

# Cosmology & Neutrinos

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HIDDe 

Hunting Invisibles: Dark sectors, Dark matter and Neutrinos



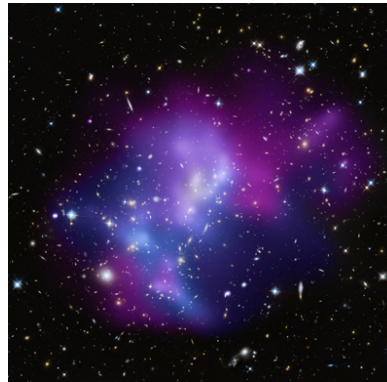
What is the main purpose of **Cosmology**?  
To study the evolution and structure of **the large scales in our universe**



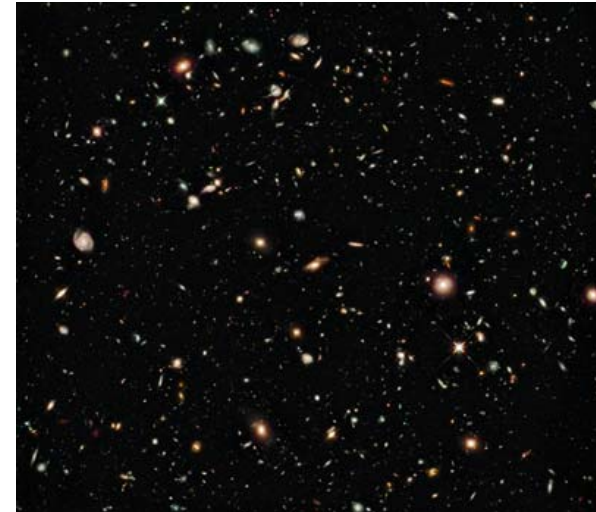
SUN  
 $2 \cdot 10^{33} \text{ g}$   
 $7 \cdot 10^{10} \text{ cm}$



Galaxies  
 $2 \cdot 10^{44} \text{ g}$   
 $10 \text{ kpc} = 3 \cdot 10^{22} \text{ cm}$



Galaxy Clusters  
 $2 \cdot 10^{47} \text{ g}$   
 $\sim \text{Mpc} = 10^{25} \text{ cm}$



The universe: our "Hubble volume"  
 $8 \cdot 10^{55} \text{ g}$   
 $3000 \text{ Mpc} = 10^{28} \text{ cm}$

What is a parsec (parallax of one arcsecond)?

A parsec (parallax of one arcsecond) is a length measure commonly used in astrophysics and cosmology. A parsec was defined as the distance at which one astronomical unit subtends an angle of one arc-second.

1 AU =  $1.5 \times 10^8$  m

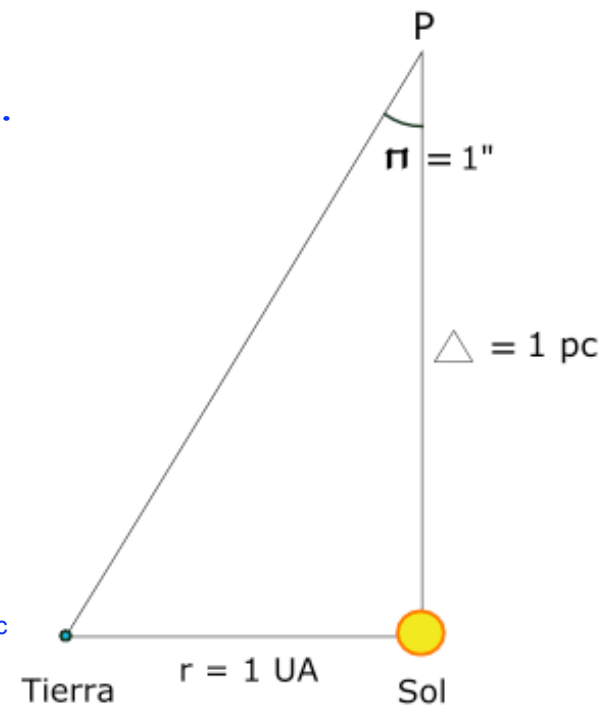
1 pc =  $3.08 \times 10^{16}$  m (3.26 light years)

A parsec amounts to go and come back from the Sun.....

### reminder of some scales

100 000 times!

- keep in mind some rough scales when considering galaxies:
  - Sun's distance from centre of Galaxy: ~ 8 kpc
  - diameter of Galaxy: ~ 30 kpc
  - nearest (non-satellite) galaxies: ~750 kpc
  - sizes of groups and clusters: 1-3 Mpc
  - nearest rich clusters: 20-100 Mpc
  - sizes of 'walls' and large-scale structure: 100's Mpc



1. INTRODUCTION: FUNDAMENTAL INGREDIENTS & THERMAL HISTORY OF THE UNIVERSE

2. NEUTRINO DECOUPLING IN THE EARLY UNIVERSE

3. BIG BANG NUCLEOSYNTHESIS &  $N_{\text{eff}}$

4. COSMOLOGY &  $N_{\text{eff}}$

5. COSMOLOGY & NEUTRINO MASSES

6. TAKE HOME MESSAGES

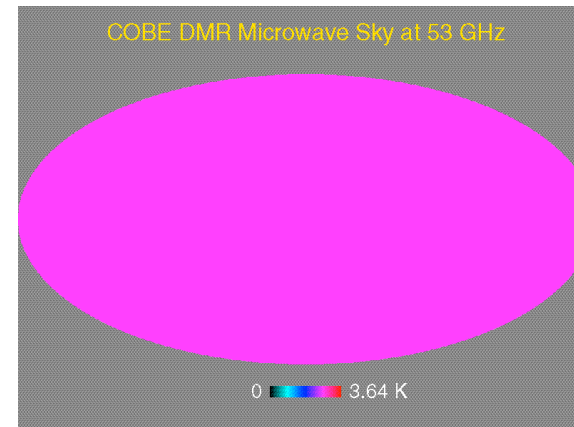
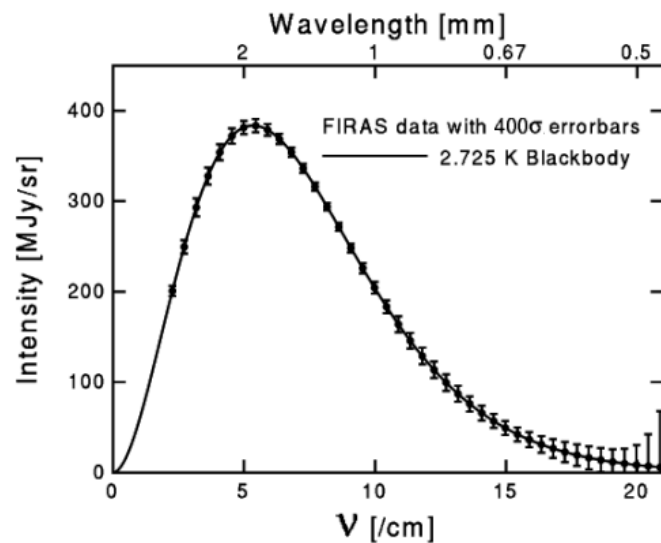
Standard Cosmology refers to FLRW Cosmology  
(FRIEDMANN LEMAITRE ROBERTSON WALKER)  
and it is based on two basic elements:

- FLRW Geometry (i.e. the metric, which determines the geodesics)
- FLRW Dynamics (Friedmann Equations, which determine the curvature of the space-time)

# FLRW GEOMETRY

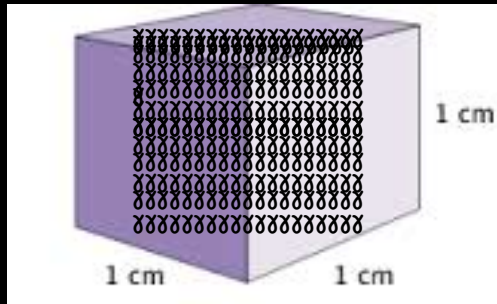
The FLRW geometry assumes that at large scales the universe is homogeneous and isotropic.

**The most robust confirmation of the isotropy of the universe** at large scales is provided by the **CMB**, the **Cosmic Microwave Background radiation** (Penzias & Wilson'64). When one measures the sky temperature in any direction, one notices that the photons have a thermal black body spectrum with a temperature of 2.725 K. This has been measured with high accuracy by the spectrophotometer FIRAS on the NASA COBE satellite. There are small fluctuations in the temperature across the sky at the level of about 1 part in 100,000  $\sim(10^{-5})$



The existence of a **CMB**, that is, a relic photon bath, was predicted by **Alpher & Herman** in 1948 while working on **BBN**. **Penzias & Wilson**, in 1965, discovered accidentally the **CMB** while working with a very sensitive radio telescope at **Bell Labs** in **New Jersey**. In 1978, **Penzias and Wilson** were awarded the **Nobel Prize for Physics** for their joint discovery of the **CMB**.

410 photons/cm<sup>3</sup>

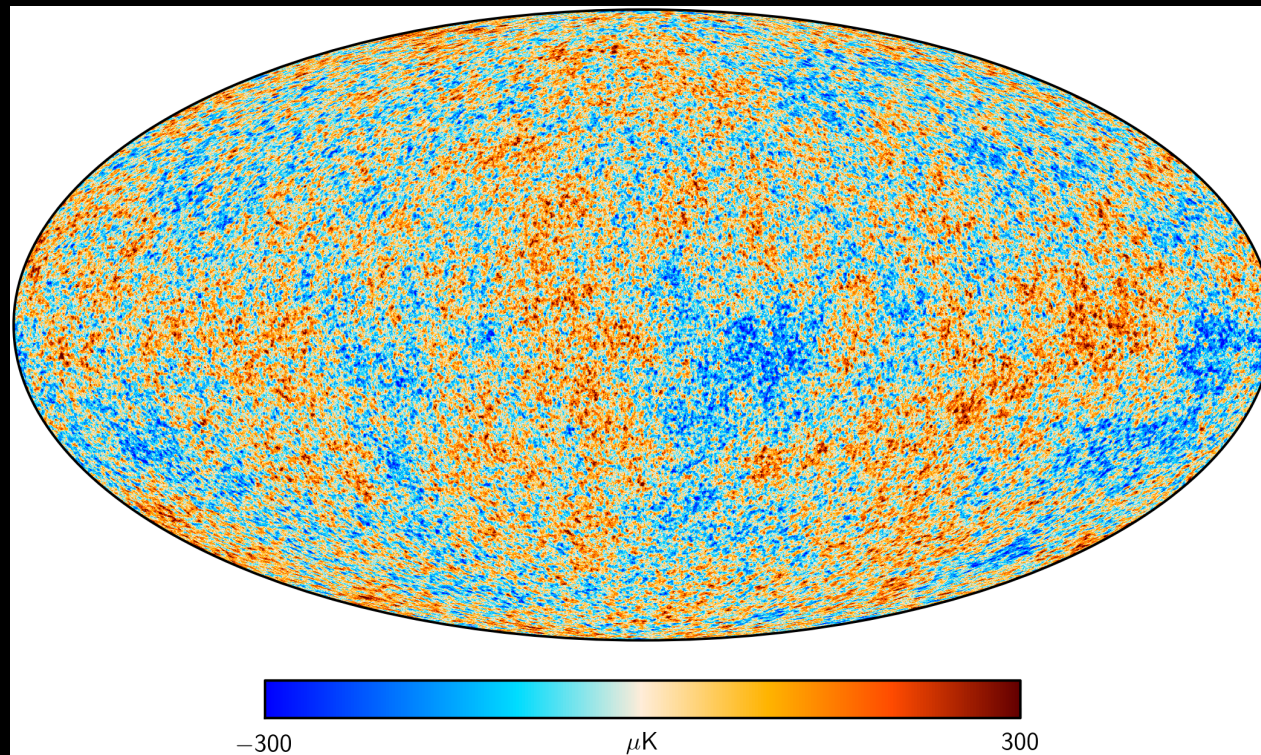


6720 photons/in<sup>3</sup>



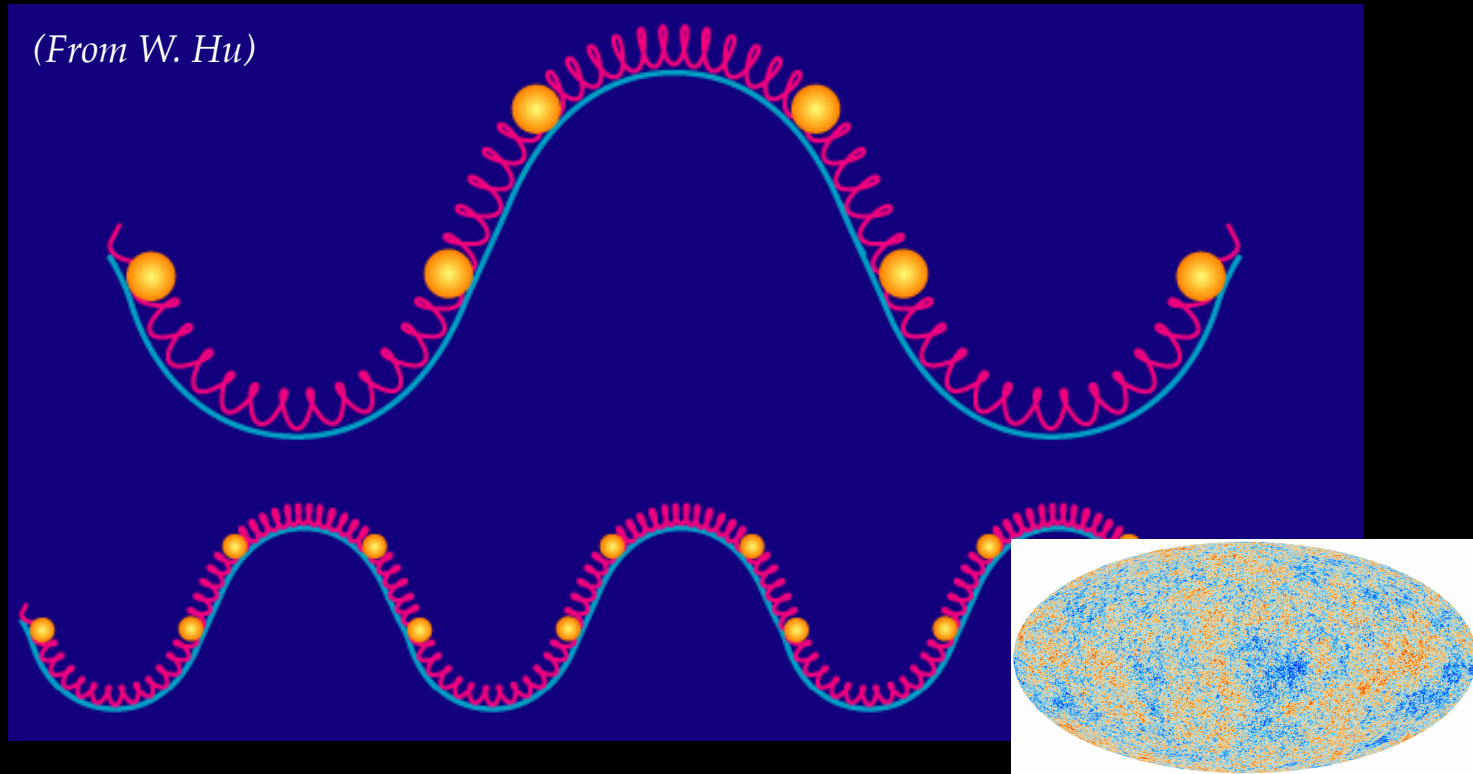


The radiation in the universe has a mean  $T \approx 2.725$  K!

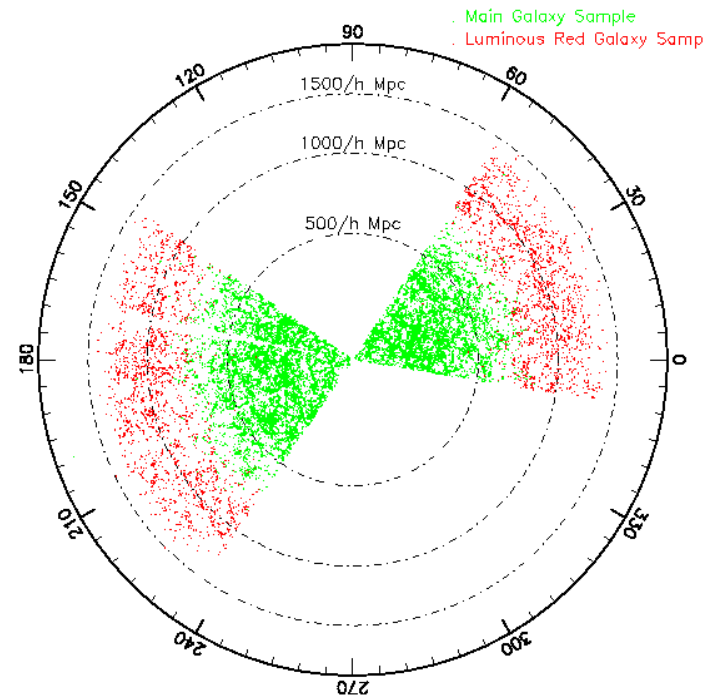
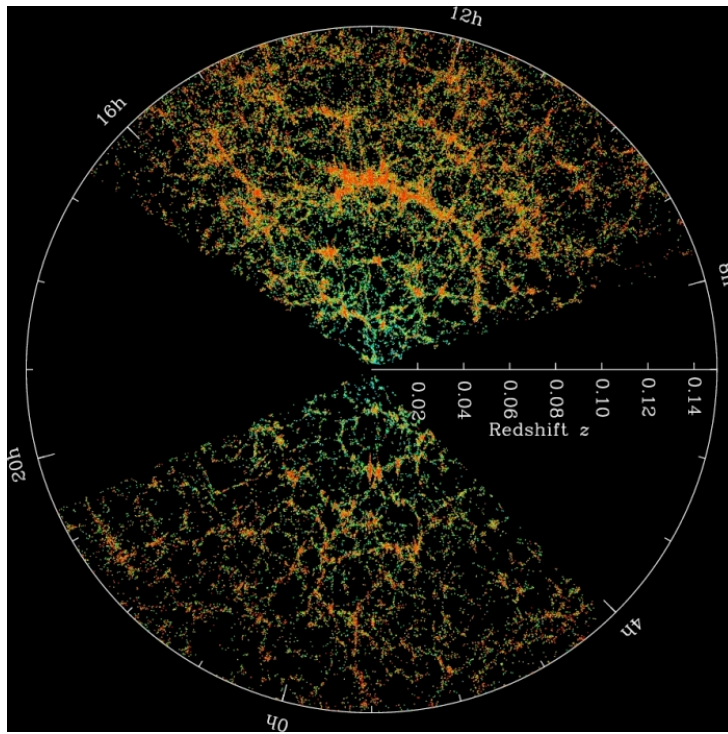


This map is just telling us how the CMB temperature varies with the angular size of patches in the sky...

The CMB fluctuations are due to the acoustic oscillations in the baryon-photon fluid before recombination.



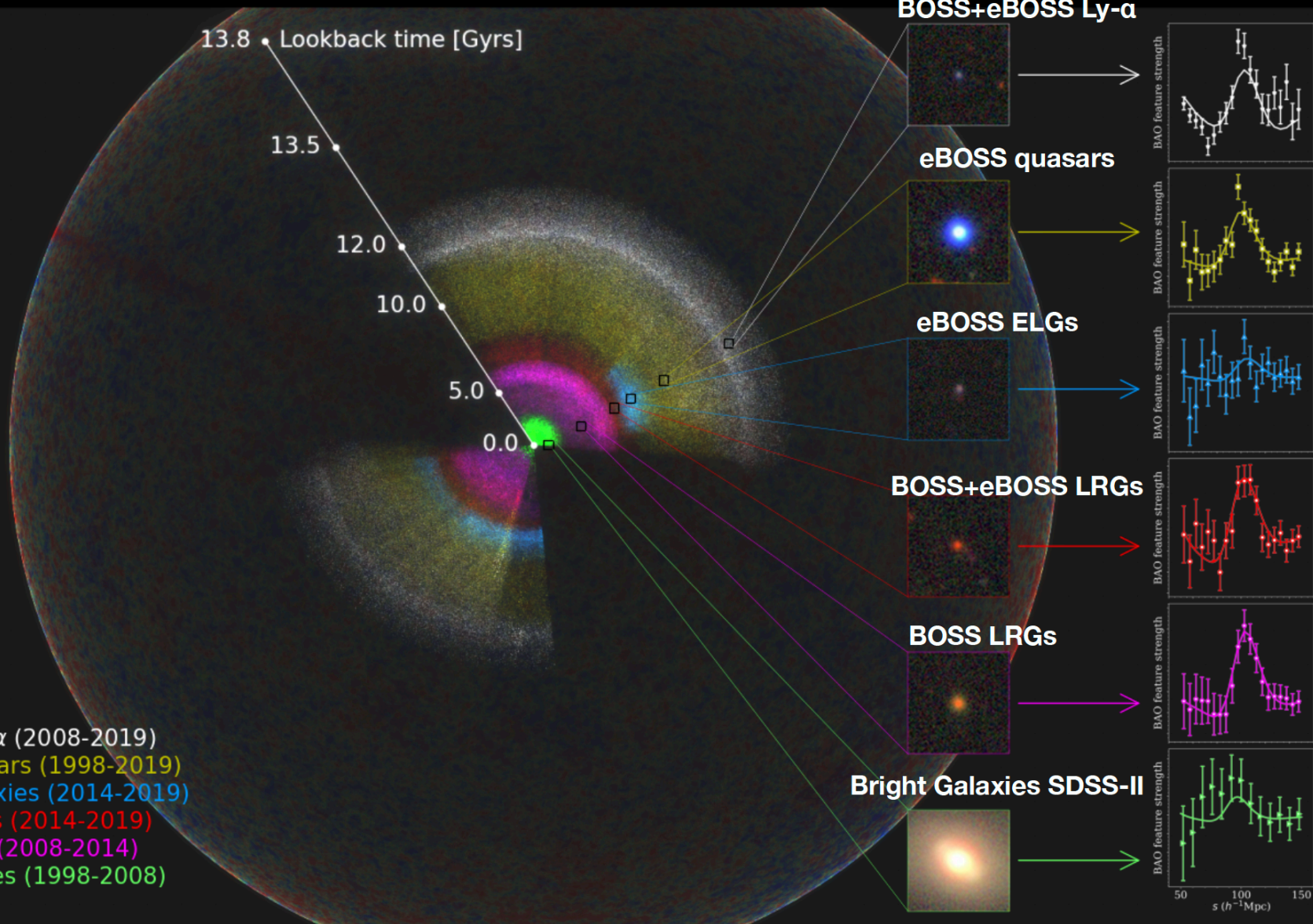
Potential wells  $\longrightarrow$  High density  $\longrightarrow$  COLD SPOTS in CMB maps  
Potential hills  $\longrightarrow$  Low density  $\longrightarrow$  HOT SPOTS in CMB maps



At distances larger than 100 Mpc, galaxy survey observations indicate that the universe is **homogeneous**, that is, galaxies and clusters of galaxies are equally distributed in the sky in all possible directions.



See animation [here](#)



eBOSS + BOSS Lyman- $\alpha$  (2008-2019)  
eBOSS + SDSS I-II Quasars (1998-2019)  
eBOSS Young Blue Galaxies (2014-2019)  
eBOSS Old Red Galaxies (2014-2019)  
BOSS Old Red Galaxies (2008-2014)  
SDSS I-II Nearby Galaxies (1998-2008)

Credit A. Raichoor



The metric  $g_{\mu\nu}$  connects the values of the coordinates to the more physical measure of the interval (proper time):

$$ds^2 = \sum_{\mu, \nu=0}^3 g_{\mu\nu} dx^\mu dx^\nu$$

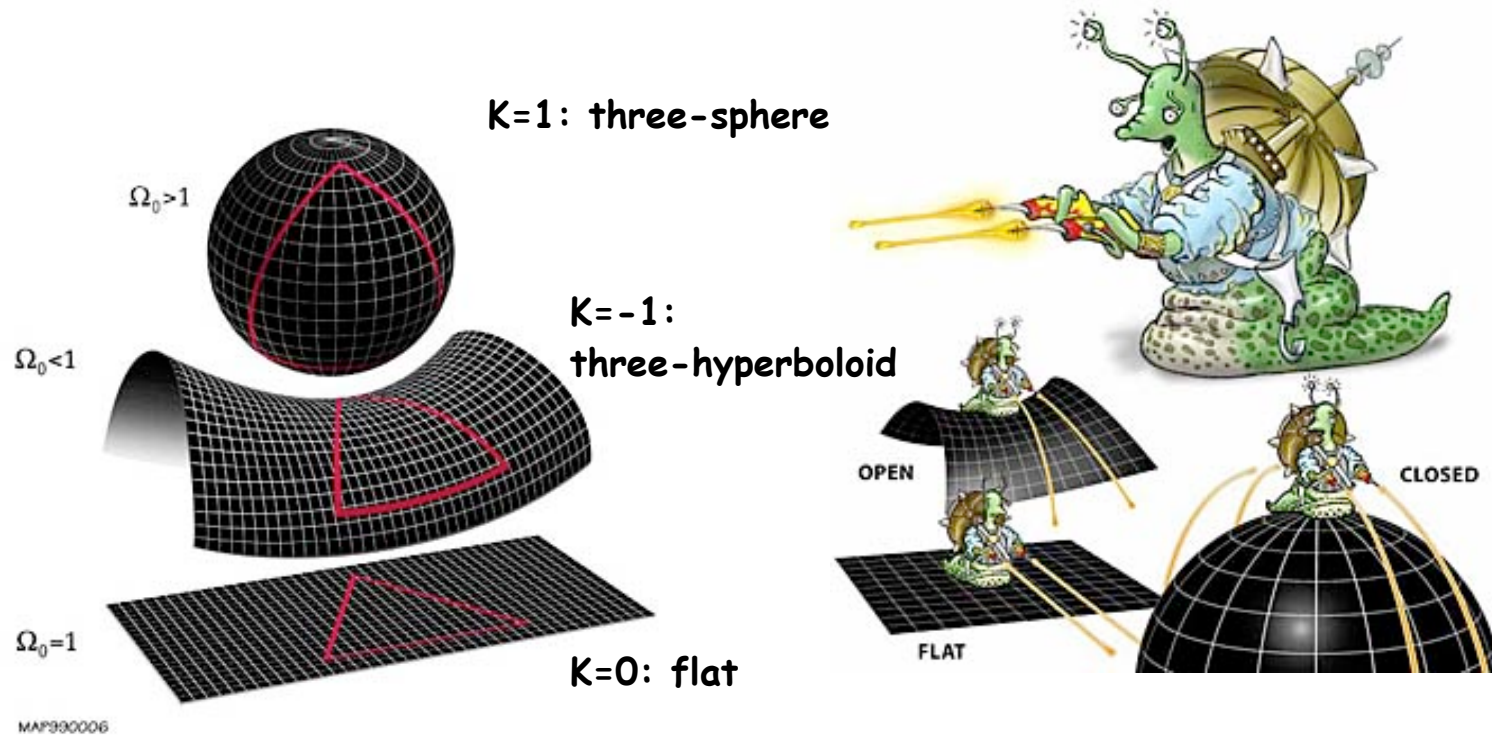
- $dx^0$  refers to the time-like component, the last three are spatial coordinates.
- $g_{\mu\nu}$  is the metric, necessarily symmetric.
- In special relativity,  $g_{\mu\nu} = \eta_{\mu\nu}$  (Minkowski metric)
- In an expanding, homogeneous and isotropic universe the metric is the FLRW one:

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right)$$

- If the universe is flat ( $K=0$ ), the FLRW metric, with  $a(t)$  the scale factor:

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix}$$

The spatial geometry depends on the curvature,  $K$ :



The FLRW metric tells us how to measure distances in each of these possible geometries.

# Geodesics

- A geodesic refers to the path followed by a particle in the absence of any forces, (in the Minkowski metric it will be a straight line):

$$\frac{d^2 \vec{x}}{dt^2} = 0$$

which should be generalised in the context of an expanding universe to:

$$\frac{d^2 x^\mu}{d\lambda^2} = -\Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda}$$

- The Christoffel symbols will be extensively used in the following

$$\Gamma^\mu_{\alpha\beta} = \frac{g^{\mu\nu}}{2} \left( \frac{\partial g_{\alpha\nu}}{\partial x^\beta} + \frac{\partial g_{\beta\nu}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\nu} \right)$$

- We can apply the geodesic equation to compute the particle's energy changes as the universe expands:

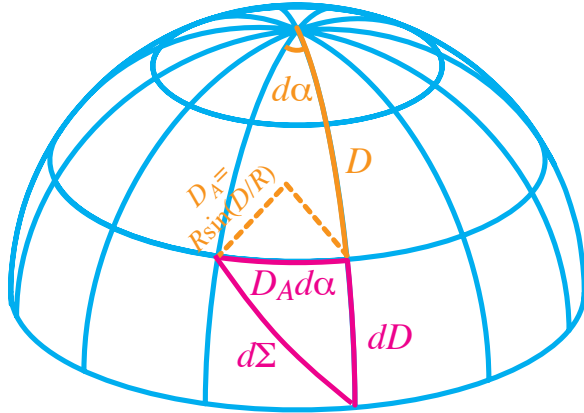
$$\frac{d}{d\lambda} = \frac{dx^0}{d\lambda} \frac{d}{dx^0} = E \frac{d}{dt} \quad P^\alpha = (E, \vec{P}) \quad P^\alpha = \frac{dx^\alpha}{d\lambda}$$

- The 0-th component of the geodesic equation reads as:

$$\frac{dE}{dt} + \frac{\dot{a}}{a} E = 0 \quad E \propto \frac{1}{a}$$



We see two type of distances:



- **Radial distance  $D$**  (photon path length)
- **Angular distance  $D_A$**  (associated to the angle subtended by an object of known physical size)

The volume element is defined as:

$$dV = D_A^2 dD d\Omega$$

Some examples of each possible distance/volume element:

- Distance to a Supernova
- Angular size of the universe at photon decoupling
- Galaxy number density

## Hubble parameter

- It provides the expansion rate of the universe as a function of time:

$$H \equiv \frac{1}{a} \frac{da}{dt} = \frac{d \ln a}{dt}$$

- The cosmic time reads as:

$$t = \int dt = \int \frac{1}{a H(a)} da$$

- The conformal time is given by:

$$\eta = \int \frac{dt}{a} = \int \frac{1}{a^2 H(a)} da$$

# Cosmological redshift

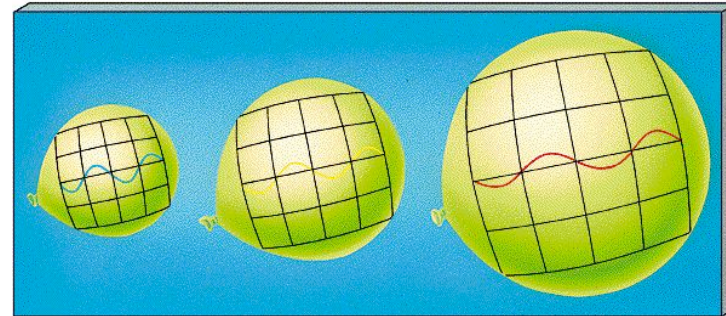
- Doppler Effect:



- **Cosmic time:** The photon wavelength is stretched with the scale factor as the universe expands.

$$\lambda = \frac{\lambda_0}{a} = (1 + z)\lambda_0$$

$$a = \frac{1}{1 + z}$$



- **If we interpret the redshift  $z$  as the Doppler effect, galaxies recede (i.e. they move further away) in an expanding universe.**

# Hubble law

- The comoving distance to an object located at redshift  $z$  reads as:

$$D(a) = \int_a^1 \frac{da'}{a'^2 H(a')} \qquad D(z) = \int_0^z \frac{dz'}{H(z')}$$

- At small redshifts,  $z \approx v/c$ .

The Hubble law can be written as:

$$\lim_{z \rightarrow 0} D(z) = \frac{z}{H(z=0)} = \frac{z}{H_0}$$

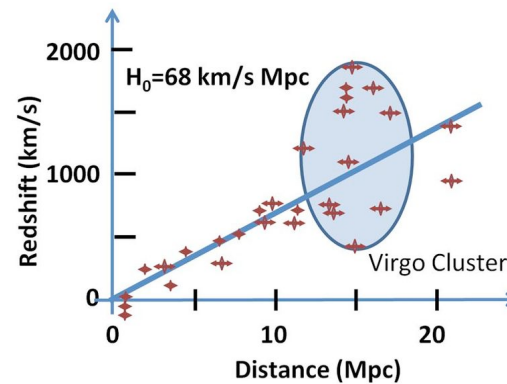
with the Hubble constant,  $H_0$ :

$$H_0 = 100h \text{ km/s/Mpc}$$

- Cosmological observations have determined that  $h \approx 0.7$



1929: Edwin Hubble measures the spectra of hundred of galaxies and notices that they are redshifted, meaning that they are moving away from our galaxy. Furthermore, the further the galaxy is located, the faster it moves away from our galaxy.



# DYNAMICS FLRW

General Relativity relates the metric with the matter and energy content in the universe. The scale factor  $a(t)$  will evolve in time accordingly to the matter-energy content of the universe.

In other words, matter and energy will tell us how the geometry of the space-time is curved via the Einstein equations.

# Einstein Equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = 8\pi T_{\mu\nu}$$

- $R_{\mu\nu}$  is the Ricci tensor, depending on the metric  $g_{\mu\nu}$  and its derivatives:

$$R_{\mu\nu} = \Gamma_{\mu\nu,\alpha}^{\alpha} - \Gamma_{\mu\alpha,\nu}^{\alpha} + \Gamma_{\beta\alpha}^{\alpha}\Gamma_{\mu\nu}^{\beta} - \Gamma_{\beta\nu}^{\alpha}\Gamma_{\mu\alpha}^{\beta}$$

(It seems tedious but there are only two components different from 0, the 00 and the ii ones)

- $\mathcal{R}$  is the Ricci scalar,  $\mathcal{R} = g^{\mu\nu}R_{\mu\nu}$ .
- $T_{\mu\nu}$  is the energy-momentum tensor.
- The Christoffel symbols:

$$\Gamma_{\alpha\beta}^{\mu} = \frac{g^{\mu\nu}}{2} \left( \frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} \right)$$

# Friedmann Equations

- First Friedmann Equation reads as:

$$H^2(a) = H_0^2 \frac{\rho(a)}{\rho_{crit}} \quad \rho_{crit} \equiv \frac{3H_0^2}{8\pi G}$$

- The second Friedmann Equation reads as:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

and it determines the accelerated processes in our universe's expansion.  
In order to have such an accelerated expansion it is required that:

$$\rho + 3p < 0$$

i.e. a negative pressure fluid!

# Energy-momentum tensor conservation

- Time evolution of the  $T_{\mu\nu}$  components
- In the absence of external forces, the energy momentum tensor is conserved.
- In an expanding universe, the energy momentum tensor conservation implies that its covariant derivative equals zero.

$$T^\mu{}_{\nu;\mu} \equiv \frac{\partial T^\mu{}_\nu}{\partial x^\mu} + \Gamma^\mu{}_{\alpha\mu} T^\alpha{}_\nu - \Gamma^\alpha{}_{\nu\mu} T^\mu{}_\alpha \quad T^\mu{}_{\nu;\mu} = 0$$

$$T^\mu{}_{0;\mu} = 0$$

$$\Gamma_{0j}^i = \Gamma_{j0}^i = \delta_j^i \frac{\dot{a}}{a}$$

$$\frac{\partial T^\mu{}_0}{\partial x^\mu} + \Gamma^\mu{}_{\alpha\mu} T^\alpha{}_0 - \Gamma^\alpha{}_{0\mu} T^\mu{}_\alpha$$

$$- \frac{\partial \rho}{\partial t} - \Gamma^\mu{}_{0\mu} \rho - \Gamma^\alpha{}_{0\mu} T^\mu{}_\alpha$$

$$\frac{\partial \rho}{\partial t} + \frac{\dot{a}}{a} (3\rho + 3p) = 0$$

$$\frac{\partial \rho}{\partial t} + 3H\rho(1 + w) = 0$$

Equation of state

- Matter (either cold dark matter or baryonic one) has zero pressure:
- Radiation is characterised by  $p=\rho/3$ :
- While dark energy should behave as:

$$\rho_m \propto a^{-3}$$

$$\rho_r \propto a^{-4}$$

$$\rho + 3p < 0$$

$$w < -1/3$$

$$\rho_{de} \propto a^{-3(1+w)}$$



# Friedmann Equations

- The first Friedmann equation can be written as:

$$H^2(a) = H_0^2 \frac{\rho(a)}{\rho_{crit}} \quad \rho_{crit} \equiv \frac{3H_0^2}{8\pi G} \quad H_0 = 100h \text{ km/s/Mpc}$$
$$\rho_{crit} = 1.879h^2 \times 10^{-29} \text{ g cm}^{-3}$$

# Friedmann Equations

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$$\rho_{crit} = 1.879h^2 \times 10^{-29} \text{ g cm}^{-3}$$

$$H^2(a) = H_0^2 (\Omega_m(a) + \Omega_r(a) + \Omega_{de}(a))$$

$$\Omega_m(a) \equiv \rho_m(a)/\rho_{crit} = \rho_{m,0}a^{-3}/\rho_{crit} = \Omega_{m,0}a^{-3} = (\Omega_{dm,0} + \Omega_{b,0})a^{-3}$$

$$\Omega_r(a) \equiv \rho_r(a)/\rho_{crit} = \rho_{r,0}a^{-4}/\rho_{crit} = \Omega_{r,0}a^{-4} = (\Omega_{\gamma,0} + \Omega_{\nu,0})a^{-4}$$

$$\Omega_{de}(a) \equiv \rho_{de}(a)/\rho_{crit} = \rho_{de,0}a^{-3(1+w)}/\rho_{crit} = \Omega_{de,0}a^{-3(1+w)}$$

- These expressions are valid for a FLAT universe. In case the universe is not flat:

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right)$$

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2}$$

$$H^2(a) = H_0^2 (\Omega_m(a) + \Omega_r(a) + \Omega_{de}(a) + \Omega_K(a))$$

$$\Omega_K(a) = -Ka^{-2}/H_0^2 = \Omega_{K,0}a^{-2}$$

## "Cosmic sum rule"

$$\Omega_{m,0} + \Omega_{r,0} + \Omega_{de,0} + \Omega_{K,0} = 1$$

- In a flat universe,  $K=0$ , therefore:

$$\Omega_{m,0} + \Omega_{r,0} + \Omega_{de,0} = 1$$

- In an open universe,  $K=-1$ , therefore the curvature contribution is positive:

$$\Omega_{m,0} + \Omega_{r,0} + \Omega_{de,0} < 1$$

- In a close universe,  $K=+1$ , therefore the curvature contribution is negative:

$$\Omega_{m,0} + \Omega_{r,0} + \Omega_{de,0} > 1$$

Current cosmological observations indicate that the universe as a geometry very, very close to the FLAT one:

$$\Omega_K = -0.037^{+0.043}_{-0.049}$$

## Radiation: photons and neutrinos

• **Photons:** The cosmic microwave background radiation temperature is 2.725 K, measured with a precision of 50 parts in a million. The energy of such a photon bath is given by the integral of the Bose-Einstein distribution times  $E=p$  (massless):

$$\rho_\gamma = 2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{p/T} - 1} p \quad x \equiv p/T \quad \rho_\gamma = \frac{8\pi T^4}{(2\pi)^3} \int_0^\infty \frac{x^3 dx}{e^x - 1} p = \frac{\pi^2}{15} T^4$$

$$\Omega_\gamma(a) = \frac{\rho_\gamma}{\rho_{crit}} = \frac{\pi^2}{15} \left( \frac{2.725 \text{ K}}{a} \right)^4 \frac{1}{\rho_{crit}} = \frac{2.47 \times 10^{-5}}{a^4 h^2} = \frac{4.75 \times 10^{-5}}{a^4}$$

• **Neutrinos:** Neutrinos are fermions and therefore follow the Fermi-Dirac statistics. As we shall soon see, neutrinos decouple from the thermal bath before electron-positron annihilation and therefore they did not share in the entropy release, being their temperature lower than that of photons:

$$\Omega_\nu(a) = \frac{\rho_\nu}{\rho_{crit}} = \frac{1.68 \times 10^{-5}}{a^4 h^2} \quad (m_\nu = 0) \quad \left( \frac{T_\nu}{T_\gamma} \right) = \left( \frac{4}{11} \right)^{1/3}$$

**But neutrinos are massive particles!**  $n_\nu(T) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{p/T} - 1} = \frac{3}{22} n_\gamma(T)$

$$\Omega_\nu(a) = \frac{m_\nu n_\nu}{\rho_{crit}} = \frac{\sum m_\nu}{94 \text{ eV} h^2} \frac{1}{a^3} \quad \xrightarrow{\text{Data tell us...}} \quad 0.0006 \lesssim \Omega_{\nu,0} h^2 \lesssim 0.0025$$

## Matter: baryons and dark matter

- **Baryons:** The baryon density can not be inferred from temperature measurements. Currently we know that:

$$\Omega_b h^2 = 0.02205^{+0.00056}_{-0.00055}$$

from the *CMB* anisotropies. Other methods to extract the present baryonic mass-energy density are light element abundances, quasar spectra or the gas population in galaxies.

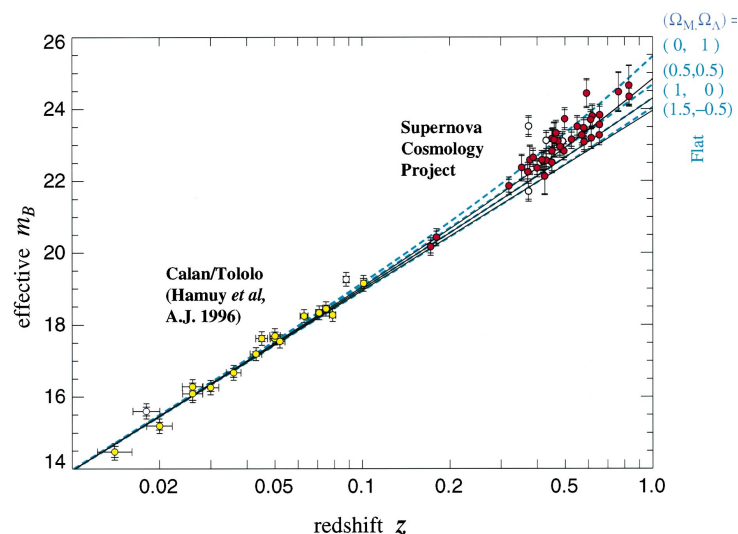
- **Dark matter**

A number of observations (galaxy rotation curves, galaxy clusters, gravitational lensing, large scale structure and the *CMB* anisotropies) indicate that the majority of the matter in the universe is unknown: dark matter!

$$\Omega_{dm} h^2 = 0.1199^{+0.0053}_{-0.0052}$$

Furthermore, observations of the large scale structure of our universe tell us that a *COLD* dark matter component provides an excellent fit to data.

# Dark energy

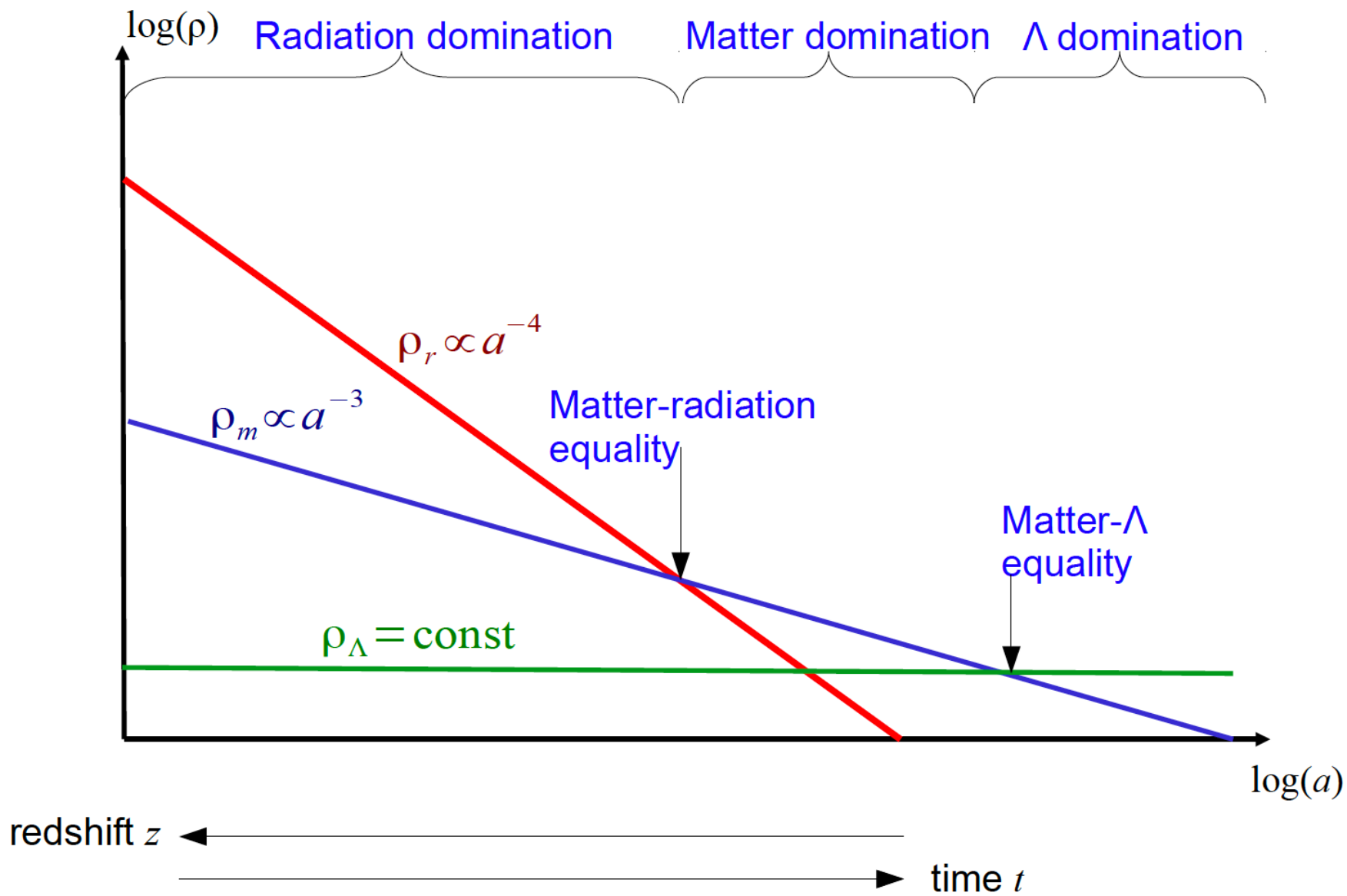


In 1998, two independent groups, observed that type Ia Supernovae were much fainter than what one would expect in a universe with only matter.

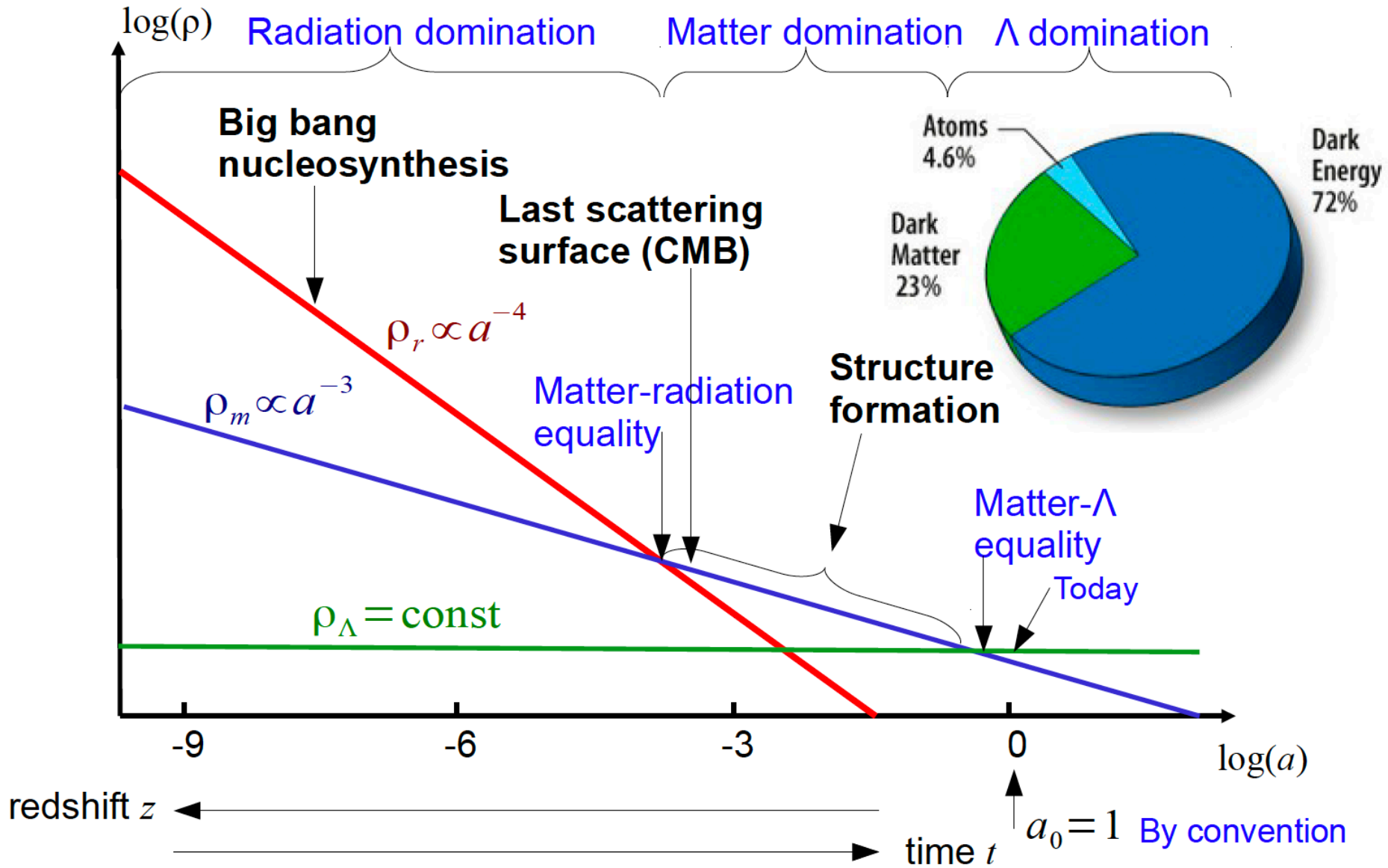
An additional ingredient was mandatory to make the universe to expand in an accelerated way!

Today the evidence for an accelerated expansion of the universe is  $4.2\sigma$ - $4.6\sigma$  with JLA SNIa data alone, and  $11.2\sigma$  in a flat universe.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad \rho + 3p < 0$$

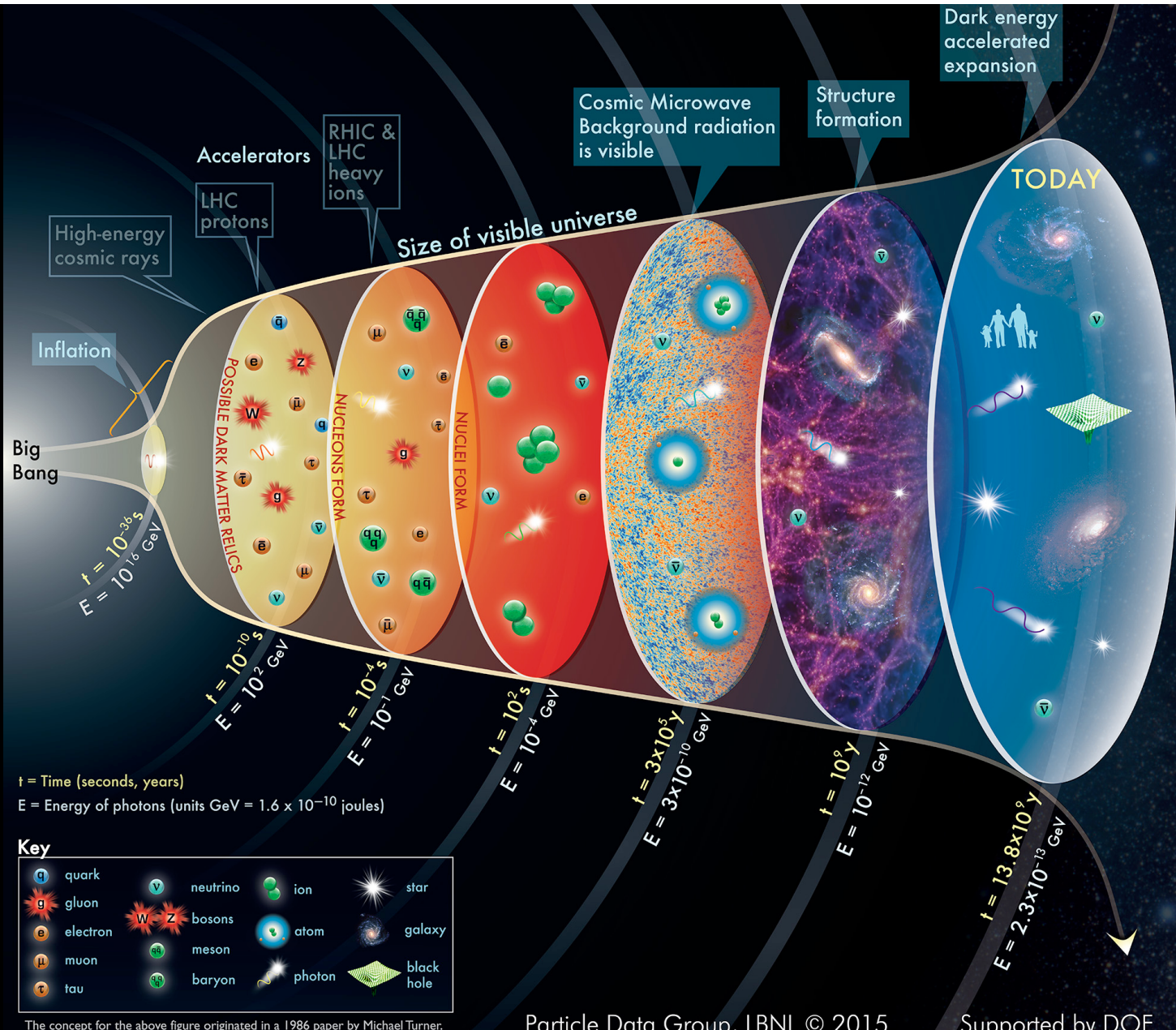


From Y. Wong



From Y. Wong





The concept for the above figure originated in a 1986 paper by Michael Turner.

Particle Data Group, IJLNL © 2015

Supported by DOE

## Particle distribution functions

- The usual way of describing particles in thermal equilibrium is via their distribution function, indicating the number of particles in the phase space with a given position  $\mathbf{x}$  and a momentum  $\mathbf{p}$ . At 0th order, we have the Bose Einstein or the Fermi-Dirac distributions:

- $f_{BE} = \frac{1}{e^{(E-\mu)/T} - 1}$        $f_{FD} = \frac{1}{e^{(E-\mu)/T} + 1}$

- The number and energy densities and the pressure read as:

$$n = \frac{g}{(2\pi)^3} \int f(\vec{x}, \vec{p}) d^3x d^3p$$

$$\rho = \frac{g}{(2\pi)^3} \int E(\vec{p}) f(\vec{x}, \vec{p}) d^3x d^3p$$

$$p = \frac{g}{(2\pi)^3} \int \frac{p^2}{3E(p)} f(\vec{x}, \vec{p}) d^3x d^3p$$

- While the entropy density is

$$s \equiv \frac{\rho + p}{T}$$

# BOLTZMANN EQUATIONS

- Throughout the universe's history, particles remain in thermal equilibrium until their interaction rate is equal or larger than the expansion rate of the universe. Then, the particle will decouple from the thermal bath. Of course this is an approximation:

$$\Gamma \gtrsim H$$

- The accurate calculation requires to solve the Boltzmann equation:

$$L f = C f$$

- where  $f$  is the distribution function,  $L$  is the Liouville operator, and  $C$  contains all the collision terms.

- In classical mechanics:  $\hat{L} = \frac{d}{dt} + \vec{v} \cdot \vec{\nabla}_x + \vec{F}/m \cdot \vec{\nabla}_v$

- The relativistic version is:  $\hat{L} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma_{\beta\gamma}^\alpha p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}$   $P^\alpha = (E, \vec{P})$   $P^\alpha = \frac{dx^\alpha}{d\lambda}$

- FRW geometry:  $\hat{L} f = E \frac{\partial f}{\partial t} - H p^2 \frac{\partial f}{\partial E}$   $\frac{dn}{dt} + 3Hn = \frac{g}{(2\pi)^3} \int C f \frac{d^3 p}{E}$

# BOLTZMANN EQUATIONS

- Simplifying the possible processes (1+2 ↔ 3+4):

In an expanding universe, the number of particles gets diluted!

In the absence of interactions,  $n \propto a^{-3}$

$$\frac{dn}{dt} + 3Hn = \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \int \frac{d^3 p_3}{(2\pi)^3 2E_3} \int \frac{d^3 p_4}{(2\pi)^3 2E_4}$$

Particle Physics

$$\times (2\pi)^4 \delta^3(p^1 + p^2 - p^3 - p^4) \delta(E^1 + E^2 - E^3 - E^4) |\mathcal{M}|^2$$

Energy-momentum tensor conservation

$$\times (f_3 f_4 - f_1 f_2)$$

Loss rate of 1 is proportional to the occupation numbers of 1 and 2

Production rate of 1 is proportional to the occupation numbers of 3 and 4

# BOLTZMANN EQUATIONS

- After defining the thermally-averaged cross section:

$$\frac{dn}{dt} + 3Hn = n_1^0 n_2^0 \langle \sigma v \rangle \left( \frac{n_3 n_4}{n_3^0 n_4^0} - \frac{n_1 n_2}{n_1^0 n_2^0} \right)$$

- where the equilibrium number densities:

$$n_i^0 \equiv g_i \int \frac{d^3 p}{(2\pi)^3} e^{-E_i/T} \left\{ \begin{array}{l} g_i (m_i T / 2\pi)^{3/2} e^{-m_i/T} \quad m_i \gg T \\ g_i \frac{T^3}{\pi^2} \quad m_i \ll T \end{array} \right.$$

## BOLTZMANN EQUATIONS

$$\frac{dn}{dt} + 3Hn = n_1^0 n_2^0 \langle \sigma v \rangle \left( \frac{n_3 n_4}{n_3^0 n_4^0} - \frac{n_1 n_2}{n_1^0 n_2^0} \right)$$

- Neutron-Proton ratio

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ n & \nu_e/e^+ & p & e^-/\bar{\nu}_e \end{array}$$

- Recombination

$$\begin{array}{cccc} e & p & H & \gamma \end{array}$$

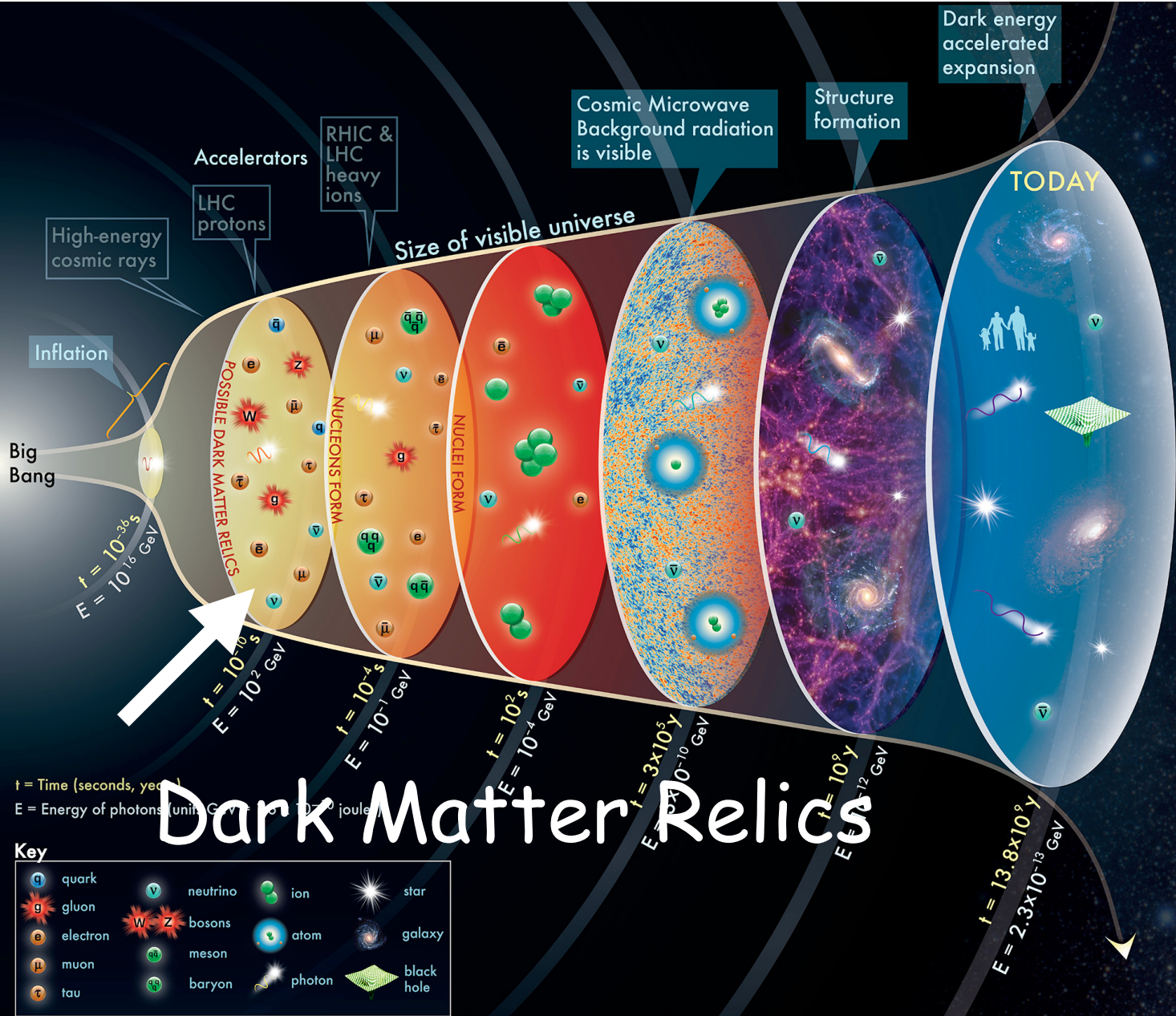
- Dark matter production

$$\begin{array}{cccc} X & X & l & l \end{array}$$

- When looking into the DM annihilating case,  $XX \leftrightarrow ll$ , 3 and 4 will not couple anymore and therefore:

$$n_3 n_4 = n_3^0 n_4^0$$

$$\frac{dn_X}{dt} + 3Hn_X = \langle \sigma v \rangle \left( (n_X^0)^2 - n_X^2 \right)$$



## HANDS-ON SESSION (I)!

- PLEASE GO TO THE WEB PAGE:

<http://www.astro.ucla.edu/%7Ewright/CosmoCalc.html>

- COMPUTE THE AGE OF THE UNIVERSE TODAY AS A FUNCTION OF THE HUBBLE CONSTANT. HOW IS THE CORRELATION? CAN YOU EXPLAIN THAT IN TERMS OF THE ANALYTICAL EXPRESSIONS?
- COMPUTE THE AGE OF THE UNIVERSE TODAY AS A FUNCTION OF THE MATTER DENSITY. HOW IS THE CORRELATION? CAN YOU EXPLAIN THAT IN TERMS OF THE ANALYTICAL EXPRESSIONS?
- COMPUTE THE DIFFERENT OBSERVABLES IN OPEN AND FLAT COSMOLOGIES, AT A REDSHIFT OF  $Z=1000$ . EXPLAIN THE DIFFERENCES!



1. INTRODUCTION: FUNDAMENTAL INGREDIENTS & THERMAL HISTORY OF THE UNIVERSE ✓

2. NEUTRINO DECOUPLING IN THE EARLY UNIVERSE

3. BIG BANG NUCLEOSYNTHESIS &  $N_{\text{eff}}$

4. COSMOLOGY &  $N_{\text{eff}}$

5. COSMOLOGY & NEUTRINO MASSES

6. TAKE HOME MESSAGES

Event	Time	Redshift	Temperature
Baryogenesis	?	?	?
EW phase transition	$2 \times 10^{-11}s$	$10^{15}$	$100GeV$
QCD phase transition	$2 \times 10^{-5}s$	$10^{12}$	$150MeV$
Neutrino decoupling	$1s$	$6 \times 10^9$	$1MeV$
Electron-positron annihilation	$6s$	$2 \times 10^9$	$500keV$
Big bang nucleosynthesis	$3min$	$4 \times 10^8$	$100keV$
Matter-radiation equality	$6 \times 10^4yrs$	3400	$.75eV$
Recombination	$2.6 - 3.8 \times 10^5yrs$	1100-1400	$.26 - .33eV$
CMB	$3.8 \times 10^5yrs$	1100	$.26eV$

## Particle decoupling in the early universe: Neutrinos

- We have seen that a very easy and straightforward hand-waving rule to compute a particle decoupling time in the early universe is:

$$\Gamma \lesssim H$$

- Neutrinos only interact via weak interactions, with a rate:

$$\Gamma_\nu = n\sigma v \simeq T^3 G_F^2 T^2 \sim G_F^2 T^5$$

- While the expansion rate of the universe is given by the Hubble factor:

$$H^2 = \frac{8\pi G}{3} \rho \sim T^4 / m_{pl}^2$$

$$\Gamma_\nu / H \sim \left( \frac{T}{1 \text{ MeV}} \right)^3$$

- Therefore neutrinos decouple from the thermal bath around 1 MeV.

They do not inherit any of the energy associated to  $e^+ e^-$  annihilations, being colder than photons:

$$T_{\nu 0} = \left(\frac{4}{11}\right)^{1/3} T_0 = 1.945 \text{ K} \rightsquigarrow 1.697 \times 10^{-4} \text{ eV}$$

## Particle decoupling in the early universe: Neutrinos

- The entropy density is:  $s \equiv \frac{\rho + p}{T}$

¿How are related the photon and the neutrino temperatures?

- Electron positron annihilation takes place **AFTER** neutrino decoupling.
- In an expanding universe the entropy density per comoving volume is conserved:

- Boson's entropy contribution:  $2\pi^2 T^3 / 45$
- Fermion's entropy contribution:  $7/8 \times 2\pi^2 T^3 / 45$

- Before electron/positron annihilation= electrons ( $g=2$ ), positrons ( $g=2$ ), neutrinos (3), antineutrinos (3) and photons ( $g=2$ ) therefore:

$$s(a_1) = 2\pi^2 T_1^3 / 45 (2 + 7/8(2 + 2 + 3 + 3))$$

- After, only neutrinos, antineutrinos and photons but at different temperature!

$$s(a_2) = 2\pi^2 / 45 (2T_\gamma^3 + 7/8(3 + 3)T_\nu^3)$$

$$s(a_1)a_1^3 = s(a_2)a_2^3 \quad a_1 T_1 = a_2 T_\nu \quad \longrightarrow \quad \left(\frac{T_\nu}{T_\gamma}\right) = \left(\frac{4}{11}\right)^{1/3}$$

## Number of neutrinos: $N_{\text{eff}}$

The total radiation in the universe can be written as:

$$\Omega_r h^2 = \left( 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}} \right) \Omega_\gamma h^2$$

Bennett et al, 2012.02726

$N_{\text{eff}} = 3.0440 \pm 0.0002$  standard scenario: electron, muon and tau neutrinos

$N_{\text{eff}} < 3.044$  (less neutrinos): Neutrino decays ?

$N_{\text{eff}} > 3.044$  (more neutrinos): Sterile neutrino species ?

*But...if they are sterile, and do not interact with other particles, how cosmologists measure them?*



*That's the dark side of the GRAVITATIONAL FORCE...*



All particle species behave as ideal gases

(ideal gas approximation)

The neutrino decoupling process is localised at  $T = T_\nu = T_d$   
the neutrino and QED sectors transit from a state of tight thermal contact  
to a state of zero thermal contact at the neutrino decoupling temperature

(instantaneous decoupling approximation)

The electron/positron sector is fully ultra-relativistic at the time of neutrino  
decoupling

$T_d/m_e \rightarrow \infty$  (ultra-relativistic approximation)

Standard-model corrections to $N_{\text{eff}}^{\text{SM}}$	Leading-digit contribution
$m_e/T_d$ correction	+0.04
$\mathcal{O}(e^2)$ FTQED correction to the QED EoS	+0.01
Non-instantaneous decoupling+spectral distortion	-0.005
$\mathcal{O}(e^3)$ FTQED correction to the QED EoS	-0.001
Flavour oscillations	+0.0005
Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

The ultra relativistic approximation:

$$T_d/m_e \rightarrow \infty$$

is not well satisfied in reality!

There will be a change in the QED plasma entropy density

$$\delta s^{\text{Rel}} = \frac{g_e}{2\pi^2 T_d} \int_0^\infty dp p^2 \left( E_e + \frac{p^2}{3E_e} \right) f_D(E_e) \Bigg|_{T_d/m_e \rightarrow \infty}^{T_d/m_e}$$

- Physically, a non-zero  $\delta N_{\text{eff}}$  arising from relaxing the ultra relativistic approximation implies that electron-positron annihilation is not a temporally localised event at  $T \sim 0.5$  MeV.



MORE AT RASMUS SLOTH SEMINAR!

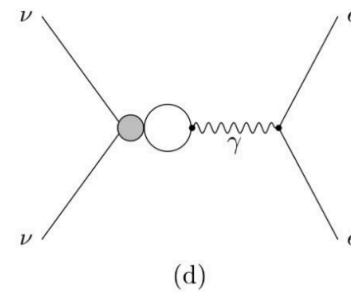
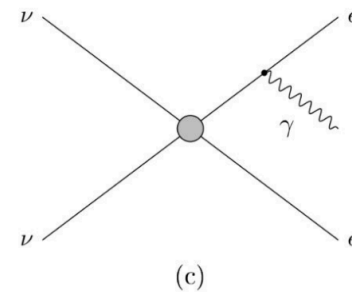
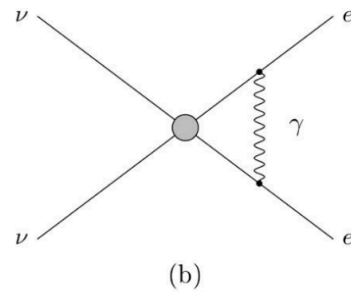
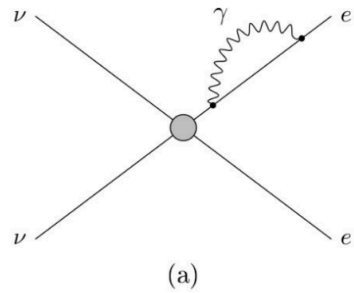


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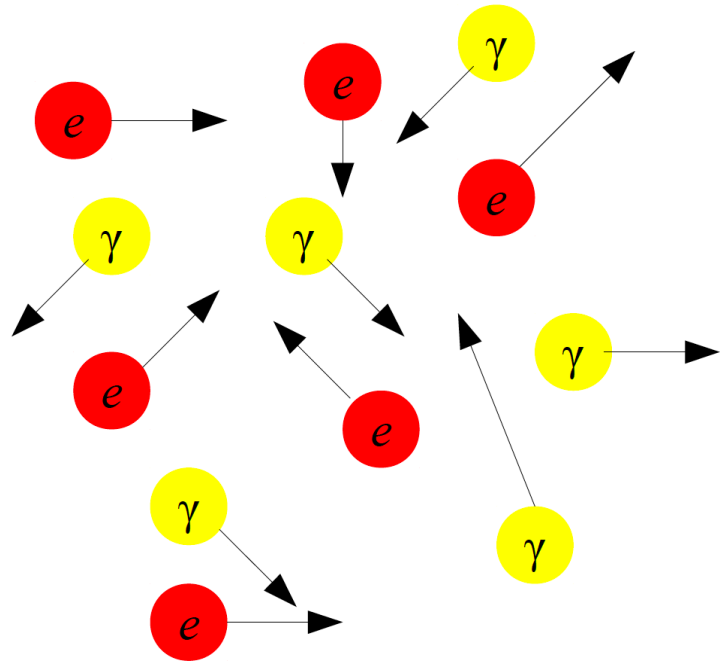
Bennett et al, 1911.04504

MORE AT RASMUS SLOTH SEMINAR!

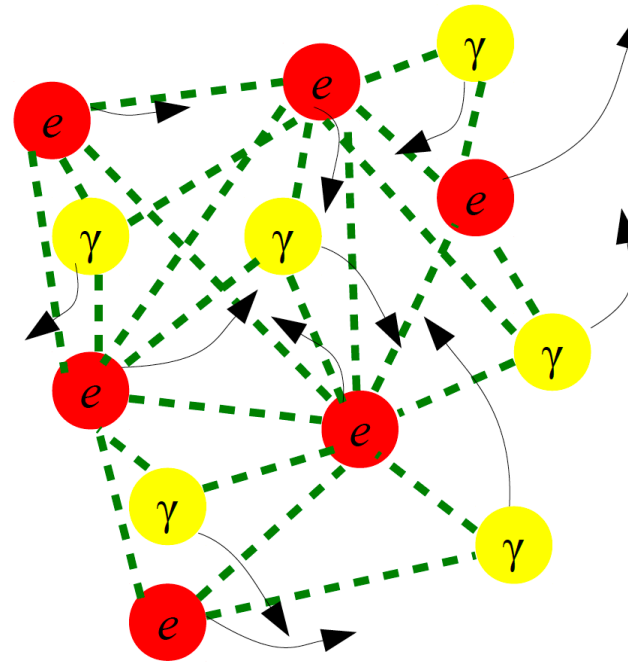
$$\ln Z^{(2)} + \ln Z^{(3)} = -\frac{1}{2} \left[ \text{Diagram 1} \right] + \frac{1}{2} \left[ \frac{1}{2} \left[ \text{Diagram 2} \right] - \frac{1}{3} \left[ \text{Diagram 3} \right] + \frac{1}{4} \left[ \text{Diagram 4} \right] + \dots \right]$$



**Ideal gas**



**+ EM interactions**



Temperature  
-dependent  
dispersion relation  
+  
Forces

**Modified QED Equation of State**

*From Y. Wong*

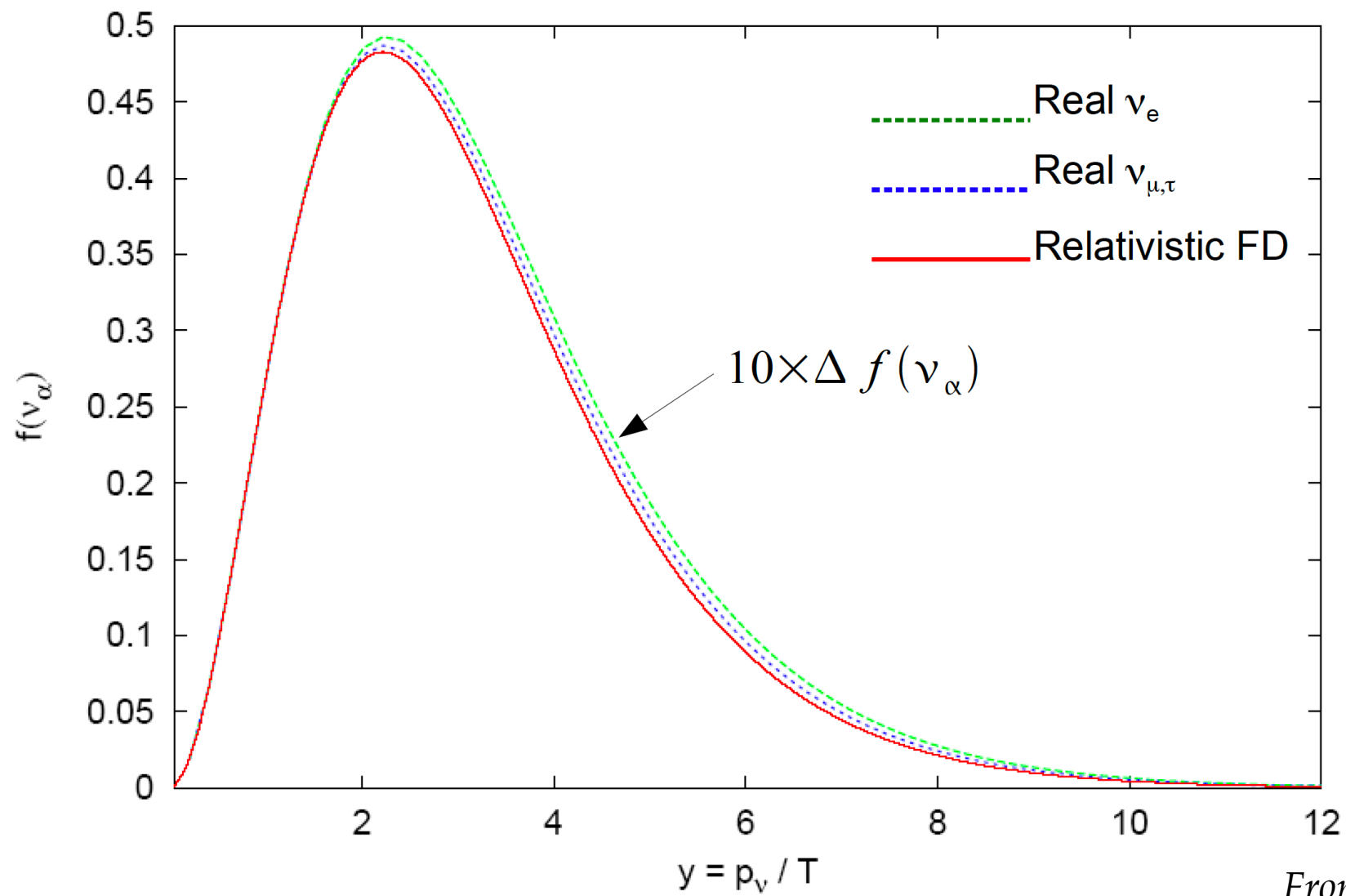
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Flavour oscillations	+0.0005
Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

Neutrino decoupling and electron/positron annihilations are processes quite close in time.

None of these two events are localised in time!

There are relic interactions between electrons, positrons and neutrinos at cosmological temperatures smaller than 1MeV

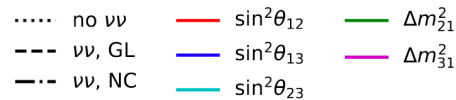
These processes are more efficient for neutrinos with larger momenta, leading to non-thermal distortions in the neutrino spectra at the percent level and a slightly smaller increase of the comoving photon temperature.



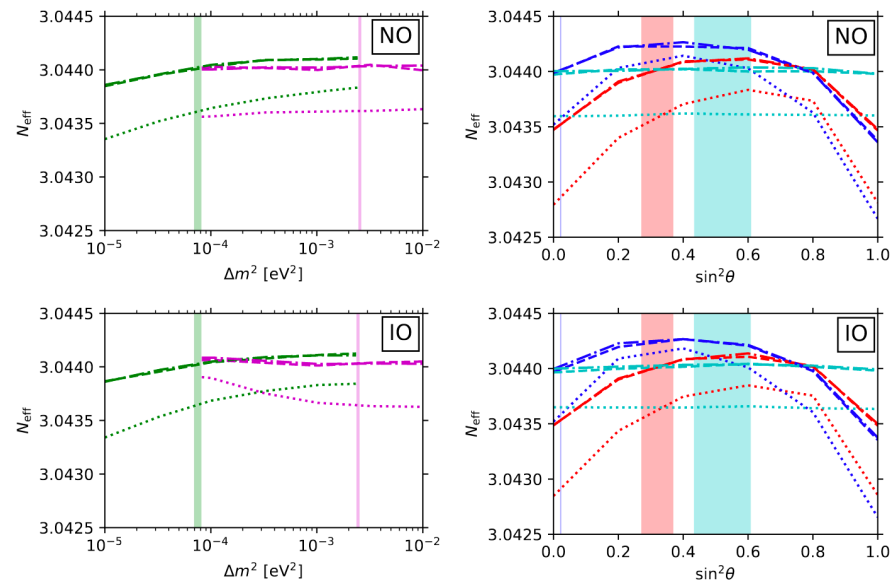
*From S. Pastor*

## 10<sup>-4</sup> Uncertainty due to measurement errors on the solar mixing angle

Standard-model corrections to $N_{\text{eff}}^{\text{SM}}$	Leading-digit contribution
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MORE AT RASMUS SLOTH SEMINAR!



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$$T_{\nu 0} = \left(\frac{4}{11}\right)^{1/3} T_0 = 1.945 \text{ K} \rightsquigarrow 1.697 \times 10^{-4} \text{ eV}$$

**If** these neutrinos are massive, their energy density, at  $T \ll m$  is

$$\rho_\nu = m_\nu n_\nu \quad n_{\nu_i}(T_{\nu 0}) \approx 56 \text{ cm}^{-3} \quad \Omega_\nu h^2 = \frac{\sum m_\nu}{93 \text{ eV}}$$

Then, demanding that massive neutrinos do not over-close the universe,  $\sum m_\nu \lesssim 45 \text{ eV}$

$$\rho_\nu = m_\nu n_\nu$$

We integrate the Fermi-Dirac distribution for the (anti)neutrinos, with 0 chemical potential

$$\begin{aligned} n_{\nu_i}(T_\nu) = n_{\nu_i^c}(T_\nu) &= \frac{1}{(2\pi)^3} \int d^3p \frac{1}{\exp(p/T_\nu) + 1} \\ &= \frac{3\zeta(3)}{4\pi^2} T_\nu^3, \\ &= \frac{3}{22} n_\gamma(T). \end{aligned}$$

$$n_{\nu_i 0} = n_{\nu_i^c 0} \equiv n_{\nu_i}(T_{\nu 0}) \approx 56 \text{ cm}^{-3}$$

$$\Omega_\nu = \frac{\rho_\nu}{\rho_{\text{crit}}} = \frac{\sum m_\nu n_\nu}{3H_0^2/8\pi G}$$

$$H_0 = 100h$$



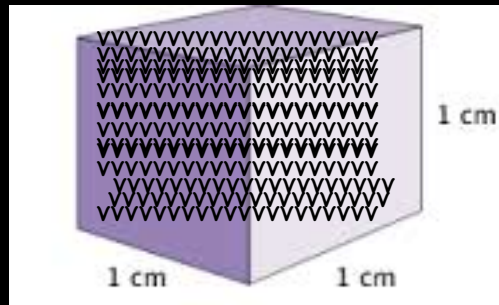
$$\Omega_\nu h^2 = \frac{\sum m_\nu}{93 \text{ eV}}$$





According to standard cosmology,  
there is a cosmic neutrino background,  
equivalent to the CMB photon background, albeit slightly colder  $T \approx 1.94$  K

**340 neutrinos/cm<sup>3</sup>**

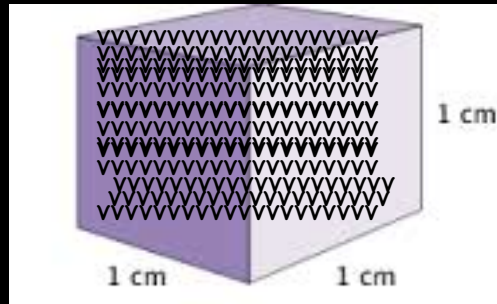


This cosmic relic neutrino background has never been detected directly.

The universe is filled with a dense flux of "relic neutrinos"  
created in the Big Bang.

This makes neutrinos the most abundant KNOWN form of...

**340 neutrinos/cm<sup>3</sup>**



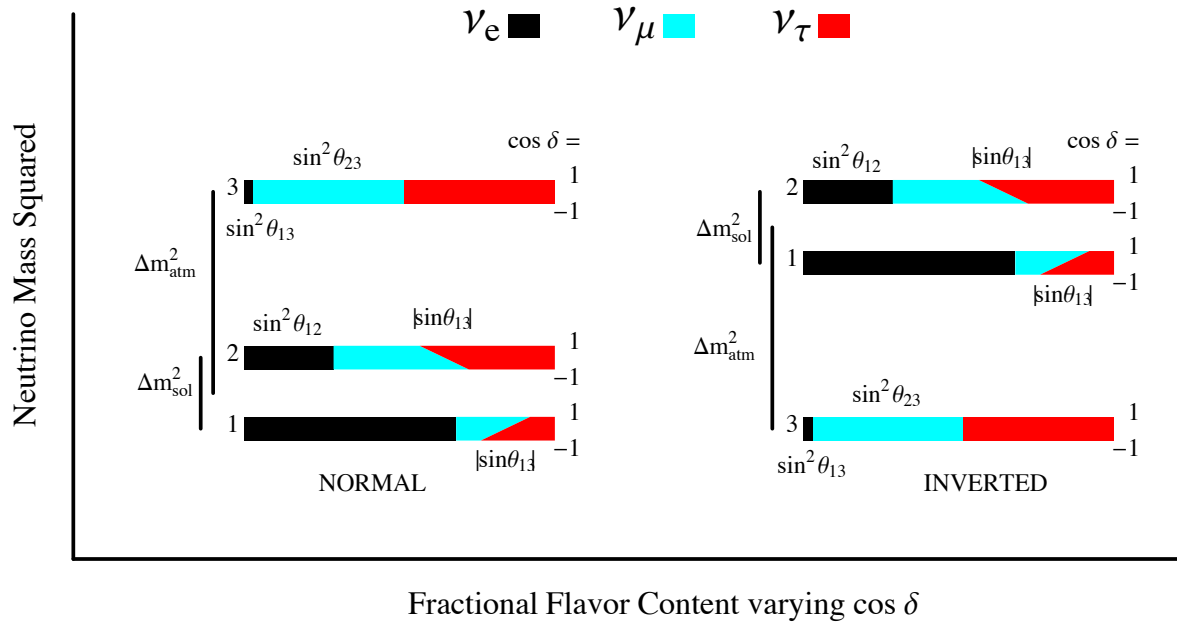
**HOT dark matter!**

According to **neutrino oscillation physics**, we know that there are at least two Dirac or Majorana **massive** neutrinos:

$$\Delta m_{12}^2 = (7.05 - 8.14) \times 10^{-5} \text{eV}^2$$

$$\Delta m_{13}^2 = (2.41 - 2.60) \times 10^{-3} \text{eV}^2$$

$$\Delta m_{32}^2 = -(2.31 - 2.51) \times 10^{-3} \text{eV}^2$$



(Mena, Parke, PRD'04)

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We are sure then that **two neutrinos have a mass above:**

$$\sqrt{\Delta m_{12}^2} \simeq 0.008 \text{ eV}$$

and that **at least one of these neutrinos has a mass larger than**

$$\sqrt{|\Delta m_{13}^2|} \simeq 0.05 \text{ eV}$$

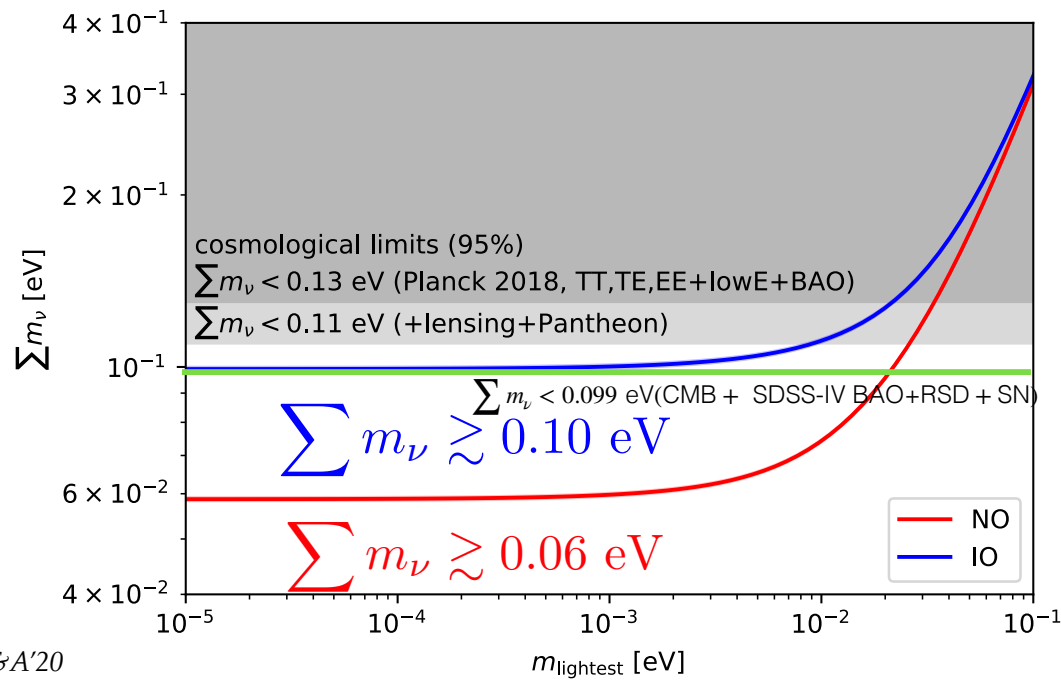
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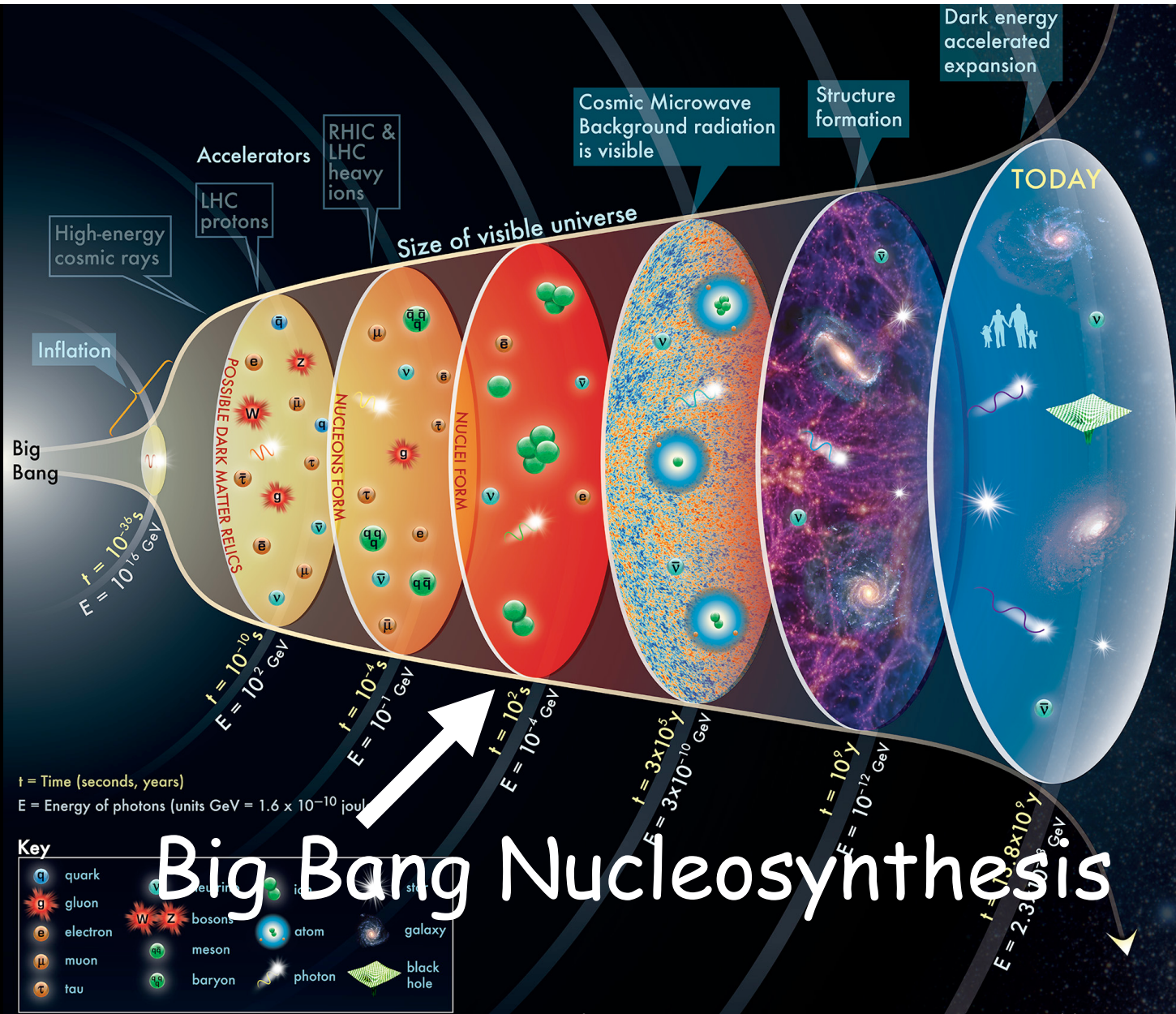
$$\Delta m_{23}^2 = -(2.31 - 2.51) \times 10^{-3} \text{eV}^2$$

which translates into a lower bound on the total neutrino mass, depending on the ordering:



1. INTRODUCTION: FUNDAMENTAL INGREDIENTS & THERMAL HISTORY OF THE UNIVERSE ✓
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Event	Time	Redshift	Temperature
Baryogenesis	?	?	?
EW phase transition	$2 \times 10^{-11} s$	$10^{15}$	$100 GeV$
QCD phase transition	$2 \times 10^{-5} s$	$10^{12}$	$150 MeV$
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# Big Bang Nucleosynthesis

The concept for the above figure originated in a 1986 paper by Michael Turner.



# Big Bang Nucleosynthesis: $N_{\text{eff}}$

BBN theory predicts the abundances of D,  $^3\text{He}$ ,  $^4\text{He}$  and  $^7\text{Li}$  which are fixed by  $t \approx 180$  s. They are observed at late times: low metallicity sites with little evolution are “ideal”.

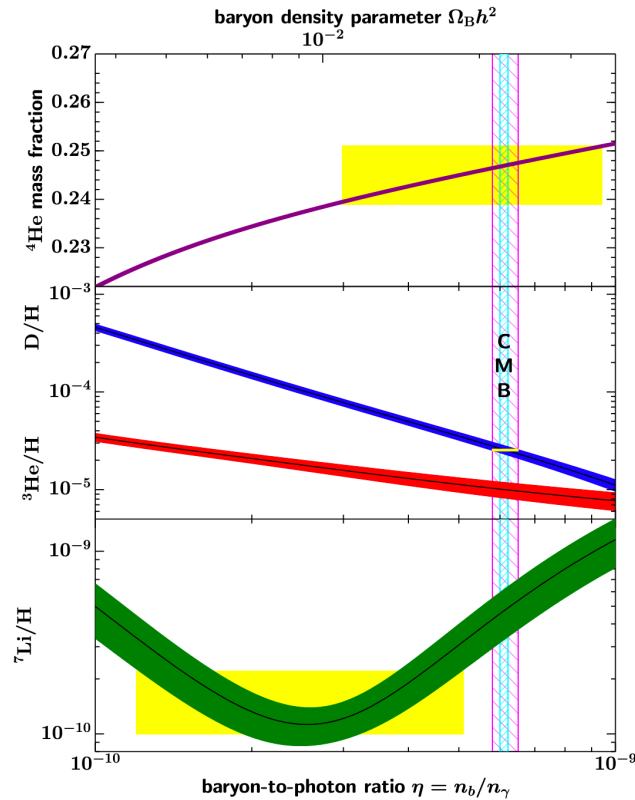


Figure 24.1: The primordial abundances of  $^4\text{He}$ , D,  $^3\text{He}$ , and  $^7\text{Li}$  as predicted by the standard model of Big-Bang nucleosynthesis — the bands show the 95% CL range [47]. Boxes indicate the observed light element abundances. The narrow vertical band indicates the CMB measure of the cosmic baryon density, while the wider band indicates the BBN D+ $^4\text{He}$  concordance range (both at 95% CL).

*P.A. Zyla et al. (Particle Data Group),  
Prog. Theor. Exp. Phys. 2020, 083C01 (2020).*

Low metallicity extragalactic HII regions.  
Produced in stars. 🤔



High z QSO absorption lines.  
Destroyed in stars. 😞



Solar system and high metallicity HII  
galactic regions.

$^3\text{He}$  not used for cosmological constraints.



Metal poor stars in our galaxy.  
Destroyed in stars and produced by  
galactic cosmic ray interactions.



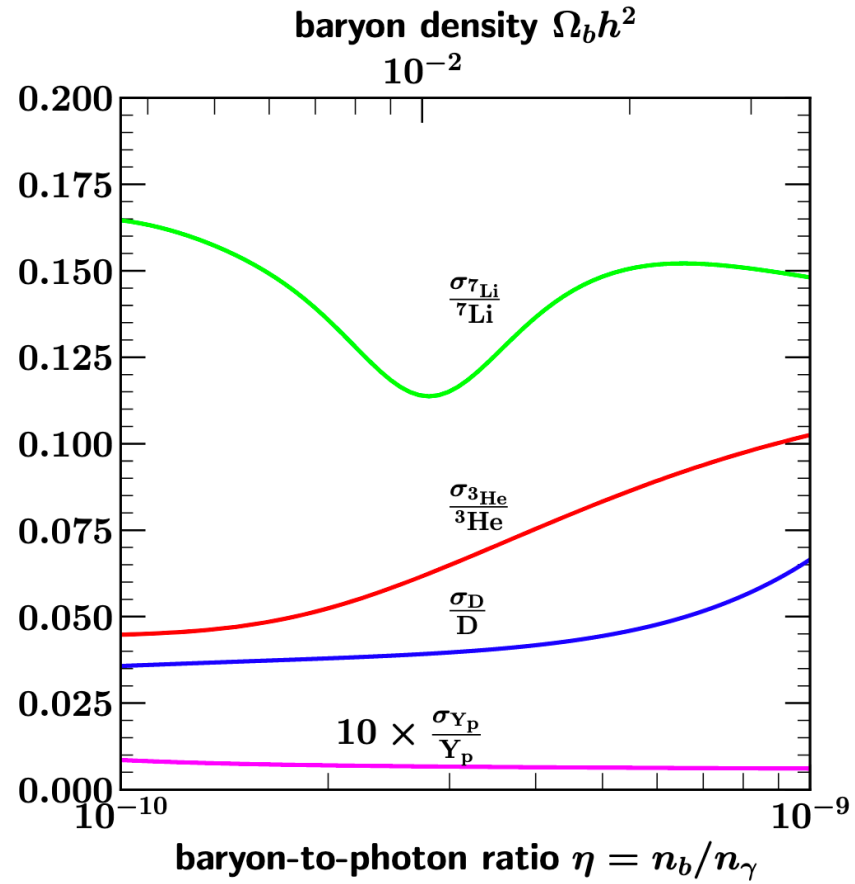


FIG. 3. Fractional uncertainties in the light element abundance predictions shown in Fig. 2. For each species  $i$ , we plot ratio of the standard deviation  $\sigma_i$  to the mean  $\mu_i$ , as a function of baryon-to-photon ratio. The relative uncertainty of the  ${}^4\text{He}$  abundance has been multiplied by a factor of 10.

# Big Bang Nucleosynthesis: $N_{\text{eff}}$

$N_{\text{eff}}$  changes the freeze out temperature of weak interactions:

$$\Gamma_{n \leftrightarrow p} \sim H$$

**MORE NEUTRINOS:**

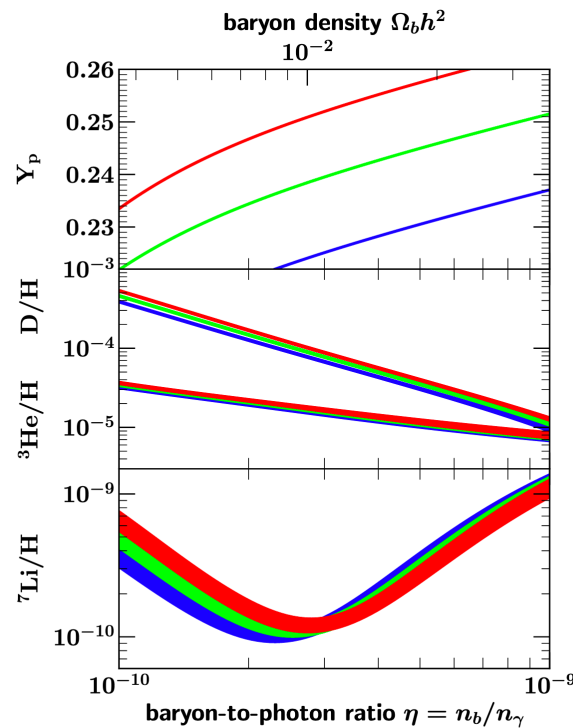
Higher  $N_{\text{eff}}$ : larger expansion rate & freeze out temperature, **MORE HELIUM 4**

$$n/p \simeq e^{-\frac{m_n - m_p}{T_{\text{freeze}}}} \quad Y_p = \frac{2(n/p)}{1 + n/p}$$

$$N_{\text{eff}} = 2$$

$$N_{\text{eff}} = 3$$

$$N_{\text{eff}} = 4$$



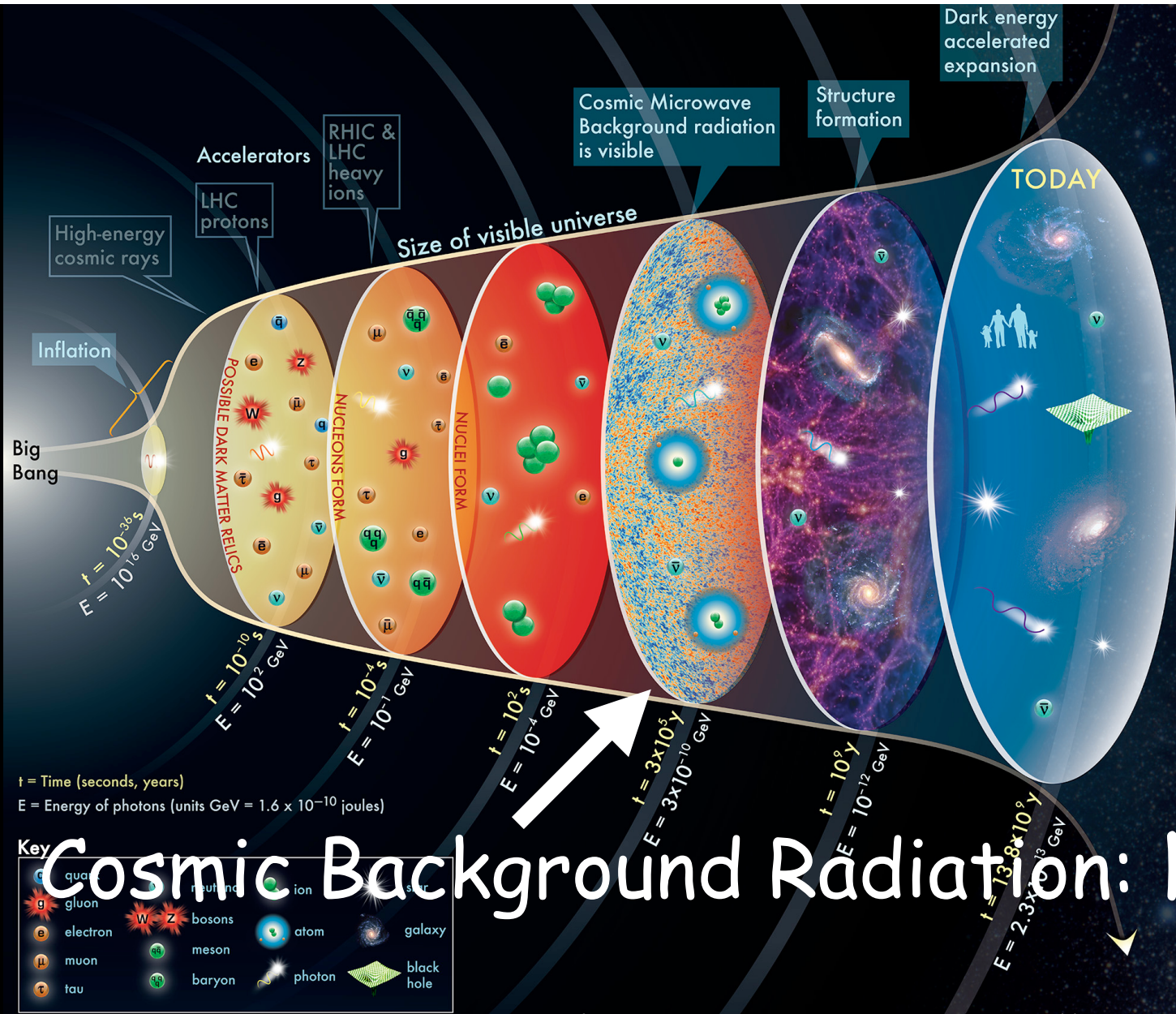
Fields, Olive, Yeh & Young JCAP '20

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Also known as “photon decoupling”, as photons started freely travel through the universe without interacting with matter and the CMB is “frozen”

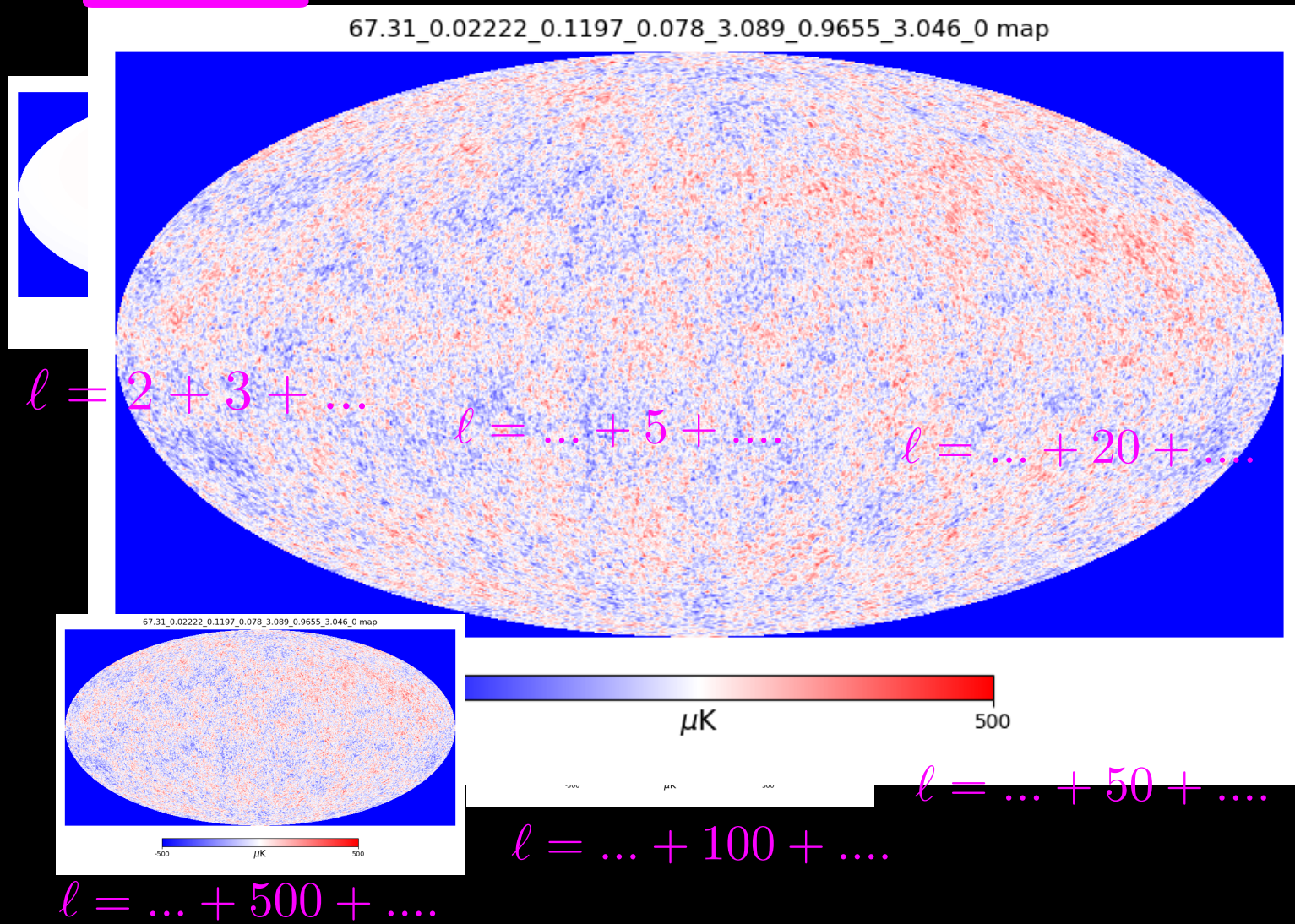
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# Cosmic Background Radiation: $N_{eff}$

The concept for the above figure originated in a 1986 paper by Michael Turner.

# CMB: $N_{\text{eff}}$





# CMB: $N_{\text{eff}}$

Spherical harmonics decomposition:

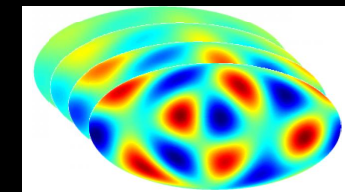
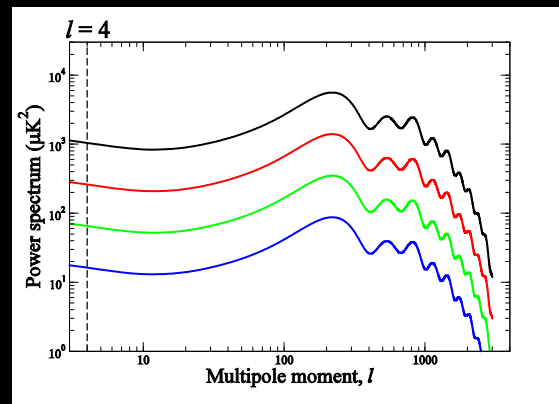
$$T(\hat{n}) = \sum_{\ell=0}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{n})$$

With expansion coefficients:

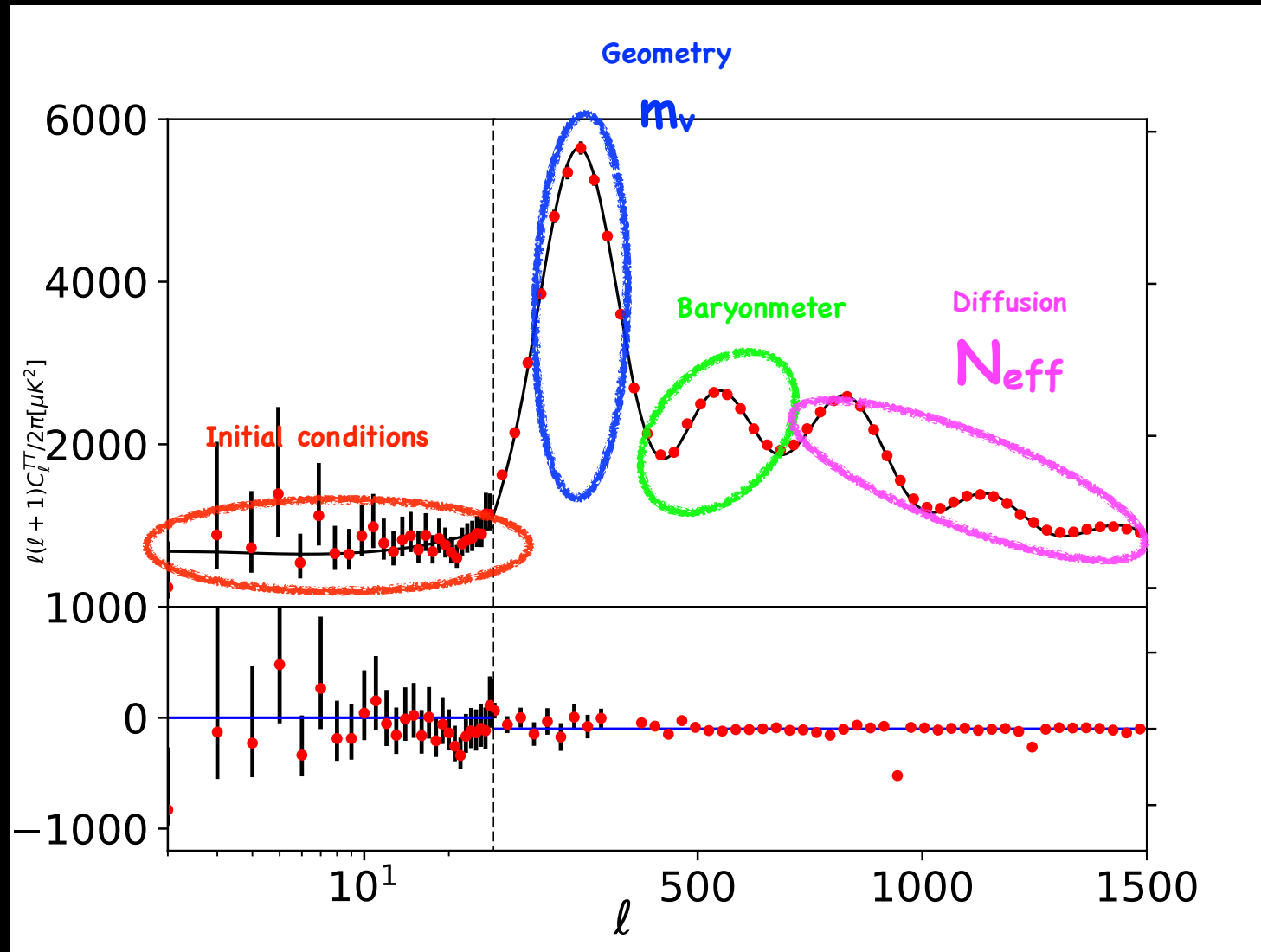
$$a_{\ell m} = \int_{4\pi} T(\hat{n}) Y_{\ell m}^*(\hat{n}) d\Omega$$

The angular power spectrum measures the amplitude of the expansion coefficients as a function of the wavelength:

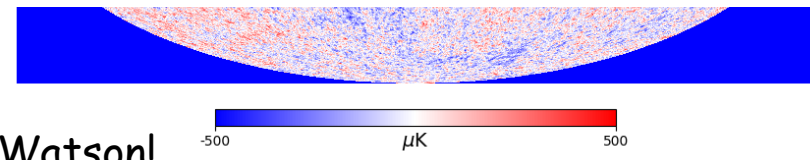
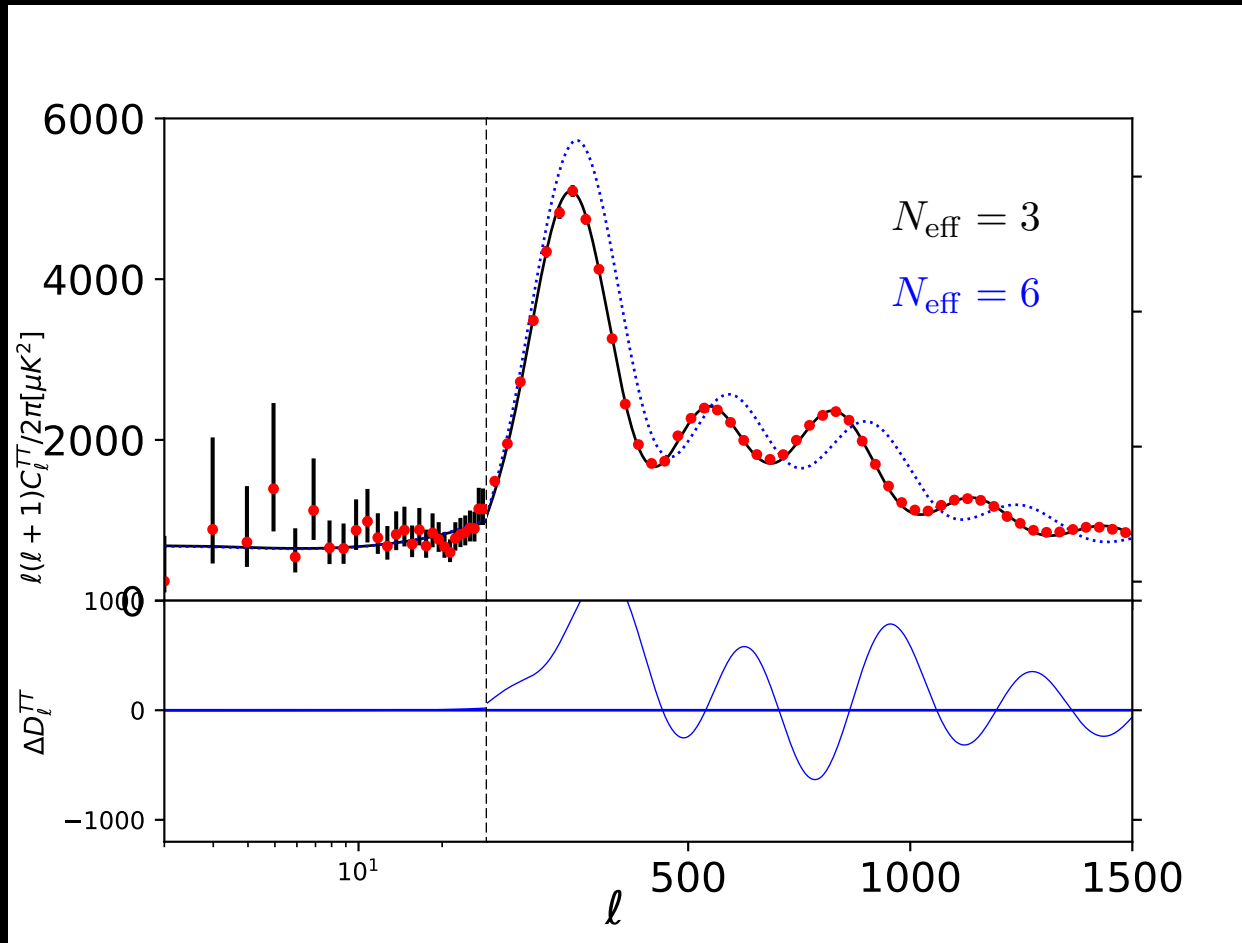
$$C_{\ell} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$



# CMB: a lot to learn about....



# CMB: $N_{\text{eff}}$

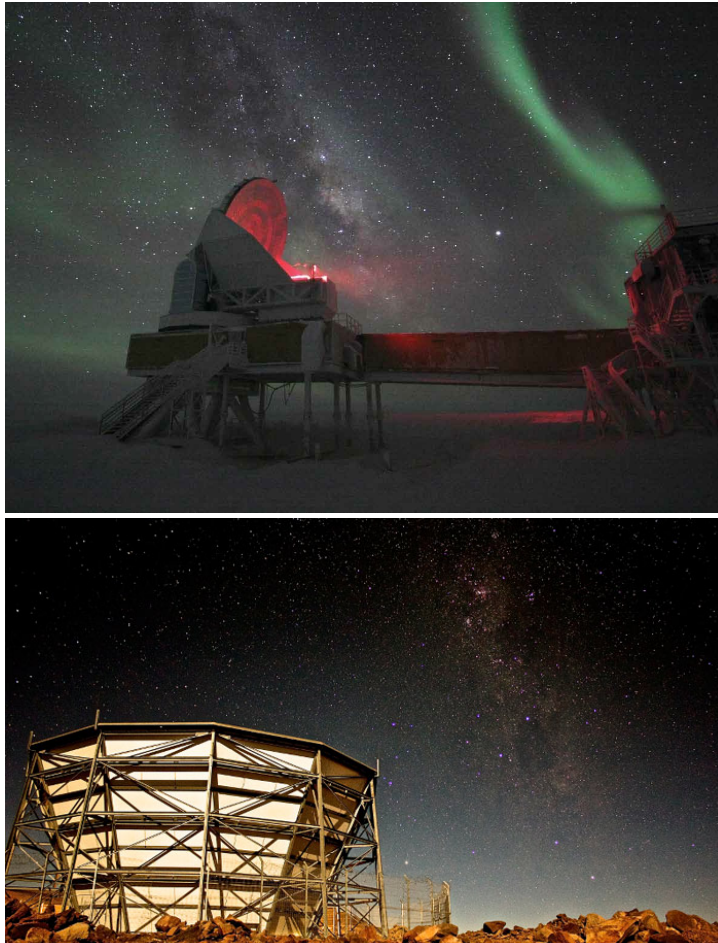


Elementary, my dear Watson!

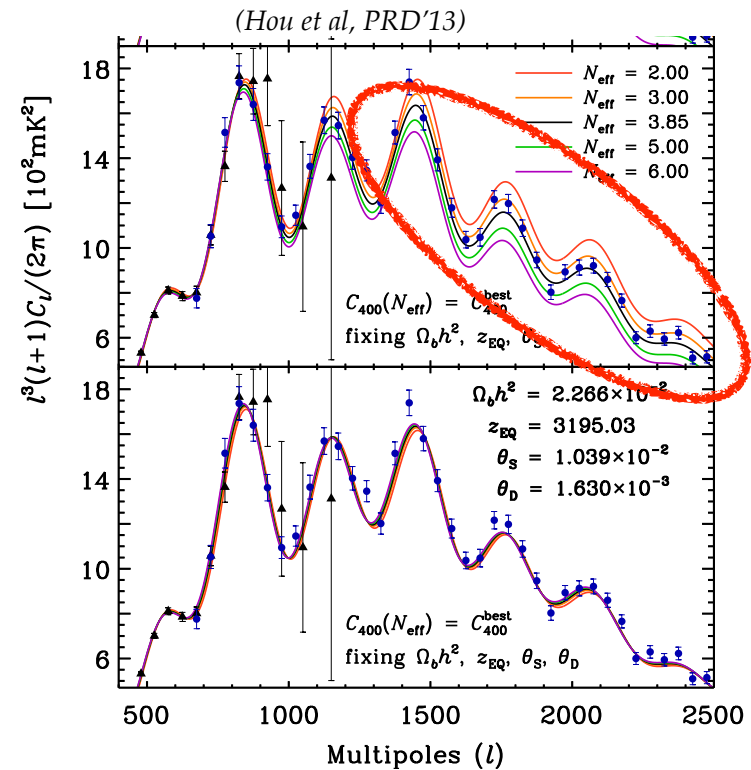
# CMB: $N_{\text{eff}}$

@Cosmic Microwave Background in the damping tail,  
measured by SPT, ACT & Planck:

Higher  $N_{\text{eff}}$  will increase the expansion rate AND  
the damping at high multipoles.

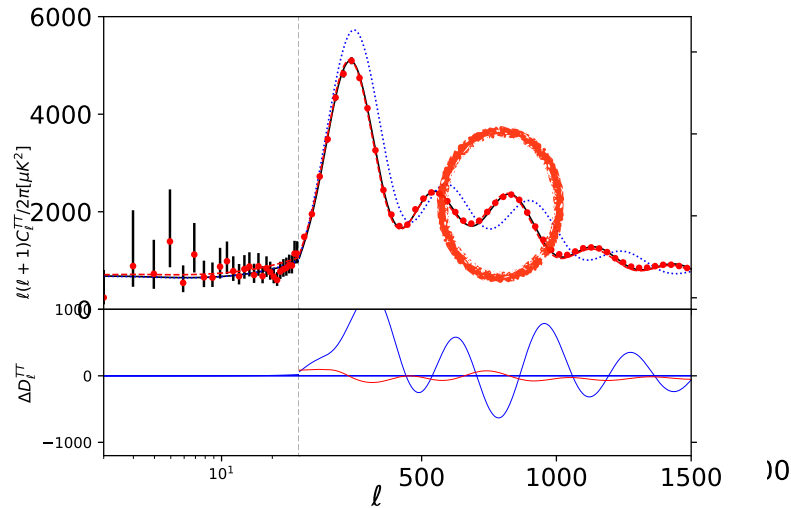


$$r_d^2 \propto \int_0^{a_*} \frac{da}{a^3 \sigma_T n_e H}$$



# CMB: $N_{\text{eff}}$

$$N_{\text{eff}} = 6 \quad N_{\text{eff}} = 3 \quad N_{\text{eff}} = 6$$

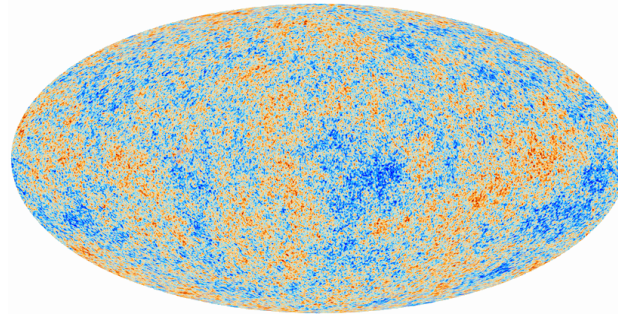


$$(\omega_b, \omega_m, h, A_s, n_s, \tau, N_{\text{eff}})$$

**Warning!**

It is elementary, Sherlock Holmes!

Only effect at  $l < 1000$  that can not be mimicked by others: anisotropic stress, around 3<sup>rd</sup> peak



Neutrinos are free-streaming particles propagating at the speed of light, faster than the sound speed in the photon fluid, suppressing the oscillation amplitude of CMB modes that entered the horizon in the radiation epoch.

# $N_{\text{eff}}$

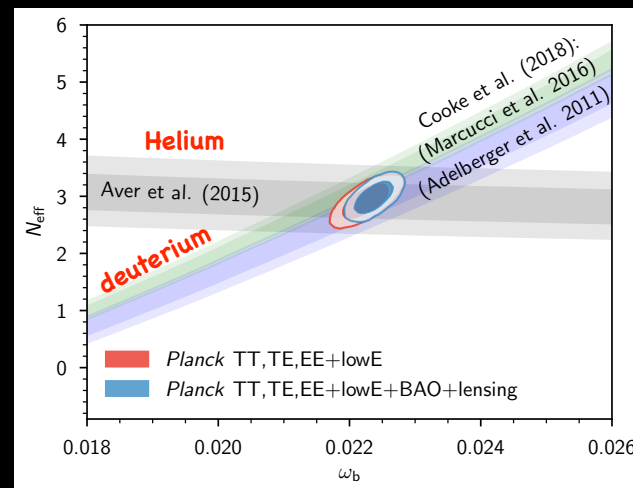
- Planck 2018 CMB temperature polarization and lensing potential data:

$$N_{\text{eff}} = 2.89^{+0.36}_{-0.38} \text{ 95\%CL}$$

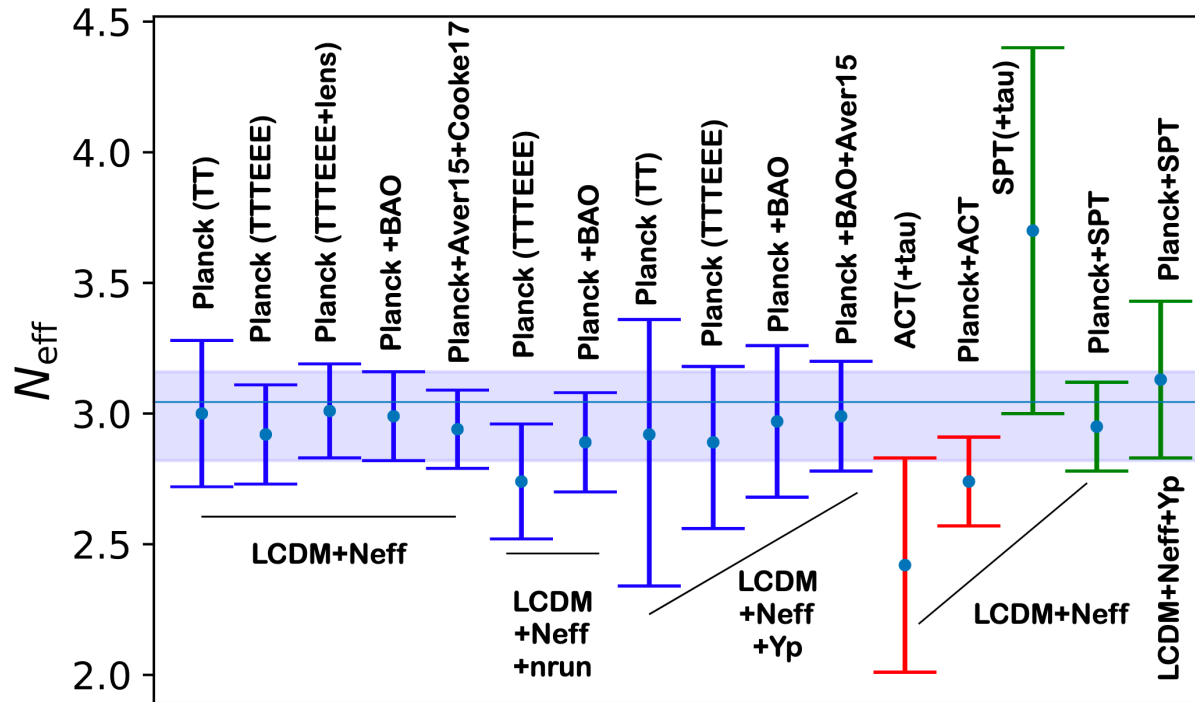
- If we add large scale structure information in the BAO shape form:

$$N_{\text{eff}} = 2.99^{+0.34}_{-0.33} \text{ 95\%CL}$$

- Perfectly consistent with BBN estimates:



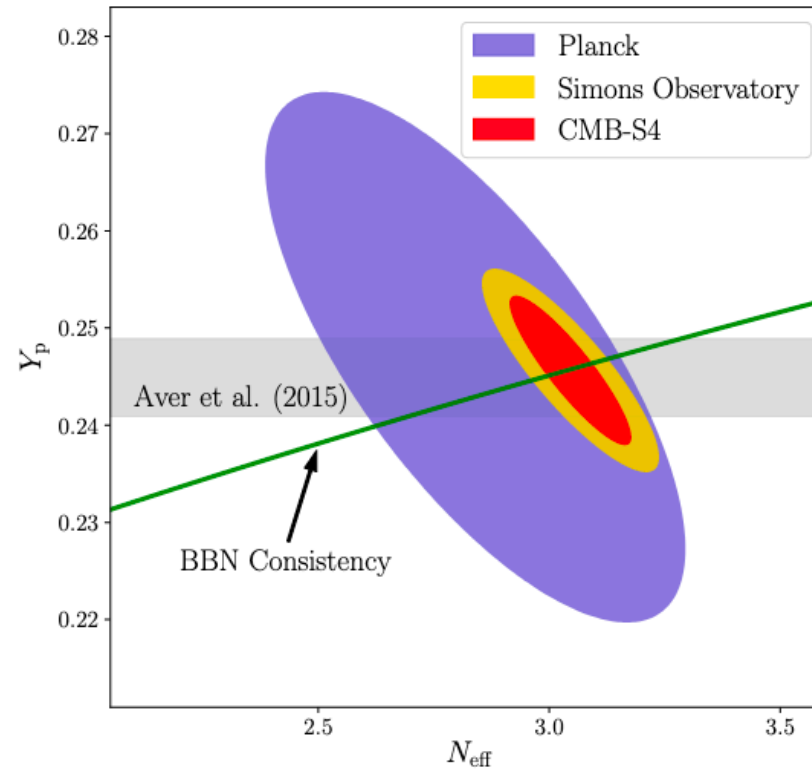
## N<sub>eff</sub>: current status



Planck collaboration, VI 2018  
 ACT Collaboration (Aiola+), 2020  
 SPT Collaboration (Dutcher+, Balkenhol+), 2021

# CMB Stage IV: $N_{\text{eff}}$

$$\Delta N_{\text{eff}} < 0.06 \text{ 95\%CL}$$



*CMB-S4 Science Case, Reference Design, and Project Plan, 1907.04473*



## HANDS-ON SESSION (II)!

- PLEASE GO TO THE WEB PAGE:

<https://alterbbn.hepforge.org/>

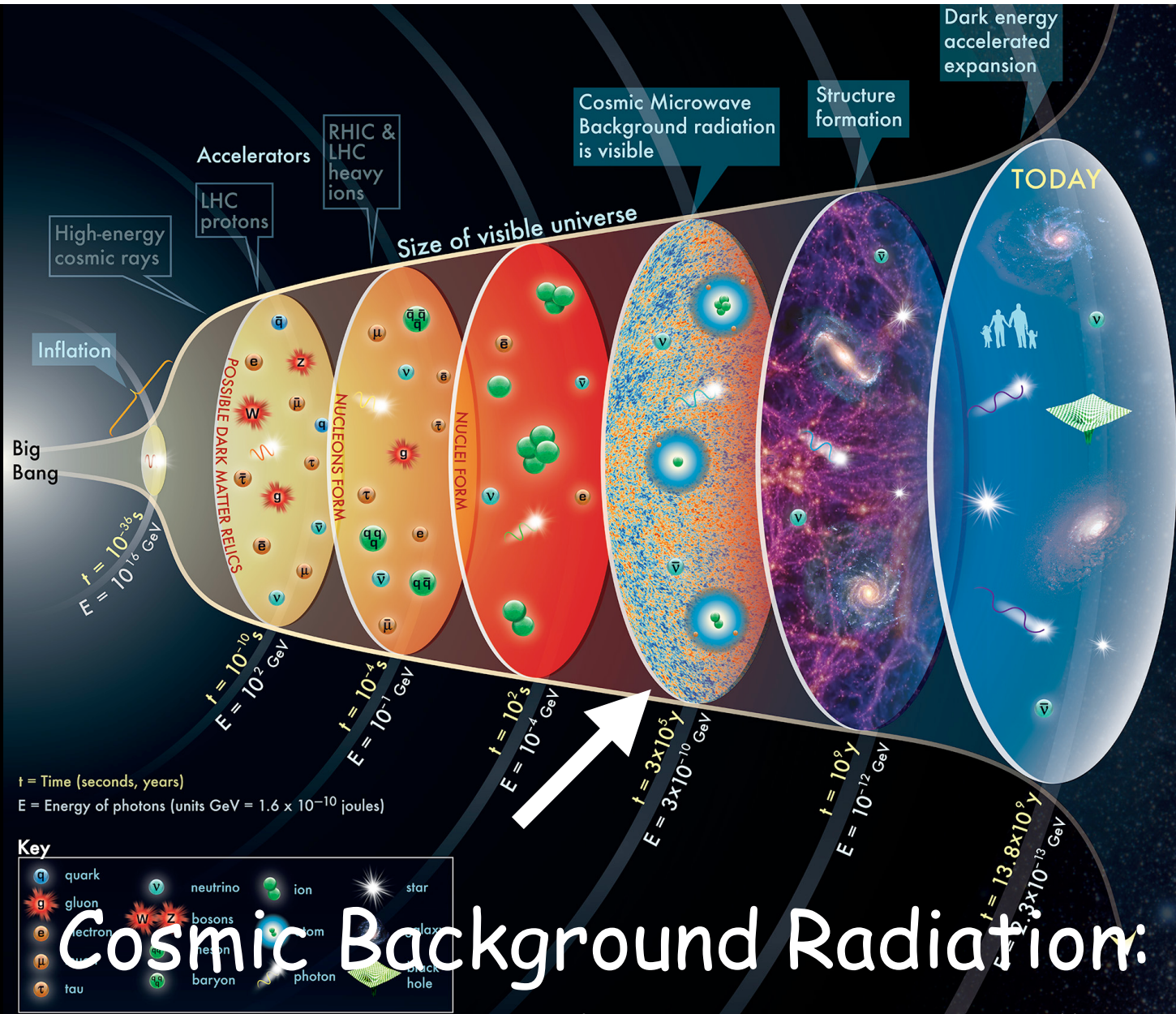
- DOWNLOAD THE CODE AND FOLLOW THE MANUAL TO INSTALL IT AND RUN IT:

<https://alterbbn.hepforge.org/manuals/alterbbn2.2.pdf>

- RUN THE CODE ALTER\_NEUTRINOS WITH THE TWO REQUIRED PARAMETERS  $N_{\text{eff}}$   $\Delta N_{\text{eff}}$
- FIX  $N_{\text{eff}}$  to 3.046 and change slightly  $\Delta N_{\text{eff}}$  from 0. to 1.

**WHICH MODELS ARE EXCLUDED BY BBN OBSERVATIONS? WHY?**

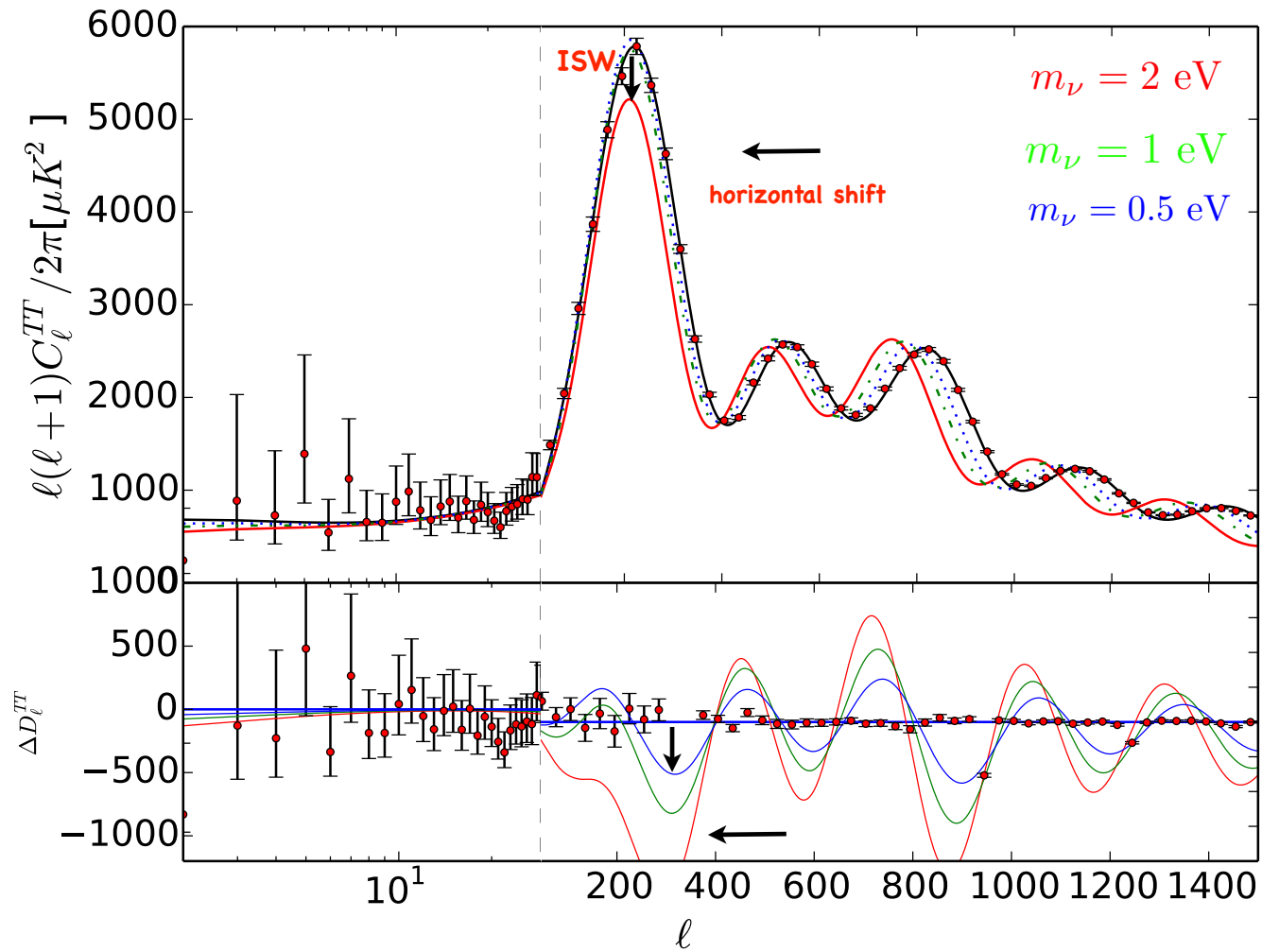
1. INTRODUCTION: FUNDAMENTAL INGREDIENTS & THERMAL HISTORY OF THE UNIVERSE ✓
2. NEUTRINO DECOUPLING IN THE EARLY UNIVERSE ✓
3. BIG BANG NUCLEOSYNTHESIS &  $N_{\text{eff}}$  ✓
4. COSMOLOGY &  $N_{\text{eff}}$  ✓
- 5. COSMOLOGY & NEUTRINO MASSES**
6. TAKE HOME MESSAGES



The concept for the above figure originated in a 1986 paper by Michael Turner.

# CMB: $\Sigma m_\nu$

@ CMB: Early Integrated Sachs Wolfe effect (ISW).  
Shift in the angular position of the peaks.



# CMB: $\Sigma m_\nu$

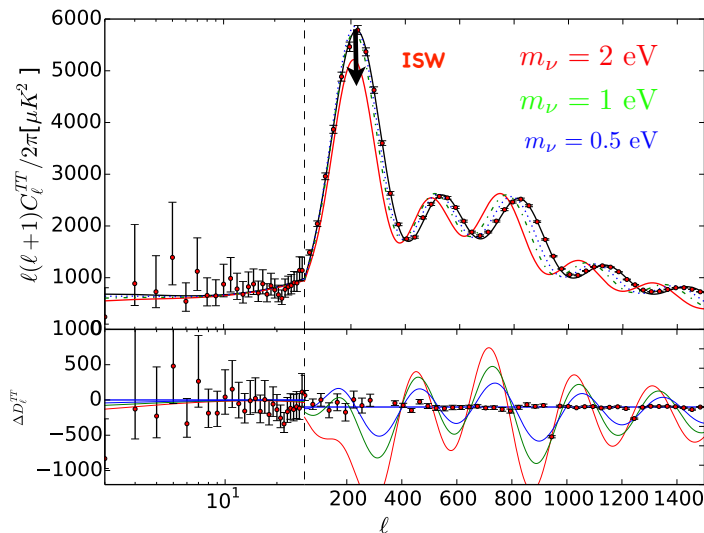
@ CMB: Early Integrated Sachs Wolfe effect (ISW)

$$\Theta(\hat{n}) = \frac{\delta T}{T}(\hat{n}) \simeq \Theta_0 + \Psi + \hat{n}(\hat{v}_e - v) + \int \dot{\Psi} + \dot{\Phi} d\eta$$

In matter domination, the gravitational potential is constant. **NO ISW effect!**

The transition **from the relativistic to the non relativistic neutrino regime** gets imprinted in the decays of the gravitational potentials near the recombination period, **contributing to the ISW effect!**

as an effect



This early ISW effect leads to a depletion of:

$$\frac{\Delta C_\ell}{C_\ell} = -(\sum m_\nu / 0.1 \text{ eV})\%$$

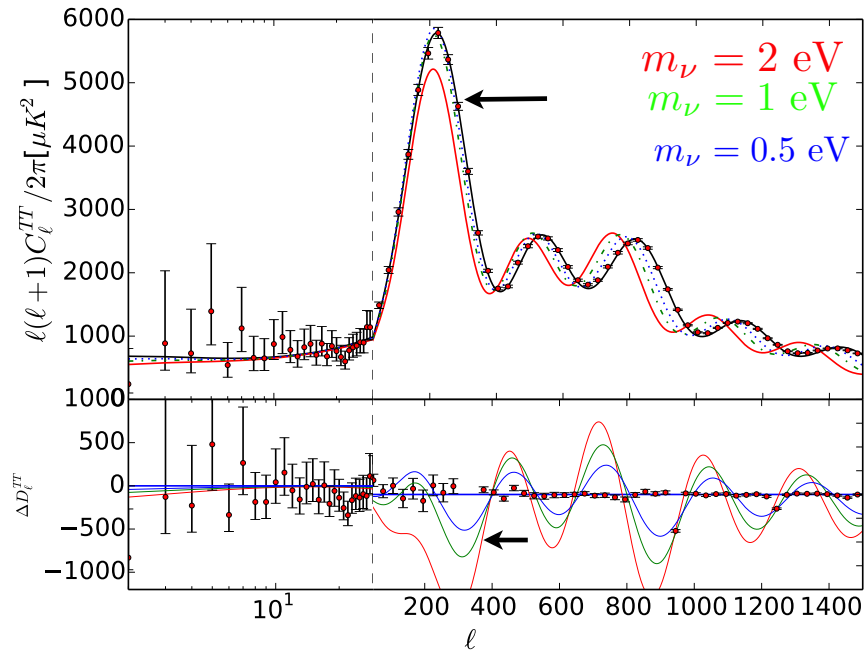
on multipoles:

$$20 < \ell < 200$$

# CMB: $\Sigma m_\nu$

@ CMB: Early Integrated Sachs Wolfe effect (ISW).

Shift in the angular position of the peaks.



$$\theta_s = \frac{r_s}{D_A}$$

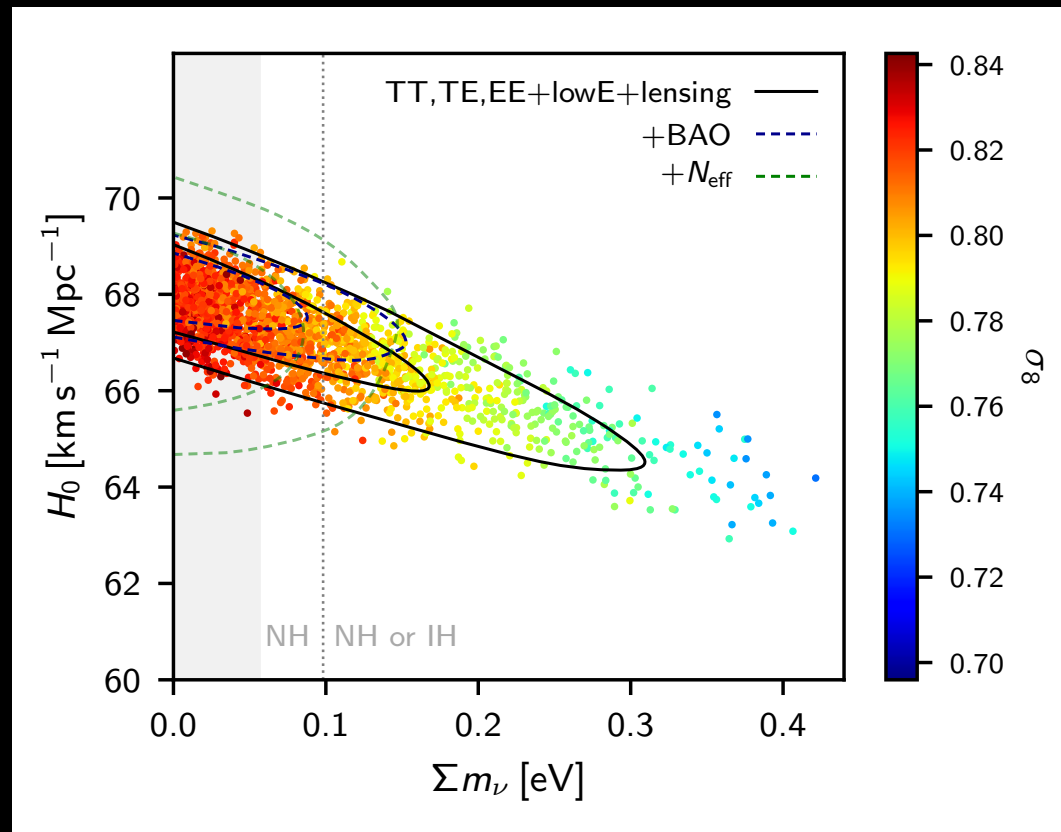
$$r_s = \int_0^{t(z_d)} c_s(1+z)dt = \frac{2}{3k_{\text{eq}}}\sqrt{\frac{6}{R_{\text{eq}}}} \ln \frac{\sqrt{1+R_d} + \sqrt{R_d+R_{\text{eq}}}}{1 + \sqrt{R_{\text{eq}}}}$$

$$D_A = \int_0^{z_{\text{rec}}} \frac{dz}{H(z)}$$

The higher the neutrino mass, the lower the angular diameter distance.

Peaks shift to lower multipoles. But this effect can be compensated with a lower Hubble constant:

**Strong degeneracy between  $\Sigma m_\nu$  and the Hubble constant  $H_0$ !**



Planck Coll. A&A'20

**Strong degeneracy between  $\Sigma m_\nu$  and the Hubble constant  $H_0$ !**

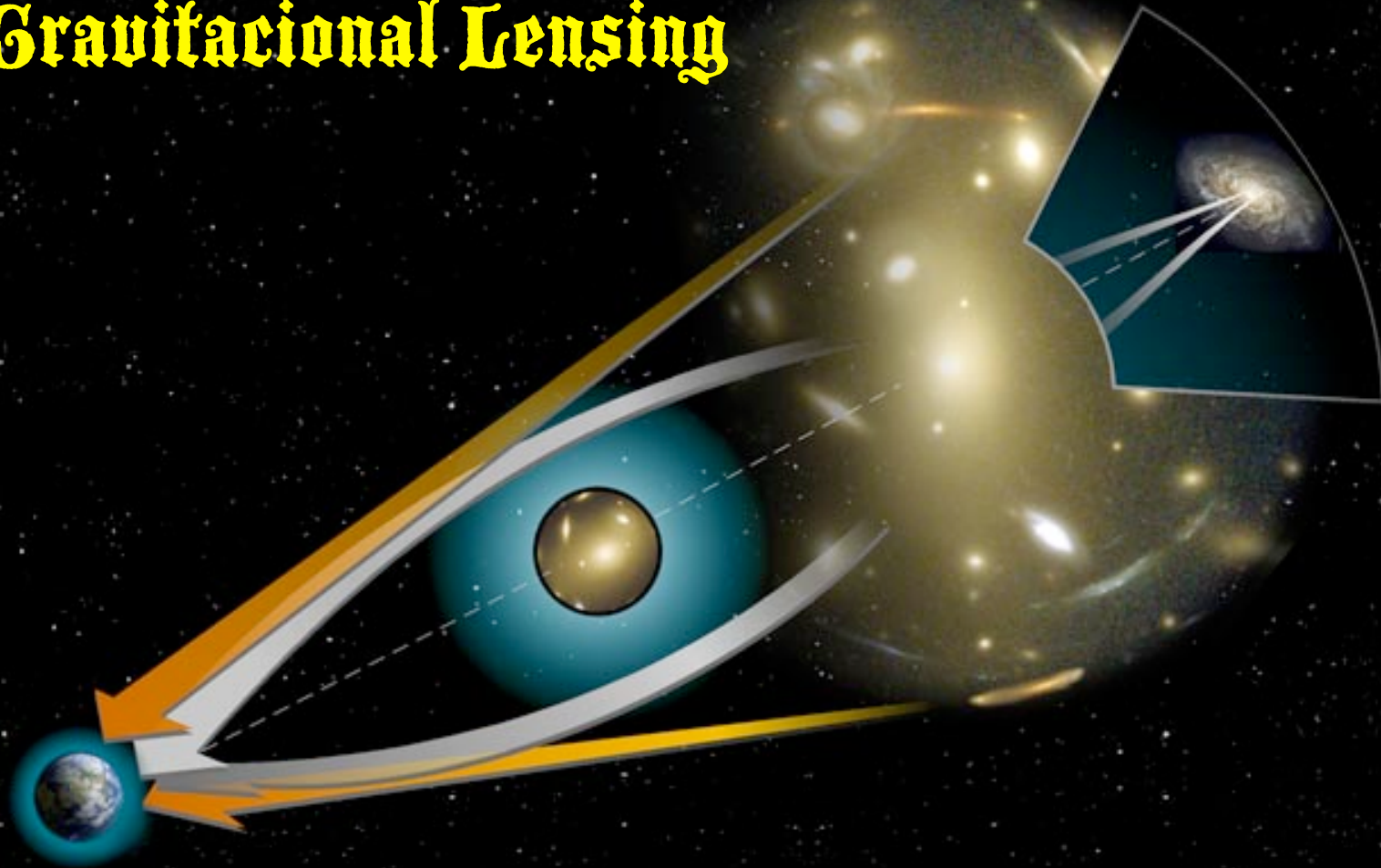
# CMB: $\Sigma m_\nu$

@ CMB: Early Integrated Sachs Wolfe effect (ISW).  
Shift in the angular position of the peaks.





# Gravitational Lensing



Einstein's relativity predicts that the presence of a massive body will curve space time, distorting the light trajectory. The shape of the background objects will change/multiplied by the presence of intervening galaxies.

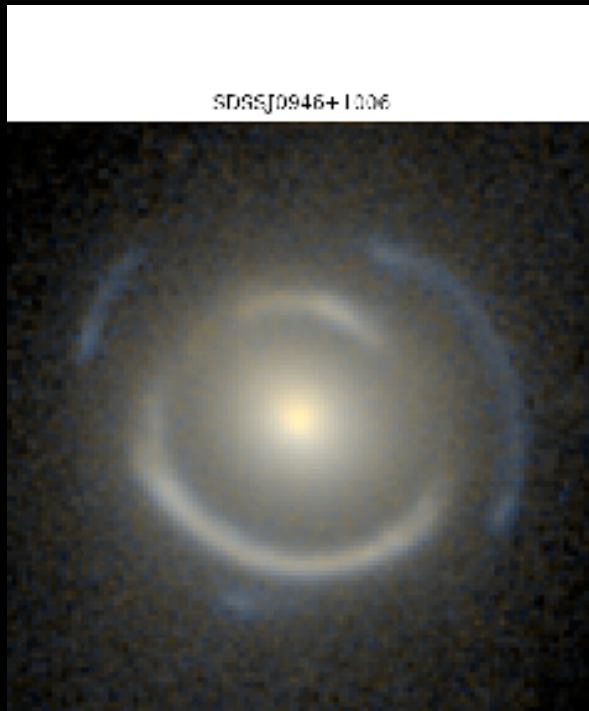
## Einstein rings: Perfect alignment: Syzygy!

Lensing Galaxy



This movie shows a spiral galaxy acting as a lense of a background quasar (Quasi-stellar radio source) moving behind the galaxy. When the alignment source-lens-observer is perfect, we see the formation of the Einstein ring!

# Gravitational Lensing



Double Einstein ring! 3 perfectly aligned galaxies (probably less than 100 cases in all the universe, and we have seen one!)

# CMB Lensing: $\Sigma m_\nu$

Lensing remaps the CMB fluctuations:

$$\Theta_{\text{lensed}}(\hat{n}) = \Theta(\hat{n} + \nabla\phi(\hat{n}))$$

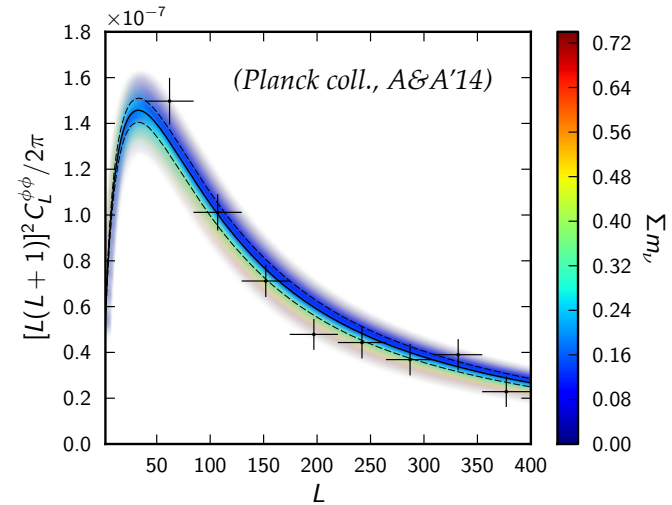
Lensing potential  $\phi$  is a measure of the integrated mass distribution back to the last scattering surface

$$\phi(\hat{n}) = -2 \int_0^{z_{\text{rec}}} \frac{dz}{H(z)} \underbrace{\Psi(z, D(z)\hat{n})}_{\text{Matter distribution}} \underbrace{\left( \frac{D(z_{\text{rec}}) - D(z)}{D(z_{\text{rec}})D(z)} \right)}_{\text{Geometry}}$$

$$C_L^{\phi\phi} = \frac{8\pi^2}{L^3} \int_0^{z_{\text{rec}}} \frac{dz}{H(z)} D(z) \left( \frac{D(z_{\text{rec}}) - D(z)}{D(z_{\text{rec}})D(z)} \right)^2 P_\Psi(z, k = L/D(z))$$

Neutrinos are hot relics with large thermal velocities, implying less clustering on small scales, reducing therefore CMB lensing!

(Kaplinghat et al PRL'03, Lesgourgues et al, PRD'06)

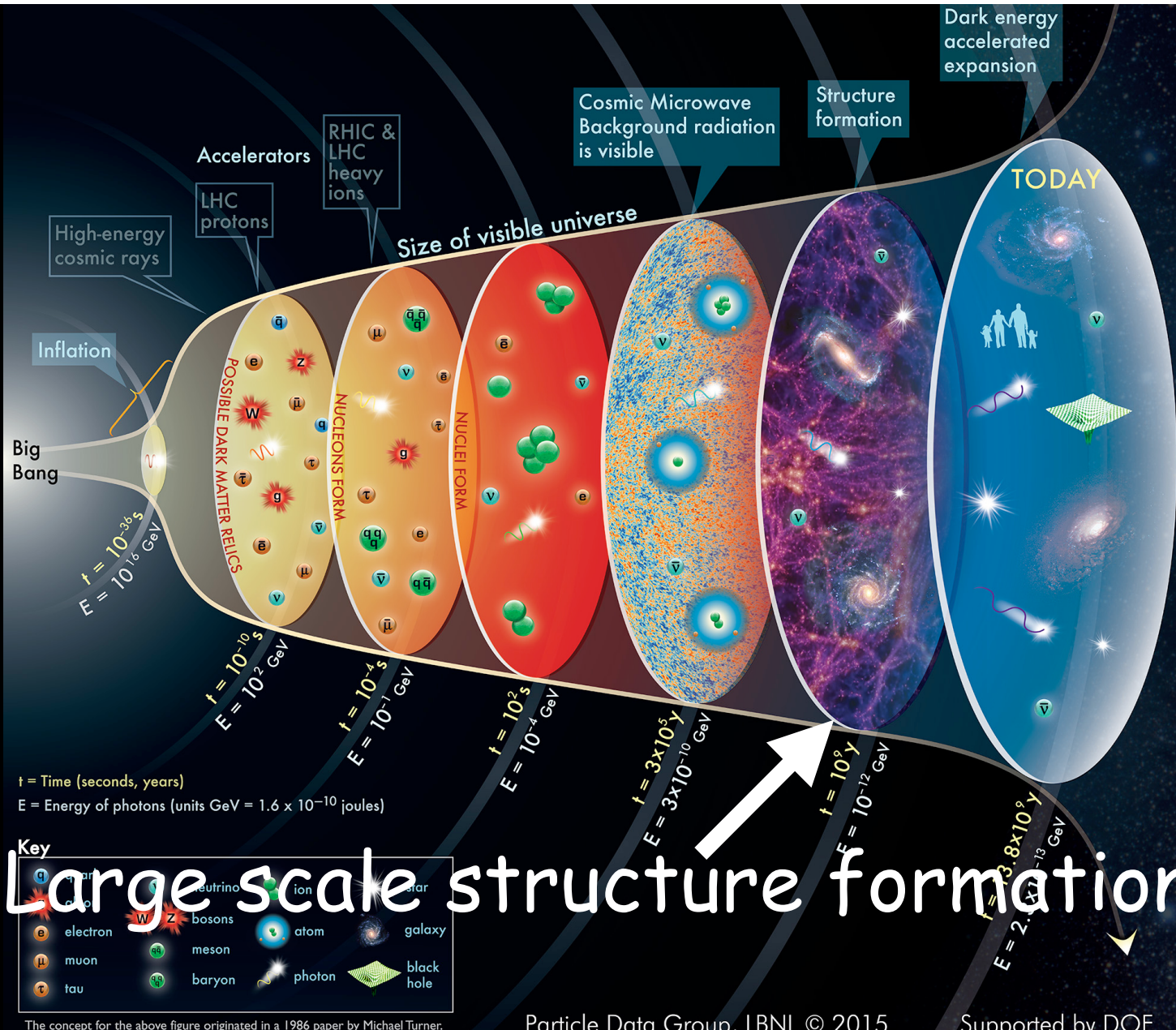


**CMB:  $\Sigma m_\nu$**

**Planck TTTEEE+lowT+lowE+lensing**

*Planck Coll. A&A'20*

$$\Sigma m_\nu < 0.24 \text{ eV } 95\% \text{CL}$$



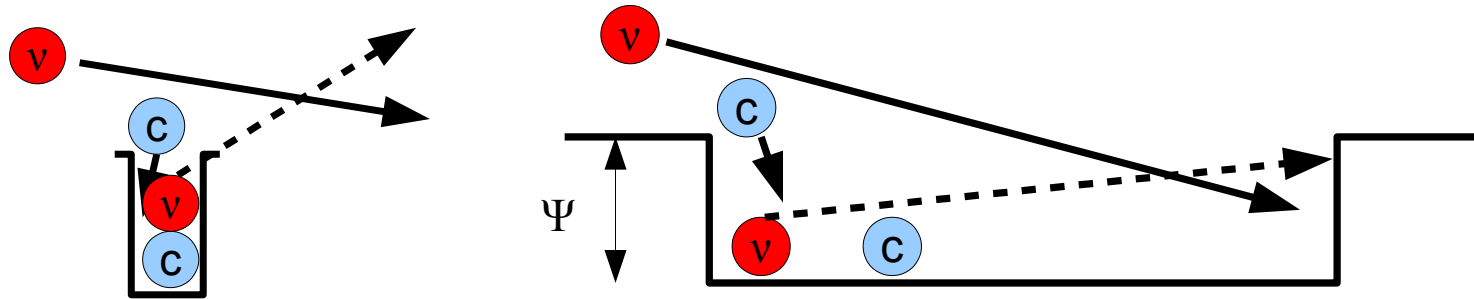
The concept for the above figure originated in a 1986 paper by Michael Turner.

# Large scale structure: $m_\nu$

Neutrino masses suppress structure formation on scales larger than their free streaming scale when they are **relativistic**. (*Bond et al PRL'80*)

Neutrinos with eV or sub-eV masses are **HOT** relics with **LARGE** thermal velocities!

Cold dark matter instead has zero velocity and therefore it clusters at any scale!



$$\lambda \ll \lambda_{fs,\nu} \rightarrow k \gg k_{fs,\nu}$$

$$\lambda \gg \lambda_{fs,\nu} \rightarrow k \ll k_{fs,\nu}$$

## Large scale structure: $m_\nu$

Growth equation for a single uncoupled fluid, linear regime, with constant sound speed:

$$\ddot{\delta} + \underbrace{2 \frac{\dot{a}}{a} \dot{\delta}}_{\text{Hubble drag}} - \underbrace{c_s^2 k^2 \frac{\delta}{a^2}}_{\text{Pressure}} = \underbrace{4\pi G \rho \delta}_{\text{Gravity}}$$

Jeans scale:

$$k_J \equiv \sqrt{\frac{4\pi G \rho}{c_s^2 (1+z)^2}}$$

$k > k_J$  no growth can occur

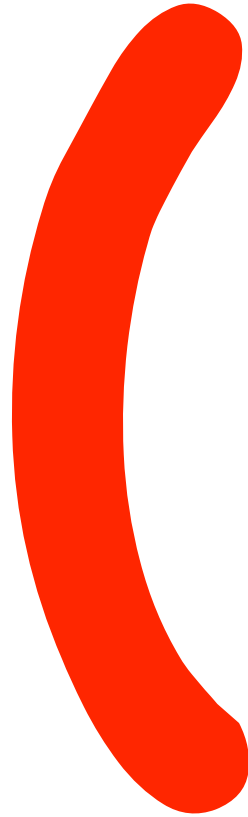
$k < k_J$  density perturbations growth

Neutrino free streaming scale:

$$k_{fs,\nu}(z) \equiv \sqrt{\frac{3}{2}} \frac{H(z)}{(1+z)\sigma_{\nu,\nu}(z)}$$



Compute the neutrino free streaming scale



$$k_{\text{fs},i}(z) \equiv \sqrt{\frac{3}{2}} \frac{H(z)}{(1+z)\sigma_{v,i}(z)},$$

$$\begin{aligned} \sigma_{v,i}^2(z) &\equiv \frac{\int \frac{d^3p}{\exp[p/T_\nu(z)]+1} \frac{p^2/m^2}{\exp[p/T_\nu(z)]+1}}{\int \frac{d^3p}{\exp[p/T_\nu(z)]+1}} = \frac{15\zeta(5)}{\zeta(3)} \frac{T_\nu^2(z)}{m_{\nu,i}^2} \\ &= \frac{15\zeta(5)}{\zeta(3)} \left(\frac{4}{11}\right)^{2/3} \frac{T_\gamma^2(0)(1+z)^2}{m_{\nu,i}^2}, \end{aligned}$$

$$k_{\text{fs},i}(z) \simeq \frac{0.677}{(1+z)^{1/2}} \left(\frac{m_{\nu,i}}{1 \text{ eV}}\right) \Omega_{\text{m}}^{1/2} h \text{ Mpc}^{-1}.$$

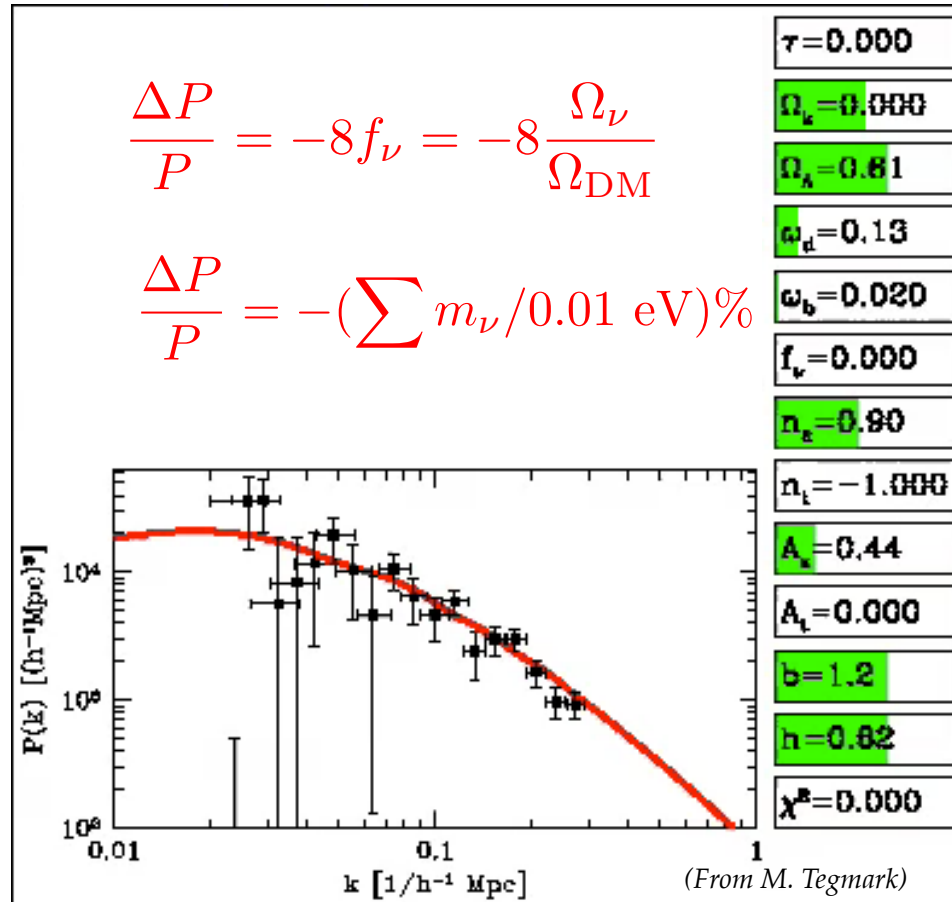


# Large scale structure: $m_\nu$

Matter power spectrum suppression:

$$\frac{\Delta P}{P} = -8f_\nu = -8\frac{\Omega_\nu}{\Omega_{DM}}$$

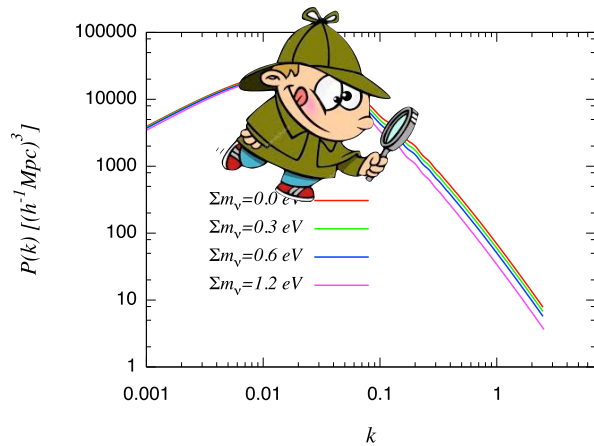
$$\frac{\Delta P}{P} = -(\sum m_\nu / 0.01 \text{ eV})\%$$



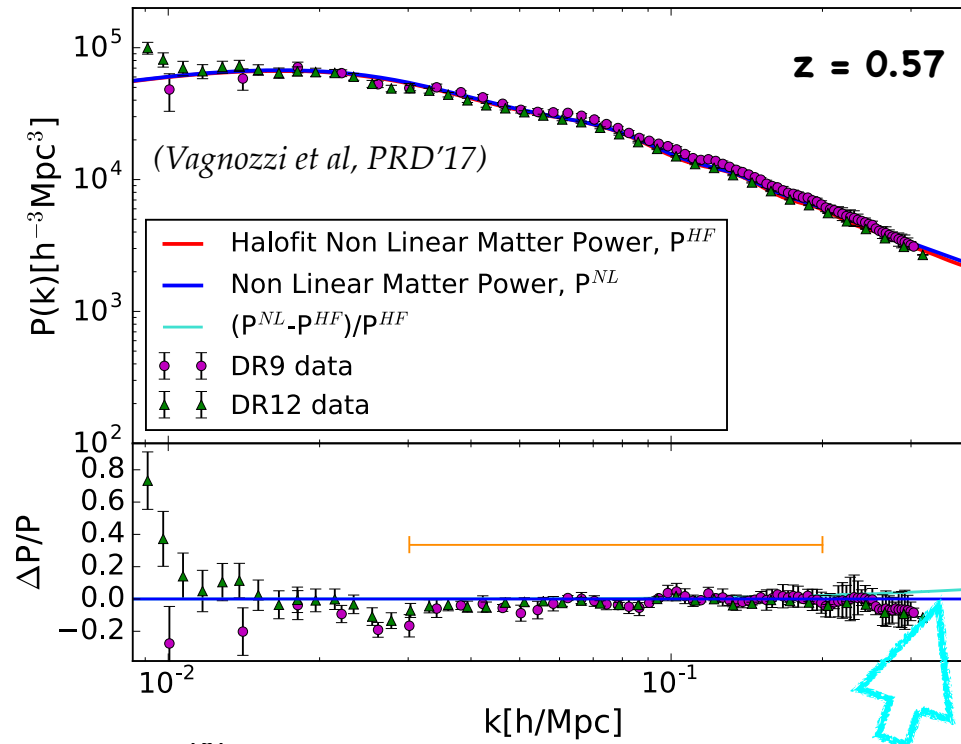
100 ↑ Small scales

# Large scale structure: $m_\nu$

@LSS: Caveats, NON-LINEARITIES



Beyond a given scale  $k_{nl}$ , linear perturbation theory breaks down!



## Large scale structure: $m_\nu$

@LSS: Caveats, BIAS!

$$P_{gg}(k, z) = \textit{bias}^2 P(k, z)$$

Galaxies are **biased** tracers of the underlying matter density field! (*Kaiser, APJ'84*)

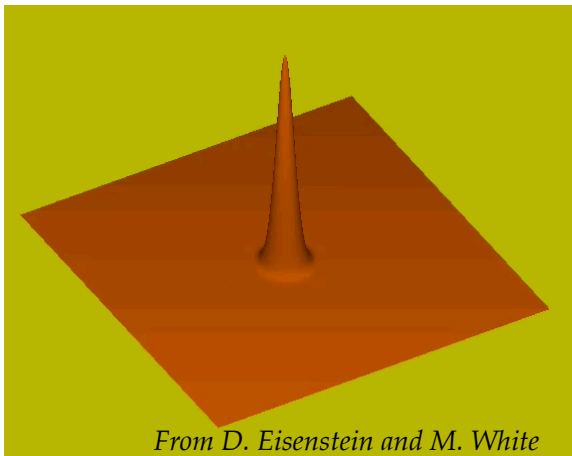
Neutrinos themselves induce a scale-dependent bias (*LoVerde & Zaldarriaga; Castorina et al*)

## Baryon Acoustic Oscillations

Photons and baryons in the early universe behave as a tightly coupled fluid, resembling acoustic waves, generated as the baryon-photon fluid is attracted and falls onto the overdensities:

$$\ddot{\delta} + [\text{Pressure} - \text{Gravity}]\delta = 0 \quad \delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \bar{\rho}(\vec{x})}{\bar{\rho}(\vec{x})}$$

The time when the baryons are “released” from the drag of the photons is known as the drag epoch. From then on photons expand freely while the acoustic waves “freeze in” the baryons at a scale given by the size of the horizon at the drag epoch:



$$R \equiv 3\rho_b/4\rho_\gamma$$

$$r_s = \int_0^{t(z_d)} c_s (1+z) dt = \frac{2}{3k_{\text{eq}}} \sqrt{\frac{6}{R_{\text{eq}}}} \ln \frac{\sqrt{1+R_d} + \sqrt{R_d + R_{\text{eq}}}}{1 + \sqrt{R_{\text{eq}}}}$$

$$r_s = 147.09 \pm 0.26 \text{ Mpc}$$

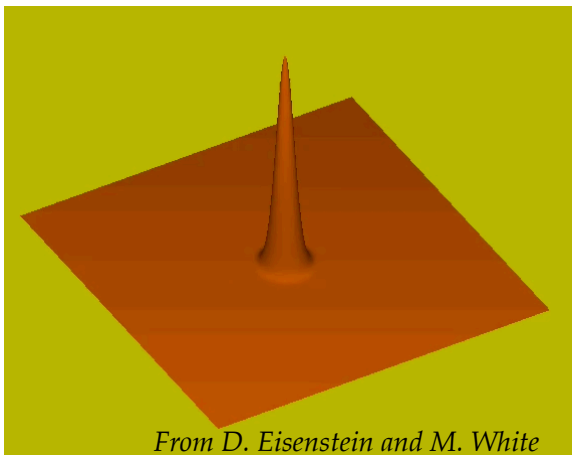
Planck Coll. A&A'20

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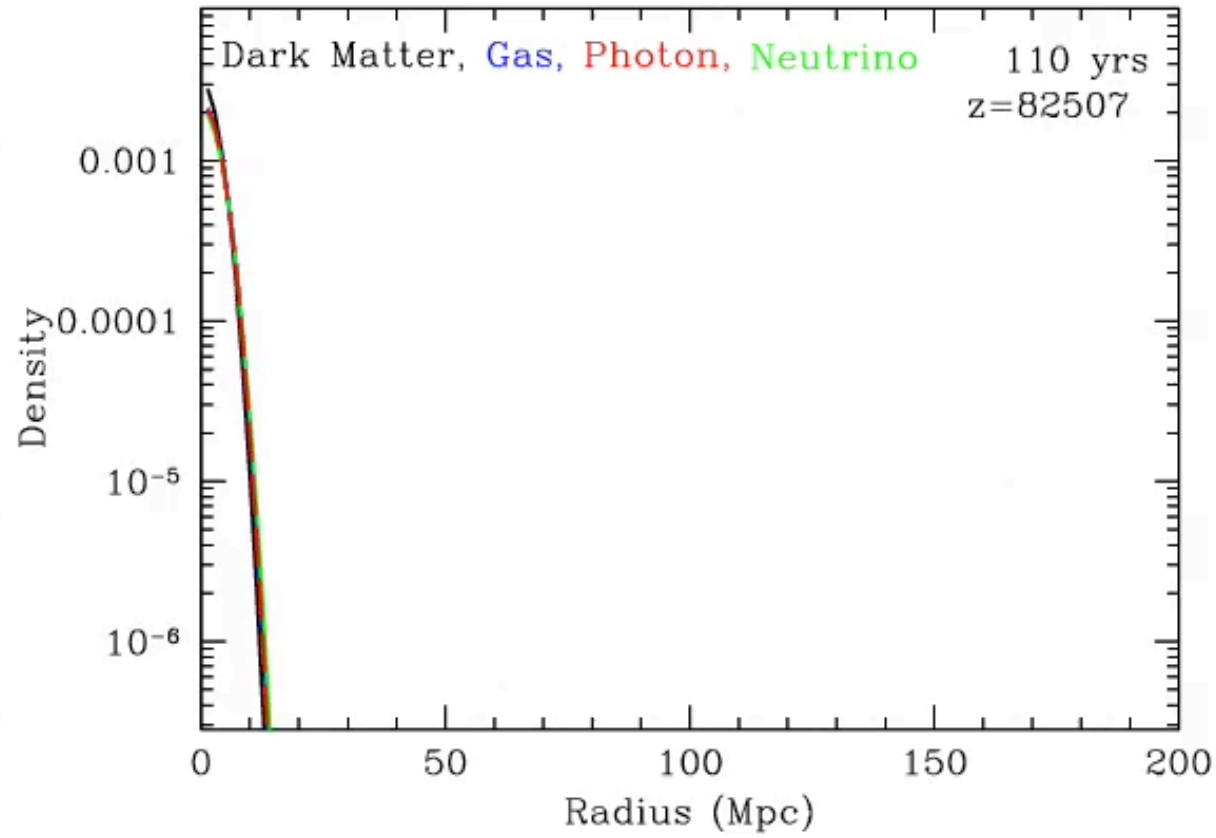
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Planck Coll. A&A'20



## Baryon Acoustic Oscillations



*From D. Eisenstein and M. White*

## Baryon Acoustic Oscillations

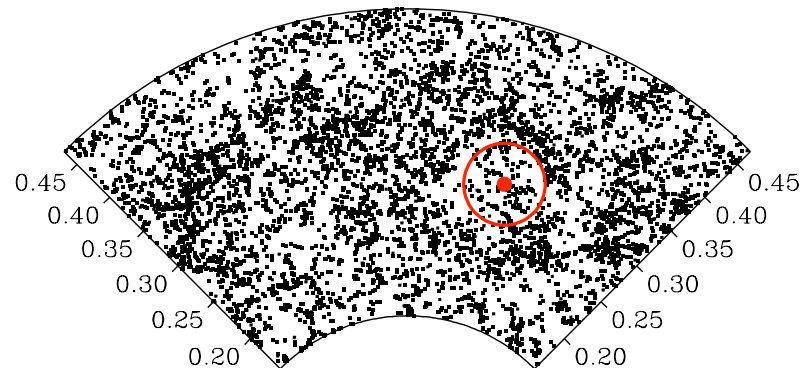
Photons and baryons in the early universe behave as a tightly coupled fluid, resembling acoustic waves, generated as the baryon-photon fluid is attracted and falls onto the overdensities:

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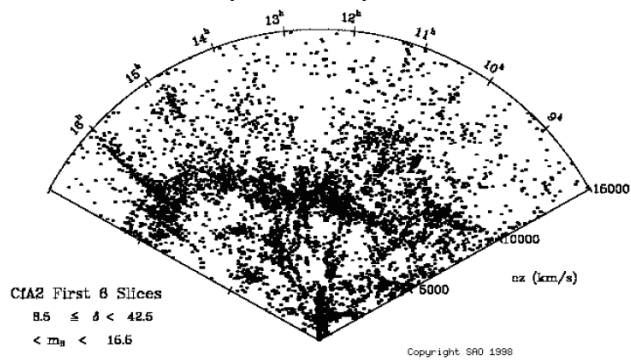
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$$r_s = 147.09 \pm 0.26 \text{ Mpc} \quad \text{Planck Coll. A\&A'20}$$

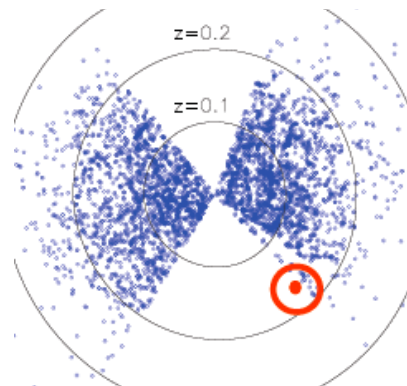
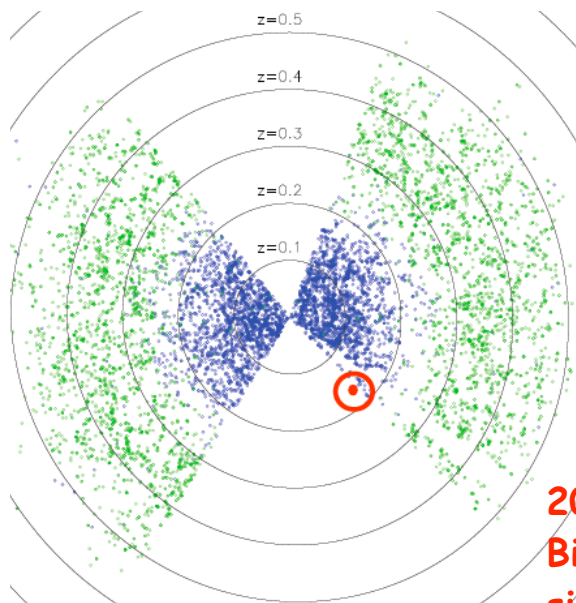
There should be a small excess in the two-point galaxy correlation function around 150 Mpc!



# 80's: Tiny surveys **Baryon Acoustic Oscillations**



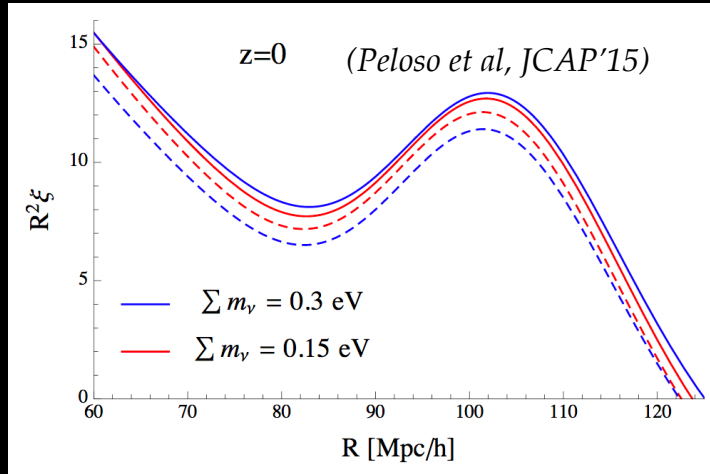
BAO scale



2000: Main galaxies @SDSS.  
Big number, but small volume

2005: Luminous Red Galaxies @ SDSS.  
Big Volume: first detection of the BAO signature

# Large scale structure: $m_\nu$



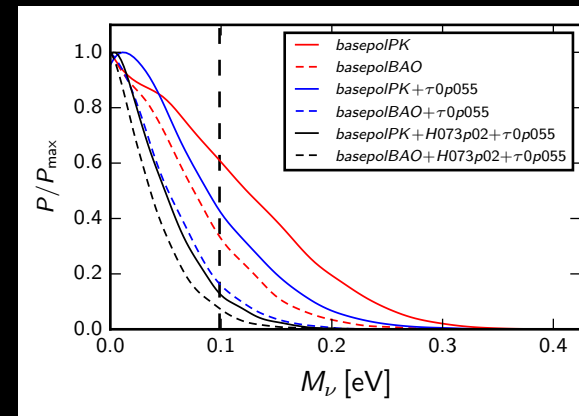
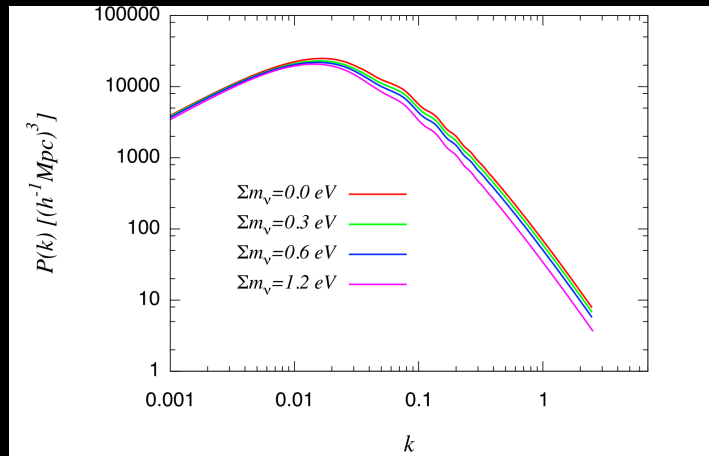
Large scale structure measurements can be interpreted either in the geometrical or shape forms

2 point correlation function

Matter power spectrum

Fourier Transform

BAO information more powerful



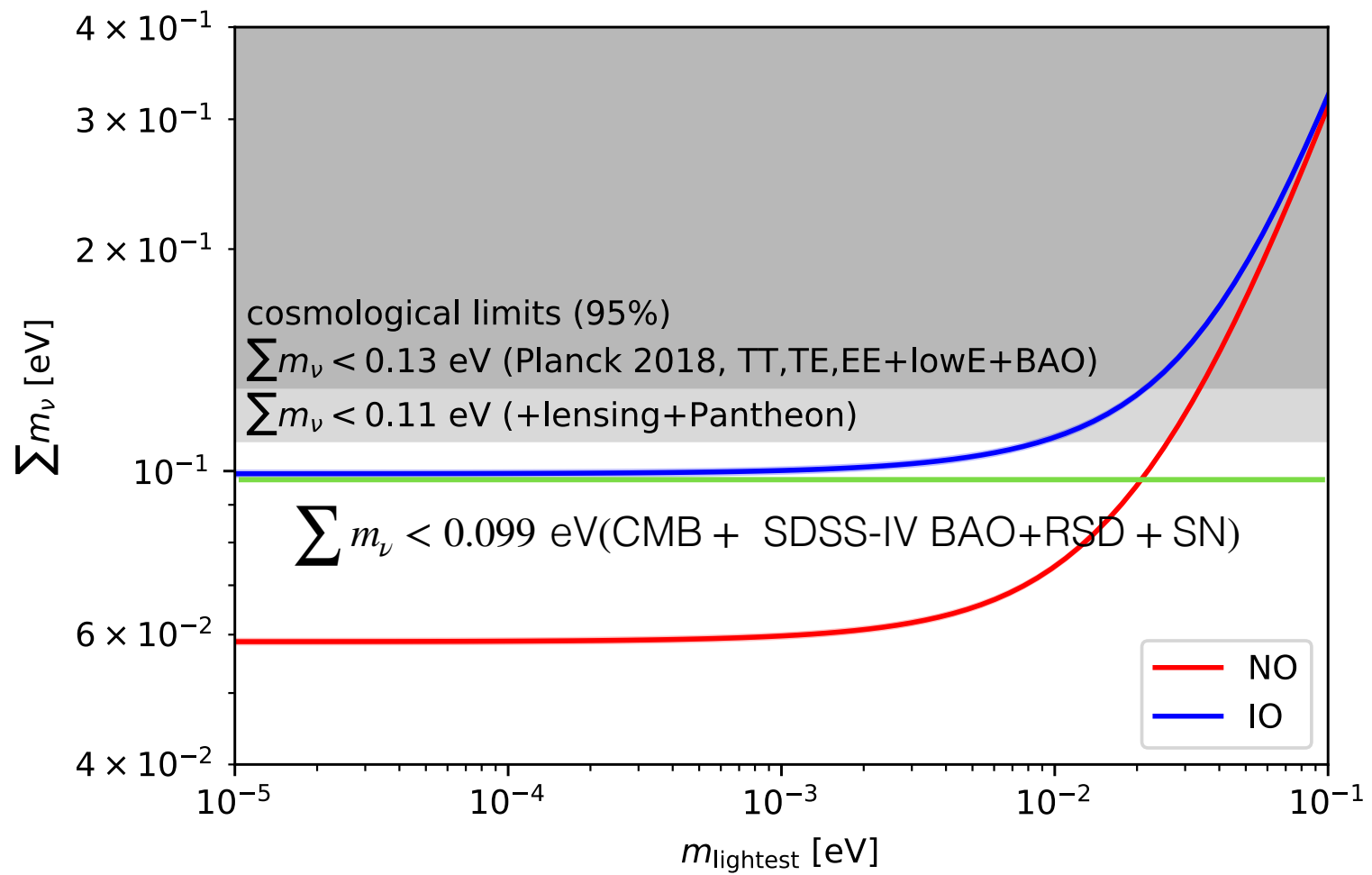
$$\Sigma m_\nu$$

- Planck 2018 CMB temperature polarization and lensing potential data:

$$\Sigma m_\nu < 0.24 \text{ eV } 95\% \text{CL}$$

- If we add large scale structure information in its BAO form

$$\Sigma m_\nu < 0.12 \text{ eV } 95\% \text{CL}$$



$\Sigma m_\nu$ 

Planck Coll. A&amp;A'20

Planck TTTEEE+lowT+lowE+lensing

$$\Sigma m_\nu < 0.24 \text{ eV } 95\% \text{CL}$$

+ BAO

$$\Sigma m_\nu < 0.12 \text{ eV } 95\% \text{CL}$$

+ BAO + SNIa

$$\Sigma m_\nu < 0.11 \text{ eV } 95\% \text{CL}$$

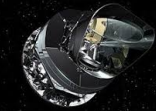
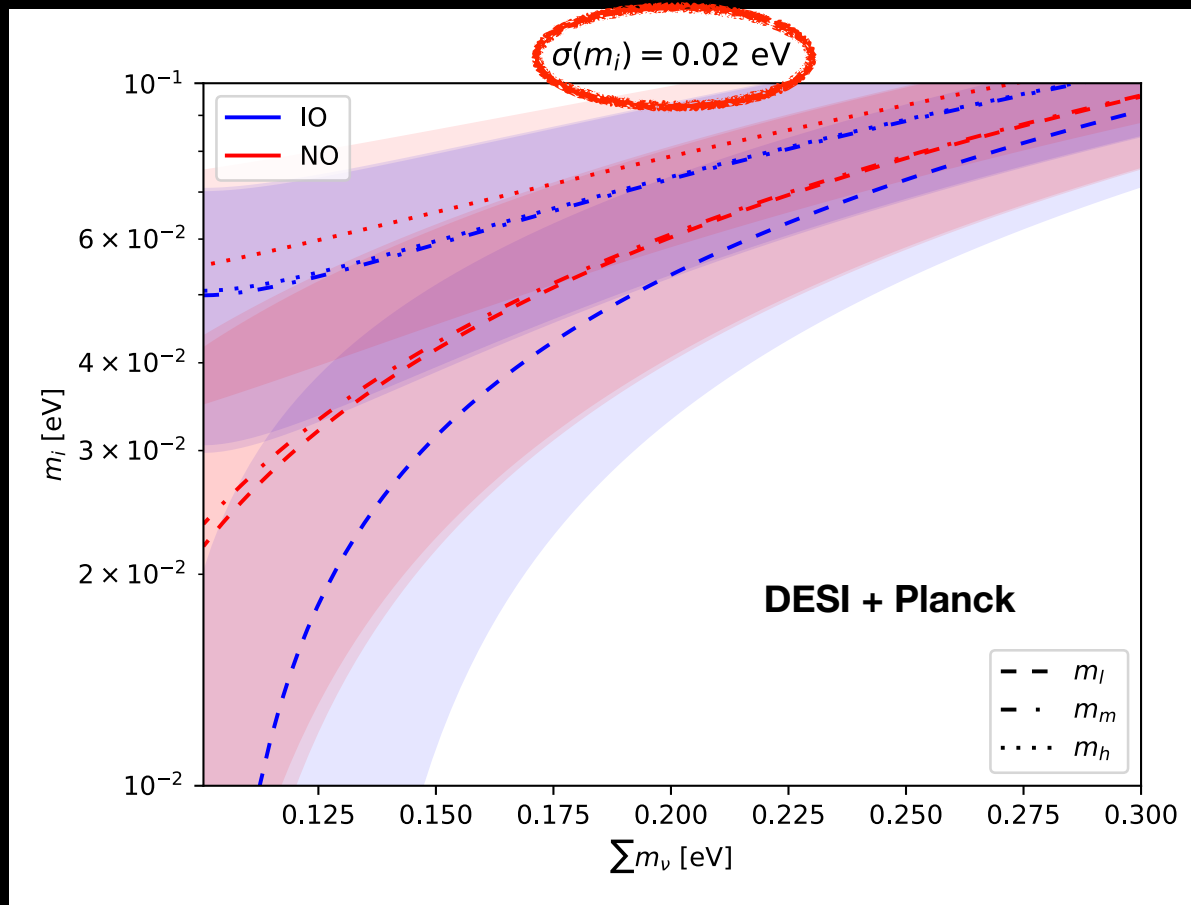
+ SDSS-IV (BAO + RSD) + SNIa

$$\Sigma m_\nu < 0.099 \text{ eV } 95\% \text{CL} \quad \text{eBOSS Coll. PRD'21}$$

+ BAO + SNIa +  $H_0 = 73.45 \pm 1.66 \text{ km/s/Mpc}$  *Riess et al, APJ'18*

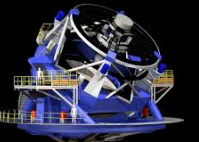
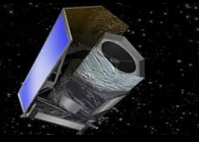
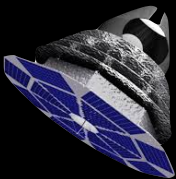
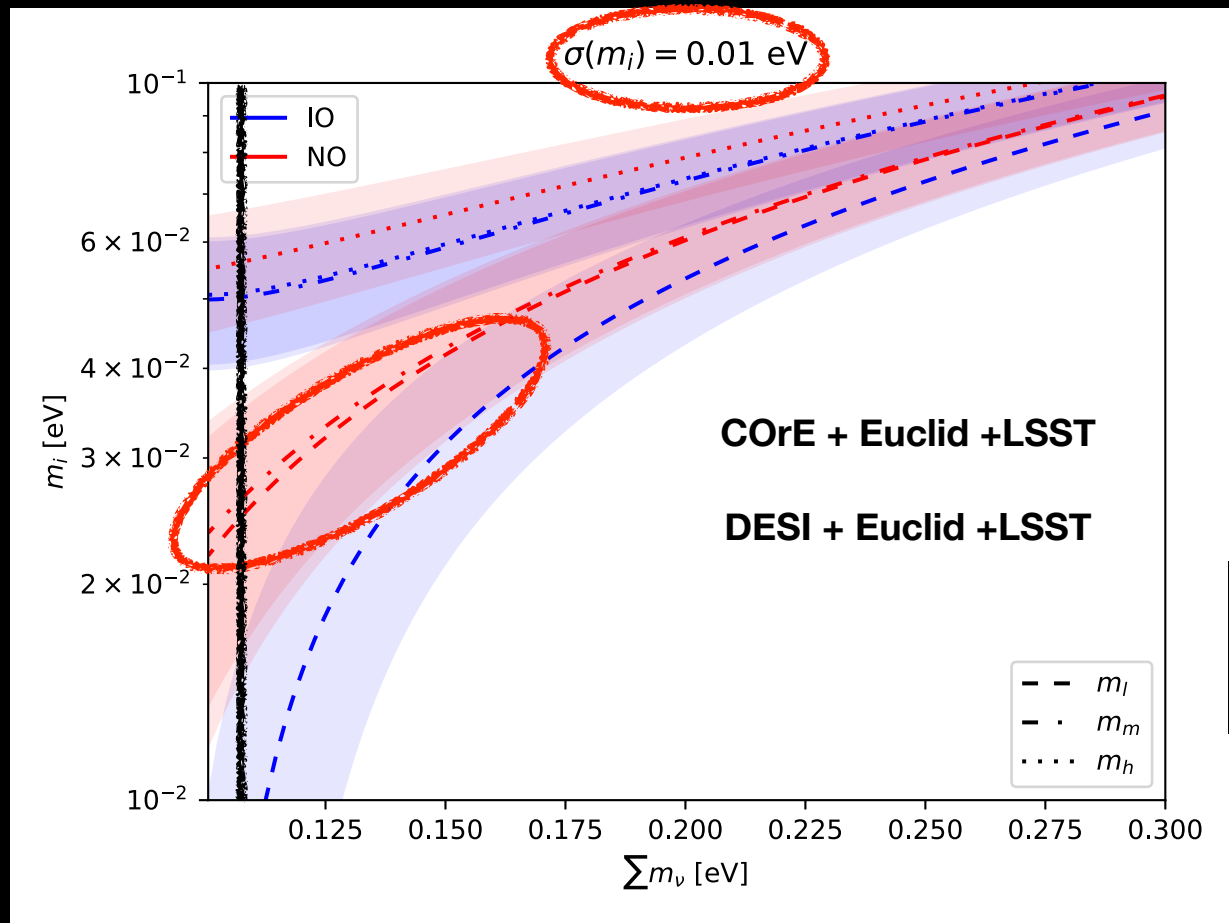
$$\Sigma m_\nu < 0.0970 \text{ eV } 95\% \text{CL}$$

- Neutrino mass ordering

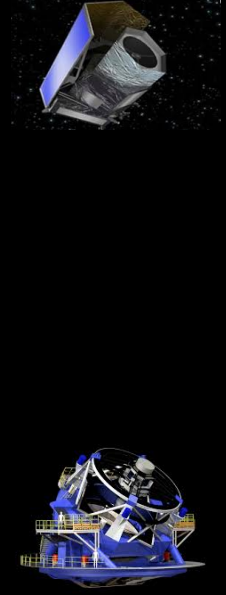
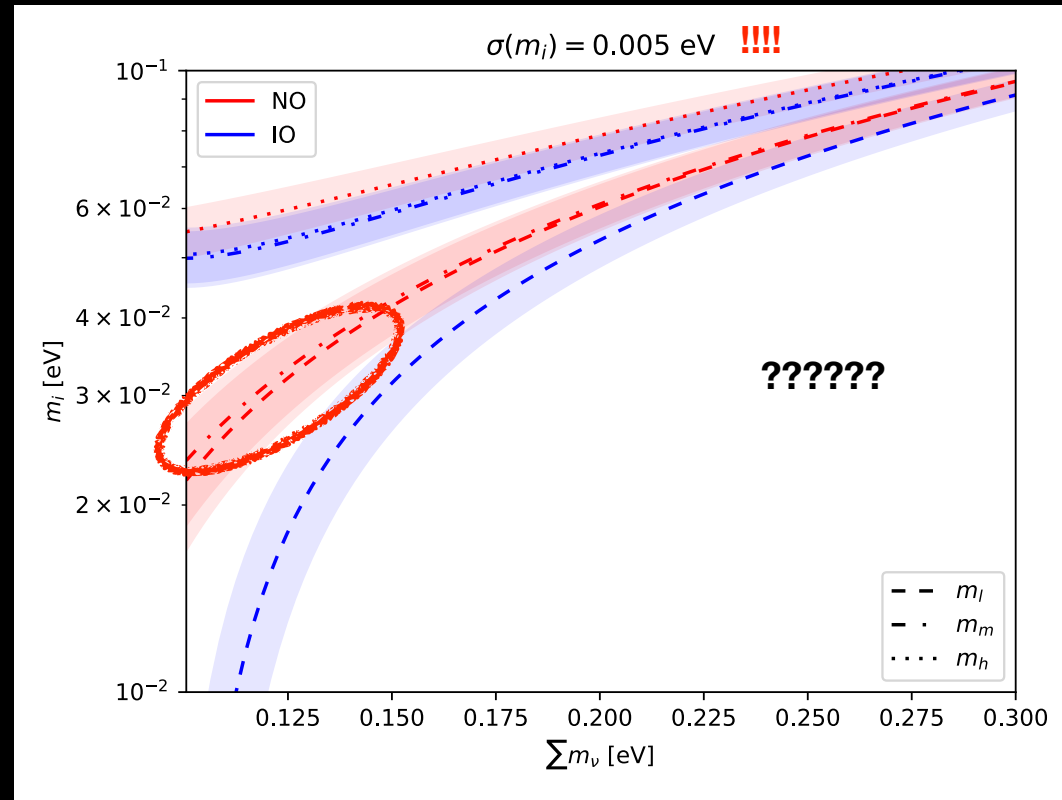
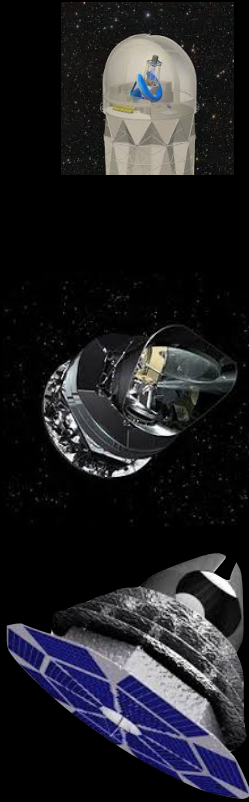




- Neutrino mass ordering

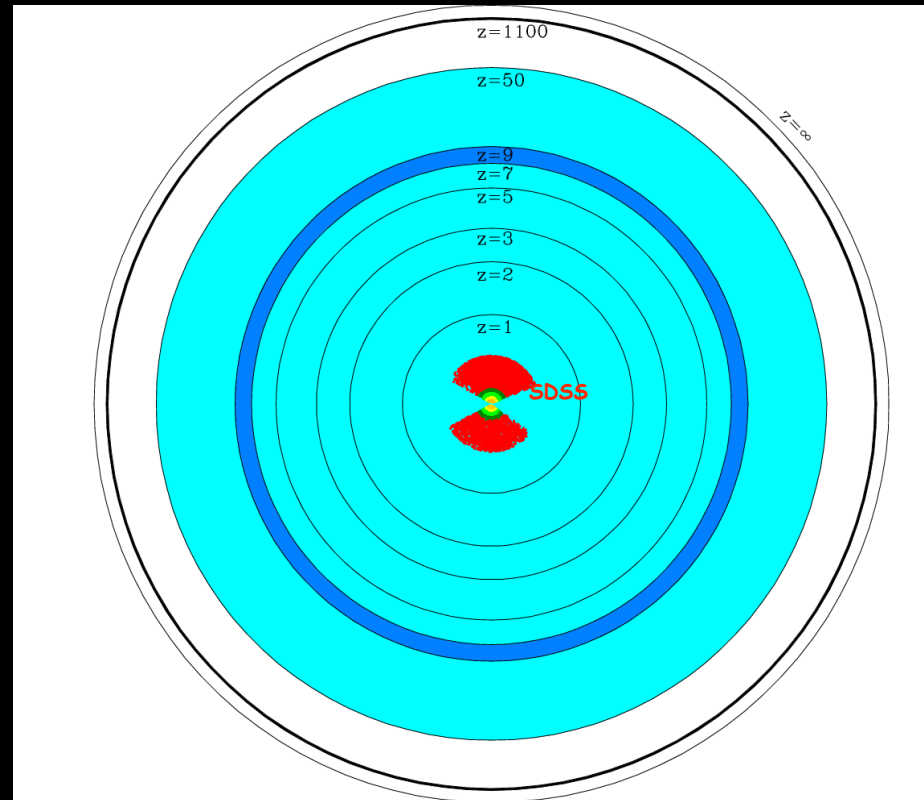


- Neutrino mass ordering



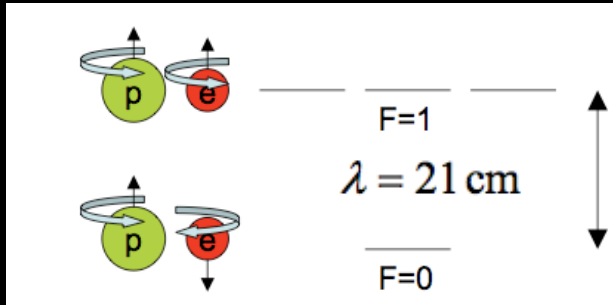
# The 21 cm universe

21 cm cosmology could be able to map most of our observable universe, whereas the CMB probes mainly a thin shell at  $z \approx 1100$  and large-scale structure maps only small volumes near the center so far.



# The 21 cm universe

From C. Hirata



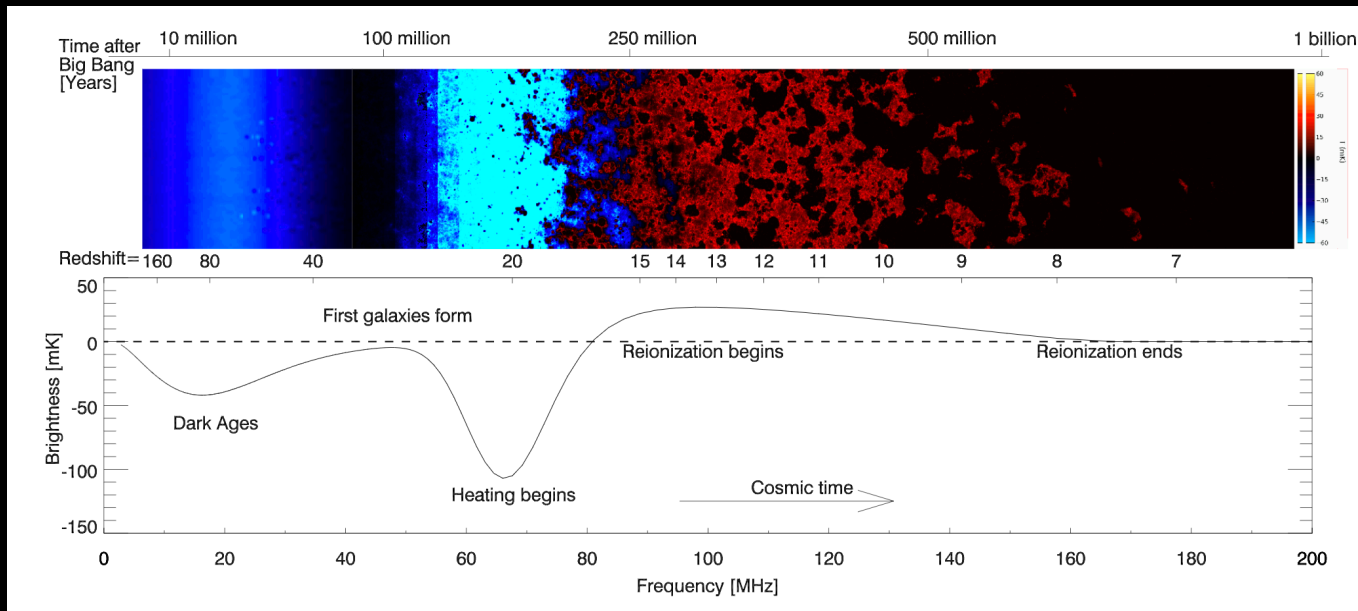
$$\Delta E = \frac{4}{3} \alpha^4 g_p \frac{m_e}{m_p} m_e c^2 = 5.9 \mu\text{eV} = h(1420\text{MHz})$$

$$\frac{n_{F=1}}{n_{F=0}} = 3e^{-\Delta E/kT_s}$$

Hyperfine transition of neutral hydrogen, that will be the **TRACER**. Can be measured in emission or absorption with respect to the in CMB emission ( $z < 10$ ) or in absorption ( $z > 10$ )

<http://homepage.sns.it/mesinger/21CMMC.html>

J. Pritchard & A. Loeb,  
Rept. Prog. Phys'12



# The 21 cm universe: Square Kilometer Array



2000 high & mid frequency dishes plus a million low-frequency antennas:  
Effective collecting area of one million  $\text{m}^2$

# The 21 cm universe: SKA



The Murchison region where the ASKAP and SKA telescopes will eventually be located, are traditional lands of the Wajarri Yamatji People, who signed an indigenous land use agreement, which protects the Aboriginal heritage.

The agreement also brought significant benefits in terms of education and infrastructure to the local peoples in what is one of the most sparsely populated regions on Earth.

Rank	Countries	Density (pop/square km)
1	Greenland (Denmark)	0.03
2	Falkland Islands (UK)	0.21
3	Pitcairn Islands (UK)	1.19
4	Mongolia	1.92
5	Namibia	2.56
6	French Guiana (France)	2.65
7	Australia	3.14
8	Iceland	3.24
9	Suriname	3.26
10	Mauritania	3.36
11	Botswana	3.48
12	Libya	3.50
13	Guyana	3.65
14	Canada	3.65
15	Niue (NZ)	6.18
16	Gabon	6.25
17	Kazakhstan	6.31
18	Central African Republic	7.42
19	Russia	8.42
20	Chad	8.78



The data collected by the SKA in a single day would take nearly two million years to play on an ipod.

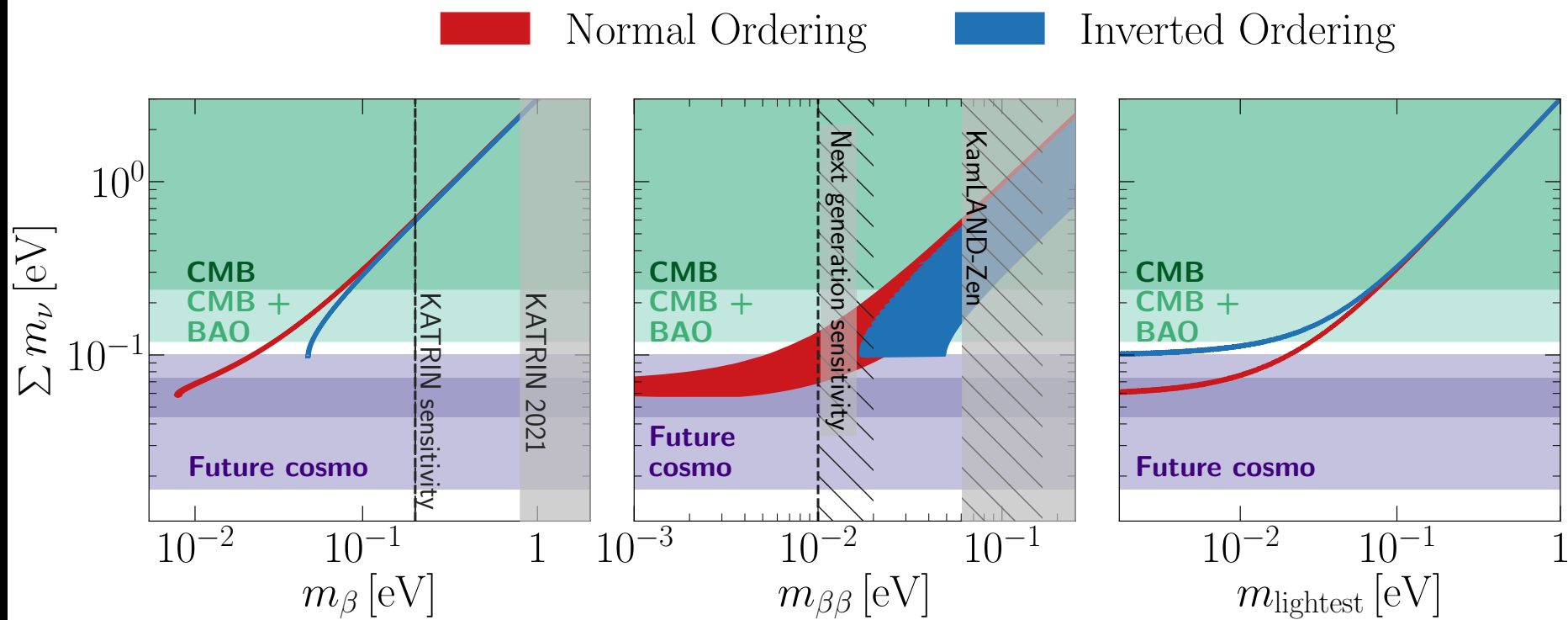
The SKA will be so sensitive that it will be able to detect an airport radar on a planet tens of light years away.

The SKA central computer will have the processing power of about one hundred million PCs.

The dishes of the SKA will produce 10 times the global internet traffic.

The SKA will use enough optical fibre to wrap twice around the Earth!





1. INTRODUCTION: FUNDAMENTAL INGREDIENTS & THERMAL HISTORY OF THE UNIVERSE ✓
2. NEUTRINO DECOUPLING IN THE EARLY UNIVERSE ✓
3. BIG BANG NUCLEOSYNTHESIS &  $N_{\text{eff}}$  ✓
4. COSMOLOGY &  $N_{\text{eff}}$  ✓
5. COSMOLOGY & NEUTRINO MASSES ✓
6. TAKE HOME MESSAGES

# The "Take Home" messages

- $\nu$  masses & abundances leave key signatures in cosmological observables.
- NO hints so far for neutrino masses or extra dark radiation species!
- $N_{\text{eff}}$  @BBN: Light element abundances ( $^4\text{He}$ ) abundances.
- $N_{\text{eff}}$  @CMB: damping tail
- $N_{\text{eff}} = 2.99^{+0.34}_{-0.33}$ , (95% CL) from 2018 Planck TTTEEE+lensing, perfectly consistent with BBN.
- Cosmology provides currently the tightest bounds to neutrino masses.
- $\nu$  masses@CMB: Early ISW, gravitational lensing
- $\nu$  masses@LSS: Free streaming
- $\sum m_\nu < 0.099 \text{ eV}$  (95%CL) from 2018 Planck TTTEEE+lensing plus RSD+BAO +SNIa data





## HANDS-ON SESSION (III)!

- PLEASE GO TO THE WEB PAGE:

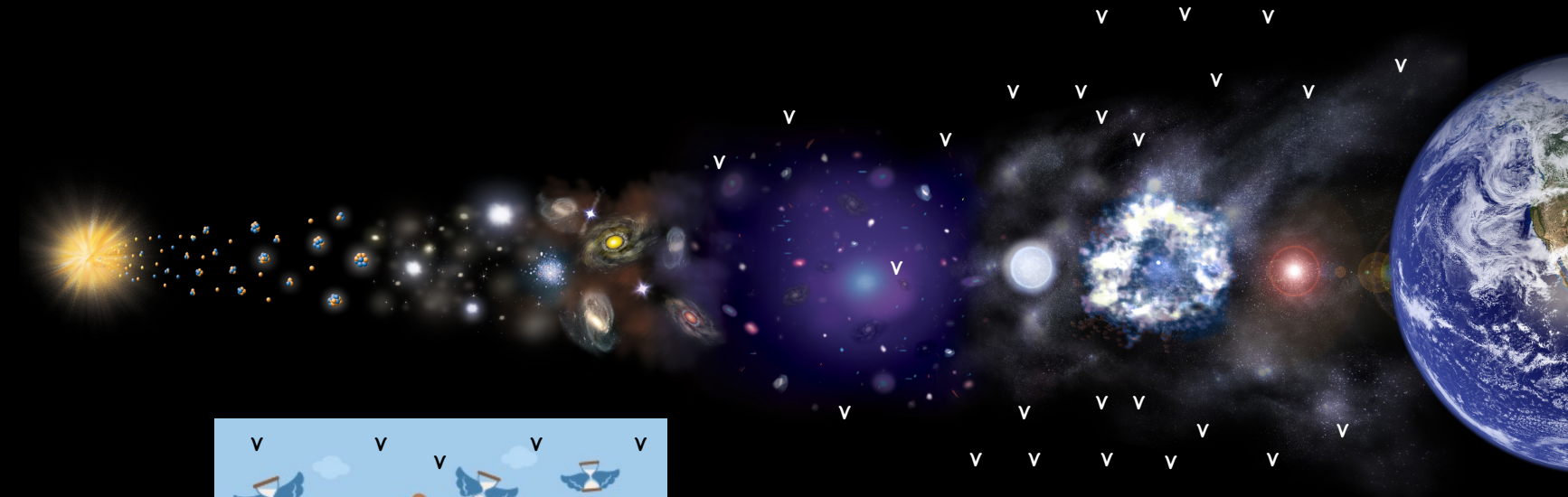
[https://lambda.gsfc.nasa.gov/toolbox/tb\\_camb\\_form.cfm](https://lambda.gsfc.nasa.gov/toolbox/tb_camb_form.cfm)

- COMPUTE THE TEMPERATURE ANISOTROPIES AND THE MATTER POWER SPECTRUM FOR

$$\sum m_\nu = 0.06, 0.1, 0.3 \text{ and } 1 \text{ eV}$$

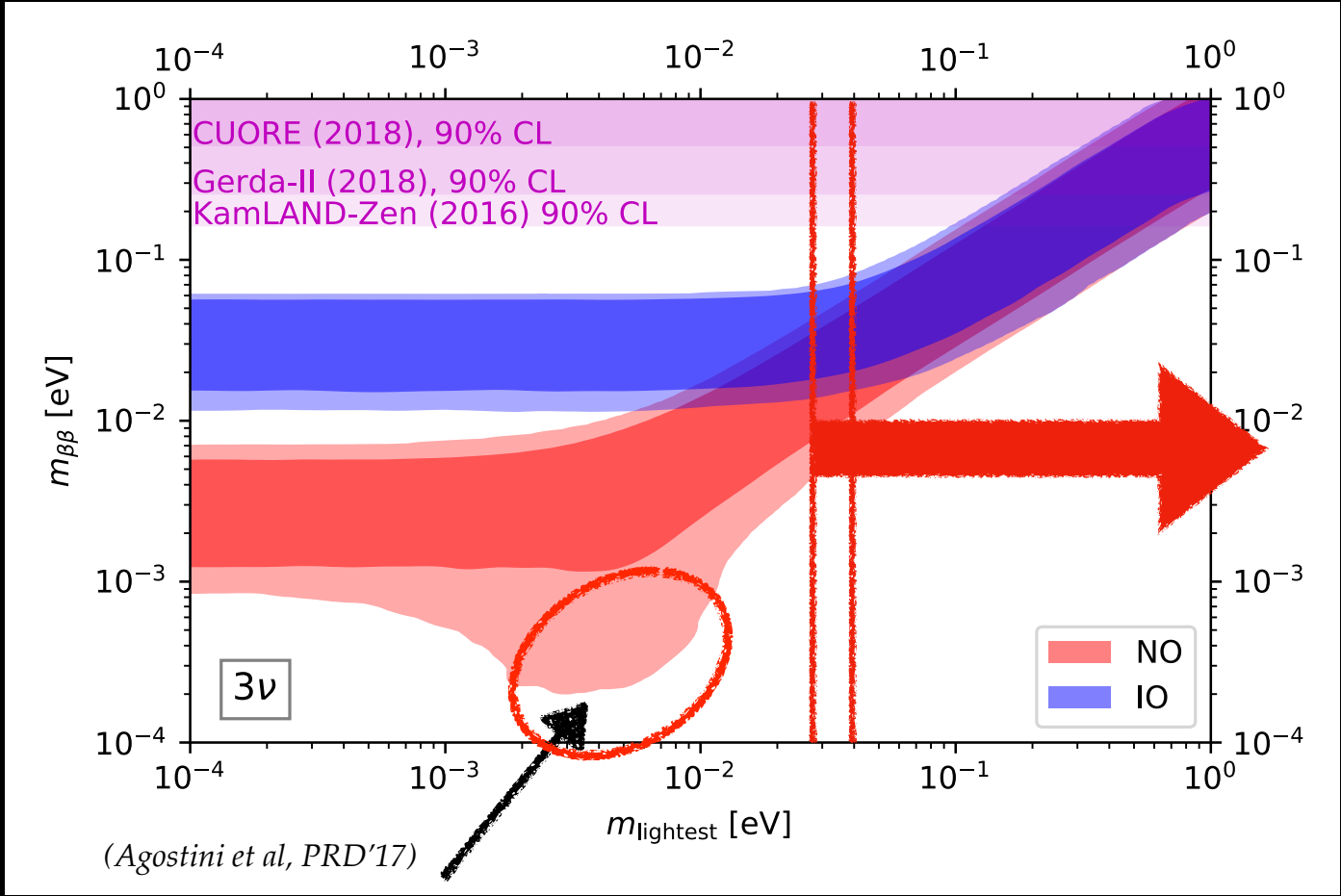
- CHANGE SOME OF THE OTHER COSMOLOGICAL PARAMETERS AND STUDY THE PARAMETER DEGENERACIES

# BACKUP SLIDES



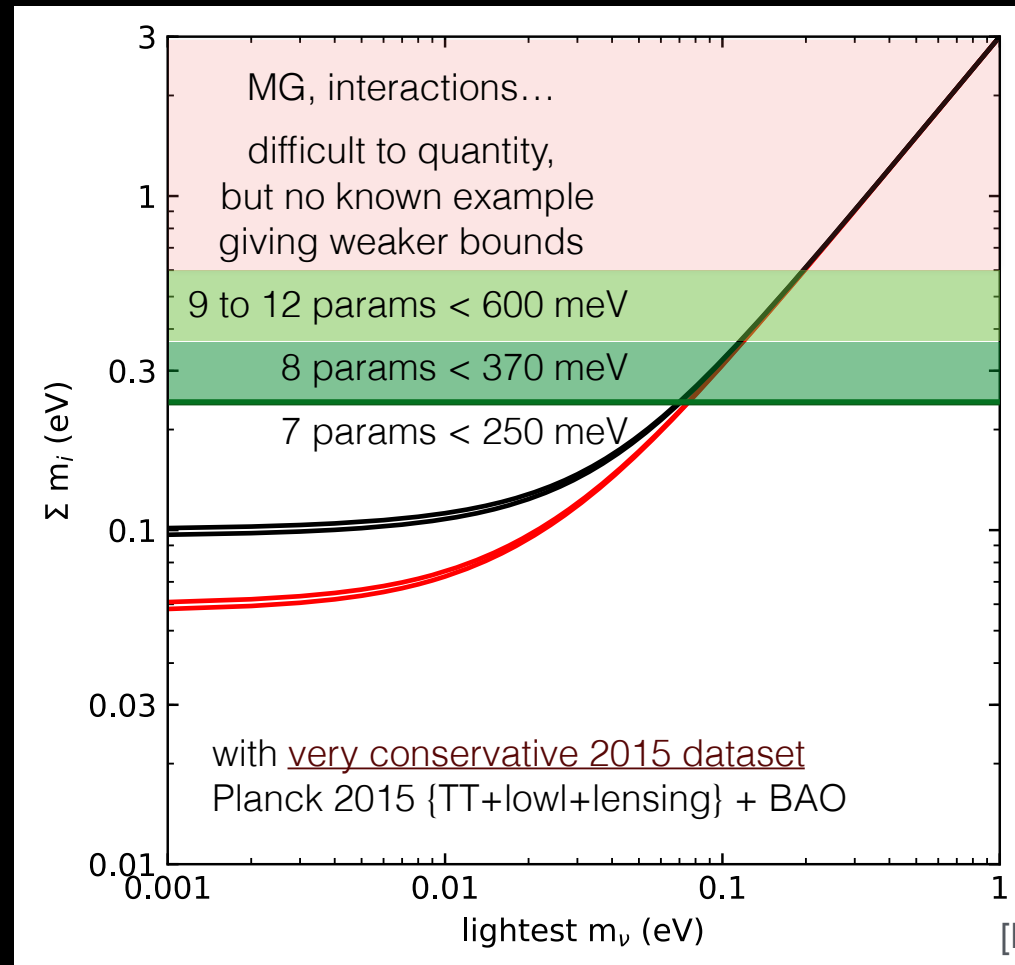
in**visibles**Plus  
elusi**ves**  
neutrinos, dark matter & dark energy physics





$m_{\beta\beta} < 2 \cdot 10^{-4}$  would require  
 some fine tuning in the Majorana phases

$$\Sigma m_\nu$$



*J. Lesgourgues, talk at Neutrino 2018*



## Methods to detect non-relativistic neutrinos: PTOLEMY

Today neutrinos have a mean temperature:

$$T_{\nu 0} = \left(\frac{4}{11}\right)^{1/3} T_0 = 1.945 \text{ K} \rightsquigarrow 1.697 \times 10^{-4} \text{ eV}$$

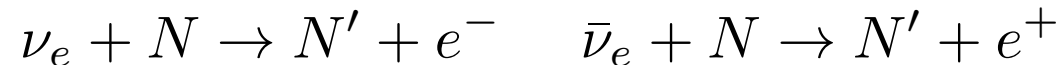
And two neutrinos have a mass above:  $\sqrt{\Delta m_{12}^2} \simeq 0.008 \text{ eV}$

at least one of these neutrinos has a mass larger than  $\sqrt{|\Delta m_{13}^2|} \simeq 0.05 \text{ eV}$

Therefore there are at least two non-relativistic neutrino states.

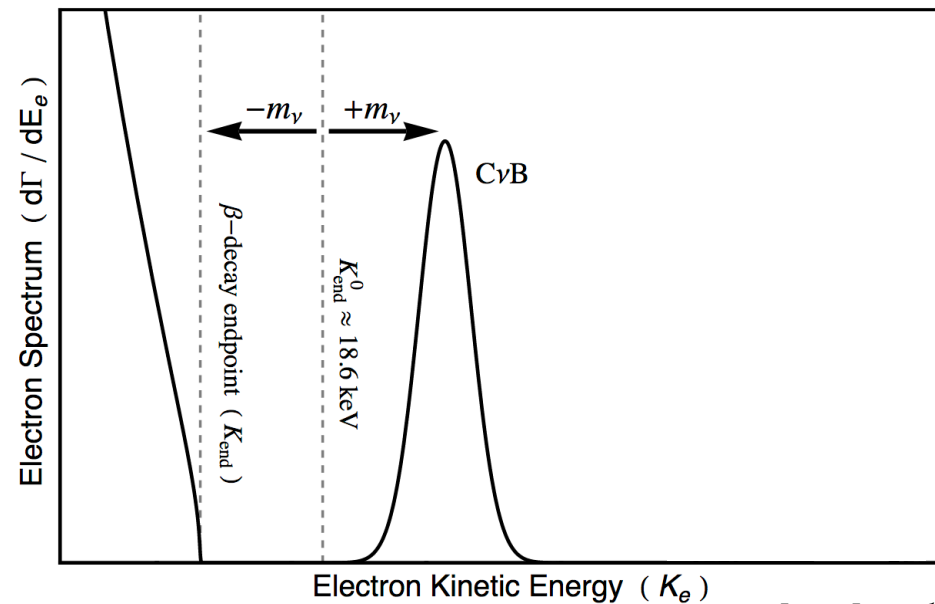
A process without energy threshold is mandatory!

### (Anti)neutrino capture on $\beta$ -decaying nuclei



$$M(N) - M(N') = Q_\beta > 0$$

## Methods to detect non-relativistic neutrinos: (Anti)neutrino capture on $\beta$ -decaying nuclei



*Long, Lunardini & Sabancillar, JCAP'14*

For finite  $m_\nu$ , the electron kinetic energy is  $Q_\beta + E_\nu \geq Q_\beta + m_\nu$ , while electrons emerging from the analogous beta decay have at most an energy  $Q_\beta - m_\nu$ , neglecting nucleus recoil energy. A minimum gap of  $2m_\nu$  is thus present and this at least in principle allows to distinguish between beta decay and NCB interaction: **GOOD ENERGY RESOLUTION!**

# PTOLEMY (PonTecorvo Observatory for Light, Early-universe, Massi Yield) @ LNGS

## PTOLEMY: A Proposal for Thermal Relic Detection of Massive Neutrinos and Directional Detection of MeV Dark Matter

E. Baracchini<sup>3</sup>, M.G. Betti<sup>11</sup>, M. Biasotti<sup>5</sup>, A. Boscá<sup>16</sup>, F. Calle<sup>16</sup>, J. Carabe-Lopez<sup>14</sup>, G. Cavoto<sup>10,11</sup>, C. Chang<sup>22,23</sup>, A.G. Cocco<sup>7</sup>, A.P. Colijn<sup>13</sup>, J. Conrad<sup>18</sup>, N. D'Ambrosio<sup>2</sup>, P.F. de Salas<sup>17</sup>, M. Faverezani<sup>6</sup>, A. Ferella<sup>18</sup>, E. Ferri<sup>6</sup>, P. Garcia-Abia<sup>14</sup>, G. Garcia Gomez-Tejedor<sup>15</sup>, S. Gariazzo<sup>17</sup>, F. Gatti<sup>5</sup>, C. Gentile<sup>25</sup>, A. Giachero<sup>6</sup>, J. Gudmundsson<sup>18</sup>, Y. Hochberg<sup>1</sup>, Y. Kahn<sup>26</sup>, M. Lisanti<sup>26</sup>, C. Mancini-Terracciano<sup>10</sup>, G. Mangano<sup>7</sup>, L.E. Marcucci<sup>9</sup>, C. Mariani<sup>11</sup>, J. Martínez<sup>16</sup>, G. Mazzitelli<sup>4</sup>, M. Messina<sup>20</sup>, A. Molinero-Vela<sup>14</sup>, E. Monticone<sup>12</sup>, A. Nucciotti<sup>6</sup>, F. Pandolfi<sup>10</sup>, S. Pastor<sup>17</sup>, J. Pedrós<sup>16</sup>, C. Pérez de los Heros<sup>19</sup>, O. Pisanti<sup>7,8</sup>, A. Polosa<sup>10,11</sup>, A. Puiu<sup>6</sup>, M. Rajteri<sup>12</sup>, R. Santorelli<sup>14</sup>, K. Schaeffner<sup>3</sup>, C.G. Tully<sup>26</sup>, Y. Raitses<sup>25</sup>, N. Rossi<sup>10</sup>, F. Zhao<sup>26</sup>, K.M. Zurek<sup>21,22</sup>

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<sup>5</sup>Università degli Studi di Genova e INFN Sezione di Genova, Genova, Italy

<sup>6</sup>Università degli Studi di Milano-Bicocca e INFN Sezione di Milano-Bicocca, Milano, Italy

<sup>7</sup>INFN Sezione di Napoli, Napoli, Italy

<sup>8</sup>Università degli Studi di Napoli Federico II, Napoli, Italy

<sup>9</sup>INFN Laboratori Nazionali di Frascati, Frascati, Italy

6 Aug 2018

# PTOLEMY (PonTecorvo Observatory for Light, Early-universe, Massive Yield) @ LNGS

The expected rate is:

$$\Gamma_{C\nu B} = [n_0(\nu_{h_R}) + n_0(\nu_{h_L})] N_T \bar{\sigma} \sum_{i=1}^3 |U_{ei}|^2 f_c(m_i).$$

For unclustered neutrinos (i.e.  $f_c = 1$ ) and 100 g of tritium, the expected number of events per year:

$$\Gamma_{C\nu B}^D \simeq 4 \text{ yr}^{-1}, \quad \Gamma_{C\nu B}^M = 2\Gamma_{C\nu B}^D \simeq 8 \text{ yr}^{-1}$$

If neutrinos are Majorana particles, the expected number of events is doubled with respect to the Dirac case.

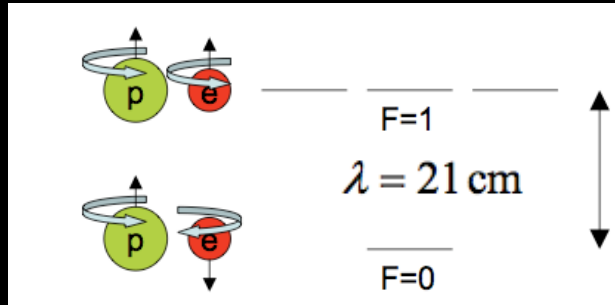
The reason is related to the fact that, during the transition from ultra-relativistic to non-relativistic particles, helicity is conserved.

The population of relic neutrinos is then composed by left- and right-helical neutrinos in the Majorana case, and only left-helical neutrinos in the Dirac case. Since the neutrino capture can only occur for left-chiral electron neutrinos, the fact that in the Majorana case the right-handed neutrinos have a left-chiral component leads to a doubled number of possible interactions.

masses (meV)	matter halo	overdensity $f_c$			$\Gamma_{C\nu B}^D$ (yr <sup>-1</sup> )			$\Gamma_{C\nu B}^M$ (yr <sup>-1</sup> )		
		{best fit   best fit + baryons   optimistic}								
any	any	no clustering			4.06			8.12		
degenerate $m_{\nu_{1,2,3}} = 150$	NFW Einasto	2.18   2.44   2.88	8.8   9.9   11.7	17.7   19.8   23.4	1.68   1.87   2.43	6.8   7.6   9.9	13.6   15.1   19.7			
minimal (IO) $m_{\nu_3} = 60$	NFW Einasto	1.15   1.18   1.21	4.07   4.08   4.08	8.15   8.15   8.16	1.09   1.12   1.18	4.07   4.07   4.08	8.14   8.14   8.15			
minimal (NO) $m_{\nu_{1,2}} = 60$	NFW Einasto	1.15   1.18   1.21	4.66   4.78   4.89	9.31   9.55   9.77	1.09   1.12   1.18	4.42   4.54   4.78	8.84   9.07   9.55			

# The 21 cm universe

From C. Hirata



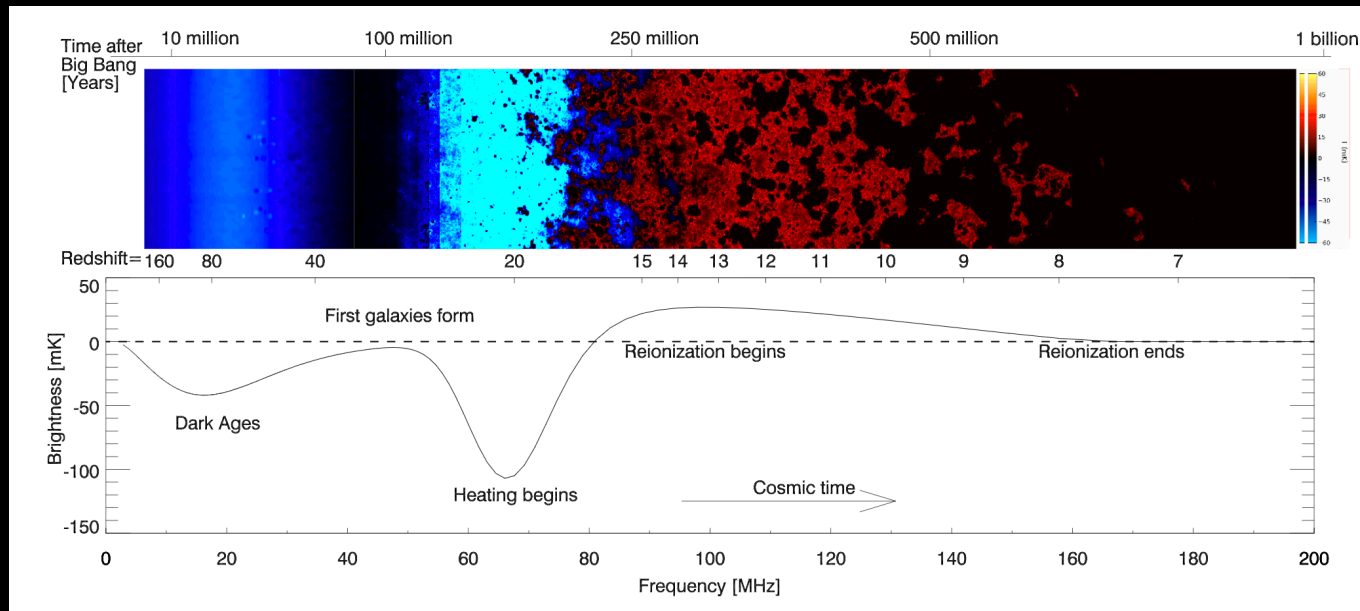
$$\Delta E = \frac{4}{3} \alpha^4 g_p \frac{m_e}{m_p} m_e c^2 = 5.9 \mu\text{eV} = h(1420\text{MHz})$$

$$\frac{n_{F=1}}{n_{F=0}} = 3e^{-\Delta E/kT_s}$$

Hyperfine transition of neutral hydrogen, that will be the **TRACER**. Can be measured in emission or absorption with respect to the in CMB emission ( $z < 10$ ) or in absorption ( $z > 10$ )

<http://homepage.sns.it/mesinger/21CMMC.html>

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One can look for  
(*sterile*) neutrinos in  
something not so  
shiny and bright....

# DARK MATTER

HOT dark matter  
HDM

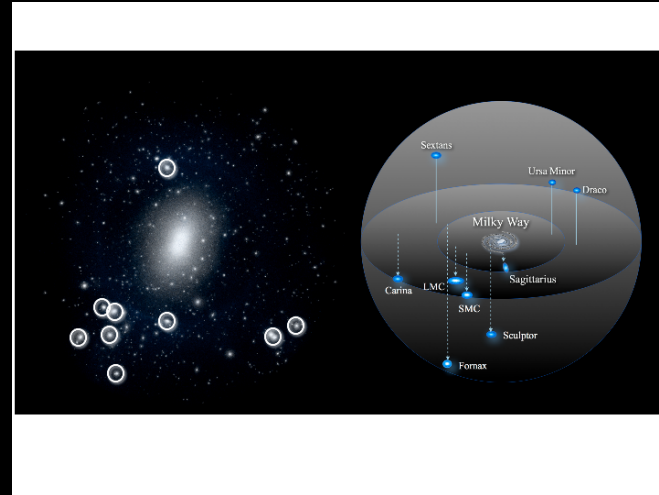
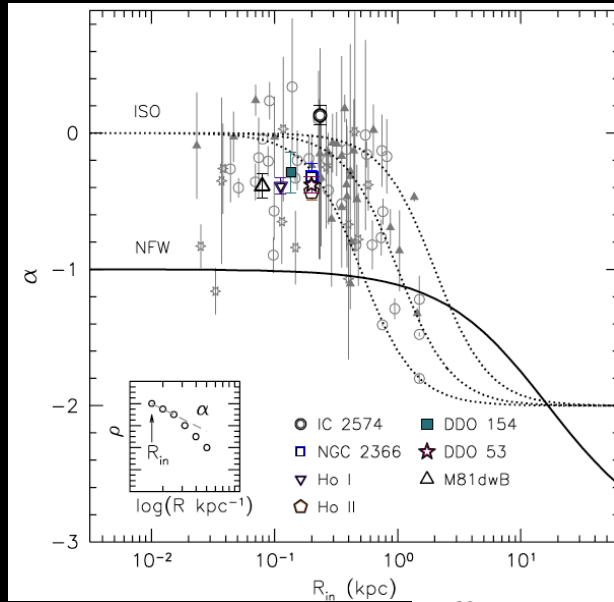
COLD dark matter  
CDM

WDM  
WARM dark matter

CREATED BY

JOSEPH DALL'AZZI

# Small scale crisis of $\Lambda$ CDM@galactic and sub-galactic scales

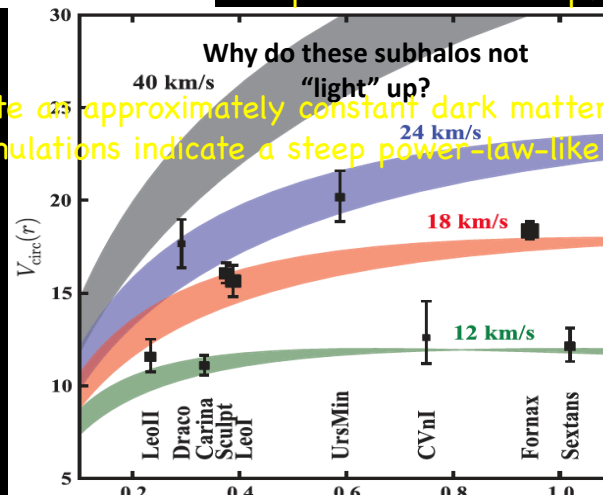


## Missing satellite problem

The predicted satellite population far exceeds the observed one

## Core/Cusp problem

Observations seem to indicate an approximately constant dark matter density in the inner parts of galaxies (core), while cosmological simulations indicate a steep power-law-like behaviour (cusp)



## Too big to fail (TBTf) problem

Massive dark subhalos are too dense to match data.  
Expected 10 subhalos in the Milky Way with  $v > 30$  km/s, only 3 known!

(Boylan et al, MNRAS'11)



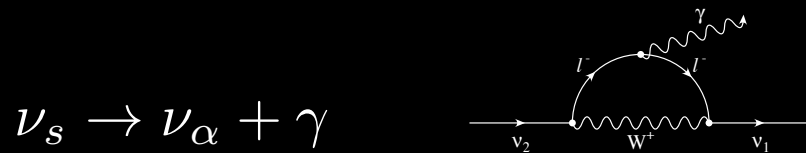
## Sterile keV (0.01 $m_e$ ) neutrino as a warm dark matter candidate?

A controversial unidentified line has been detected at with a significance  $> 3\sigma$  in two independent samples of X-ray clusters with XMM-Newton.

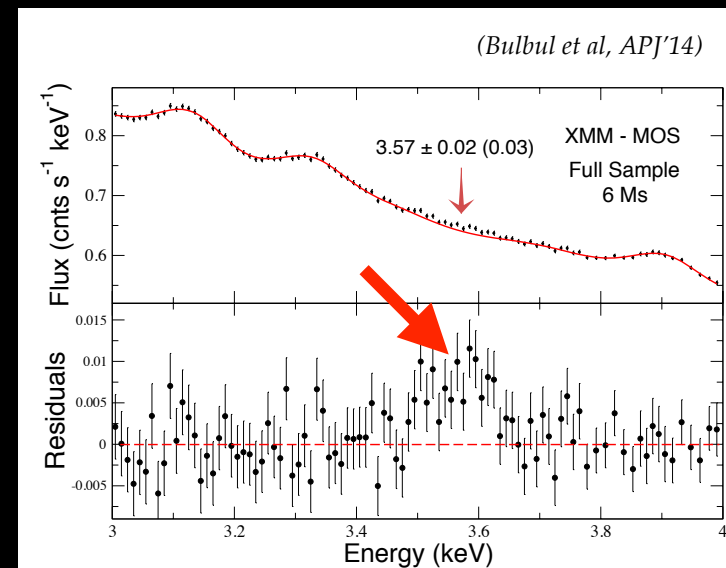
It is independently seen by the same group in the Perseus Cluster with Chandra data.

(Bulbul et al, APJ'14)

An independent group finds a line at the same energy toward Andromeda and Perseus with XMM-Newton, with a combined statistical evidence of  $4.4\sigma$ . (Boyarisky et al, PRL'14)

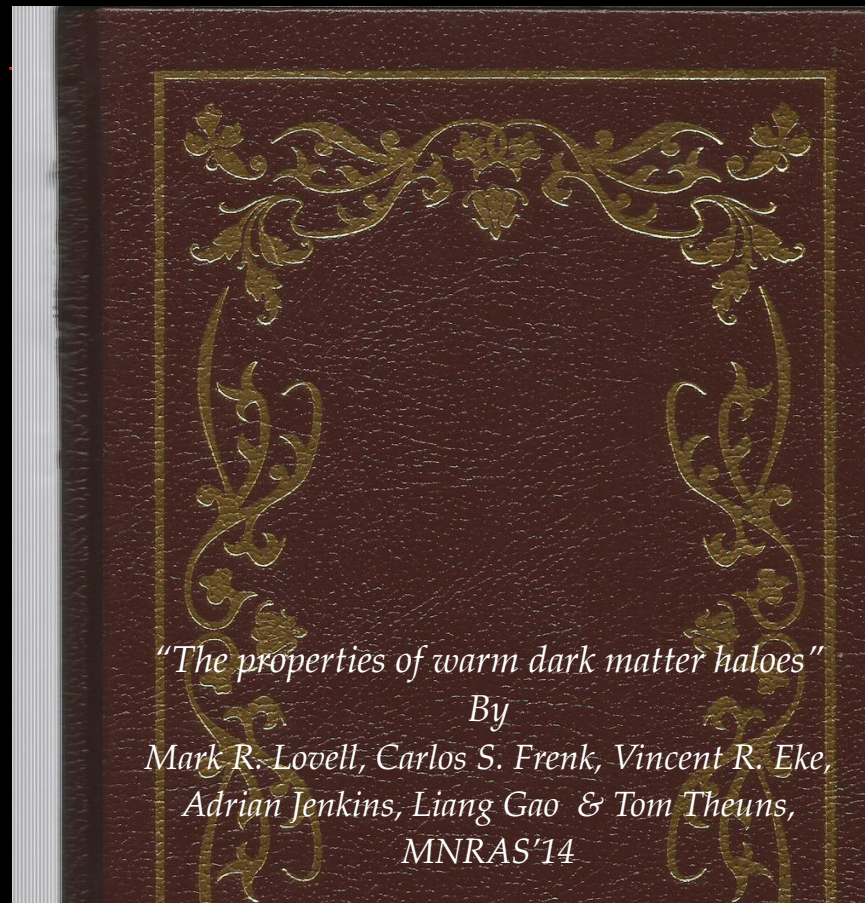


$$m_s = 2E = 7.1 \text{ keV}$$



## Sterile keV ( $0.01 m_e$ ) neutrino as a warm dark matter candidate?

WDM leads to an identical large scale structure pattern than CDM, but very different subhaloes abundance, structure and dynamics: the free streaming of a keV sterile neutrino will reduce power at the small scales, delaying structure formation and lowering the haloes concentration.



## Sterile keV ( $0.01 m_e$ ) neutrino as a warm dark matter candidate?

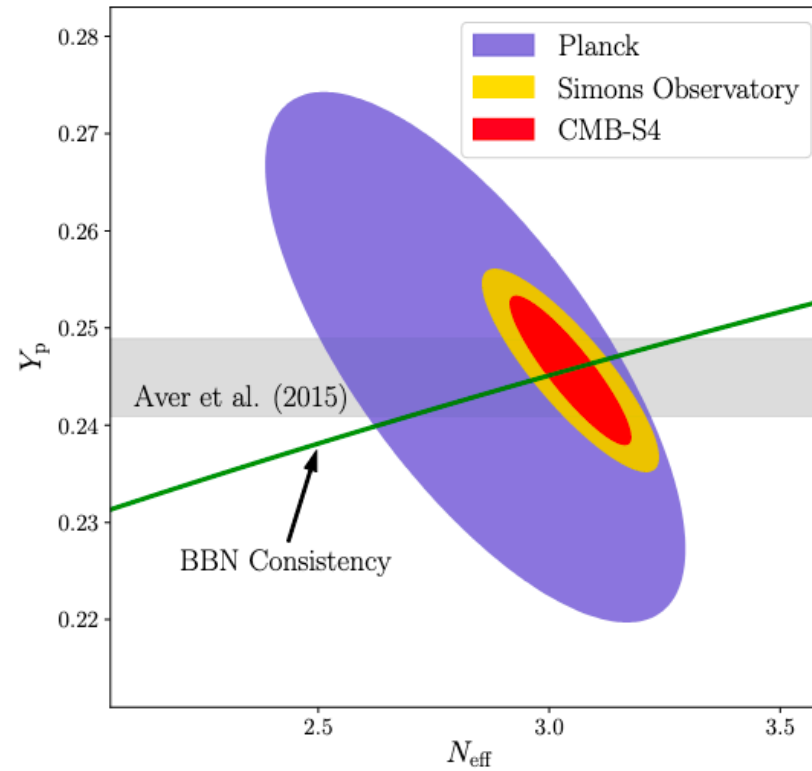
WDM leads to an identical large scale structure pattern than CDM, but very different subhaloes abundance, structure and dynamics: the free streaming of a keV sterile neutrino will reduce power at the small scales, delaying structure formation and lowering the haloes concentration.

Simulations have shown that WDM can solve/alleviate the small scale crisis of  $\Lambda$ CDM



# CMB Stage IV: $N_{\text{eff}}$

$$\Delta N_{\text{eff}} < 0.06 \text{ 95\%CL}$$



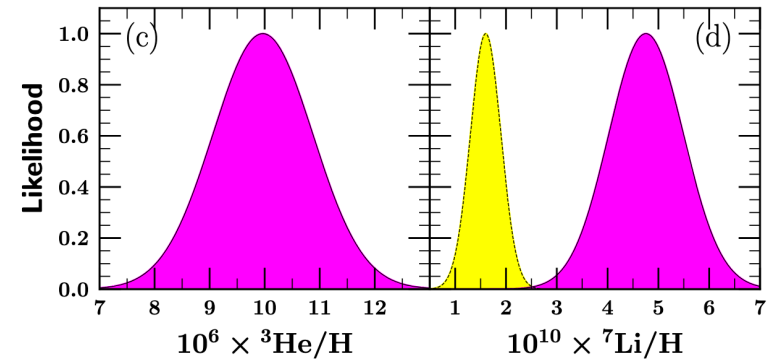
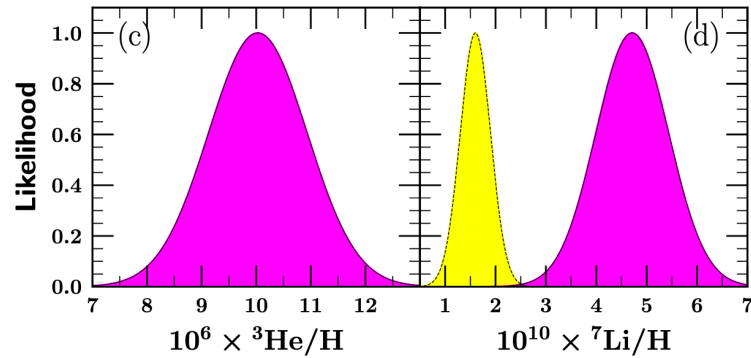
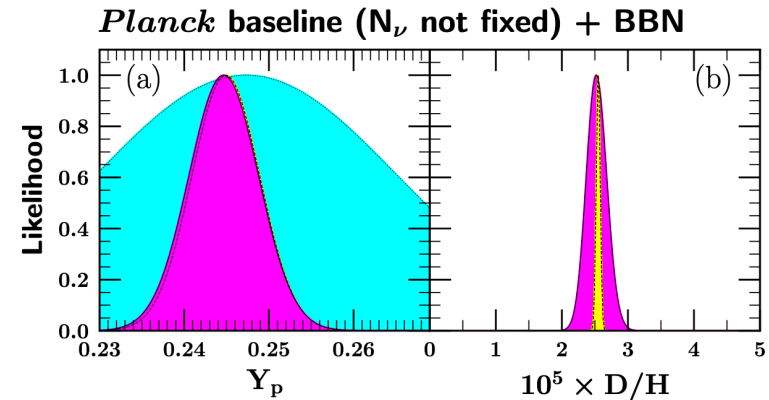
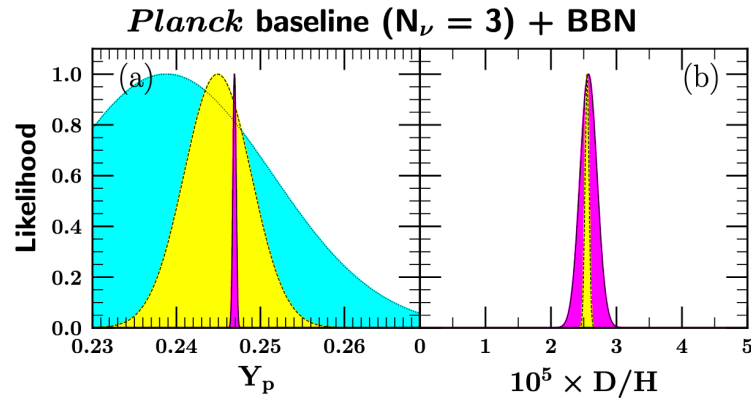
*CMB-S4 Science Case, Reference Design, and Project Plan, 1907.04473*

# CMB: $N_{\text{eff}}$

$$Y_p = 0.24691 \pm 0.00018 \quad (0.24691)$$

$$Y_p = 0.24465 \pm 0.00410 \quad (0.24498)$$

*Fields, Olive, Yeh & Young JCAP '20*

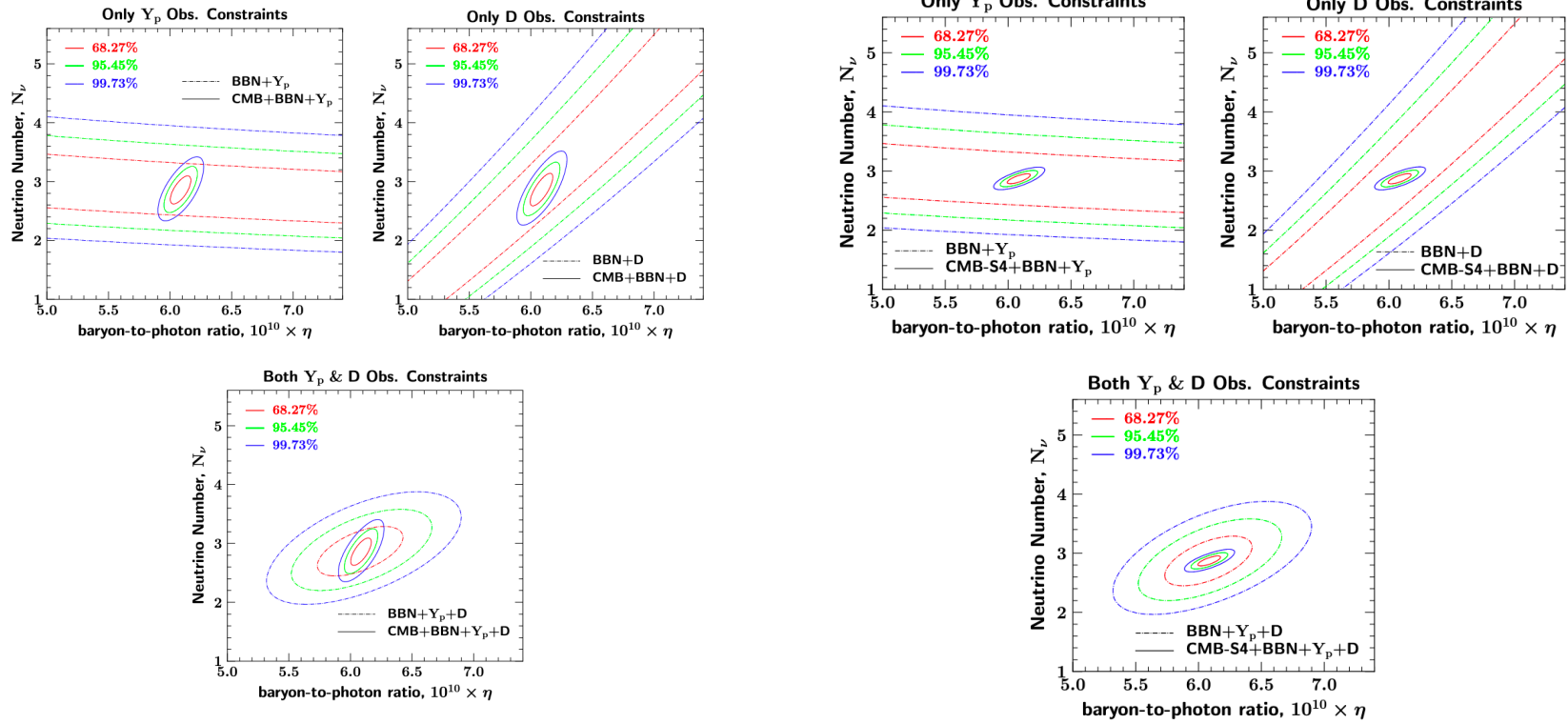


BBN+ CMB

Astronomical measurements

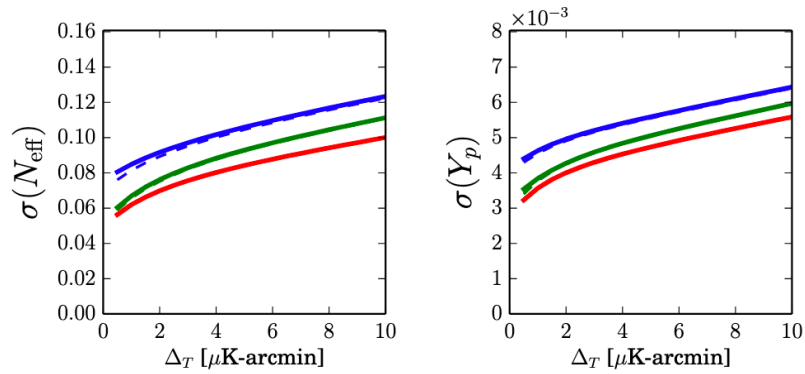
CMB He determinations

# CMB Stage IV: $N_{\text{eff}}$



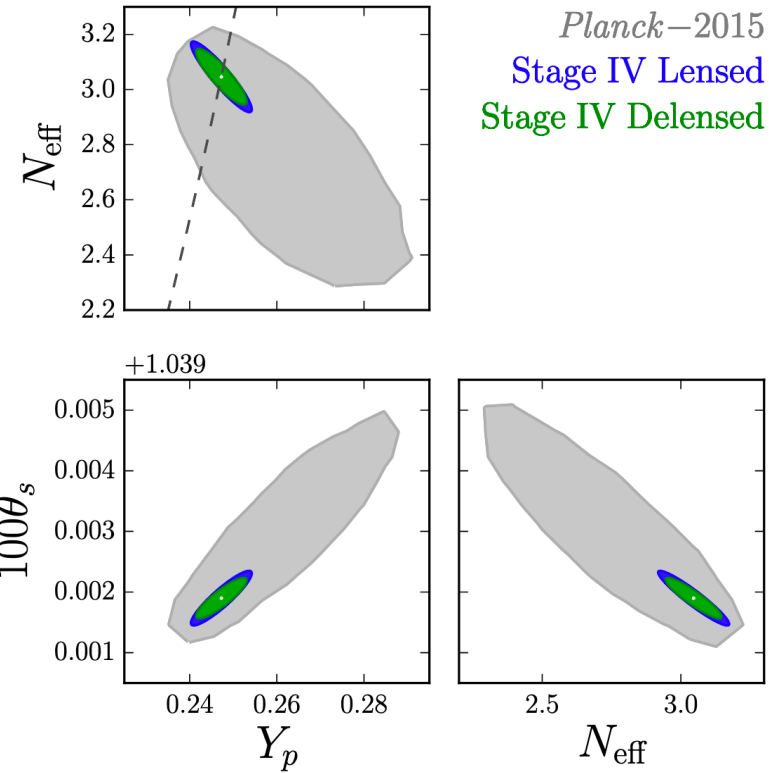
*Fields, Olive, Yeh & Young JCAP '20*

# CMB Stage IV: $N_{\text{eff}}$



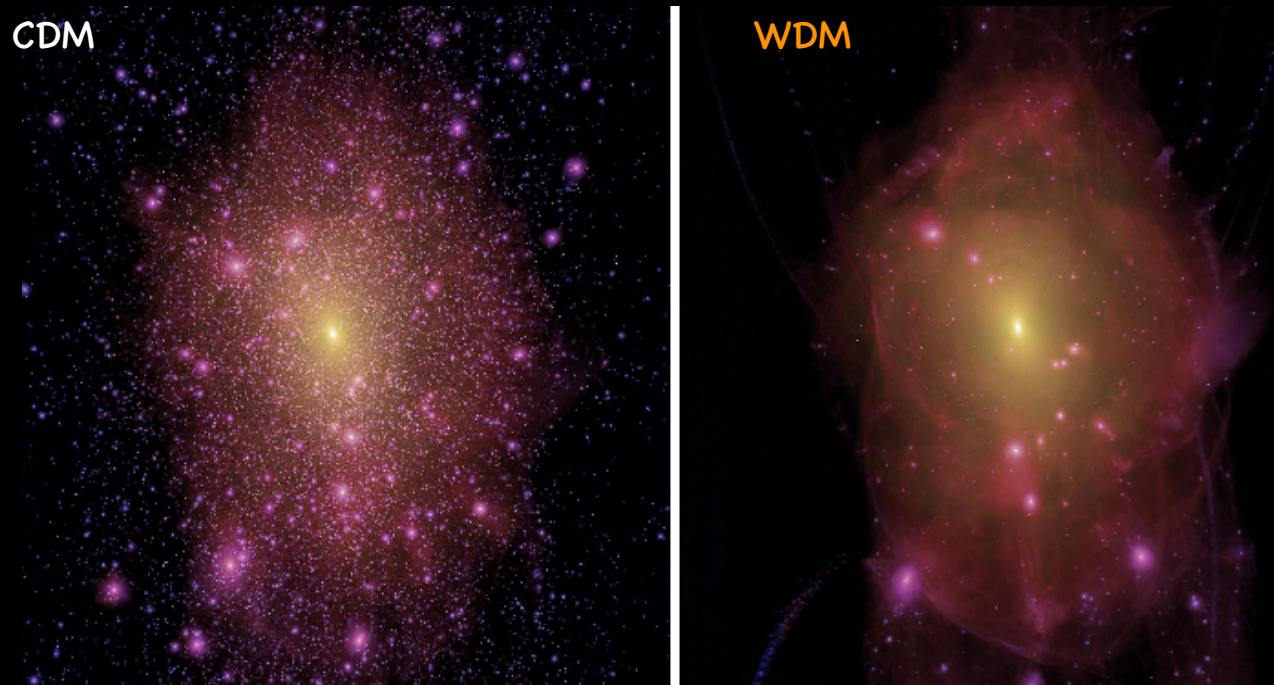
- Non-Gaussian Delensed
- - Gaussian Delensed
- Non-Gaussian Lensed
- - Gaussian Lensed
- Unlensed

*Green, Meyers & van Engelen, JCAP'17*



Sterile keV ( $0.01 m_e$ ) neutrino as a **warm** dark matter candidate?

**WDM could reconcile theory with observations!**



*"The Haloes of Bright Satellite Galaxies in a Warm Dark Matter Universe", Mark R. Lovell, Vincent R. Eke, Carlos S. Frenk, Liang Gao, Adrian Jenkins, Tom Theuns, Jie Wang, D.M. White, Alexey Boyarsky & Oleg Ruchayskiy MNRAS'12*

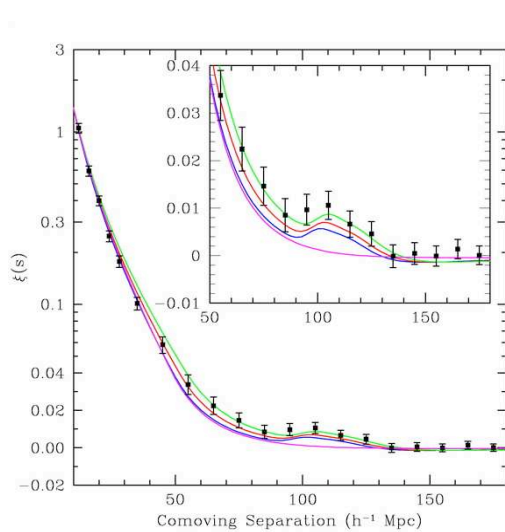
*"The properties of warm dark matter haloes", Mark R. Lovell, Carlos S. Frenk, Vincent R. Eke, Adrian Jenkins, Liang Gao & Tom Theuns, MNRAS'14*



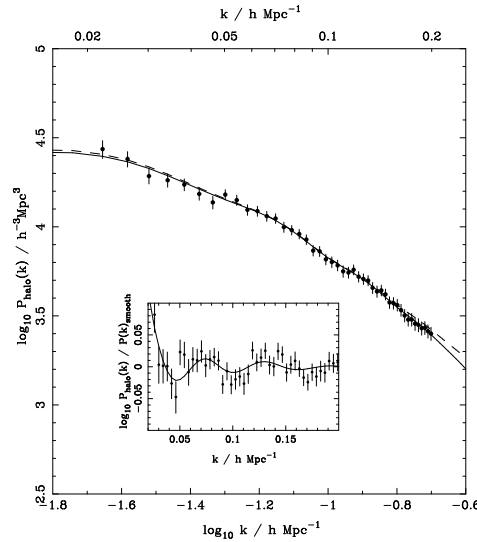
Los catálogos de galaxias miden la función de correlación:

$$\xi(\vec{r}) \equiv \langle \delta(\vec{x})\delta(\vec{x} + \vec{r}) \rangle_{\text{Volume}} \quad \langle \tilde{\delta}(\vec{k})\tilde{\delta}(\vec{k}') \rangle_{\text{Volume}} = (2\pi)^3 P(k)\delta^3(\vec{k} - \vec{k}')$$

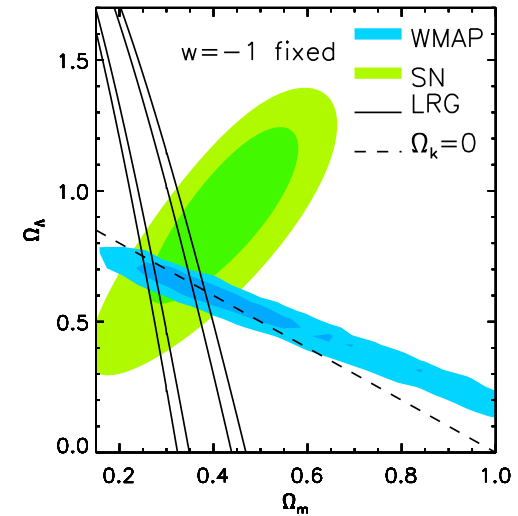
$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \bar{\rho}(\vec{x})}{\bar{\rho}(\vec{x})} \quad \tilde{\delta}(\vec{k}) \equiv \int d^3\vec{r} e^{i\vec{k}\vec{r}} \delta(\vec{r})$$



Eisenstein et al'05

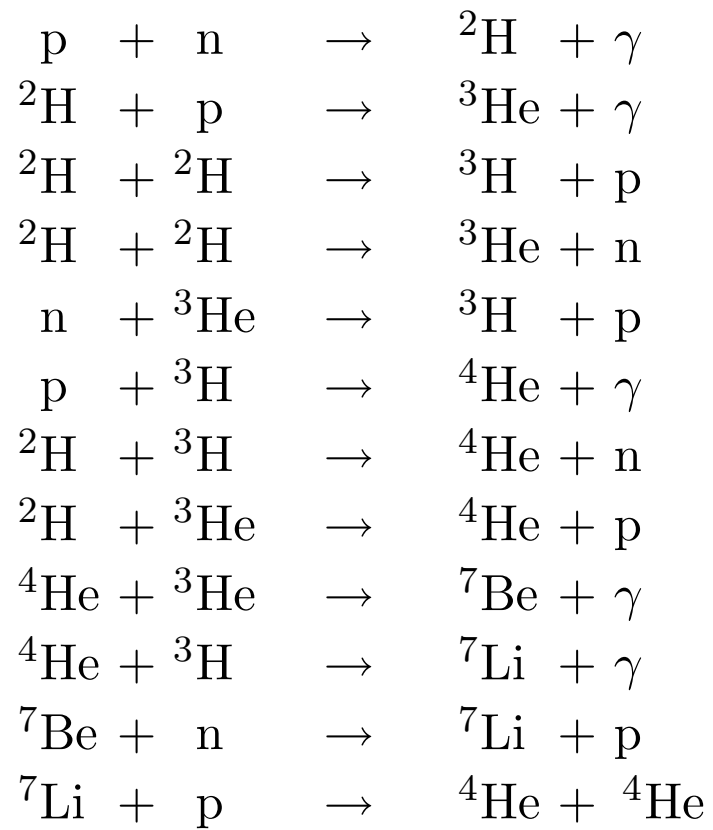


Reid et al'09



SSDS 2005: Primera detección de la señal BAO (3.4s) (47000 LRGs, 4000 deg<sup>2</sup> , z=0.35)

SDSS II 2009: 110 000 LRGs, 8000 deg<sup>2</sup> , z=0.35.



# BOLTZMANN EQUATIONS

- The non-relativistic Boltzmann equation: the Liouville operator is just the total time derivative

$$\frac{f(\mathbf{x} + \frac{\mathbf{p}}{m}dt, \mathbf{p} + \mathbf{F}dt, t + dt) - f(\mathbf{x}, \mathbf{p}, t)}{dt} = \frac{\partial}{\partial t}f(\mathbf{x}, \mathbf{p}, t) + \frac{p^i}{m} \frac{\partial}{\partial x^i} f(\mathbf{x}, \mathbf{p}, t) + F^i \frac{\partial}{\partial p^i} f(\mathbf{x}, \mathbf{p}, t)$$

$$L_{NR} = \frac{\partial}{\partial t} + \frac{dx^i}{dt} \frac{\partial}{\partial x^i} + \frac{dp^i}{dt} \frac{\partial}{\partial p^i} = \frac{d}{dt}$$

$$\hat{L} = \frac{d}{dt} + \vec{v} \cdot \vec{\nabla}_x + \vec{F}/m \cdot \vec{\nabla}_v$$

# BOLTZMANN EQUATIONS

- The relativistic version is:

$$\hat{L} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma_{\beta\gamma}^\alpha p^\beta p^\gamma \frac{\partial}{\partial p^\alpha} \quad P^\alpha = (E, \vec{P}) \quad P^\alpha = \frac{dx^\alpha}{d\lambda}$$

- FLRW geometry, the only non-vanishing component is  $\alpha = 0$ :

$$\hat{L}f = E \frac{\partial f}{\partial t} - Hp^2 \frac{\partial f}{\partial E}$$

- We can also write the Boltzmann equation in terms of the number density:

$$n_A = 4\pi \int dp p^2 f_A(E, t)$$

- Dividing by the energy and integrating over the momentum:

$$4\pi \int dp p^2 \frac{\hat{L}[f_A]}{E} = \frac{dn_A}{dt} - H4\pi \int dp \frac{p^4}{E} \frac{\partial f_A}{\partial E} = \frac{dn_A}{dt} + H4\pi \int dp \frac{\partial(p^3)}{\partial p} f_A$$

$$\frac{dn}{dt} + 3Hn = \frac{g}{(2\pi)^3} \int C f \frac{d^3p}{E}$$

Event	Time	Redshift	Temperature
Baryogenesis	?	?	?
EW phase transition	$2 \times 10^{-11}s$	$10^{15}$	$100GeV$
QCD phase transition	$2 \times 10^{-5}s$	$10^{12}$	$150MeV$
Neutrino decoupling	$1s$	$6 \times 10^9$	$1MeV$
Electron-positron annihilation	$6s$	$2 \times 10^9$	$500keV$
Big bang nucleosynthesis	$3min$	$4 \times 10^8$	$100keV$
Matter-radiation equality	$6 \times 10^4yrs$	3400	$.75eV$
Recombination	$2.6 - 3.8 \times 10^5yrs$	1100-1400	$.26 - .33eV$
CMB	$3.8 \times 10^5yrs$	1100	$.26eV$

# BOLTZMANN EQUATIONS

- At temperatures smaller than  $E - \mu$ :  $f(E) \rightarrow e^{\mu/T} e^{-E/T}$

- Therefore :

$$f_3 f_4 - f_1 f_2 \rightarrow e^{-(E_1 + E_2)/T} \left( e^{(\mu_3 + \mu_4)/T} - e^{(\mu_1 + \mu_2)/T} \right)$$

- Using the following definitions for the number density and the equilibrium number density of species as:

$$n_i = g_i e^{\mu_i/T} \int \frac{d^3 p}{(2\pi)^3} e^{-E_i/T} \quad n_i^{(0)} \equiv g_i \int \frac{d^3 p}{(2\pi)^3} e^{-E_i/T}$$

- Using these two expressions:

$$f_3 f_4 - f_1 f_2 \rightarrow e^{-(E_1 + E_2)/T} \left( \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right)$$

# BOLTZMANN EQUATIONS

- Defining the thermally averaged cross-section as:

$$\langle \sigma v \rangle \equiv e^{-(E_1+E_2)/T} \left( \frac{1}{n_1^{(0)} n_2^{(0)}} \right) \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \int \frac{d^3 p_3}{(2\pi)^3 2E_3} \int \frac{d^3 p_4}{(2\pi)^3 2E_4} \\ \times (2\pi)^4 \delta^3(p^1 + p^2 - p^3 - p^4) \delta(E^1 + E^2 - E^3 - E^4) |\mathcal{M}|^2$$

# Einstein Equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = 8\pi T_{\mu\nu}$$

- $R_{\mu\nu}$  is the Ricci tensor, depending on the metric  $g_{\mu\nu}$  and its derivatives:

$$R_{\mu\nu} = \Gamma_{\mu\nu,\alpha}^{\alpha} - \Gamma_{\mu\alpha,\nu}^{\alpha} + \Gamma_{\beta\alpha}^{\alpha}\Gamma_{\mu\nu}^{\beta} - \Gamma_{\beta\nu}^{\alpha}\Gamma_{\mu\alpha}^{\beta}$$

(It seems tedious but there are only two components different from 0, the 00 and the ii ones)

- $\mathcal{R}$  is the Ricci scalar,  $\mathcal{R}=g^{\mu\nu}R_{\mu\nu}$ .
- $T_{\mu\nu}$  is the energy-momentum tensor.
- The Christoffel symbols:

$$\Gamma_{\alpha\beta}^{\mu} = \frac{g^{\mu\nu}}{2} \left( \frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} \right)$$



## Einstein Equations

$$\Gamma_{\alpha\beta}^{\mu} = \frac{g^{\mu\nu}}{2} \left( \frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} \right) \quad g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix}$$

$$\Gamma_{\alpha\beta}^0 = -\frac{1}{2} \left( \frac{\partial g_{\alpha 0}}{\partial x^{\beta}} + \frac{\partial g_{\beta 0}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^0} \right) = \frac{1}{2} \left( \frac{\partial g_{\alpha\beta}}{\partial x^0} \right)$$

• EXERCISE, Check that:

$$\Gamma_{00}^0 = 0$$

$$\Gamma_{0i}^0 = \Gamma_{i0}^0 = 0$$

$$\Gamma_{ij}^0 = \delta_{ij} \dot{a} a$$

$$\Gamma_{0j}^i = \Gamma_{j0}^i = \delta_j^i \frac{\dot{a}}{a}$$

# Einstein Equations

$$\Gamma_{ij}^0 = \delta_{ij} \dot{a} a$$

$$\Gamma_{0j}^i = \Gamma_{j0}^i = \delta_j^i \frac{\dot{a}}{a}$$

- Lets compute the 00 component for the Einstein equations:

$$R_{00} - \frac{1}{2} g_{00} \mathcal{R} = 8\pi G T_{00}$$

$$R_{00} = \Gamma_{00,\alpha}^\alpha - \Gamma_{0\alpha,0}^\alpha + \Gamma_{\beta\alpha}^\alpha \Gamma_{00}^\beta - \Gamma_{\beta 0}^\alpha \Gamma_{0\alpha}^\beta$$

- But we know that  $\Gamma_{\alpha 0}^\alpha = 0$ , therefore:

$$R_{00} = -\Gamma_{0i,0}^i - \Gamma_{j0}^i \Gamma_{0i}^j = -\frac{\partial}{\partial t} \left( \frac{\dot{a}}{a} \right) \delta_{ii} - \left( \frac{\dot{a}}{a} \right)^2 \delta_{ii} = -3 \left( \frac{\ddot{a}}{a} - \left( \frac{\dot{a}}{a} \right)^2 \right) - 3 \left( \frac{\dot{a}}{a} \right)^2 = -3 \frac{\ddot{a}}{a}$$

- EXERCISE, Check that:

$$R_{ij} = \delta_{ij} (2\dot{a}^2 + a\ddot{a})$$

- And consequently:

$$\mathcal{R} \equiv g^{\mu\nu} R_{\mu\nu} = -R_{00} + \frac{1}{a^2} R_{ii} = 6 \left( \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 \right)$$

- Finally we find that:

$$3 \left( \frac{\dot{a}}{a} \right)^2 \leftarrow \boxed{R_{00} - \frac{1}{2} g_{00} \mathcal{R} = 8\pi G T_{00}}$$

# Einstein Equations

$$3 \left( \frac{\dot{a}}{a} \right)^2$$

$$R_{00} - \frac{1}{2}g_{00}\mathcal{R} = 8\pi GT_{00}$$

- $T_{\mu\nu}$  is the energy-momentum tensor, that in the case of a isotropic perfect fluid:

$$T^{\mu}_{\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

$$3 \left( \frac{\dot{a}}{a} \right)^2 = 8\pi G\rho$$

$$H^2(a) = \frac{8\pi G}{3}\rho \quad \text{Friedmann Equation (1)}$$

- Exercise! From:

$$R_{ij} - \frac{1}{2}g_{ij}\mathcal{R} = 8\pi GT_{ij}$$

- Derive the Friedmann Equation (2):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$