Lecture 2

Wednesday, 7 July 2021 09:49

2. Neutrino Mixing and Oscillations

2.1 Three Flavors of Neutrinos

$$\mathcal{L} \supset \sum_{\alpha=e,\mu,\tau} \Big[\overline{\nu_{\alpha,L}}\, \partial\!\!\!/\, \nu_{\alpha L} + \frac{g}{\sqrt{2}} \left(W^\mu \, \overline{\nu_{\alpha L}} \gamma_\mu e_{\alpha L} + h.c. \right) + \frac{g}{2\cos\theta_W} Z^\mu \, \overline{\nu_{\alpha L}} \gamma_\mu \nu_{\alpha L} \Big]$$

$$+ \left(\sum_{\alpha,\beta} \frac{1}{2} m_{\alpha\beta} \overline{(\nu_{\alpha L})^c} \, \nu_{\beta L} + h.c. \right)$$

- $(m_{\alpha\beta})$ in general off-diagonal
- Diagonalization: $\boxed{\underbrace{\nu_{\alpha L}}_{\text{flavor eigenstates}} = \underbrace{U_{\alpha j}}_{\text{mixing matrix (unitary)}} \underbrace{\nu_{jL}}_{\text{mass eigenstates}}}$

with $U^T m U = \text{diag}(m_1, m_2, m_3)$

This only works because $(m_{\alpha\beta})$ is symmetric.

- Note: For Dirac ν: $\mathcal{L} \supset \sum_{\alpha,\beta} m_{\alpha\beta} \overline{\nu_{\alpha L}} \nu_{\beta R}$

↳ the trafo is: $\nu_{\alpha L} = U_{\alpha j} \nu_{jL}$

$$\nu_{\beta R} = V_{\beta j} \nu_{jR}$$

$$U^\dagger m V = \text{diag}(m_1, m_2, m_3)$$

works for general complex $(m_{\alpha\beta})$

- In the mass basis

$$\mathcal{L} \supset \sum_j \left[\overline{\nu_{jL}} \, \partial\!\!\!/ \, \nu_{jL} \right]$$
$$+ \sum_\alpha \frac{g}{\sqrt{2}} \left(W^\mu \, \overline{\nu_{jL}} \, U^*_{\alpha j} \, \gamma_\mu \, e_{\alpha L} + h.c. \right)$$
$$+ \frac{g}{2\cos\theta_W} \sum_j Z^\mu \, \overline{\nu_{jL}} \, \gamma_\mu \, \nu_{jL}$$
$$+ \sum_j \frac{1}{2} m_j \, \overline{(\nu_{jL})^c} \, \nu_{jL}$$

↳ CC ν interaction involves superposition of mass eigenstates

2.2 Neutrino Oscillations

Produced in CC interaction

$$|\nu_\alpha\rangle = U^*_{\alpha j} \, |\nu_j\rangle$$

CC detection maps the neutrino onto flavor eigenstate

$$\langle \nu_\beta | = U_{\beta j} \, \langle \nu_j |$$

$$\hookrightarrow \mathcal{A} = \langle \nu_\beta | \nu_\alpha(t) \rangle$$

$$= \sum_j U^*_{\alpha j} \, U_{\beta j} \, \underbrace{e^{-iE_j T + i p_j L}}_{\text{time and space evolution of QM plane wave state}}$$

$$P(\nu_\alpha \to \nu_\beta) = |\mathcal{A}|^2$$

$$P(\nu_\alpha \to \nu_\beta) = |A|^2$$

$$= \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \cdot \underbrace{e^{-i(E_j - E_k)T + i(p_j - p_k)L}}_{\text{interference!}}$$

Note: States with different E and p can interfere only if E- and p-uncertainties are larger than $E_j - E_k$, $p_j - p_k$

Typically, we do not know T precisely because uncertainty in ν production time is much larger than $(E_j - E_k)^{-1}$.

$$\hookrightarrow \bar{P}(\nu_\alpha \to \nu_\beta) = \underbrace{\frac{1}{N}}_{\text{normalization factor}} \int dT \, |A|^2$$

$$= \frac{1}{N} \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta j} \cdot 2\pi \, \delta(E_j - E_k)$$

$$\cdot \underbrace{\exp\left[i\left(\sqrt{E^2 - m_j^2} - \sqrt{E^2 - m_k^2}\right) L\right]}_{\simeq \exp\left[-i \frac{\Delta m^2_{jk}}{2E} L\right]}$$

with $\Delta m^2_{jk} \equiv m_j^2 - m_k^2$

$$\left[\begin{array}{l} |\nu_\alpha\rangle = a^+_\alpha |0\rangle \\ \nu_\alpha = \int \frac{d^3p}{(2\pi)^3 2E} \left(b^+ e^{ipx} - a e^{-ipx}\right) \end{array} \right.$$

Consider 2 flavors:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\hookrightarrow \quad P(\nu_\alpha \to \nu_\beta) \overset{2\,fl.}{\simeq} \underbrace{\sin^2 2\theta}_{\text{osc. amplitude}} \; \underbrace{\sin^2 \frac{\Delta m^2 L}{4E}}_{\text{oscillation term}}$$

osc. length

$$\frac{\Delta m^2 L_{osc}}{4E} = \pi$$

$\Delta m^2_{21} \sim 8 \cdot 10^{-5} \rightarrow L_{osc} \sim 60\,km$

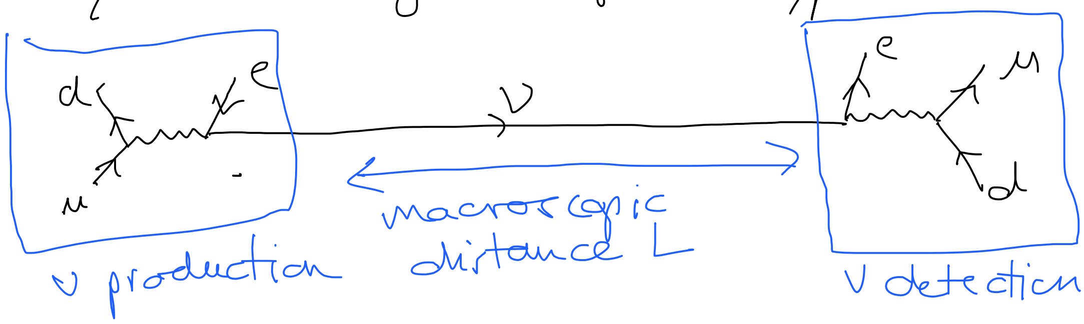
$\Delta m^2_{31} \sim 2 \cdot 10^{-3} \rightarrow L_{osc} \sim 1\,km$

@ $E \sim O(MeV) \Rightarrow L_{osc} = \frac{4\pi E}{\Delta m^2}$

mixing angle θ controls osc. amplitude

Δm^2 controls osc. length

Note: in QFT, ν osc. can be described by a Feynman diagram of the type



treat all external particles as wave packets.

2.3 3-flavor oscillations

mixing matrix is unitary 3×3 matrix

general parameterization:

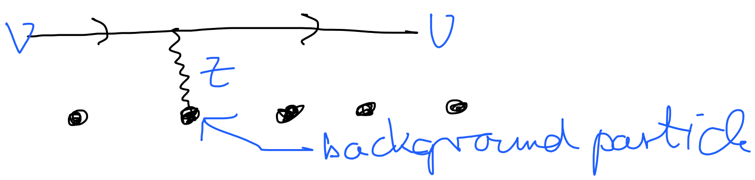
$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & \end{pmatrix}$$

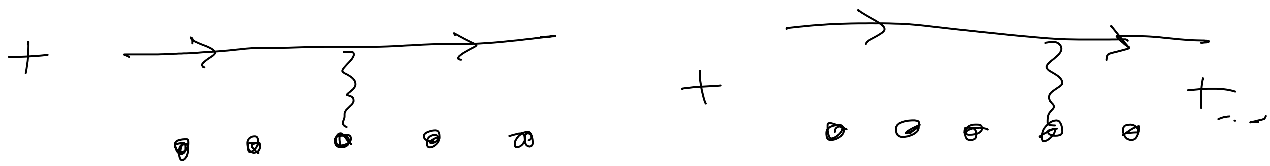
$\sin\theta_{23} \qquad -\cos\theta_{23}$

Where phase factors have been absorbed into redefinitions of states.

2.4 Neutrino oscillations in matter

Coherent forward scattering



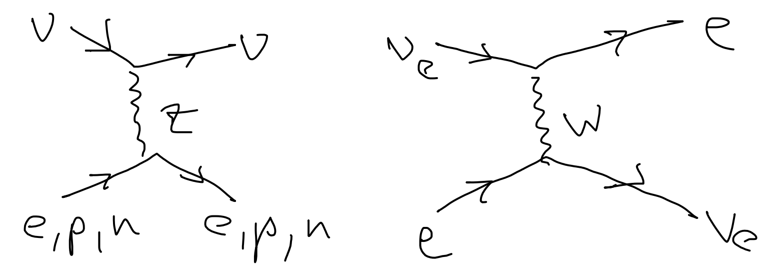


Exchange of W or Z boson without momentum change means that all background contribute coherently.

[Analogy: Photon travelling through medium

↳ $|\mathcal{M}|^2 \sim N^2 G_F^2$

↑ density of scatterers



for any ν_α for ν_e

quantum states of incoming particles identical to those of outgoing ones for coh. forward scattering

$$H_{eff} = \frac{G_F}{\sqrt{2}} [\bar{e}\gamma^\mu(1-\gamma^5)\nu_e][\bar{\nu}_e\gamma_\mu(1-\gamma^5)e]$$

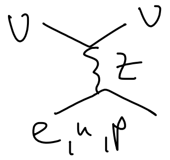
$$\overset{\text{Fierz}}{=} \frac{G_F}{\sqrt{2}} \left[\bar{e}\gamma^\mu(1-\gamma^5)e\right]\left[\bar{\nu}_e\gamma_\mu(1-\gamma^5)\nu_e\right]$$

Treat e as fixed background field
↳ take expectation value:

$$\langle H_{eff}\rangle = \frac{G_F}{\sqrt{2}} \underbrace{\langle \bar{e}\gamma^\mu(1-\gamma^5)e\rangle}_{= \begin{cases} n_e & \text{for } \mu=0 \\ 0 & \text{for } \mu=1,2,3 \end{cases}} [\bar{\nu}_e\gamma_\mu(1-\gamma^5)\nu_e]$$

$$= \underbrace{\sqrt{2}G_F n_e}_{=V_{CC}} \quad \bar{\nu}_{eL}\gamma^0\nu_{eL}$$

CC MSW potential

Similarly for 

: $V_{NC} = -\frac{1}{2}\sqrt{2}G_F n_n$

In the derivation of $P(\nu_\alpha \to \nu_\beta)$, we had factors of the form e^{ipL}.

$$\phi \equiv p\cdot L = \sqrt{(\hat{H}-\hat{V})^2 - \hat{M}^2}$$

$\hat{H}$: 2x2 matrix; $\hat{V}$: MSW potentials 2x2; $\hat{M}^2$: 2x2 mass matrix

$$\overset{\hat{V},\,\hat{M}\text{ small}}{\simeq} \hat{H} - \frac{\hat{M}^2}{2\hat{H}} - \hat{V}$$

Diagonalize
$$\hat{H} - \frac{\hat{M}^2}{2\hat{H}} - \hat{V} = E\cdot\mathbb{1} - U\begin{pmatrix}\frac{m_1^2}{2E} & \\ & \frac{m_2^2}{2E}\end{pmatrix}U^\dagger - \begin{pmatrix}V_{CC}+V_{NC} & \\ & V_{NC}\end{pmatrix}$$

Result:

$$\sin^2 2\theta_{eff} = \frac{\sin 2\theta \, \frac{\Delta m}{2E}}{\sqrt{\left(\sqrt{2}G_F n_e - \frac{\Delta m^2}{2E}\cos 2\theta\right)^2 + \left(\frac{\Delta m^2}{2E}\right)^2 \sin^2 2\theta}}$$

$$\frac{\Delta m^2_{eff}}{2E} = \sqrt{\left(\sqrt{2}G_F n_e - \frac{\Delta m^2}{2E}\cos 2\theta\right)^2 + \left(\frac{\Delta m^2}{2E}\right)^2 \sin^2 2\theta}$$

At the MSW resonance:

$$\frac{\Delta m^2}{2E}\cos 2\theta = \sqrt{2}G_F n_e$$

the effective mixing angle is maximal.