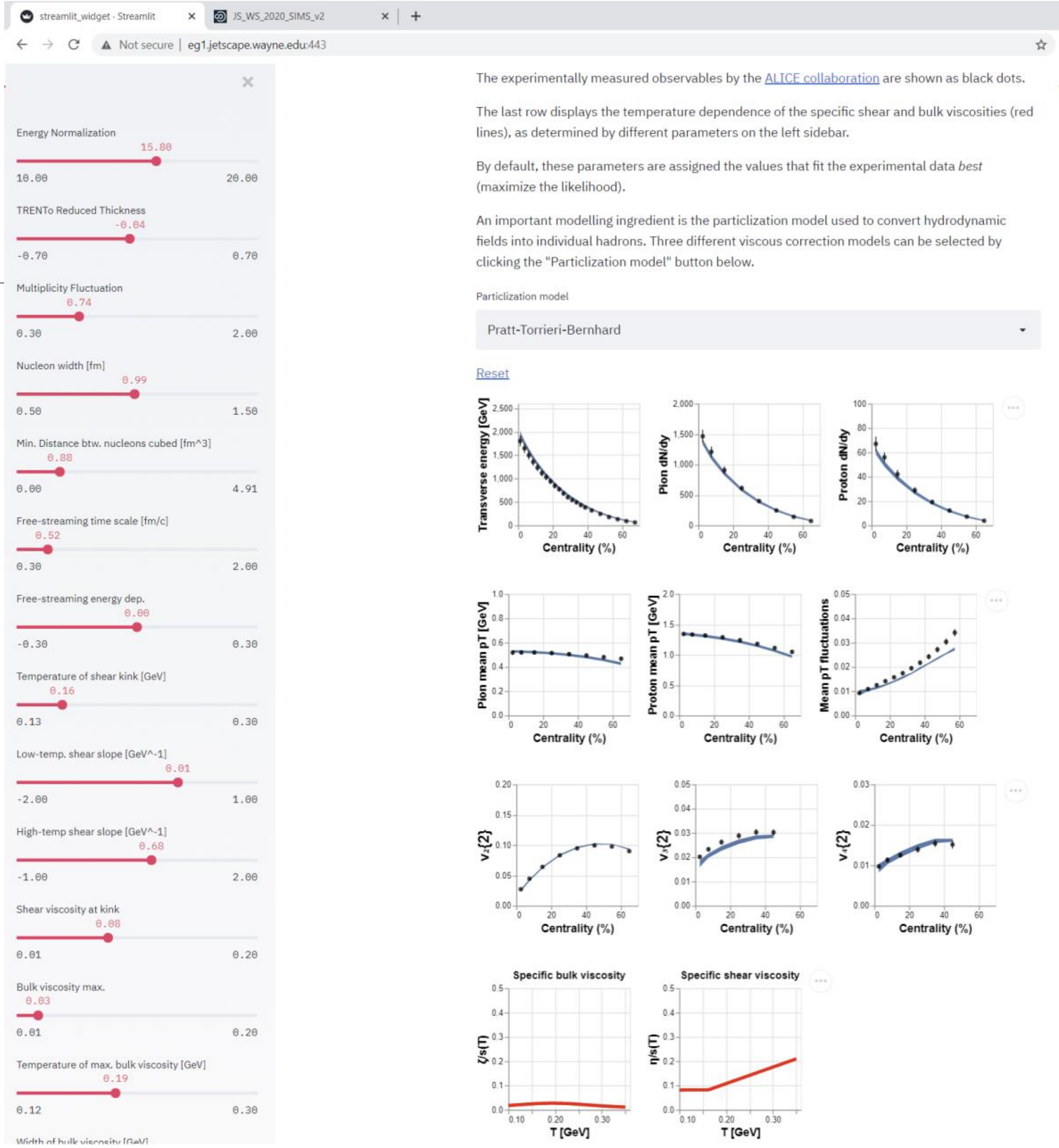
Jonah E. Bernhard, Bayesian parameter estimation for relativistic heavy-ion collisions (PhD thesis, 2018)
Scott Moreland, Initial conditions of bulk matter in ultrarelativistic nuclear collisions (PhD thesis, 2019)

Correlations among the parameters

1. Interesting correlation between free streaming time and τ_π

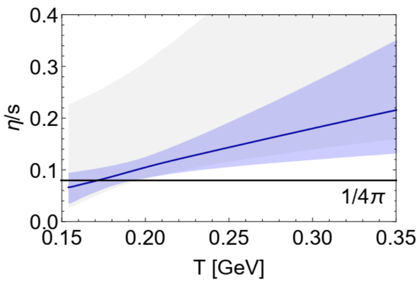


Comparison with JETSCAPE

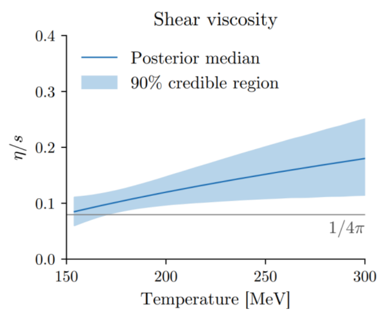


Results seem to be in relatively good agreement. Data is quite consistent without a sizeable bulk viscosity.

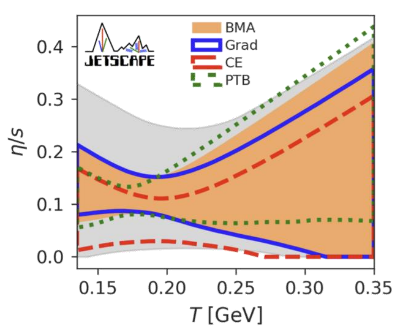
Posterior distributions – shear viscosity



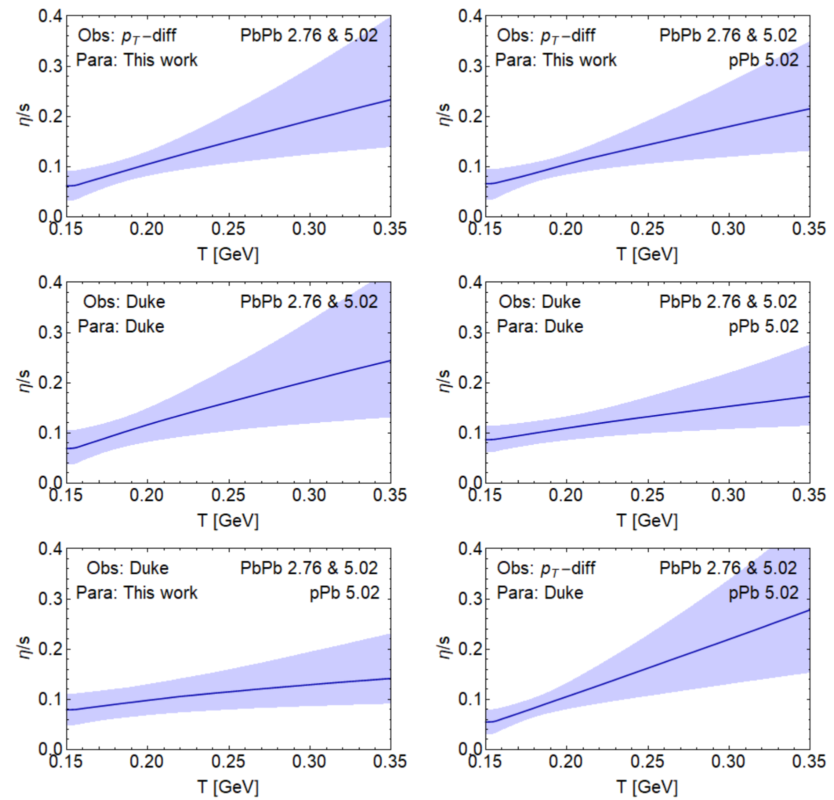
Current work (2020)



J. Bernhard, S. Moreland and S. Bass, Nature Physics (2019)



JETSCAPE (2020)



Closure test: viscosities

1. Closure test works well for both viscosities
2. Most sensitive to low-T region