Mass hierarchy and CP violation in neutrino oscillation experiments

Reference: S. Cao, A. Nath, T. V. Ngoc, Ng. K. Francis, N. T. Hong Van, P. T. Quyen, PHYS. REV. D 103, 112010 (2021)

In collaboration with IFIRSE neutrino group, ICISE, Quy Nhon, Vietnam

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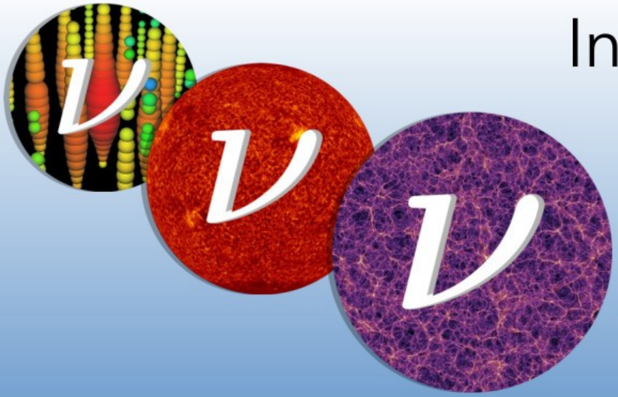
July 7, 2021



International PhD Summer School on Neutrinos

Here, There & Everywhere

July 5-9, 2021
Niels Bohr Institute, Copenhagen

Neutrino in the Standard Model

NEUTRINOS:

- very light (~10^{-6} electron mass)
- Spin = 1/2, electrically neutral leptons
- Most abundant fermions in the universe (~336 cosmic neutrinos/cc)
- Within the Standard model, neutrinos are massless.

*Super-Kamiokande experiment (Japan)
led by Prof Takaki Kajita
*Sudbury Neutrino Observatory (Canada)
led by Prof Arthur B McDonald

-- confirm neutrinos have mass by measuring **neutrino oscillations**.

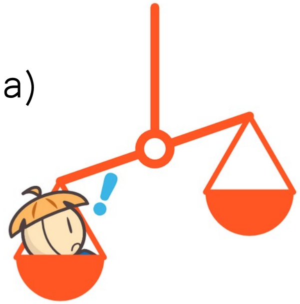
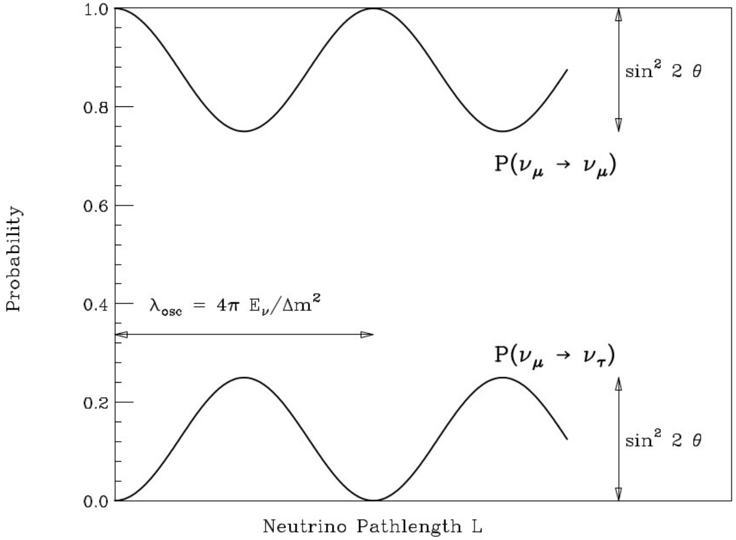


*Image courtesy: NEUTRINO MANGA

Neutrino Oscillation

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U^{\mathrm{PMNS}} \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} \quad \text{where} \quad U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\mathrm{CP}}} & c_{13}c_{23} \end{pmatrix}$$

Mass eigen states		Mass eigen values
v_1	●	m_1
v_2	●	m_2
v_3	●	m_3



Neutrino Oscillation

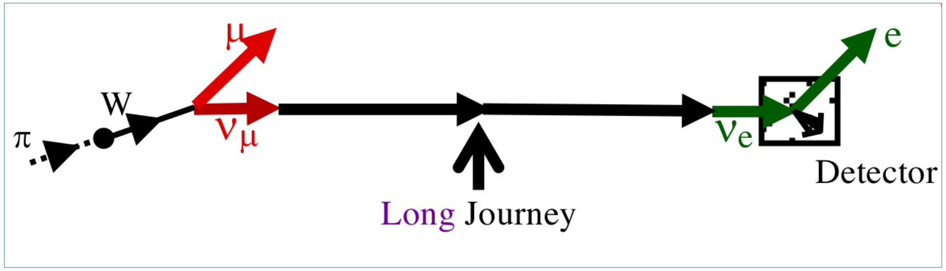


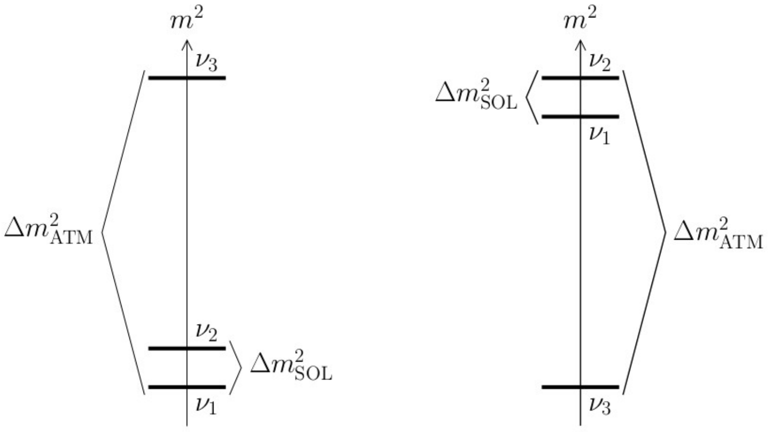
***Image**: Boris Kayser’s talk on neutrino phenomenology, VSoN 2018

Oscillation Probability

Mass-squared differences

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U^*_{\alpha k}\, U_{\beta k}\, U_{\alpha j}\, U^*_{\beta j}\, \exp\left(-i \frac{\boxed{\Delta m^2_{kj}}\, L}{2E}\right)$$

Mass Hierarchy (MH) & Leptonic CP Violation



NORMAL
HIERARCHY

INVERTED
HIERARCHY

Is $m_1<m_2<m_3$ (normal) or $m_3<m_1<m_2$ (inverted) the truth?
i.e. What is the sign of Δm^2_{31}?

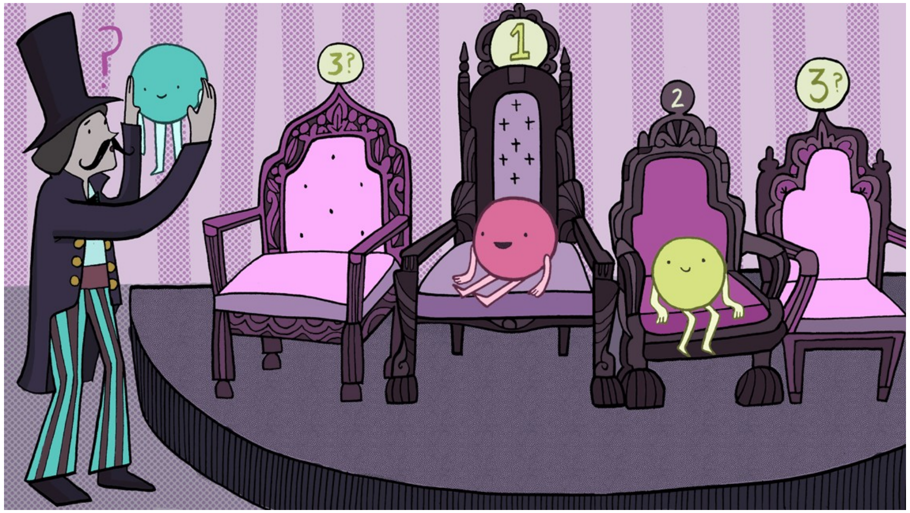
Are the oscillation probabilities $P_{\mu e} = \bar{P}_{\mu e}$?

$A_{CP} = P_{\mu e} - \bar{P}_{\mu e}$

We define CP Asymmetry in vacuum, as

$A_{CP} = 16\, s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta \sin\Delta_{21} \Delta_{31} \Delta_{31}$

Is $\delta_{CP} \neq n\pi$ where n=0,1,2 ?



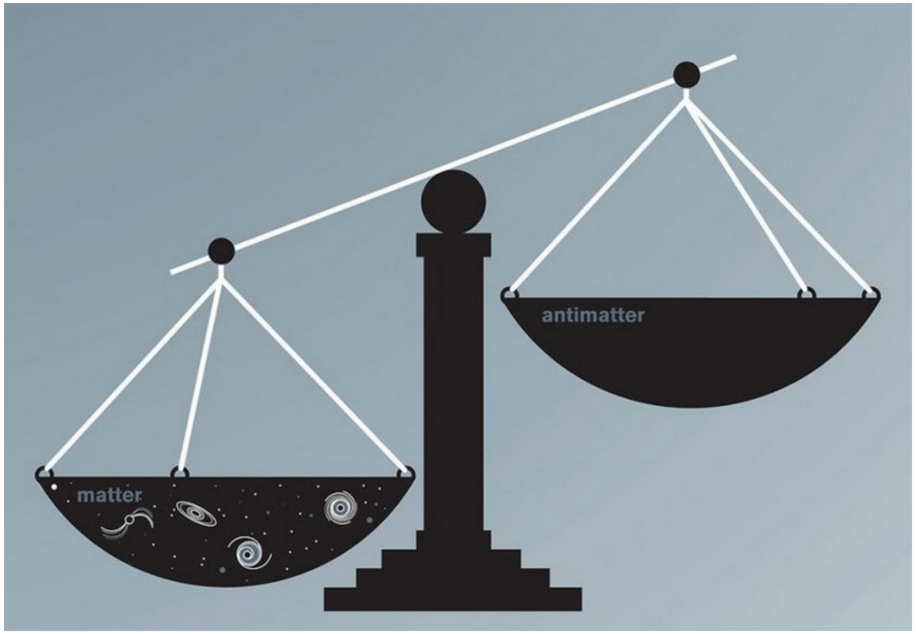


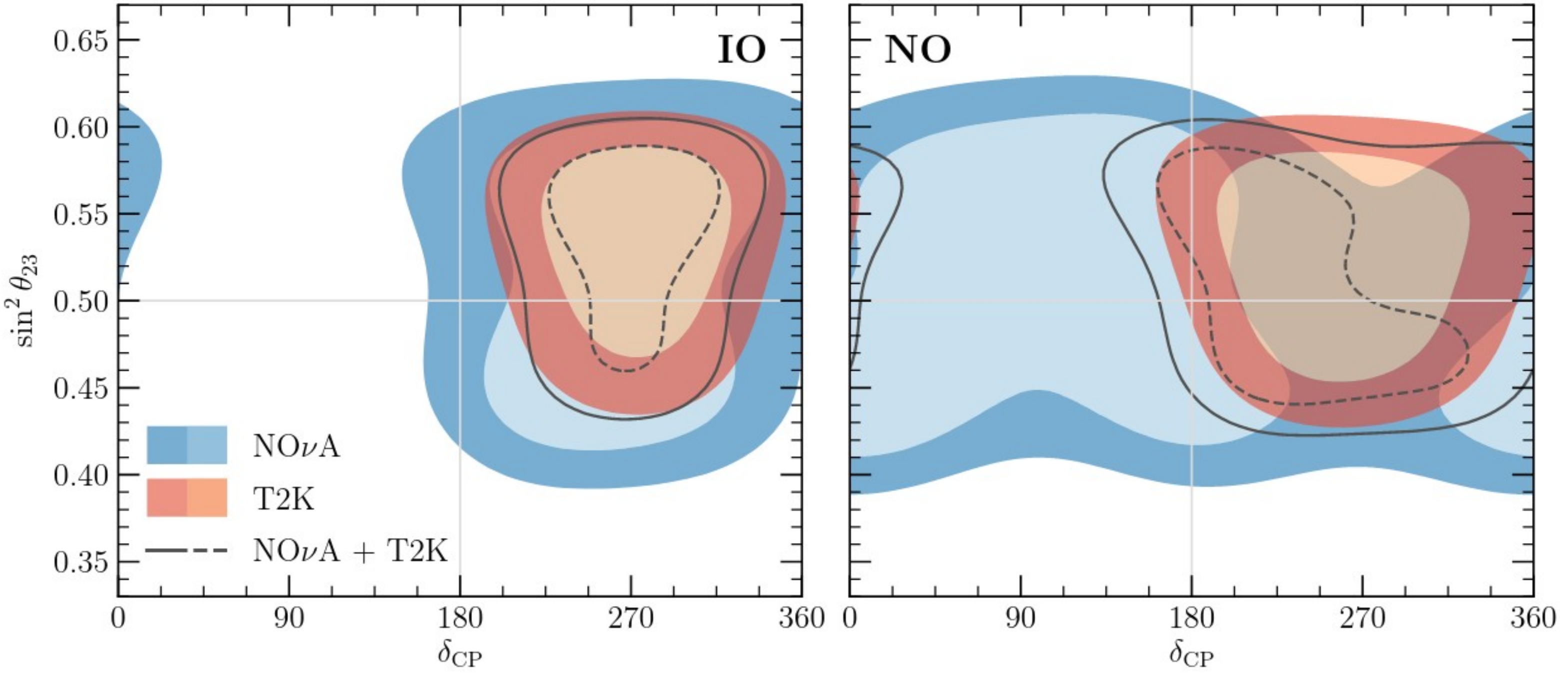
Image Courtesy: Symmetry Magazine

See Prof. Kopp’s lecture on Neutrino Theory and Phenomenology

Present status of MH and CPV

1) In NuFiT 5.0, IH is disfavored with $\Delta\chi^2$= 7.3 (2.7σ) compared to $\Delta\chi^2$= 10.4 (3.2σ) in NuFIT 4.1.

2) The best fit for the complex phase is at δ_{CP}= 195°. If we restrict to IH, the best fit of δ_{CP} remains close to maximal CP violation, with CP conservationbeing disfavored at around 3σ.



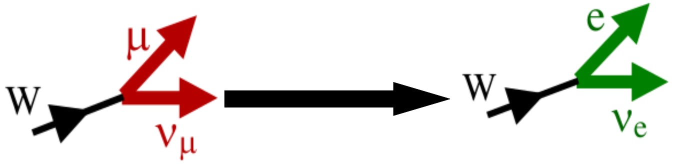
JHEP09(2020)178

Methodology

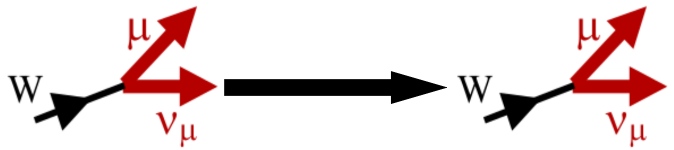
We simulate the statistical significance of --

- TWO accelerator-based Long Baseline (LBL) experiments,
  1) T2K-II : the extended run of the ongoing Tokai-To-Kamioka experiment based in Japan.
  2) NOνA-II: the extended run of NuMI Off-Axis Neutrino Appearance Experiment, based in the USA.
- ONE reactor-based Medium Baseline experiment,
  1) JUNO: Jiangmen Underground Neutrino Observatory, based in China.

Transition/Appearance Sample ($\boldsymbol{\nu}_\mu \longrightarrow \nu_e$):

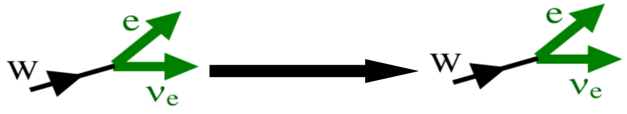


Survival/Disappearance Sample ($\boldsymbol{\nu}_\mu \longrightarrow \boldsymbol{\nu}_\mu$):



T2K-II/NO**ν**A-II

Survival/Disappearance Sample ($\bar{\boldsymbol{\nu}}_e \longrightarrow \bar{\boldsymbol{\nu}}_e$):

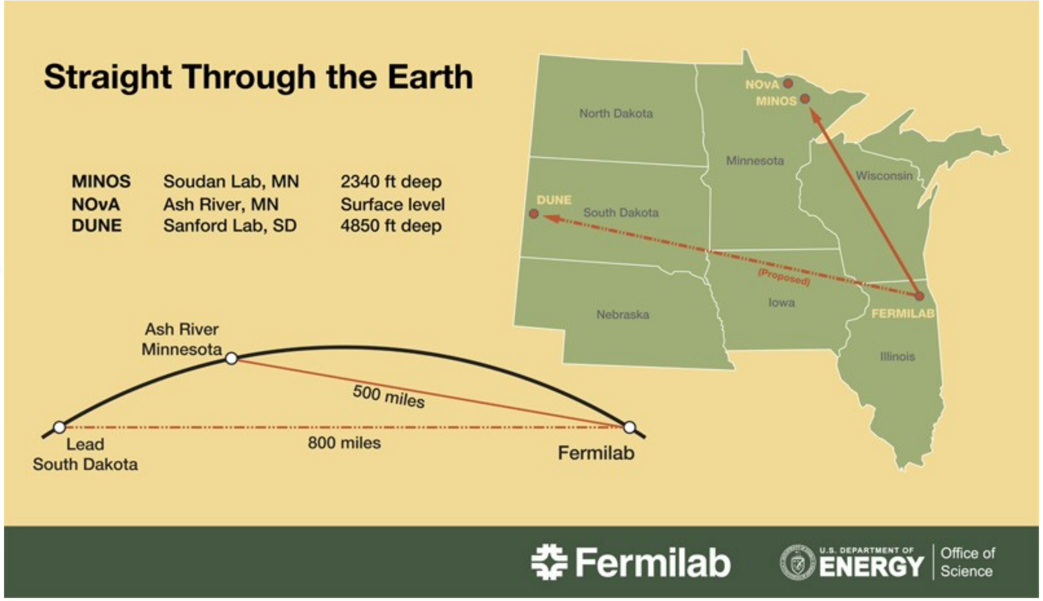


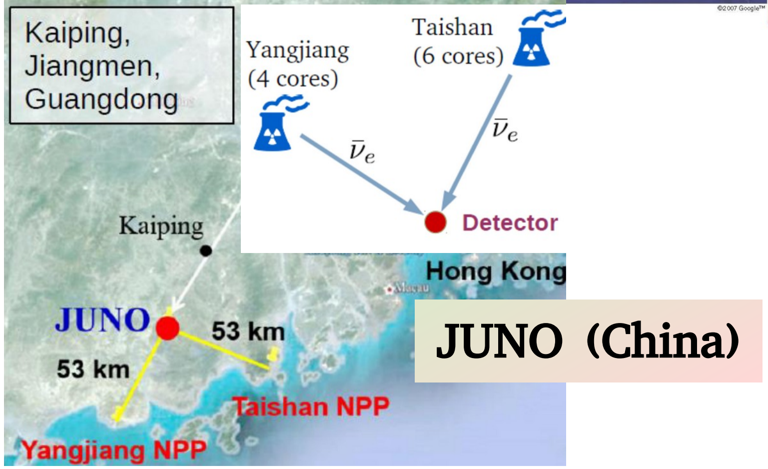
JUNO

General Long Baseline Experiment Simulator (GLoBES)* is used for simulating the experiments, calculating the statistical significance.

* *10.1016/j.cpc.2007.05.004*

Experiment Specifications

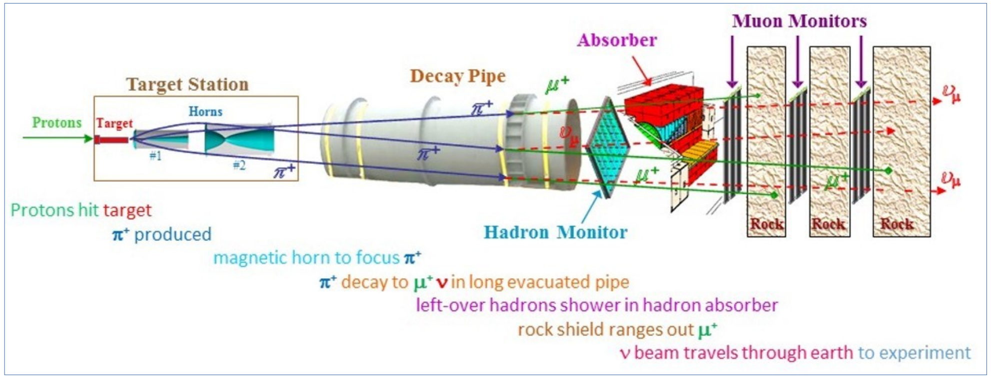




Parameters	T2K-II	NOνA	JUNO
Exposure (POT)	20×10^{21}	7.2×10^{21}	6 yrs. @ 36 GW-th
Baseline (km)	295	810	52.5
Energy peak/range	~0.6 GeV	~2.0 GeV	1-8 MeV
(Far) Det. Type	WC	LS	LS
(Far) Det. Mass	50 kt	14kt	20kt

$$\bar{\nu}_e + p \rightarrow e^+ + n \text{ (REACTOR)}$$

$$\begin{aligned} \pi^+ \rightarrow \quad & \mu^+ + \nu_\mu \text{ (ACCELERATOR)} \\ & \downarrow \\ & e^+ + \nu_e + \overline{\nu}_\mu \end{aligned}$$



Protons hit target
π^+ produced
magnetic horn to focus π^+
π^+ decay to $\mu^+ \nu$ in long evacuated pipe
left-over hadrons shower in hadron absorber
rock shield ranges out μ^+
ν beam travels through earth to experiment

Motivation

1) ν_μ ($\bar{\nu}_\mu$) disappearance provides a precise measurement of the atmospheric neutrino parameters, $\sin^2 2\theta_{23}$ and Δm^2_{31} ,
2) ν_e ($\bar{\nu}_e$) appearance rates are driven by $\sin^2 2\theta_{13}$ and are sensitive to δ_{CP} and the MH.
3) $\bar{\nu}_e$ disappearance is driven by both solar and atmospheric neutrino mass-squared splittings.

- The determination of MH and CPV in accelerator based long baseline neutrino experiments suffers due to the presence of degeneracies**[1]**.
  - CP degeneracy: (δ_{CP},θ_{13}) ambiguity
  - MH degeneracy: sgn(Δm^2_{31}) ambiguity
  - Octant degenracy: (θ_{23},$\pi/2$-θ_{23}) ambiguity
- This implies different sets of parameters giving equally good fit to the data.

[1] Phys. Rev. D 65, 073023 (2002)

Determination of MH and CPV depend on the ability to resolve the parameter degeneracies among δ_{CP}, the sign of Δm^2_{31}, θ_{13}, and θ_{23}.

- Combining the data samples of the experiments T2K-II, NoνA-II and JUNO would enhance the CPV search and the MH determination since the JUNO sensitivity to the MH has no ambiguity to δ_{CP}.
- To further enhance the CPV search, one can break the δ_{CP}-θ_{13} degeneracy by using the constraint of θ_{13} from reactor-based short-baseline neutrino experiments.

Results (MH)

To estimate quantitatively the sensitivity of the experiment(s) to the MH determination,

- we calculate the statistical significance $\sqrt{\Delta\chi^2}$ to exclude the inverted MH given the null hypothesis is a normal MH.
- The sensitivity is calculated as a function of true δ_{CP} since for the accelerator LBL experiments, the capability to determine the MH depends on the values of the CP-violating phase δ_{CP}.

The oscillation parameters are based on NuFit 4.1:

$\mathrm{Sin}^2\theta_{12}$ = 0.310,

Δm^2_{21} = $7.39\mathrm{x}10^{-5}$ eV^2,

$\mathrm{Sin}^2\theta_{23}$ = 0.5, 0.43 and 0.6,

$|\Delta m^2_{31}|$ = $2.523\mathrm{x}10^{-3}$ $\mathrm{eV}^{2,}$

$\mathrm{Sin}^2\theta_{13}$ = 0.02241

Results (MH)

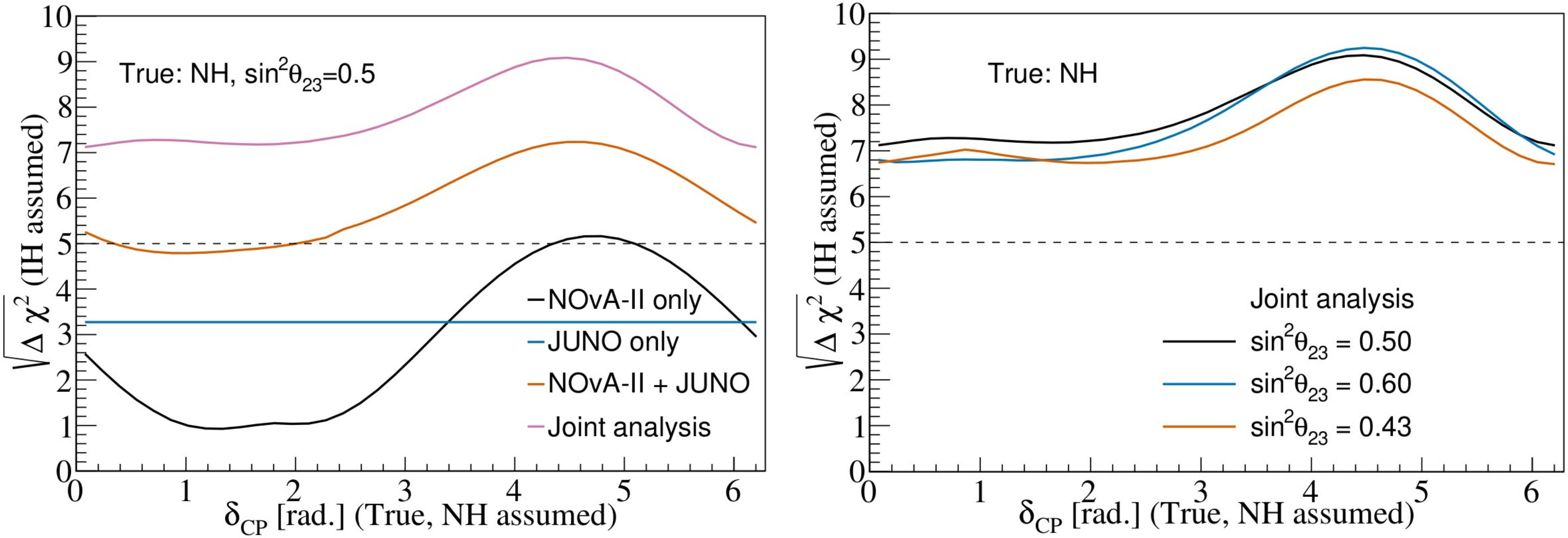


Figure 1: Mass hierarchy sensitivities as function of true $\boldsymbol{\delta}_{CP}$ for --

various experimental setups :

Left plot*

(i) JUNO only (at $\sin^2\theta_{23}$=0.5);

(ii) NOνA-II only (at $\sin^2\theta_{23}$=0.5);

(iii) a joint of JUNO and NOνA-II (at $\sin^2\theta_{23}$=0.5);

(iv) a joint of JUNO, NOνA-II, T2K-II and θ_{13} constraints from Reactor-Short Baseline (R-SBL) experiment (at $\sin^2\theta_{23}$=0.5);

Right plot*

A joint of JUNO, NOνA-II, T2K-II and θ_{13} constraints from R-SBL experiment (for different θ_{23}).

Methodology Contd. (CPV)

- The statistical significance $\sqrt{\Delta\chi^2}$ for excluding the CP-conserving values ($\delta_{CP} = 0,\pi$).
- For the minimization of χ^2 over the Mass Hierarchy (MH) options, we consider two cases:
  - → MH is known and normal as the truth value,
  - → MH is unknown.

Results (CPV)

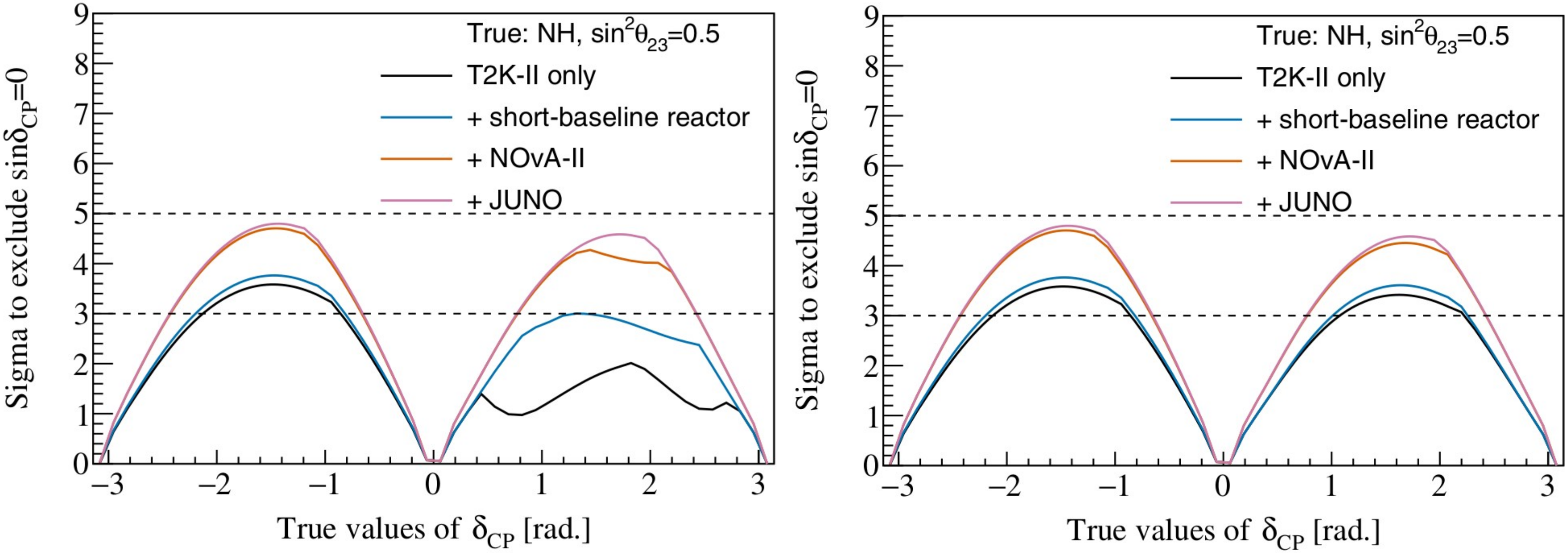


Figure 3: CP violation sensitivities as function of true $\boldsymbol{\delta}_{CP}$ for *various experimental setups* :

(i) T2K-II only (at $\sin^2\theta_{23}$=0.5);

(ii) a joint of T2K-II and NOνA-II (at $\sin^2\theta_{23}$=0.5);

(iii) a joint of JUNO, NOνA-II, T2K-II and θ_{13} constraints from Reactor-Short Baseline (R-SBL) experiment (at $\sin^2\theta_{23}$=0.5);

Left (Right) plot is with MH assumed to be unknown (known) in the analysis.

Results (CPV)

Value of $\sin^2\theta_{23}$	0.43	0.50	0.60
Fraction of true δ_{CP} values (%), NH	61.6	54.6	53.3
Fraction of true δ_{CP} values (%), IH	61.7	57.2	54.2

Table. Fractional region of δ_{CP}, depending on $\sin^2\theta_{23}$, can be explored with 3σ or higher significance.

Highlights

1) It is expected that the MH sensitivity of JUNO is more than 3σ C.L. and does not depend on δ_{CP}.

2) Resolving the MH by combining T2K-II, NOνA-II, and JUNO by 2027 is thus very encouraging, given the standard interaction of 3 active neutrinos is the reality.

3) With the combined analysis of T2K-II, NOνA-II, and JUNO, it is expected that more than half of the δ_{CP} values can be excluded with more than a 3σ C.L.

Thank you for your attention.

Results (MH)

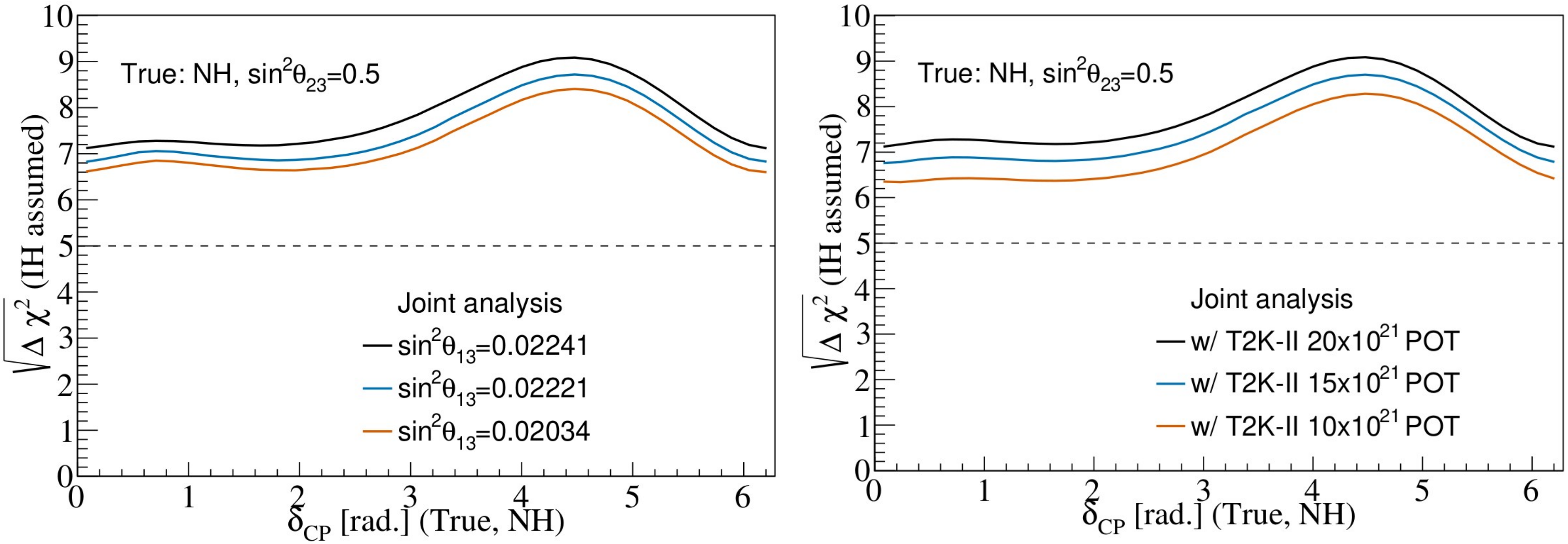


Figure 2. The effects of θ_{13} (left) and T2K runtime (right) on the sensitivity of neutrino mass hierarchy determination are studied. The plots use a combined sensitivity of all considering experiments. **$\sin^2\theta_{13}$= 0.02241** is the best fit obtained with NuFIT 4.1, $\sin^2\theta_{13}$= 0.02221 is with NuFIT 5.0. $\sin^2\theta_{13}$= 0.02034 is 3σ lower limit.

Results (CPV)

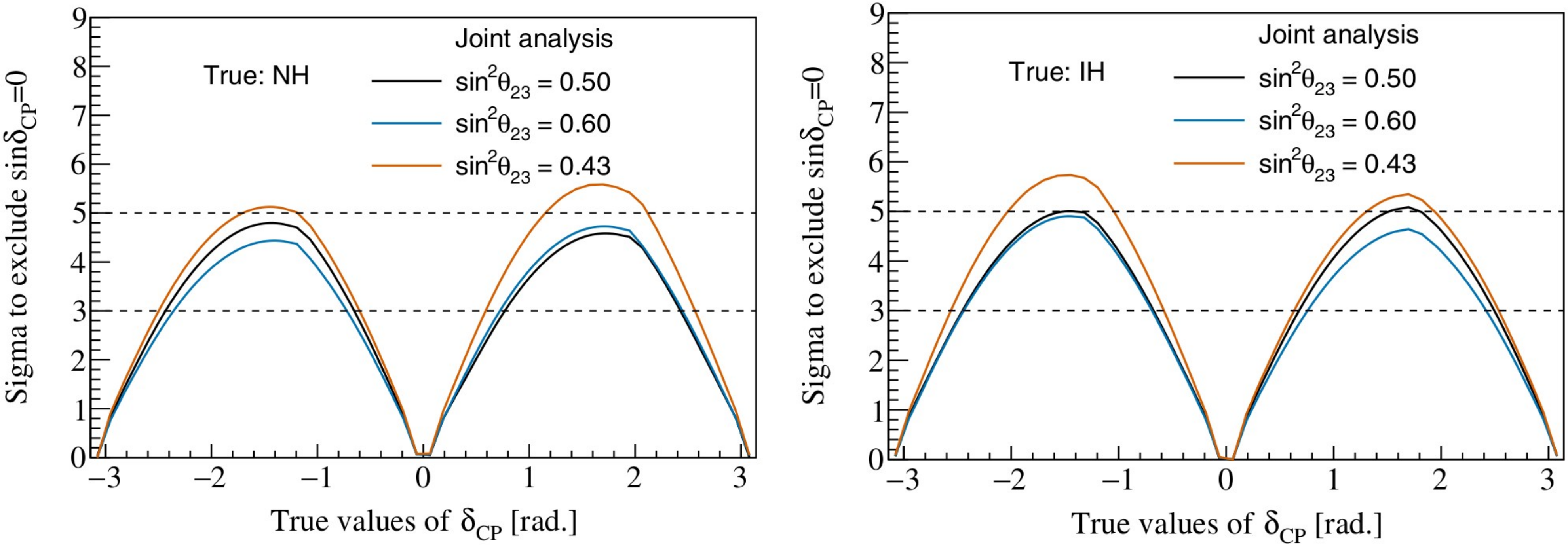


Figure 4: CPV sensitivity as a function of the true value of δ_{CP} obtained with a joint analysis of all considered experiments at different true $\sin^2\theta_{23}$ values (0.43, 0.5, 0.6). The left (right) plot is with the normal (inverted) MH, respectively.

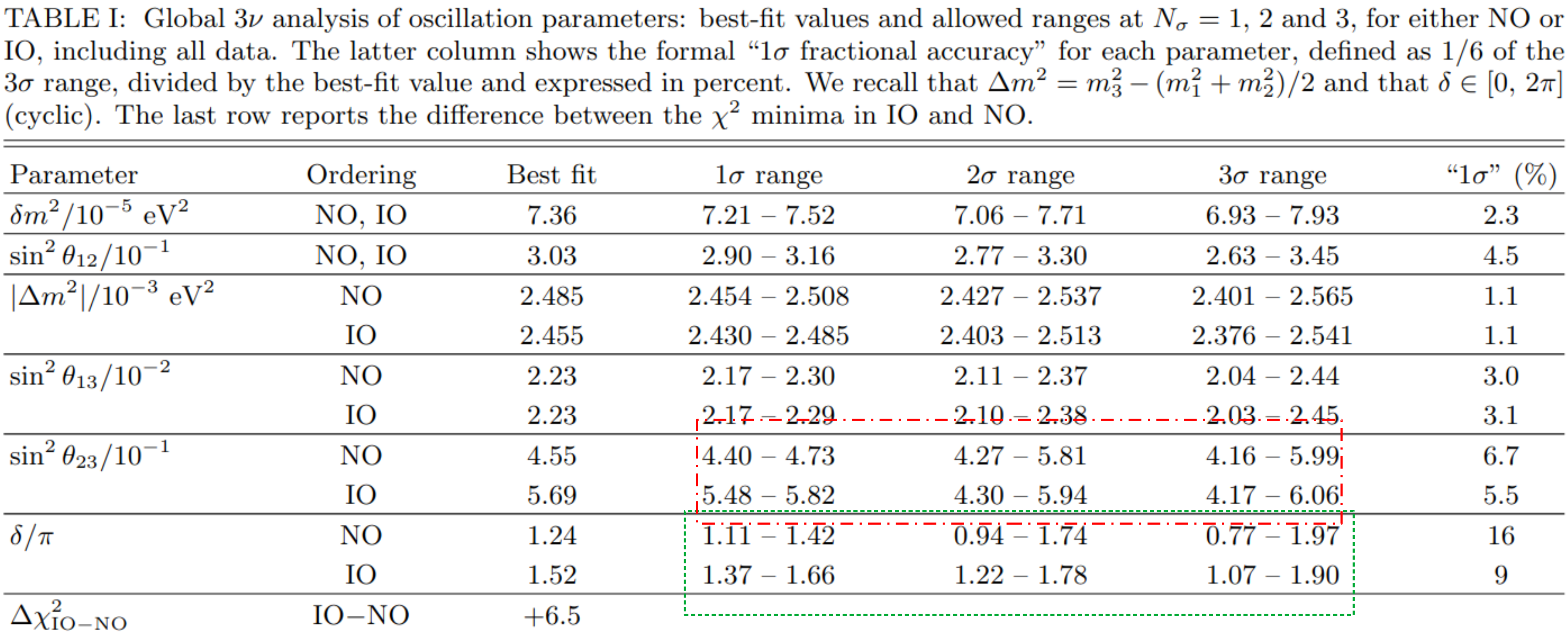


TABLE I: Global 3ν analysis of oscillation parameters: best-fit values and allowed ranges at $N_\sigma = 1$, 2 and 3, for either NO or IO, including all data. The latter column shows the formal "1σ fractional accuracy" for each parameter, defined as 1/6 of the 3σ range, divided by the best-fit value and expressed in percent. We recall that $\Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$ and that $\delta \in [0, 2\pi]$ (cyclic). The last row reports the difference between the χ^2 minima in IO and NO.

Parameter	Ordering	Best fit	1σ range	2σ range	3σ range	"1σ" (%)
$\delta m^2/10^{-5}$ eV^2	NO, IO	7.36	7.21 – 7.52	7.06 – 7.71	6.93 – 7.93	2.3
$\sin^2\theta_{12}/10^{-1}$	NO, IO	3.03	2.90 – 3.16	2.77 – 3.30	2.63 – 3.45	4.5
$\lvert\Delta m^2\rvert/10^{-3}$ eV^2	NO	2.485	2.454 – 2.508	2.427 – 2.537	2.401 – 2.565	1.1
	IO	2.455	2.430 – 2.485	2.403 – 2.513	2.376 – 2.541	1.1
$\sin^2\theta_{13}/10^{-2}$	NO	2.23	2.17 – 2.30	2.11 – 2.37	2.04 – 2.44	3.0
	IO	2.23	2.17 – 2.29	2.10 – 2.38	2.03 – 2.45	3.1
$\sin^2\theta_{23}/10^{-1}$	NO	4.55	4.40 – 4.73	4.27 – 5.81	4.16 – 5.99	6.7
	IO	5.69	5.48 – 5.82	4.30 – 5.94	4.17 – 6.06	5.5
δ/π	NO	1.24	1.11 – 1.42	0.94 – 1.74	0.77 – 1.97	16
	IO	1.52	1.37 – 1.66	1.22 – 1.78	1.07 – 1.90	9
$\Delta\chi^2_{\mathrm{IO-NO}}$	IO−NO	+6.5				

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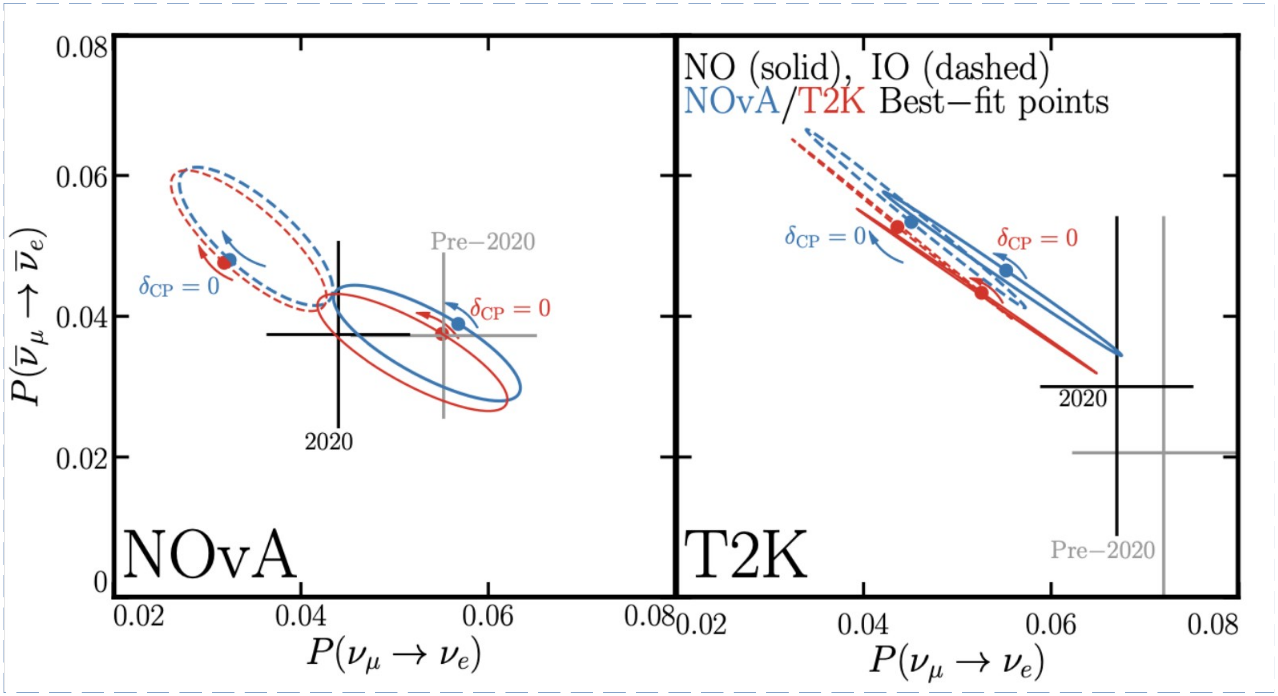


Extra Slides

- For JUNO, the mass hierarchy resolving is less sensitive to the truth of mass hierarchy since the

  dominate factor is the separation power between two oscillation frequencies driven by $|\Delta m^2_{31}|$ and

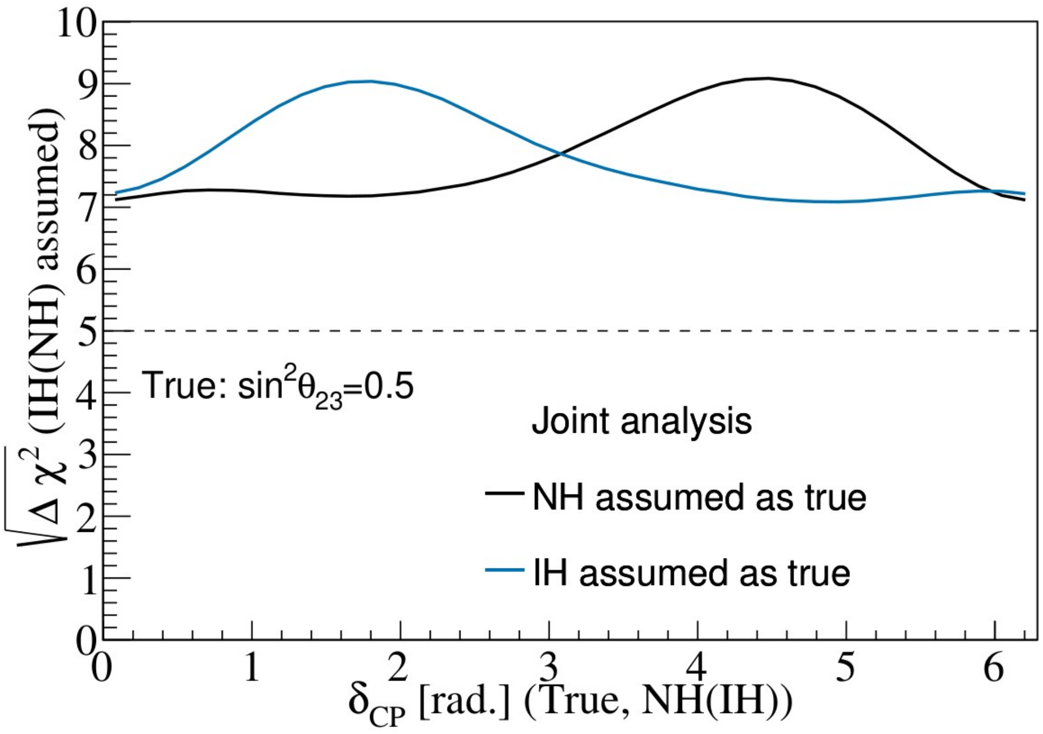
  $|\Delta m^2_{32}|$.

- The mass hierarchy sensitivity with the accelerator-based LBL neutrino experiments such as T2K-II and NOνA-II comes from the matter effect in the appearance $\nu_\mu \to \nu_e$ ($\bar{\nu}_\mu \to \bar{\nu}_e$) probabilities. These probabilities depends on the mixing angle θ_{13} and δ_{CP} .

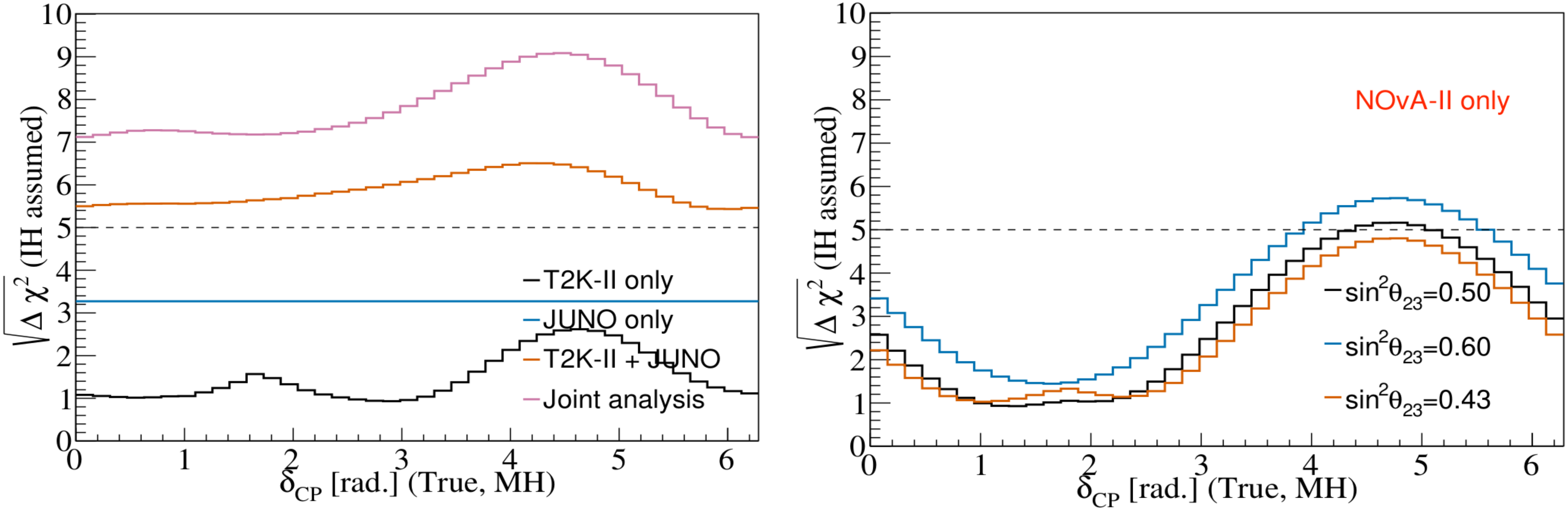
- For relation between mass hierarchy sensitivity and CP phase, the appearance probability has a CP term which is proportional to

$$\cos[(\Delta m^2_{31} L/4E)+\text{sign}_\nu \delta_{CP}]$$

  where sign_ν = +1 for neutrinos and = -1 for anti-neutrinos. This leads to a dependence of mass hierarchy resolving as function of δ_{CP}.

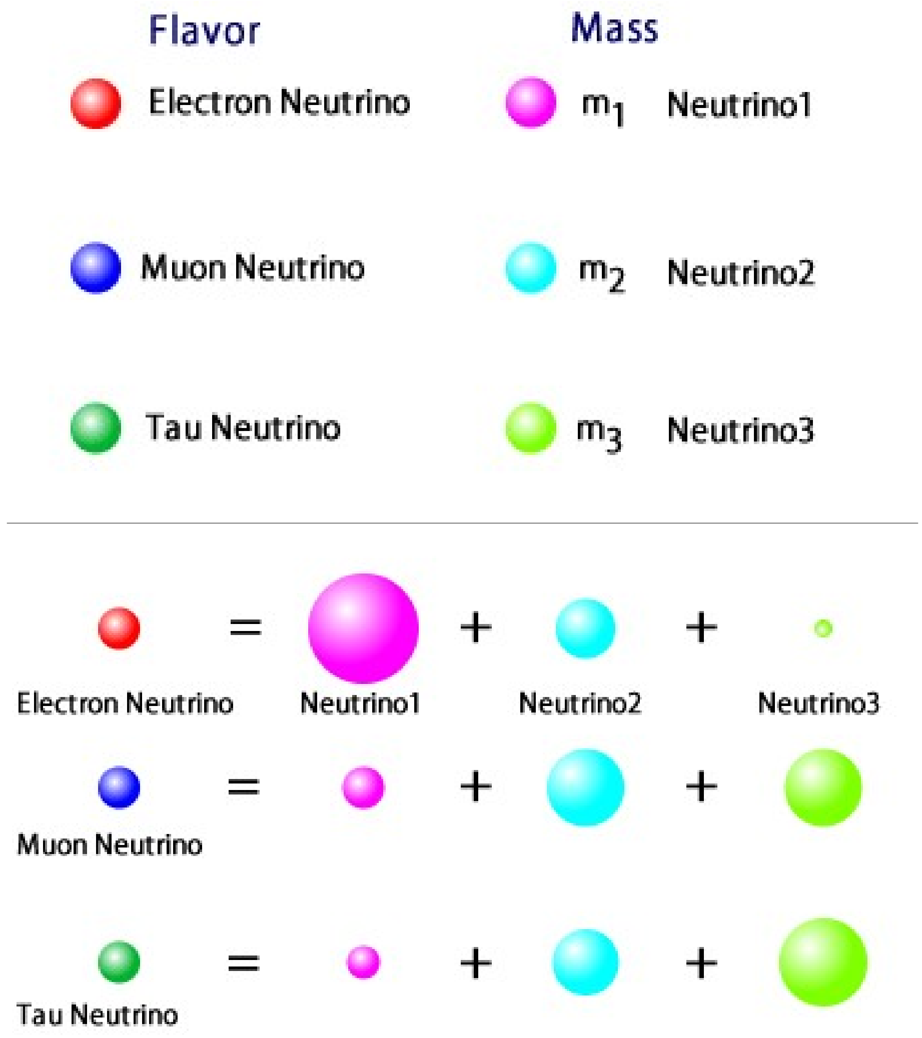


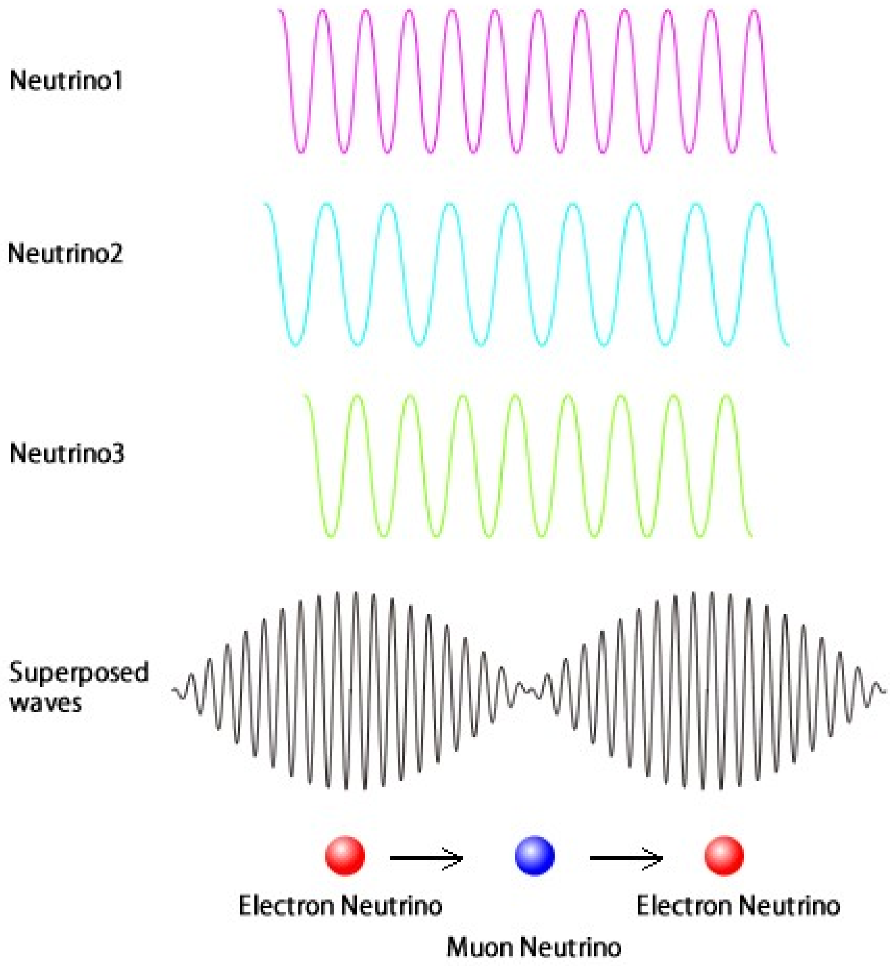
Results (MH)



1) **Figure 3.** Same as Figure 1, but showing indiviual sensitivity of T2K-II (left) and that of NOνA-II for both octants and maximal mixing of θ_{23}(right).

A sketch of Neutrino Oscillation





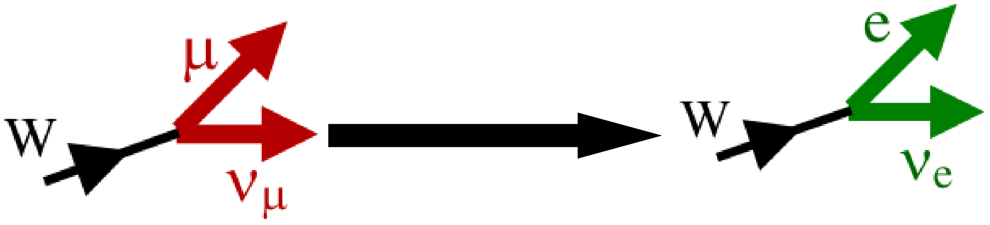
The oscillation probability of neutrinos (anti-neutrinos) in vacuum is:

$$P\left(\overset{(-)}{\nu}_{\alpha} \rightarrow \overset{(-)}{\nu}_{\beta}\right) =$$

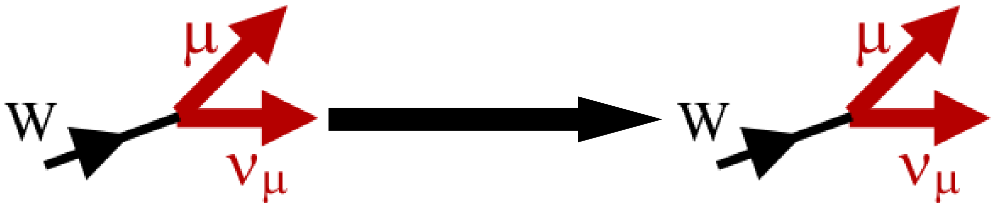
$$= \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left(U^{*}_{\alpha i} U_{\beta i} U_{\alpha j} U^{*}_{\beta j}\right) \sin^{2}\left(\Delta m^{2}_{ij} \frac{L}{4E}\right)$$

$$\underset{(-)}{+} 2\sum_{i>j} \mathrm{Im}\left(U^{*}_{\alpha i} U_{\beta i} U_{\alpha j} U^{*}_{\beta j}\right) \sin\left(\Delta m^{2}_{ij} \frac{L}{2E}\right)$$

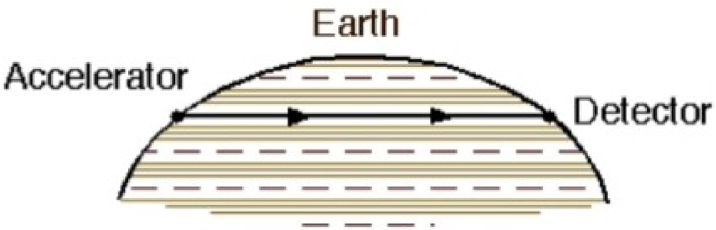
Transition/Appearance Probability :



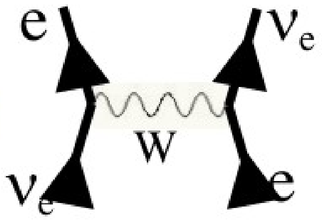
Survival/Disappearance Probability :



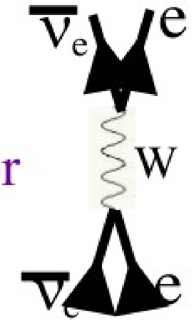
Neutrino Oscillation (matter)



involves



or



Coherent forward scattering via this W-exchange interaction leads to an extra interaction potential energy —

$$V_W = \begin{cases} +\sqrt{2} G_F N_e, & \nu_e \\ -\sqrt{2} G_F N_e, & \overline{\nu_e} \end{cases}$$

Fermi constant ————— Electron density

Oscillation Probabiility in matter

$$
\begin{aligned}
P(\nu_\mu \to \nu_e) = 4c_{13}^2 s_{13}^2 s_{23}^2 \cdot \sin^2 \Delta_{31} \\
&+8c_{13}^2 s_{12} s_{13} s_{23} (c_{12} c_{23} \cos\delta_{CP} - s_{12} s_{13} s_{23}) \cdot \cos\Delta_{32} \cdot \sin\Delta_{31} \cdot \sin\Delta_{21} \\
&-8c_{13}^2 c_{12} c_{23} s_{12} s_{13} s_{23} \sin\delta_{CP} \cdot \sin\Delta_{32} \cdot \sin\Delta_{31} \cdot \sin\Delta_{21} \\
&+4s_{12}^2 c_{13}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - 2c_{12} c_{23} s_{12} s_{23} s_{13} \cos\delta_{CP}) \cdot \sin^2\Delta_{21} \\
&-8c_{13}^2 s_{13}^2 s_{23}^2 \cdot \frac{aL}{4E_\nu} (1 - 2s_{13}^2) \cdot \cos\Delta_{32} \cdot \sin\Delta_{31} \\
&+8c_{13}^2 s_{13}^2 s_{23}^2 \frac{a}{\Delta m_{31}^2} (1 - 2s_{13}^2) \cdot \sin^2\Delta_{31},
\end{aligned}
$$

$\Delta_{ij} = \Delta m^2_{ij}\, L/4E_\nu$

Matter Term, $a = 2\sqrt{2} G_F n_e E_\nu = 7.56\times10^{-5}[\mathrm{eV}^2]\times\rho[\mathrm{g/cm}^3]\times E_\nu[\mathrm{GeV}]$

The corresponding probability for a anti-($\nu_\mu \to \nu_e$) transition is obtained by:

replacing **$\delta_{CP} \to -\delta_{CP}$** and **$a \to -a$**

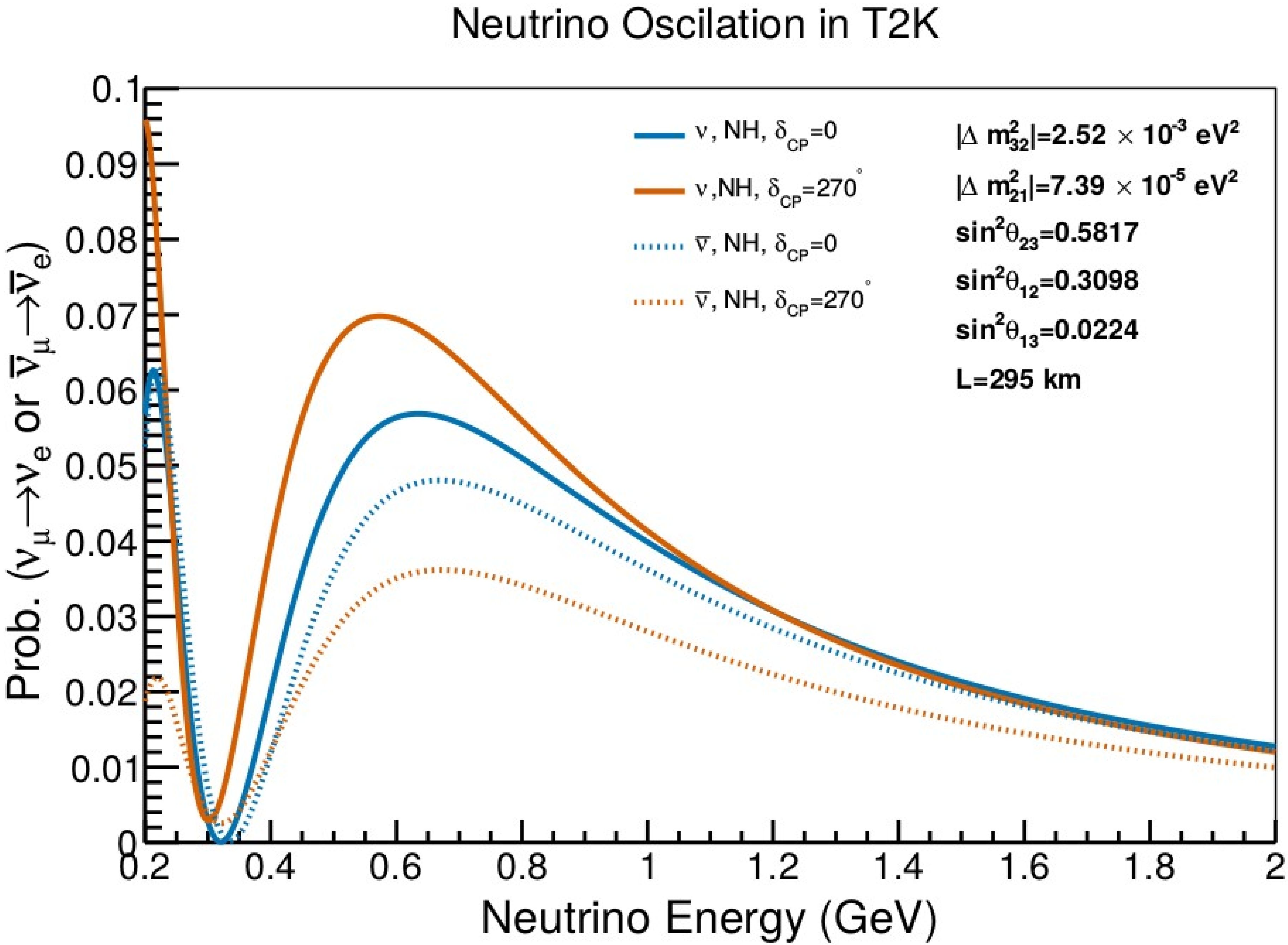
Oscillation Probabiility in matter

$$\begin{aligned} P(\nu_\mu \to \nu_e) = \; & 4c_{13}^2 s_{13}^2 s_{23}^2 \cdot \sin^2\Delta_{31} \\ & \boxed{+8c_{13}^2 s_{12} s_{13} s_{23} (c_{12} c_{23} \cos\delta_{CP}} - s_{12} s_{13} s_{23}) \cdot \cos\Delta_{32} \cdot \sin\Delta_{31} \cdot \sin\Delta_{21} \\ & \boxed{-8c_{13}^2 c_{12} c_{23} s_{12} s_{13} s_{23} \sin\delta_{CP} \cdot \sin\Delta_{32} \cdot \sin\Delta_{31} \cdot \sin\Delta_{21}} \\ & +4s_{12}^2 c_{13}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - \boxed{2c_{12} c_{23} s_{12} s_{23} s_{13} \cos\delta_{CP}) \cdot \sin^2\Delta_{21}} \\ & \boxed{\begin{aligned} & -8c_{13}^2 s_{13}^2 s_{23}^2 \cdot \frac{aL}{4E_\nu}(1-2s_{13}^2) \cdot \cos\Delta_{32} \cdot \sin\Delta_{31} \\ & +8c_{13}^2 s_{13}^2 s_{23}^2 \frac{a}{\Delta m_{31}^2}(1-2s_{13}^2) \cdot \sin^2\Delta_{31}, \end{aligned}} \end{aligned}$$

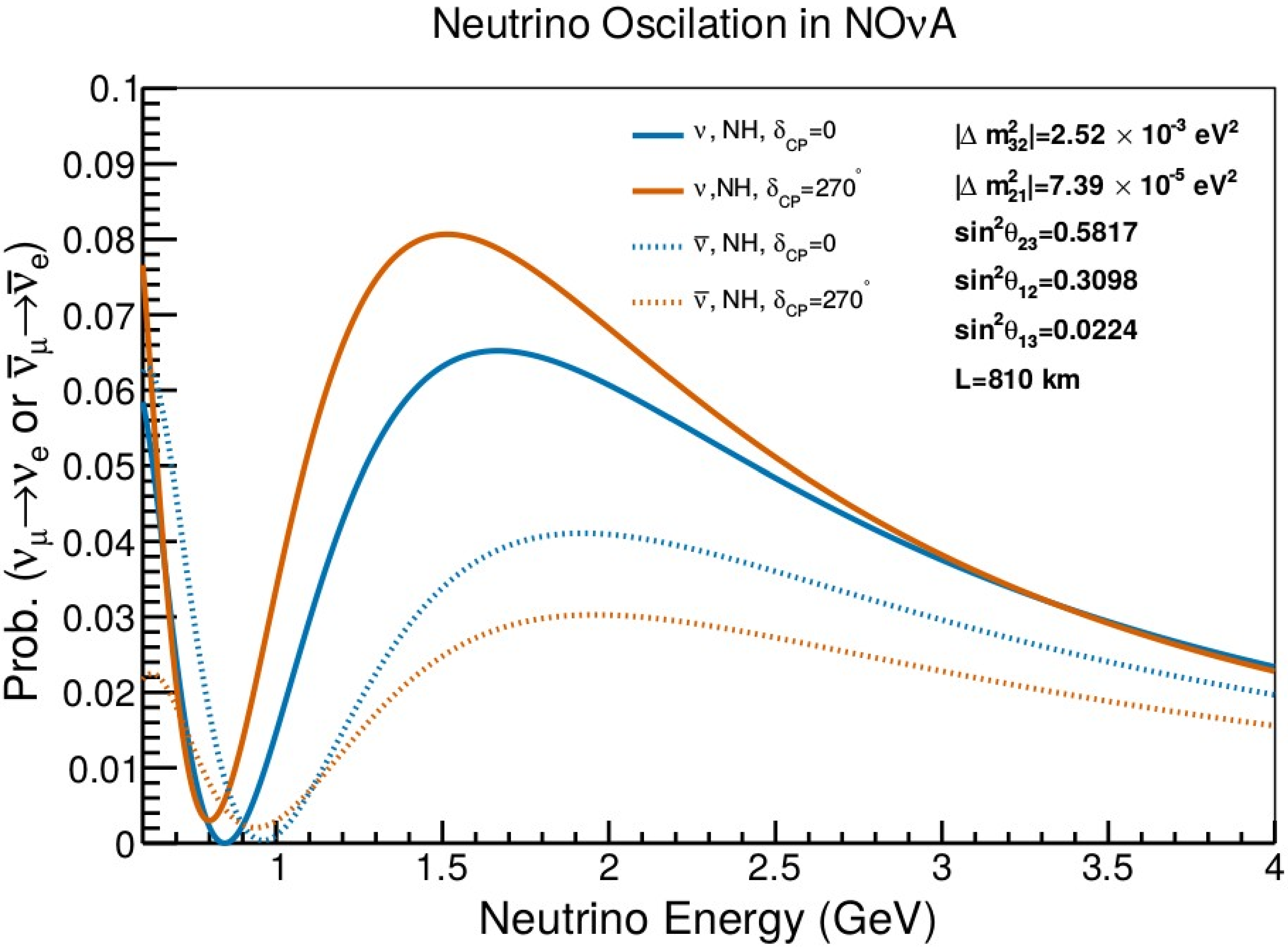
🟥 = CP-even terms 🟩 = CP-odd term 🟦 = due to matter effects

- In the leptonic mixing, CP symmetry is violated by the phase $\delta_{\text{CP.}}$
- From the above expression, δ_{CP}=0 also produces CP asymmetry due to CP-even terms. It is because the matter effect produces a fake asymmetry *(as the Earth is composed of e^- , p^+ & n, not their anti-particle)*.
- It is, therefore, important to experimentally separate the effects of the Earth matter and natural CP-violation. This will allow to get information about the dirac CP violation phase in U_{PMNS}.

T2K (Tokai to Kamioka, Japan)



NOνA (NuMI Off-Axis Neutrino Appearance, USA)



Event Spectra (Appearance, T2K-II)

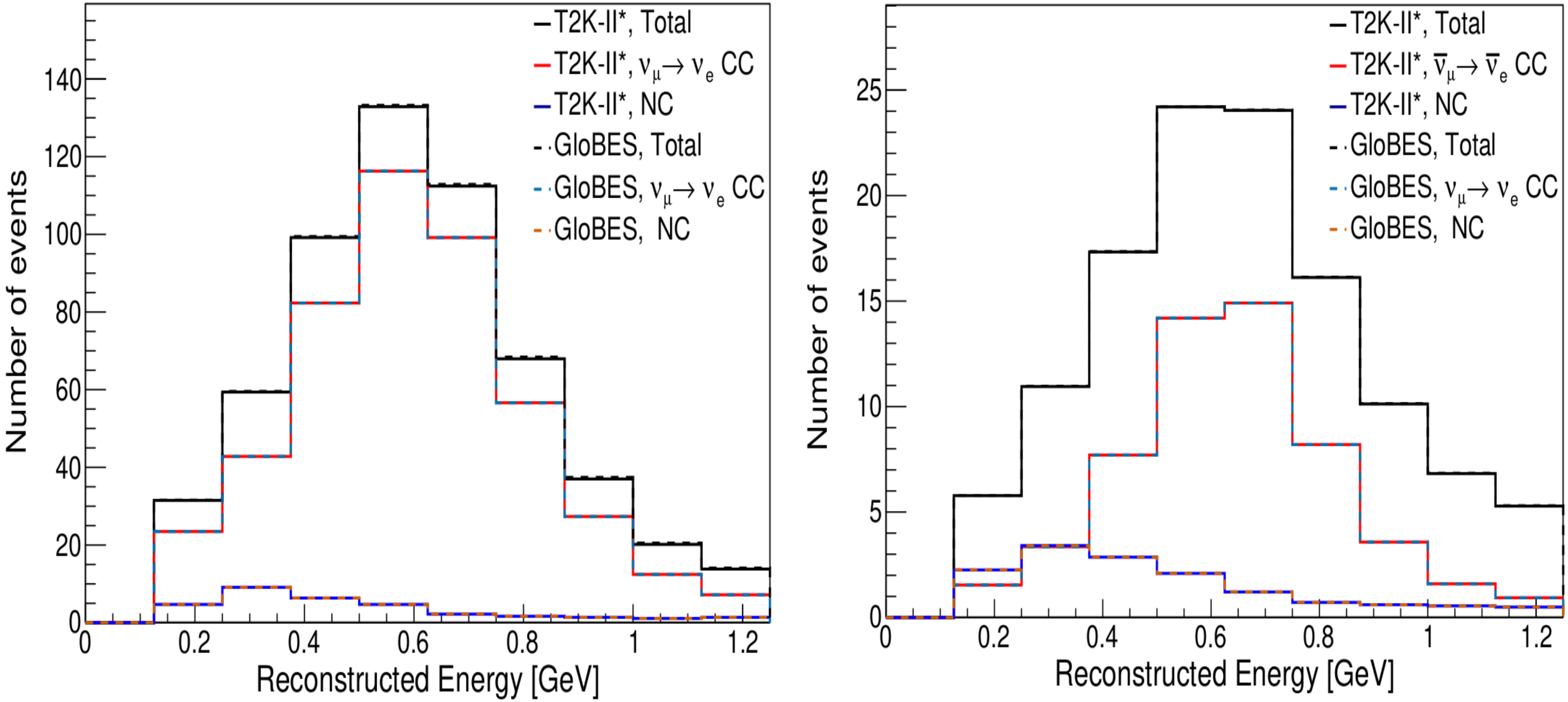


Figure 1. *Left* : ν-mode, *Right*: anti-ν mode. Oscillation parameters as in [2]

[2] PRD 96 (2017) 092006 [1707.01048]

Event Spectra (Disappearance, T2K-II)

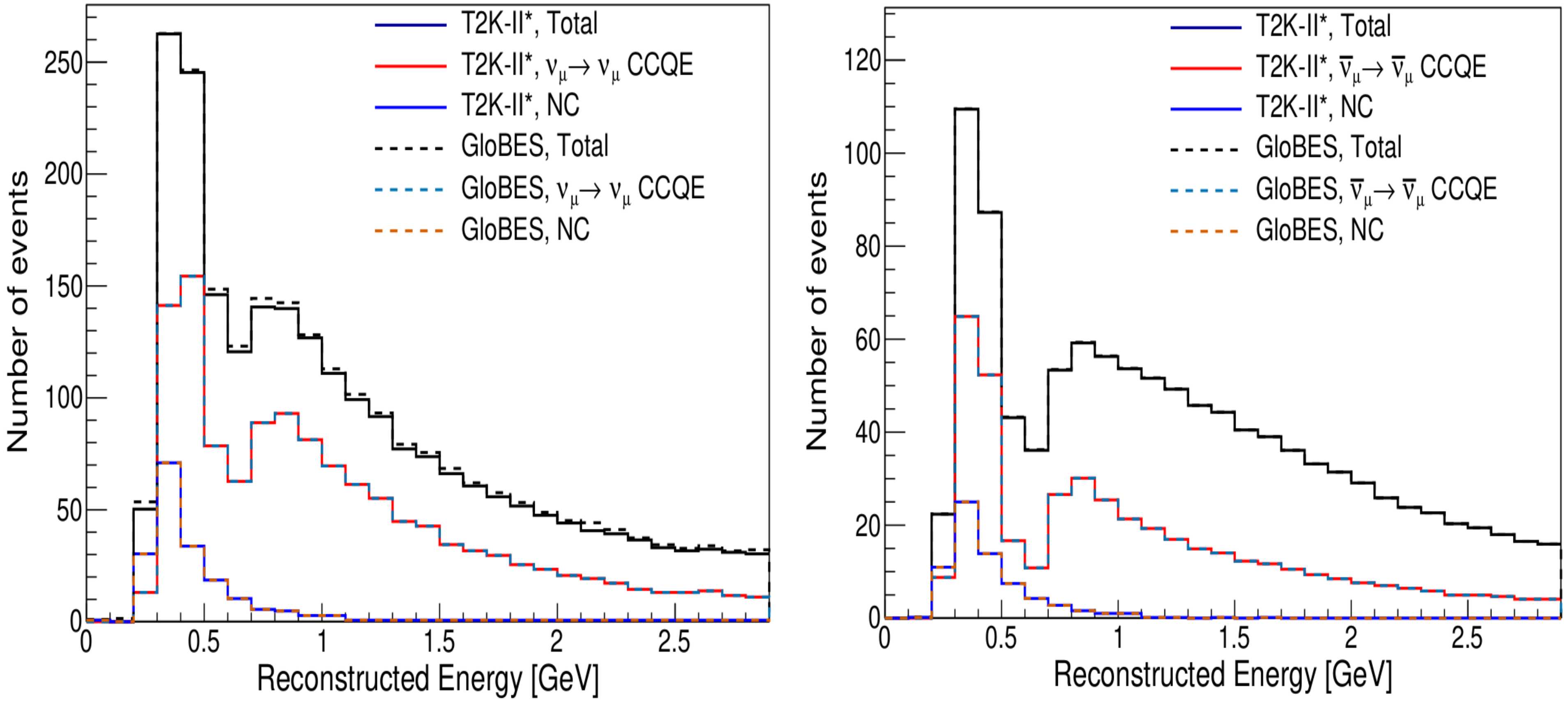


Figure 2. *Left* : ν-mode, *Right*: anti-ν mode. Oscillation parameters as in [2]

Event Spectra (Appearance, NOvA-II)

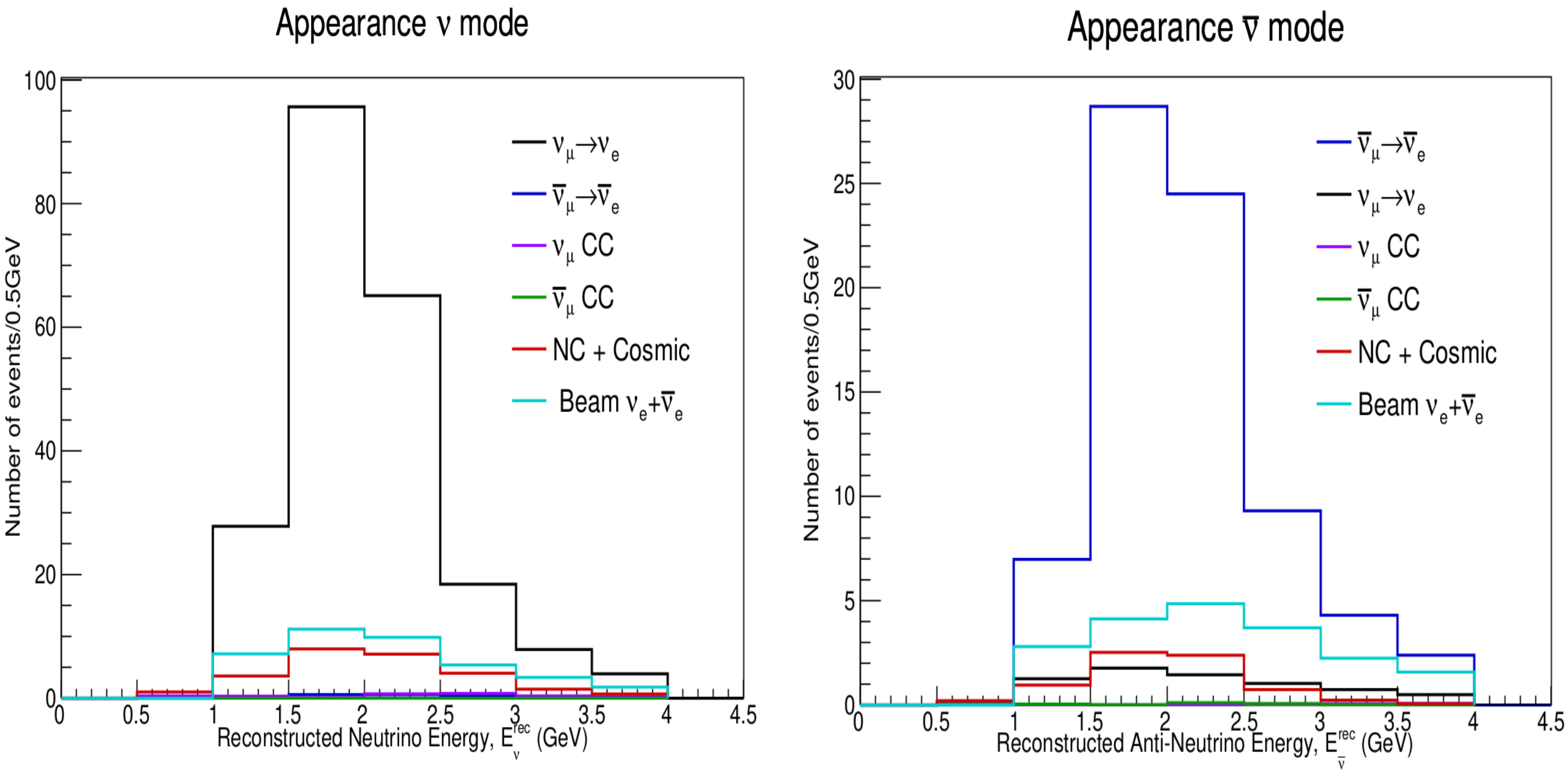


Figure 3. *Left* : ν-mode, *Right*: anti-ν mode.

Oscillation parameters as in Slide 10, with normal MH and **δ_{CP}=0**

Event Rates (Disappearance, NOvA-II)

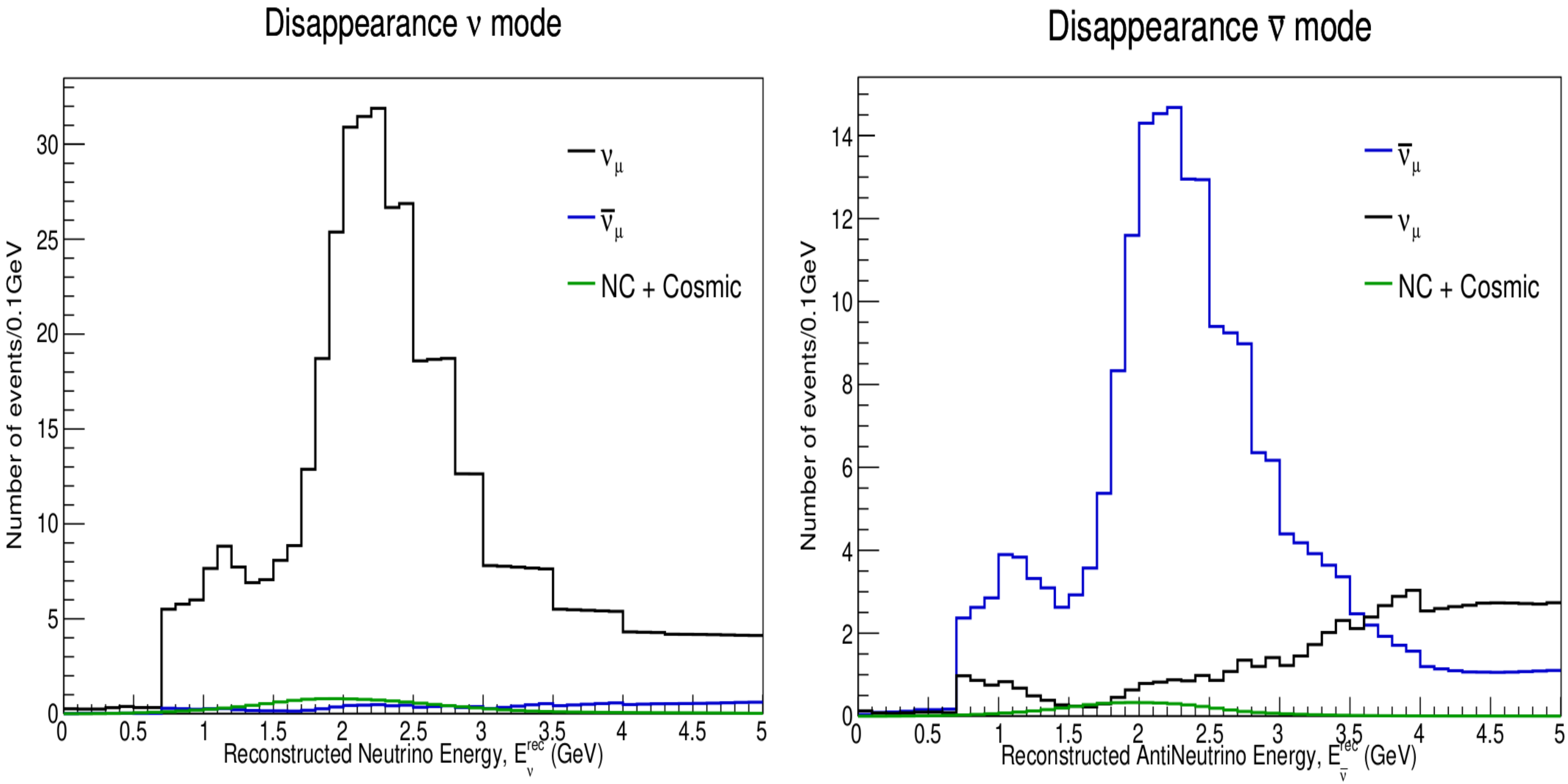
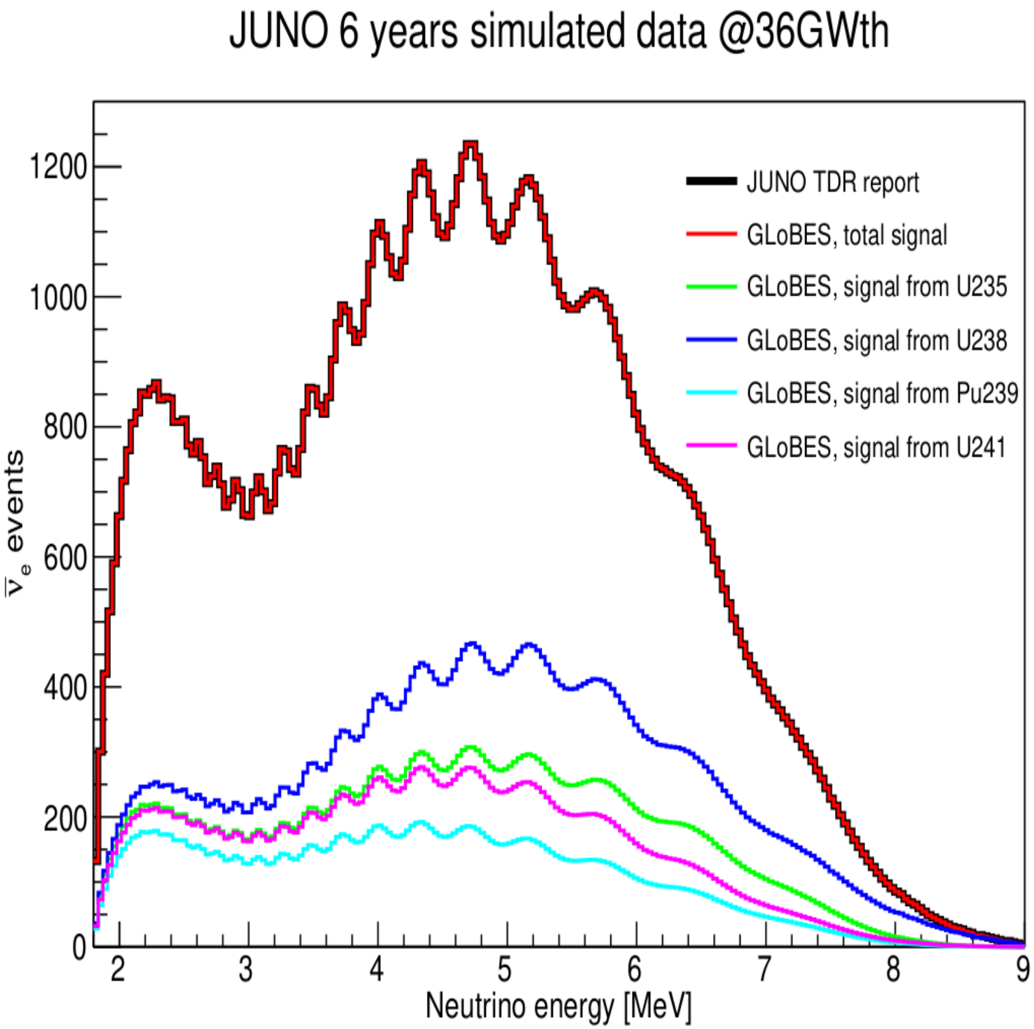


Figure 4. *Left* : ν-mode, *Right*: anti-ν mode.

Oscillation parameters as in Slide 10, with normal MH and **δ_{CP}=0**

Event Spectra (JUNO)



Characteristics	Inputs
Baseline	52.5 km
Density	2.8 gcc^{-1} [4]
Detector type	Liquid Scintillator
Detector mass	20 kton
$\bar{\nu}_e$ Detection Efficiency	73%
Running time	6 years
Thermal power	36 GW
Energy resolution	3% / $\sqrt{E}$ (MeV)
Energy window	1.8-9 MeV
Number of bins	200

Table II. JUNO specifications

Figure 5. Event rates calculated at oscillation parameters as in [3].

[3] 1508.07166 [4] PLB 803 (2020) 135354

$$P_{(\bar{\nu}_e \to \bar{\nu}_e)} = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Phi_{21} - \sin^2 2\theta_{13}\left(\cos^2\theta_{12}\sin^2\Phi_{31} + \sin^2\theta_{12}\sin^2\Phi_{32}\right)$$