

# Correlating anisotropic flow with isotropic flow in heavy-ion collisions

Nuclear phenomenology at high energy  
beyond the quark-gluon plasma

by

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to appear as a review in EPJA  
with J. Jia and Y. Zhou

- **HEAVY-ION COLLISIONS: QGP**

- Particle multiplicities ( $dN/d\eta$ ).
- Anisotropic flow ( $V_n$ ).
- Isotropic flow ( $\langle p_t \rangle$ )
- Status of soft sector.

- **PHENOMENOLOGY BEYOND THE QGP**

- Correlating  $V_n$  with  $\langle p_t \rangle$  (at fixed  $dN/d\eta$ ).
- Strong magnetic fields.
- Nuclear deformation.
- Primordial momentum anisotropies.

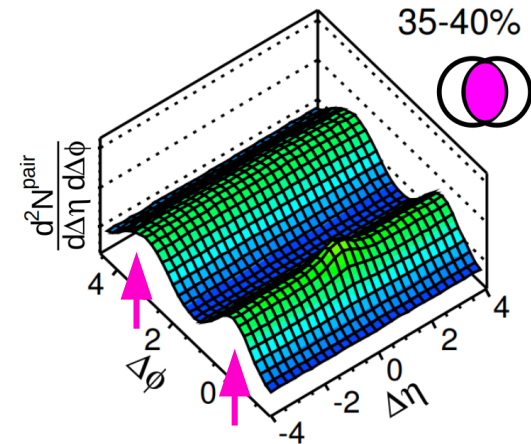
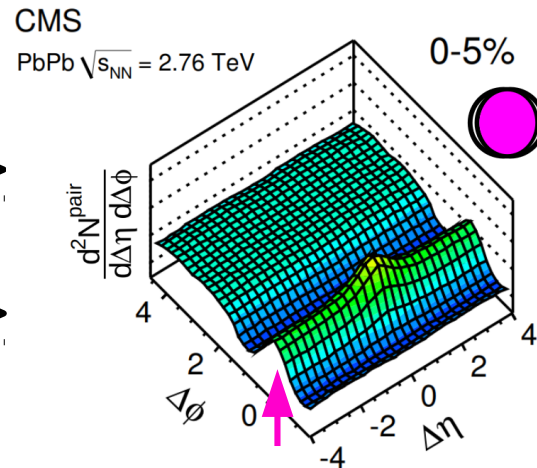
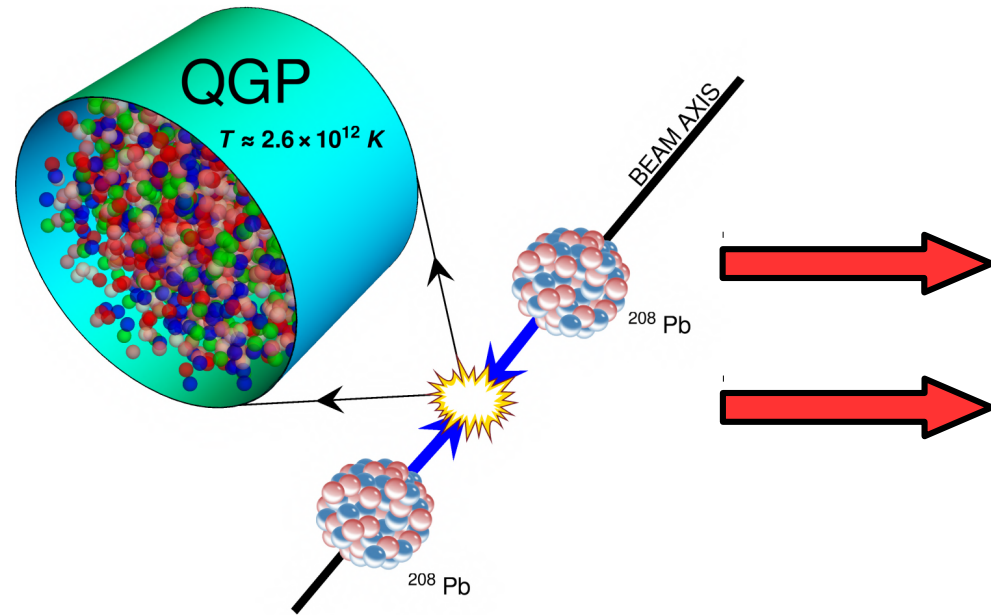
- **OUTLOOK**

- The tip of the iceberg.

# HEAVY-ION COLLISIONS: EMERGENT PHENOMENA AT HIGH ENERGY.

- Particle density is huge:  $1$  to  $10 \text{ fm}^{-3}$ . Nuclear matter:  $0.16 \text{ fm}^{-3}$ .
- Regular patterns in data: **collective phenomena.**

“More is different”  
[Anderson, 1972]



[CMS Collaboration, [1201.3158](#)]

[Gardim, Giacalone,  
Luzum, Ollitrault, [1908.09728](#)]

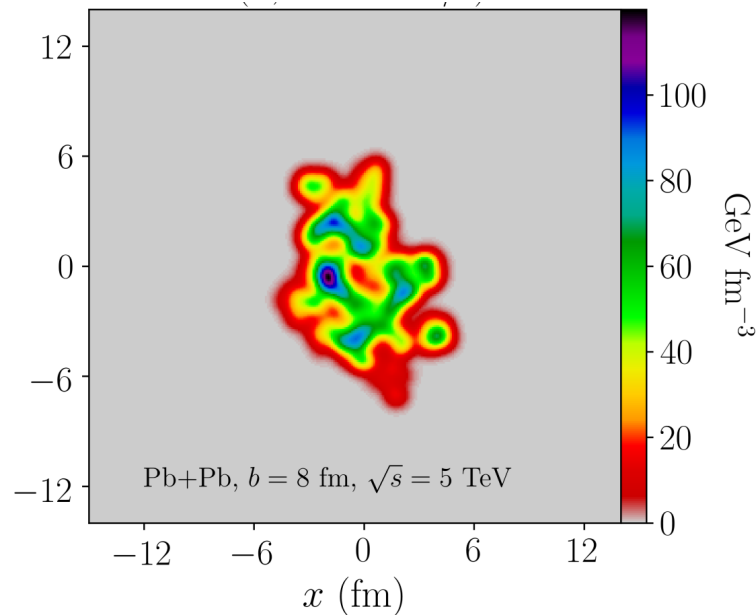
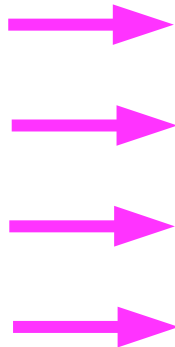
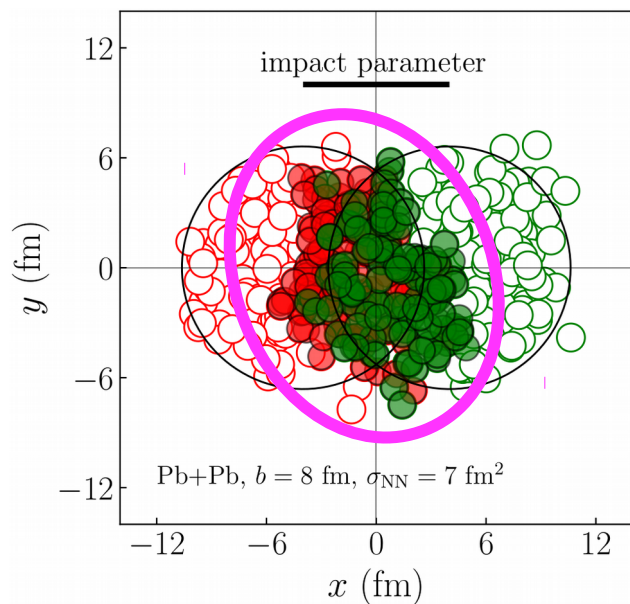
**System size  $\gg$  “mean free path”. Equilibration on time scale of QCD,  $\sim 1$  fm/c.**

[Schlichting, Teaney, [1908.02113](#)] [Berges, Mazeliauskas, Spaliński, Venugopalan, [2005.12299](#)]

$\implies$  **Effective description: relativistic fluid.**

[Romatschke & Romatschke, [1712.05815](#)]

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \text{small viscous corrections } (\eta/s, \zeta/s, \dots) + \partial_\mu T^{\mu\nu} = 0$$

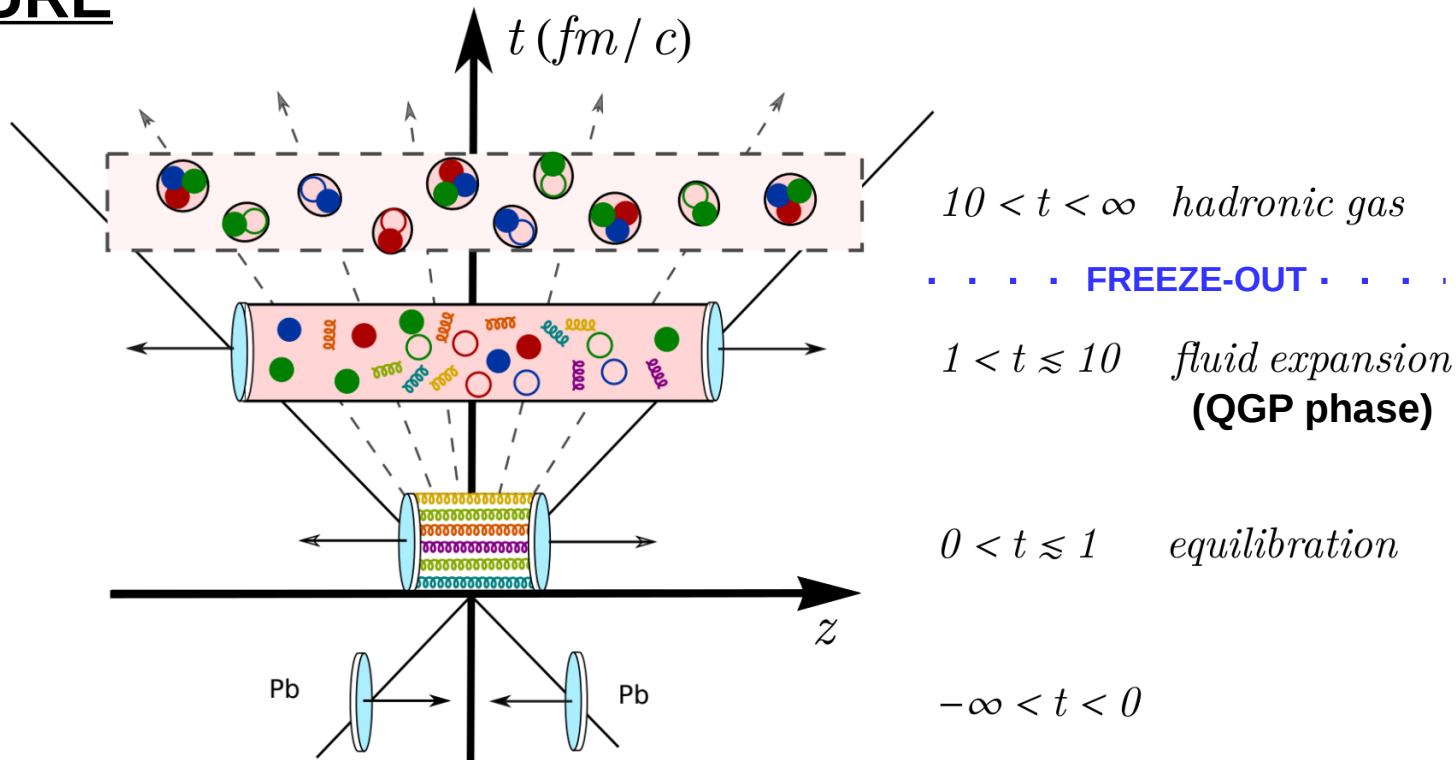


**Equation of state from lattice QCD ( $T > 156$  MeV). Large number of **DOF** ( $\sim 40$ ): QGP.**

[HotQCD collaboration, [1407.6387](#)]

# BIG PICTURE

(courtesy A. Mazeliauskas)



$t = \infty$  →
 $\frac{dN}{d^3\mathbf{p}}$  or  $\frac{dN}{dyd^2\mathbf{p}_t}$ 
**Hadron spectrum in momentum space.**

Bulk of produced  $O(10^3-10^4)$  particles is soft,  $p_t < 2 \text{ GeV}$ .

**Particle yields are nearly independent of rapidity.**

Fundamental quantities in the soft sector:

**1 - event multiplicity**

$$N = \int_{\mathbf{p}_t} \frac{dN}{d^2\mathbf{p}_t}$$

**2 - anisotropic flow**

$$V_n = \frac{1}{N} \int_{\mathbf{p}_t} \frac{dN}{d^2\mathbf{p}_t} e^{-in\phi_p}$$

**3 - average momentum**

$$\langle p_t \rangle = \frac{1}{N} \int_{\mathbf{p}_t} p_t \frac{dN}{d^2\mathbf{p}_t}$$

**Basics of QGP pheno: origin of these quantities.**

[Giacalone, **2101.00168**]

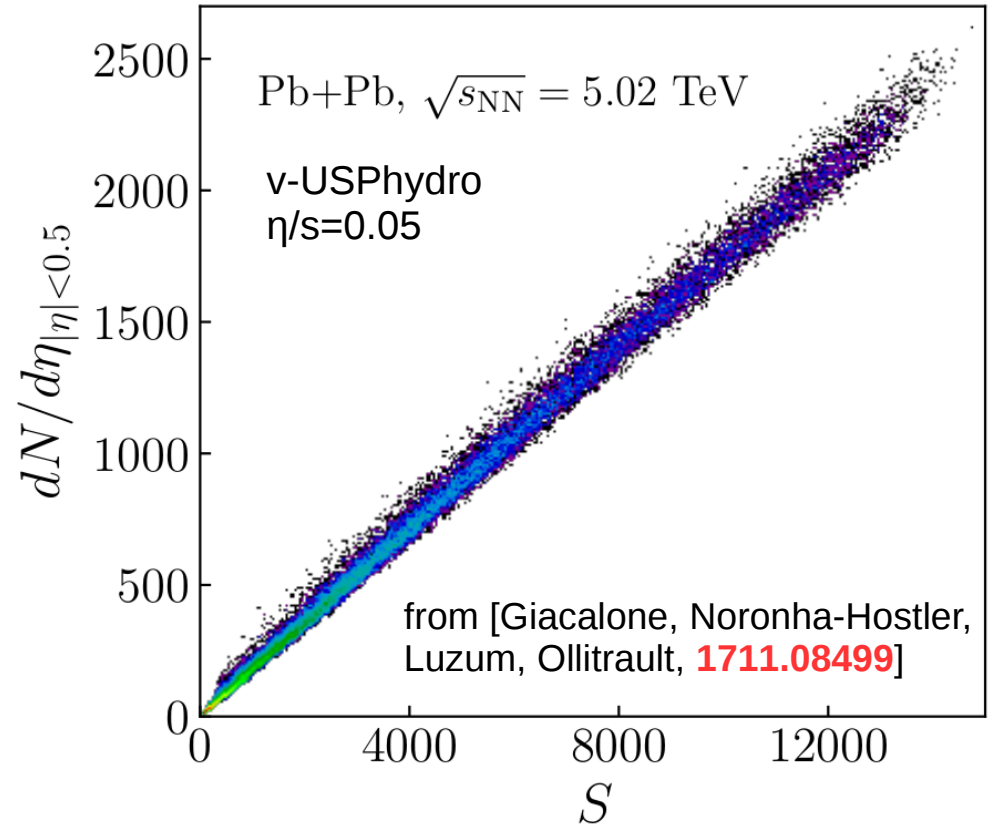
# 1 - event multiplicity.

Underlying physics is an ideal gas of massless particles.

The expansion is nearly isentropic.

**Entropy,  $S$ , proportional to the number of detected hadrons:**

$$S \propto N \text{ (dN/d}\eta\text{)}$$



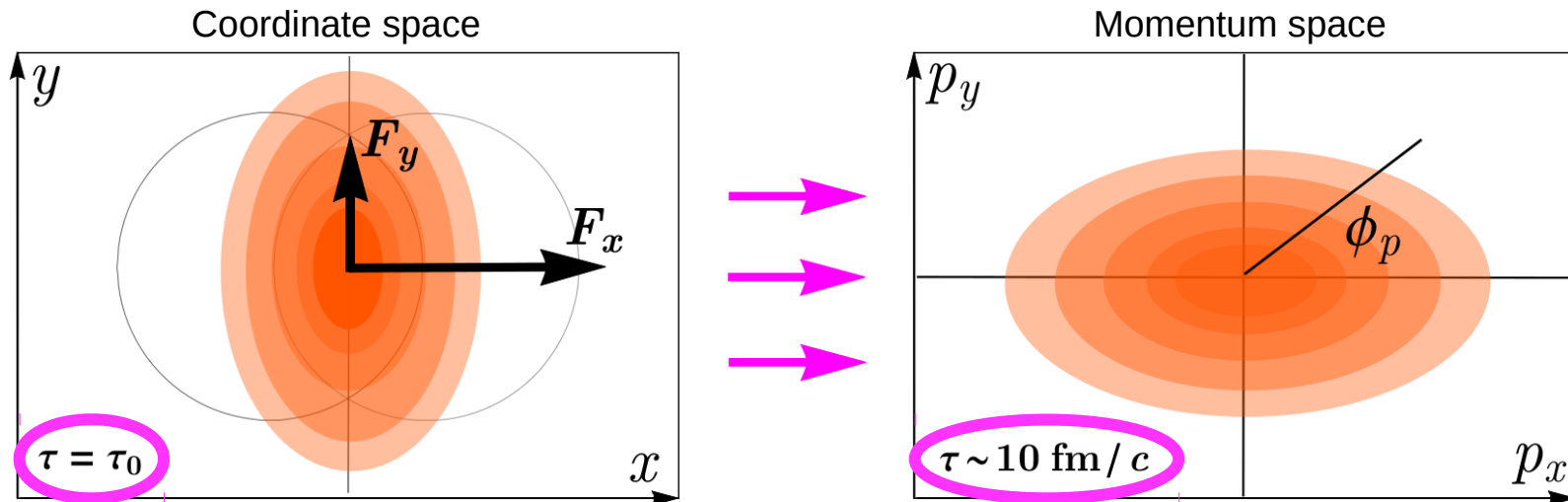
## 2 - anisotropic flow.

Azimuthal anisotropy of particle emission.  
**Elliptic flow**, the 2<sup>nd</sup> harmonic.

$$\longrightarrow V_2 = \frac{1}{N} \int_{\mathbf{p}_t} \frac{dN}{d^2\mathbf{p}_t} e^{-i2\phi_p}$$

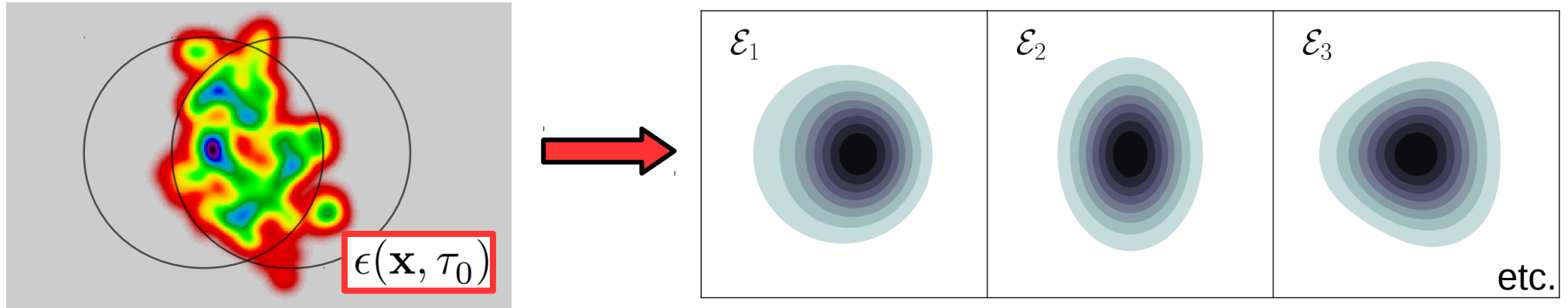
**Geometric origin:** shape-flow transmutation at finite impact parameter.

[Ollitrault, 1992]





**More than ellipse:** due to fluctuations, all multipoles are nonzero.

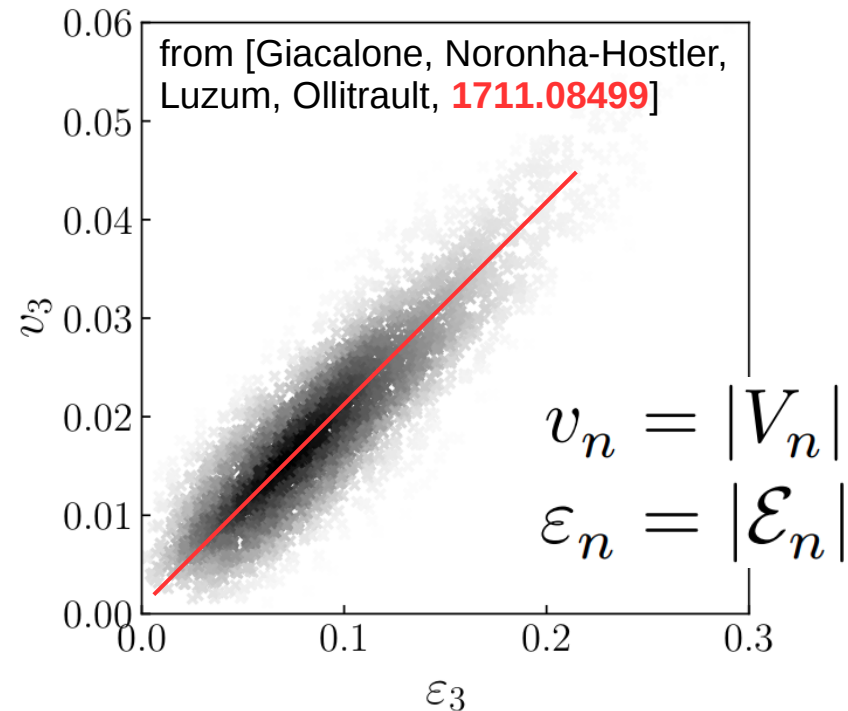
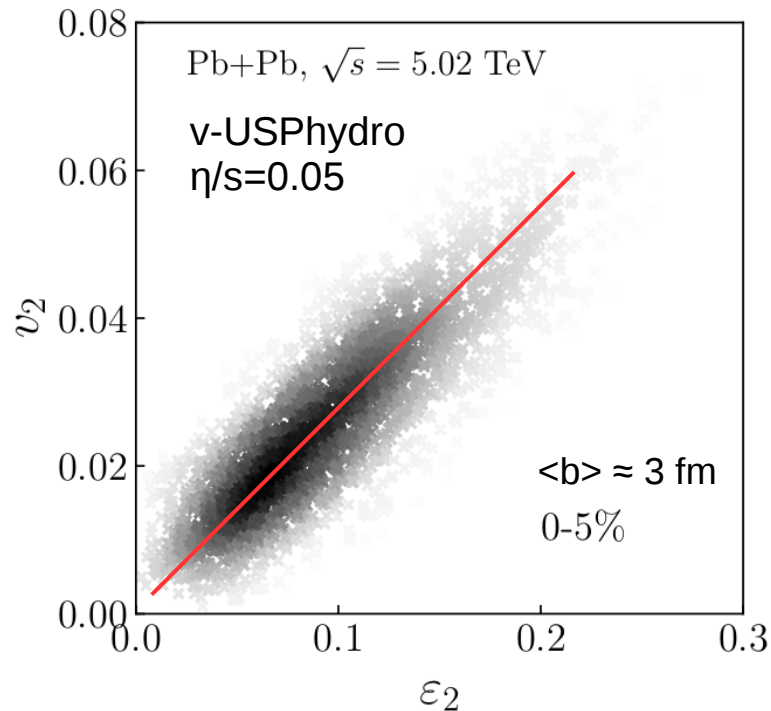


Deformations in two dimensions identified by [Teaney, Yan, [1010.1876](#)]:

$$\mathcal{E}_n = - \frac{\int r dr d\phi r^n e^{in\phi} \epsilon(r, \phi)}{\int r dr d\phi r^n \epsilon(r, \phi)}$$

Each  $\mathcal{E}_n$  in the initial state leads to  $V_n$  in the final state.

A simple relation:  $V_n \propto \mathcal{E}_n$



Explains experimental data in both large and small systems.

**The importance of initial conditions.** [Giacalone, Noronha-Hostler, Ollitrault, [1702.01730](#)]

### 3 – average momentum (isotropic flow)

Mean transverse momentum is the “energy per particle”.

$$\longrightarrow \langle p_t \rangle = \frac{1}{N} \int_{\mathbf{p}_t} p_t \frac{dN}{d^2\mathbf{p}_t}$$

Energy per particle in the ideal gas:

$$p \simeq E = 3T$$

Therefore in a heavy-ion collision we expect:

$$\langle p_t \rangle \simeq 3T$$

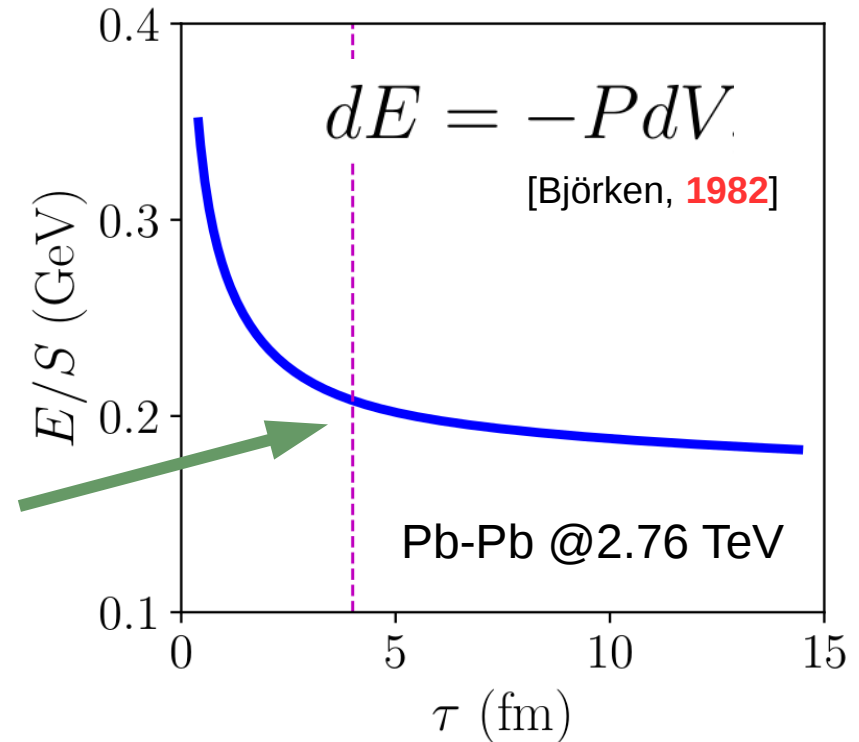
**where T is the temperature at the end of cooling.**

Verified in hydrodynamic simulations.

[Gardim, Giacalone, Luzum, Ollitrault, **1908.09728**]

#### Application:

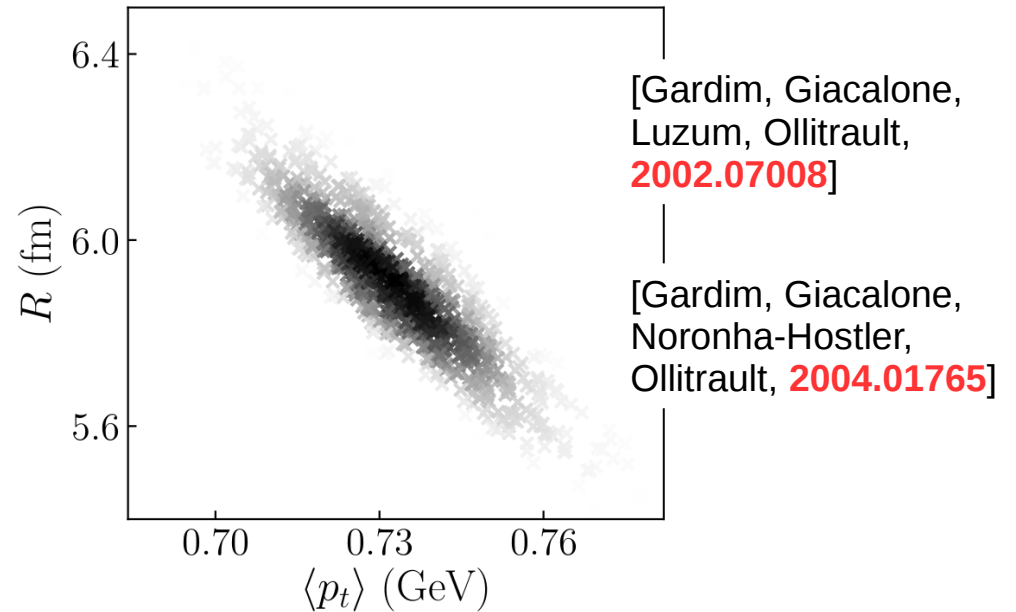
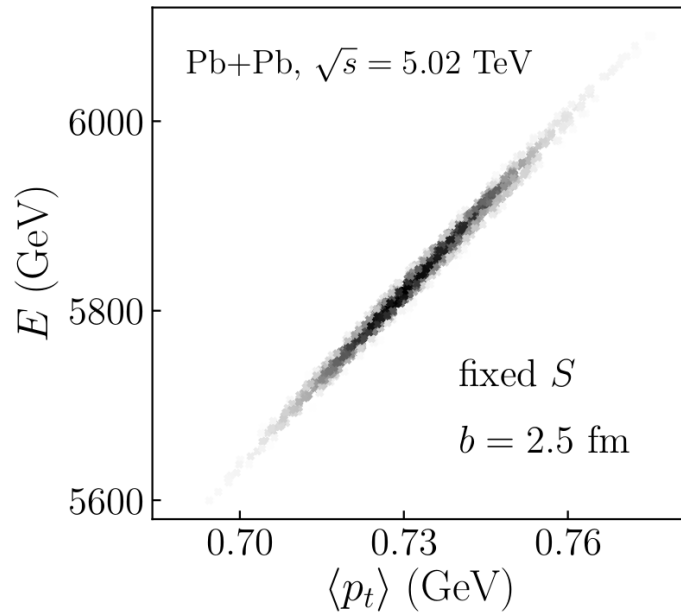
ALICE measures  $\langle p_t \rangle \approx 680 \text{ MeV}$  in 0-5%. The temperature is  $\langle p_t \rangle / 3 \approx 226 \text{ MeV} \sim 2.6 \times 10^{12} \text{ K}$ .



# Origin of $\langle p_t \rangle$ fluctuations (fixed multiplicity).

$$c_s^2 = \frac{dP}{d\epsilon} = \frac{d \ln T}{d \ln s} \xrightarrow{\langle p_t \rangle \sim T} \frac{d\langle p_t \rangle}{\langle p_t \rangle} \propto \frac{dE}{E} \quad \frac{d\langle p_t \rangle}{\langle p_t \rangle} \propto -\frac{dR}{R}$$


(E=initial energy, R=initial radius)



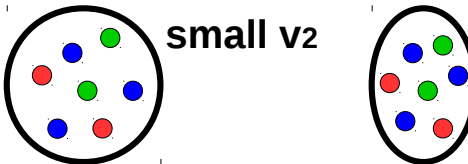
**Proportionality factors depend on the equation of state.**

# SUMMARY OF INTRODUCTION: QGP BASICS

- The **number of detected hadrons** is a measure of the **entropy**.

$$S \propto N$$


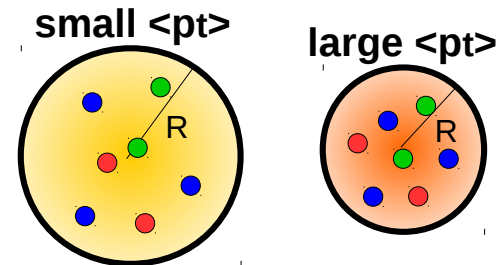
- Anisotropic flow coefficients** ( $V_n$ ) are a hydrodynamic response to the **initial spatial anisotropies** ( $\mathcal{E}_n$ ).

$$V_n \propto \mathcal{E}_n$$


- $\langle p_t \rangle$  depends on the temperature reached in the QGP. Its **fluctuations probe the thermodynamics**.

$$\frac{d\langle p_t \rangle}{\langle p_t \rangle} \propto \frac{dE}{E}$$

$$\frac{d\langle p_t \rangle}{\langle p_t \rangle} \propto -\frac{dR}{R}$$



## Status of the field. I think there are three main directions:

- **Clarifying the origin and limits of applicability of this picture.**

[Berges, Mazeliauskas, Spaliński, Venugopalan, [2005.12299](#)]

- **Refining the picture and pinning down viscosity/initial conditions/EOS.**

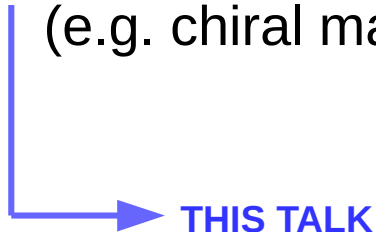
[Trajectum, [2010.15130](#), [2010.15134](#)] [JETSCAPE Collaboration, [2011.01430](#), [2010.03928](#)]

[Devetak, Dubla, Floerchinger, Grossi, Masciocchi, Mazeliauskas, Selyuzhenkov, [1909.10485](#)]

[Gardim, Giacalone, Ollitrault, [1909.11609](#)]

- **Use the established picture to reveal new phenomena at high energy.**  
(e.g. chiral magnetic effect, CGC, hydrodynamics with spin, nuclear structure)

[Giacalone, Jia, Zhou, [2108.xxxxx](#)]



# Nuclear phenomena at high energy with multi-particle correlations.

Breakthrough idea:

$$\langle v_n^2 \delta[p_t] \rangle \equiv \left\langle \frac{\int_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3} (p_1 - \langle\langle p \rangle\rangle) e^{in(\phi_2 - \phi_3)} \frac{dN}{d^2 \mathbf{p}_1 d^2 \mathbf{p}_2 d^2 \mathbf{p}_3}}{\int_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3} \frac{dN}{d^2 \mathbf{p}_1 d^2 \mathbf{p}_2 d^2 \mathbf{p}_3}} \right\rangle$$

First apparition as a byproduct of a principal component analysis.

[Mazeliauskas, Teaney, [1509.07492](#)]

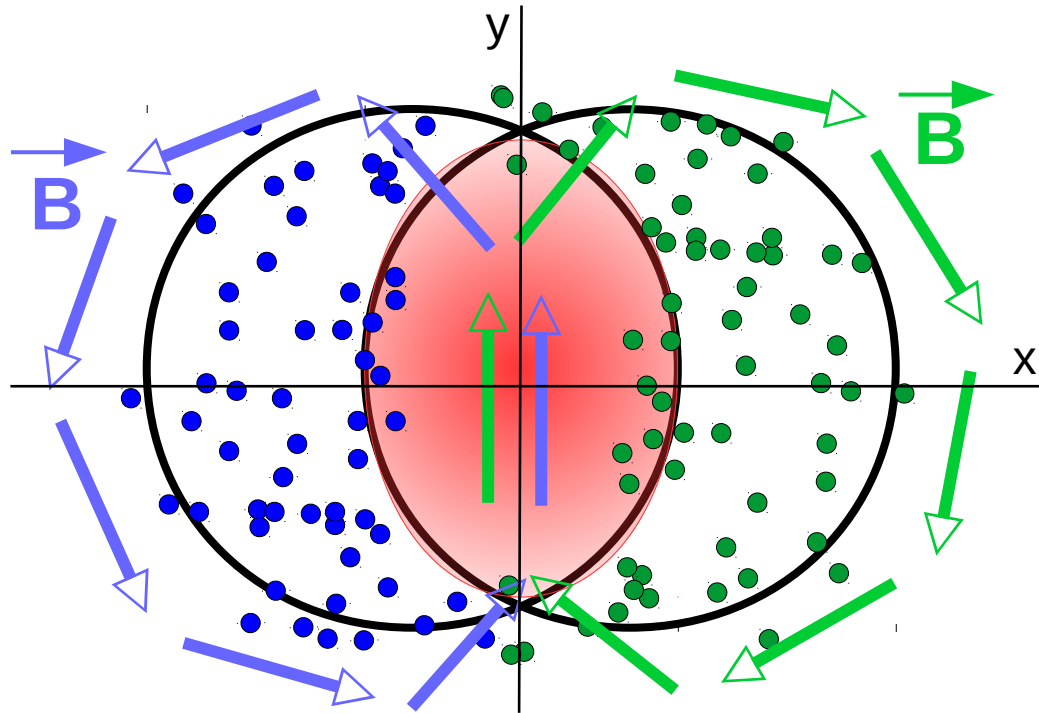
Božek's formulation as a **Pearson correlation coefficient**:

$$\rho(v_n^2, [p_t]) = \frac{\langle \delta v_n^2 \delta[p_t] \rangle}{\sqrt{\langle (\delta v_n^2)^2 \rangle \langle (\delta[p_t])^2 \rangle}}$$

[Božek, [1601.04513](#)]

With  $\delta o = o - \langle o \rangle$  at fixed multiplicity (entropy).

# #1 – Electromagnetic fields.



– Coherent B field of **spectator protons** over interaction region.

– Strong field  $|\mathbf{B}| \sim 10^{14}$  T may yield parity-violating effects (CME).

[Li, Wang, [2002.10397](#)]

– Involves **charge-dependent dipolar flows**.

$$\langle \cos(\phi_1^\pm - \phi_2^\pm) \rangle$$

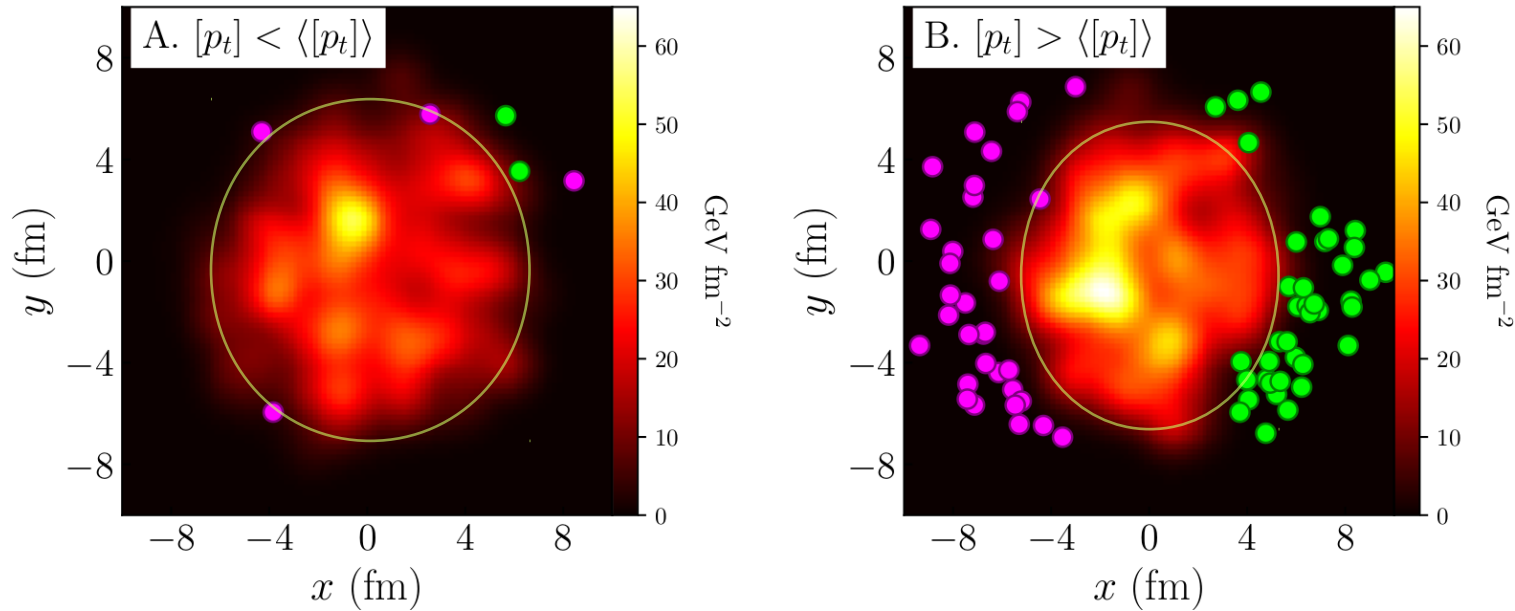
[Oliva, [2007.00560](#)]

– **Experimental evidence missing.**  
But we can use  $\langle pt \rangle$ !



Select two **central events** (2-3%) at the **same multiplicity** but **different [pt]**.

Isentropic transformation of the QGP which increases  $T$  and reduces  $R$ .



| event                         | A     | B     |
|-------------------------------|-------|-------|
| $N_{ch}$                      | 2813  | 2791  |
| $b$ (fm)                      | 0.49  | 4.41  |
| $N_s$                         | 6     | 72    |
| $[p_t]/\langle [p_t] \rangle$ | 0.976 | 1.028 |
| Isentropic transformation     |       |       |
| $R$ (fm)                      | 4.53  | 4.03  |
| $\varepsilon_2$               | 0.073 | 0.153 |

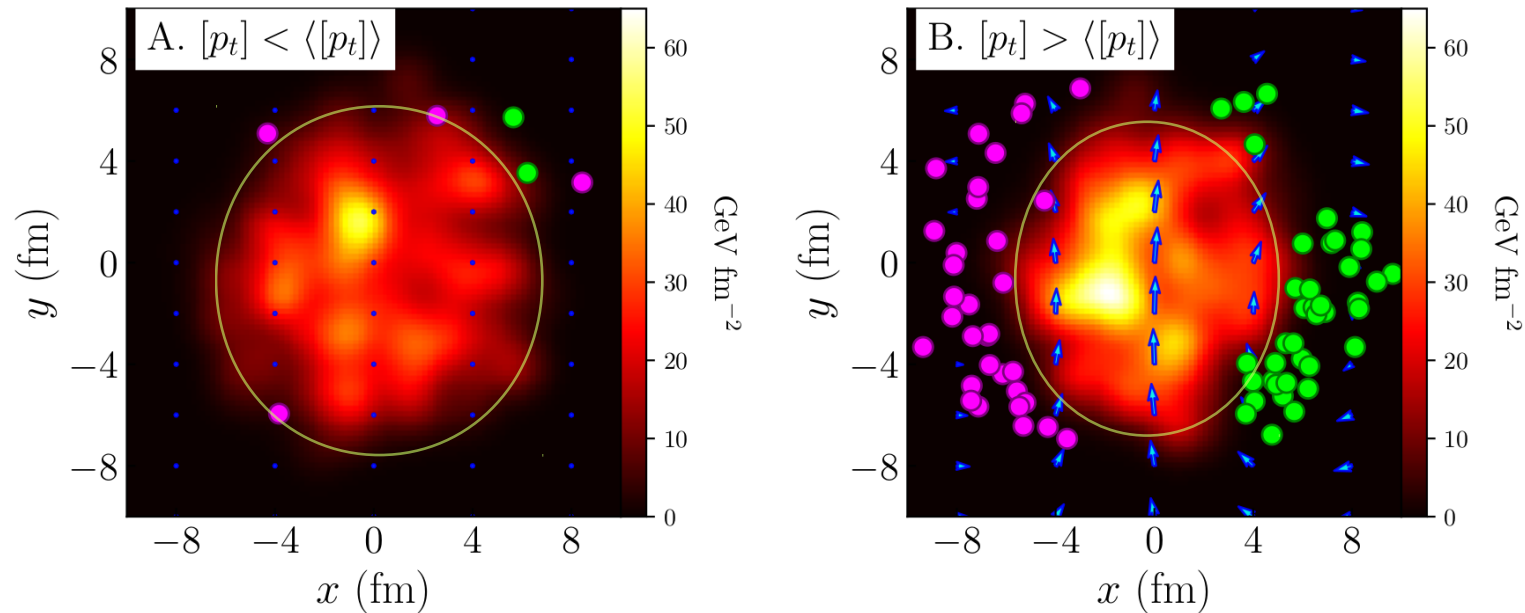
[Giacalone, [2006.06269](#)]: strong correlation between  $[pt]$  and  $\langle N_s \rangle$ .  
What about the magnetic field?

# Event-by-event calculation of $\langle B_y \rangle$ within the framework of:

[Gürsoy, Kharzeev, Rajagopal, **1401.3805**]

[Gürsoy, Kharzeev, Marcus, Rajagopal, Shen, **1806.05288**]

Consider only B field over overlap area:  $\langle B_y \rangle = \frac{1}{E} \int d^2\mathbf{x} B_y(\mathbf{x}) e(\mathbf{x})$



| event                           | A     | B     |
|---------------------------------|-------|-------|
| $N_{\text{ch}}$                 | 2813  | 2791  |
| $b$ (fm)                        | 0.49  | 4.41  |
| $N_s$                           | 6     | 72    |
| $[p_t] / \langle [p_t] \rangle$ | 0.976 | 1.028 |
| $\langle B_y \rangle / m_\pi^2$ | 0.013 | 0.151 |
| $R$ (fm)                        | 4.53  | 4.03  |
| $\varepsilon_2$                 | 0.073 | 0.153 |

**The idea works!** Increase  $[p_t]$  and the B field appears!

# AN OPTIMAL EVENT-SHAPE ENGINEERING BASED ON [pt].

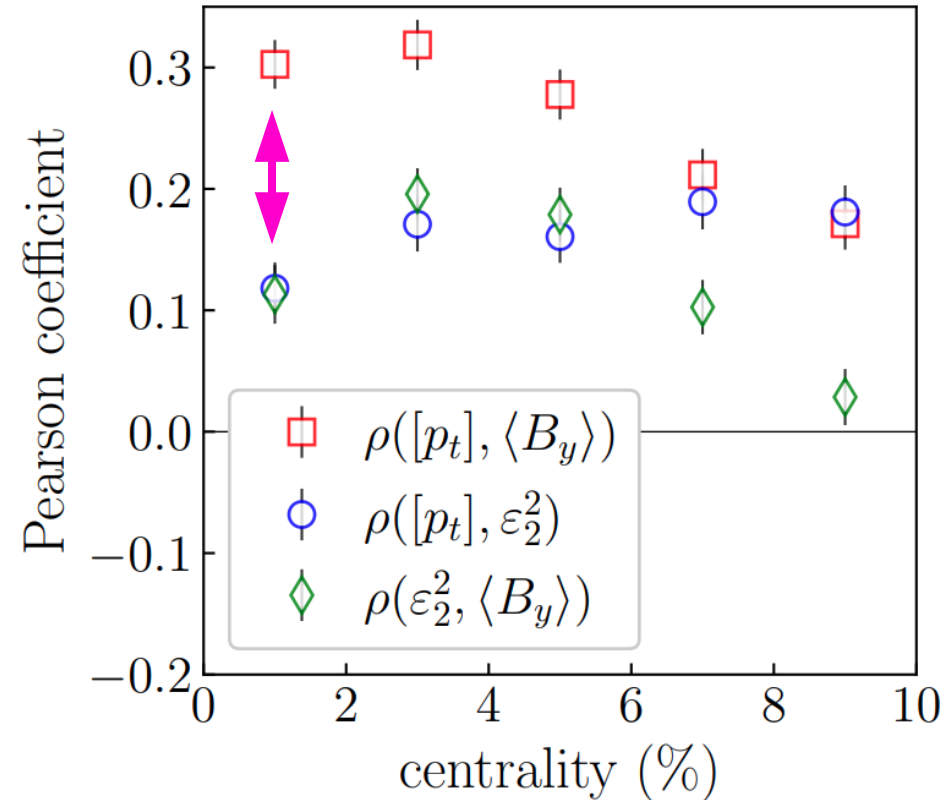
[Giacalone, Shen, [2104.01890](#)]

$$- \rho([p_t], \langle B_y \rangle) > \rho(\varepsilon_2^2, \langle B_y \rangle)$$

Better handle on  $\langle B_y \rangle$  than usual  
ESE based on  $q_2$  vectors. ;-)

$$- \rho([p_t], \langle B_y \rangle) > \rho([p_t], \varepsilon_2^2)$$

Increases  $\langle B_y \rangle$  more than  $v_2$ .  
Important for CME background.



## OBSERVABLE NATURALLY SENSITIVE TO THE B FIELD:

$$\left\langle \delta \langle p_t \rangle \cos (\phi_1^\pm - \phi_2^\pm) \right\rangle$$

[Giacalone, [2006.06269](#)]

[Giacalone, Shen, [2104.01890](#)]

– Correlation between charge-dependent  $v_1$  and  $\langle p_t \rangle$  at fixed multiplicity.

– Can be turned into a Pearson coefficient:

$$\rho^\pm (\langle p_t \rangle, v_1^\pm) = \frac{\left\langle \delta \langle p_t \rangle \cos (\phi_1^\pm - \phi_2^\pm) \right\rangle}{\sqrt{\left\langle (\delta \langle p_t \rangle)^2 \right\rangle (g^\pm / v_2)}} \rightarrow \left\langle \cos (\phi_1^\pm + \phi_2^\pm - 2\phi_3) \right\rangle$$

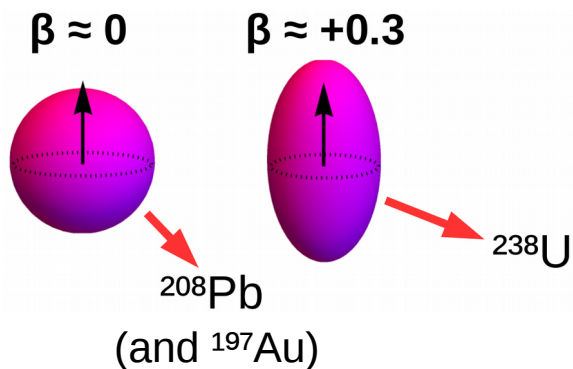
## #2 – Nuclear structure: deformation.

Majority of nuclei have an intrinsic quadrupole moment:

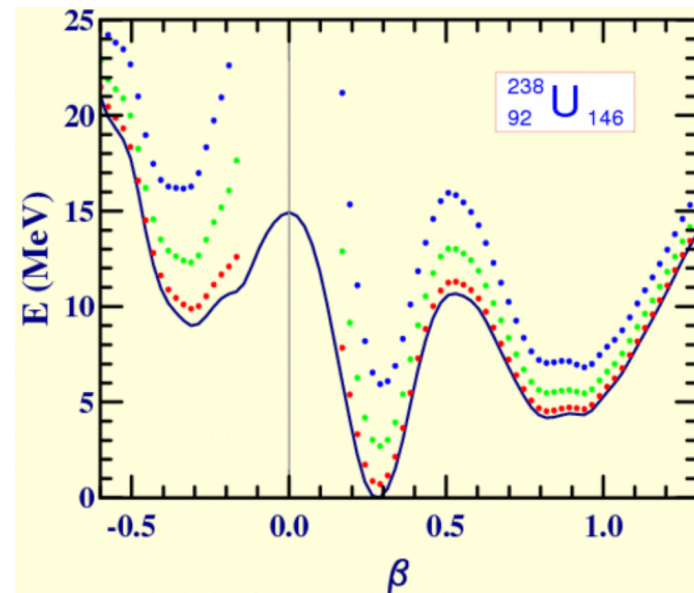
$$Q_2 \propto \langle Y_2^0(\Theta, \Phi) r^2 \rangle \neq 0$$

Deformation quantified by a coefficient:

$$\beta \propto \frac{Q_2}{\langle r^2 \rangle}$$



**Hartree-Fock approach:**  
deformed configurations  
are energetically favored.

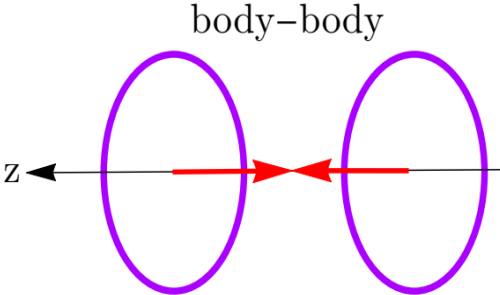
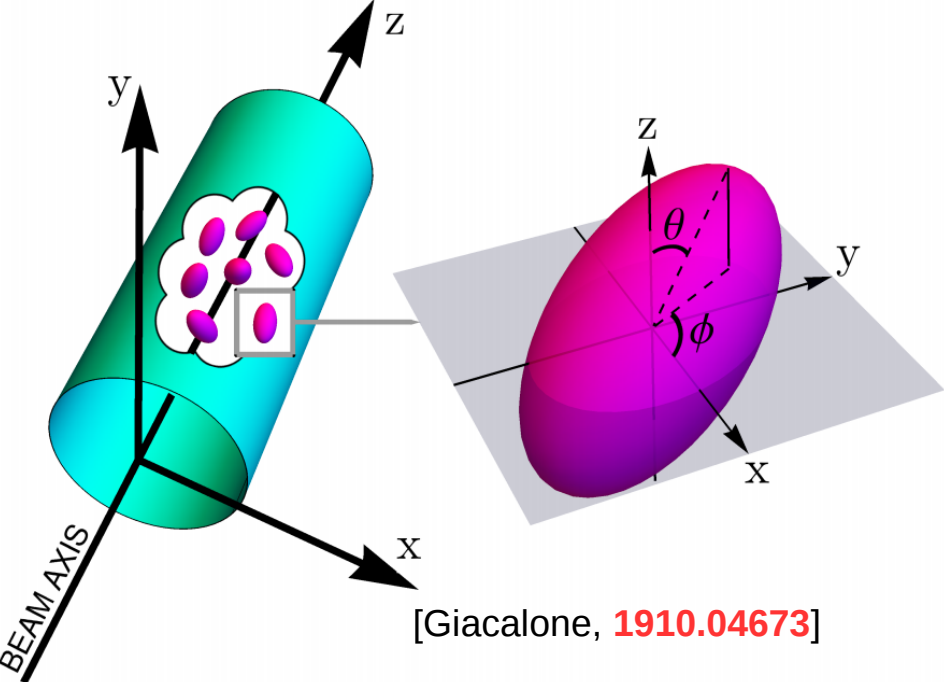


**Rotational model:**  
intrinsically deformed object  
with a random orientation.

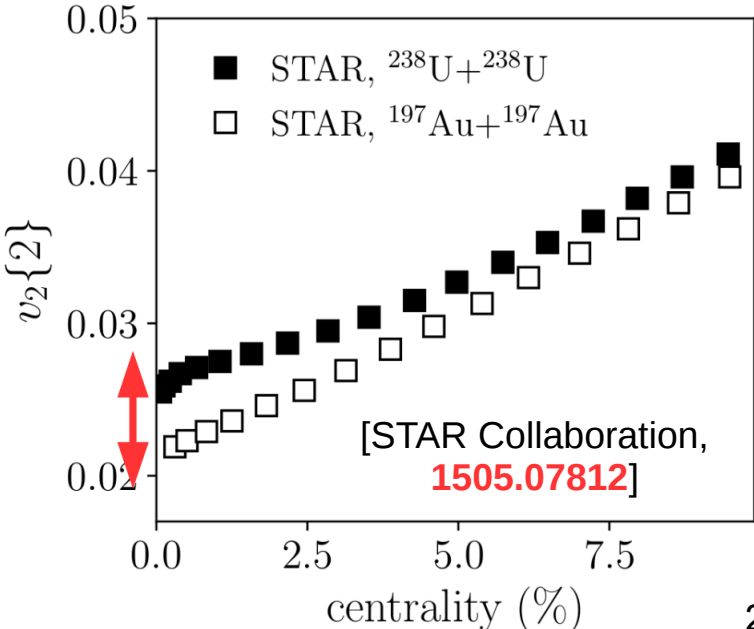
[Bohr, Mottelson 1957]

# Nuclear structure and heavy-ion collisions: an inevitable marriage.

## Deformed nuclei in the beampipe.

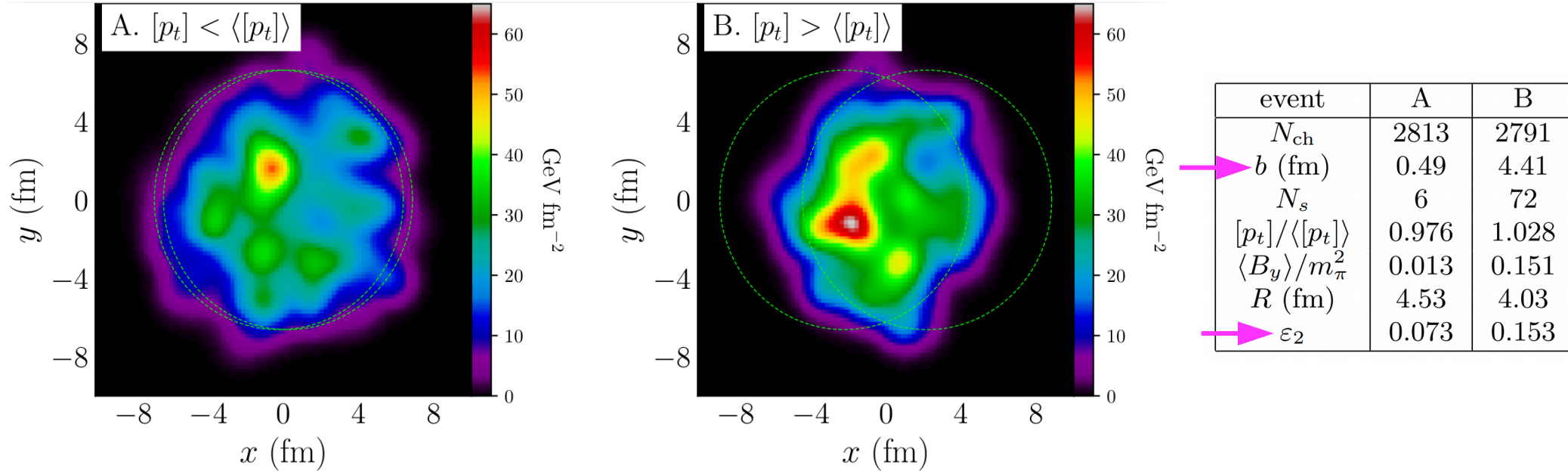


**Prediction:** excess  $v_2$  in central U+U collisions. Observed at RHIC.



We can use  $\langle p_t \rangle$  to reveal nuclear deformation. Back to spherical nuclei.

Select two **central events** (2-3%) at the **same multiplicity** but **different  $[p_t]$** .

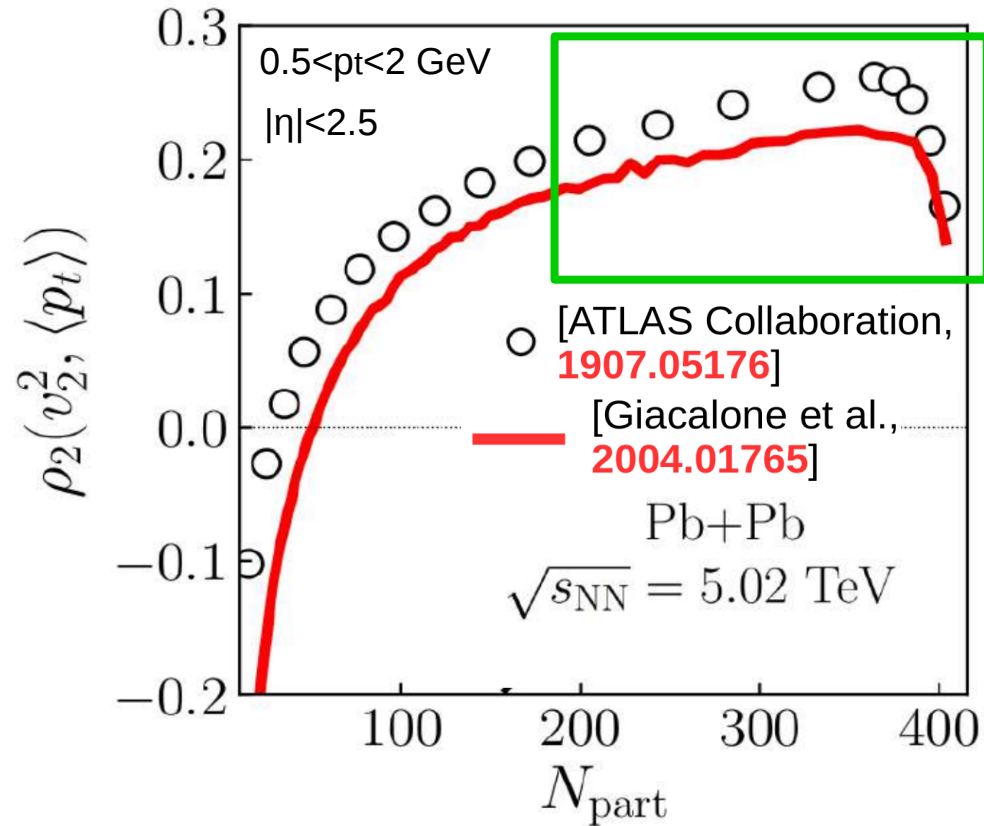


Increasing  $\langle p_t \rangle$  increases the impact parameter and eccentricity!

**Prediction.** In central heavy-ion collisions:

$$\rho(v_2^2, [p_t]) > 0$$

Prediction verified at LHC. Correlation is positive in central collisions.

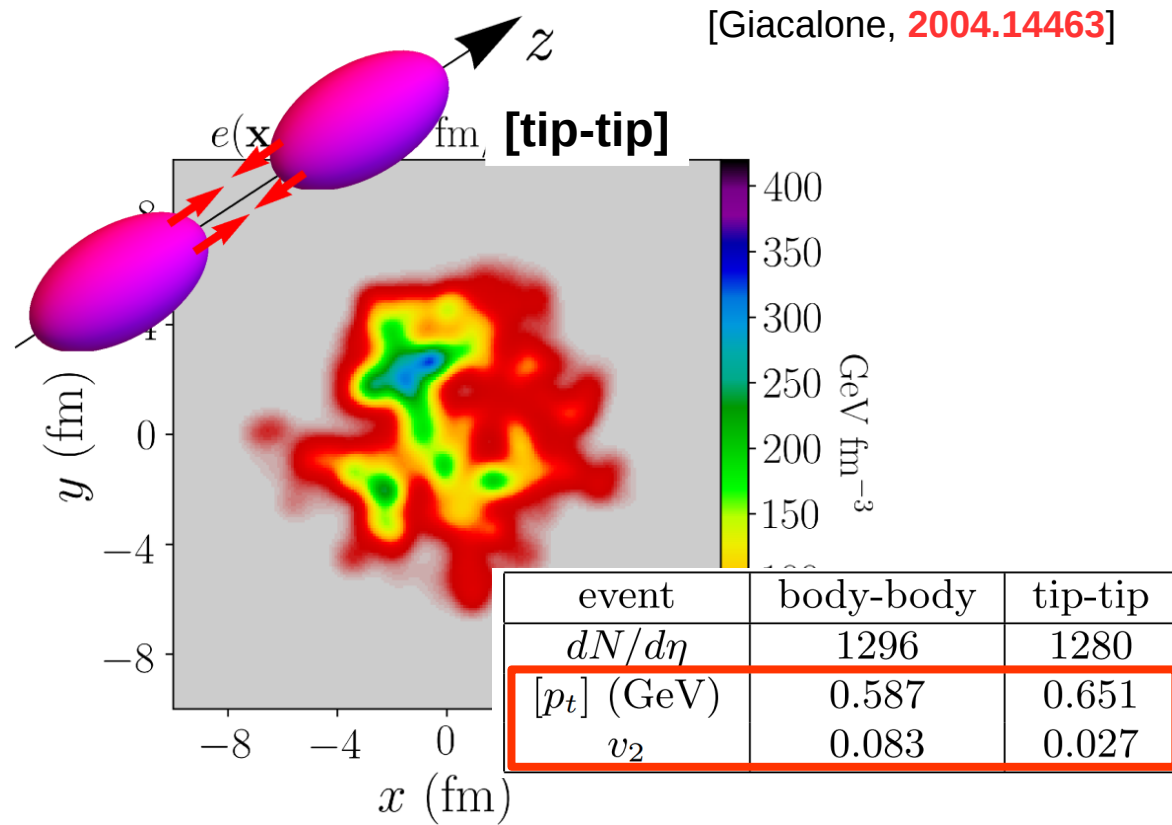
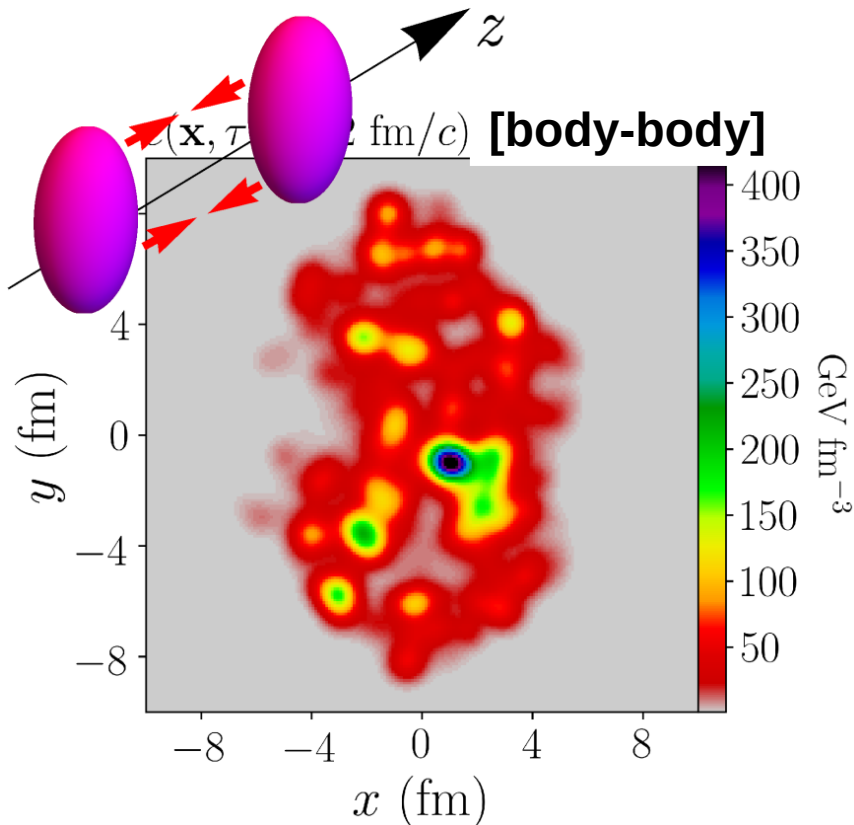




# WHAT IF THE COLLIDING NUCLEI ARE DEFORMED?

[Giacalone, [1910.04673](#)]

[Giacalone, [2004.14463](#)]

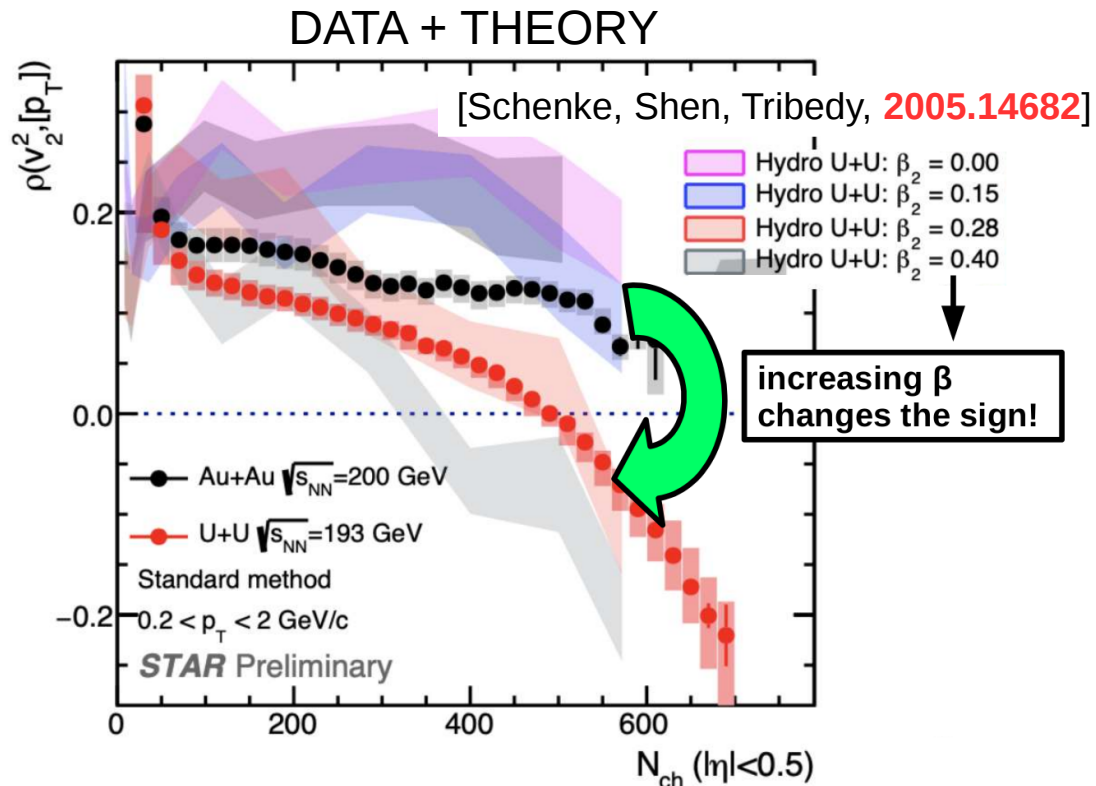
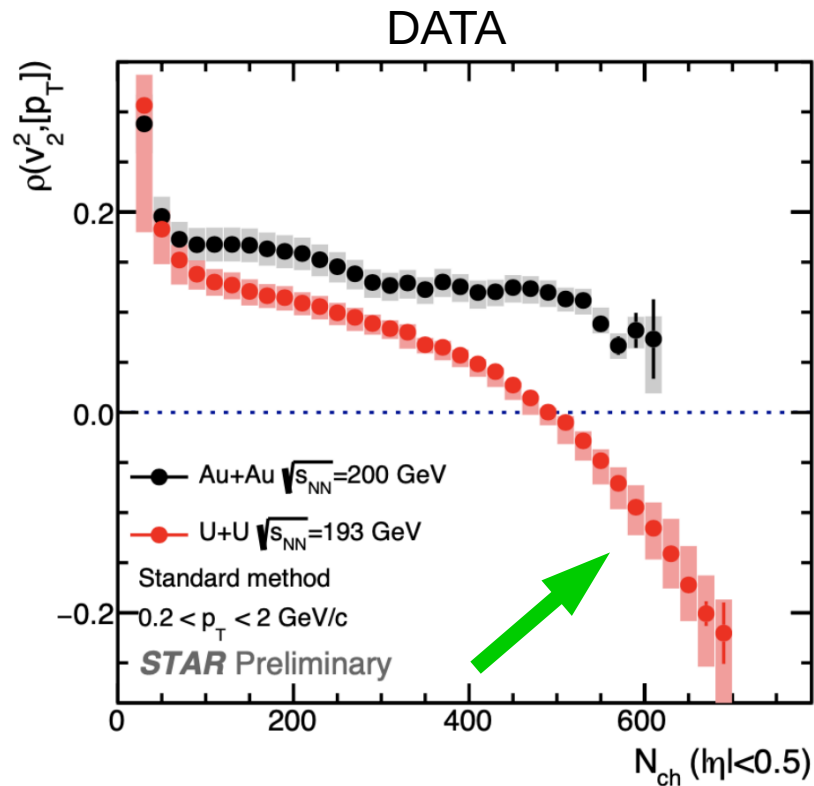


Body-Body: small  $\langle p_t \rangle$ , large  $v_2$ . Tip-tip: large  $\langle p_t \rangle$ , small  $v_2$ .

**Prediction!** In central collisions of deformed heavy ions:  $\rho(v_2^2, [p_t]) < 0$  <sub>25</sub>

# Spectacular confirmation at RHIC.

Correlation is positive in Au+Au, and negative in U+U.



Nuclear deformation interpretation confirmed by hydro calculations:  $\beta \approx 0.3$

# IMPLICATIONS

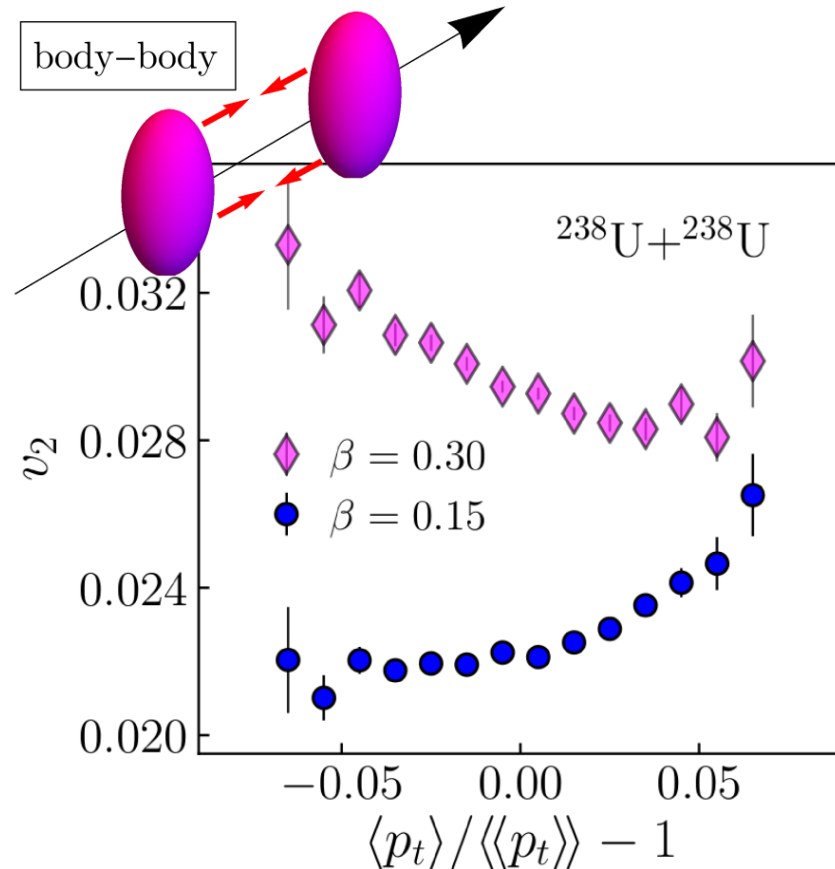
## - NUCLEAR PHYSICS ACROSS ENERGY SCALES

High-energy experiments test the low-energy structure. Consistency for  $^{238}\text{U}$ . Inconsistent results for  $^{197}\text{Au}$ ,  $^{129}\text{Xe}$ . New puzzles.

[Giacalone, Jia, Zhang [2105.01638](#)]

## - BODY-BODY COLLISIONS?

“Body-body collisions” only a useful simplification: the nucleus is  $J=0$  (i.e. spherical!). We reveal long-range correlations in nuclei, not only for the charge density probed at low energies.

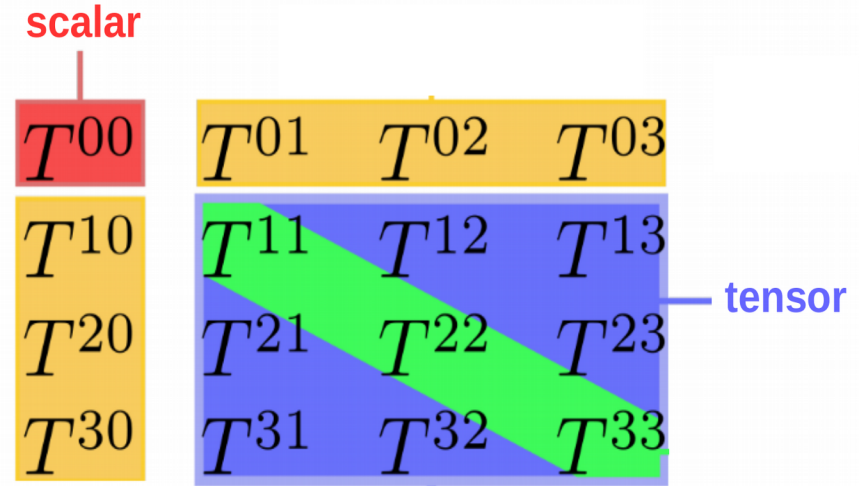


### #3 – Primordial momentum anisotropies.

Going beyond  $F = -\nabla P$ .

“MOMENTUM” ANISOTROPY

$$\mathcal{E}_{2p} \propto \langle T^{xx} - T^{yy} + 2iT^{xy} \rangle$$



[Sousa, Luzum, Noronha, [2002.12735](#)]

Engendered by pre-equilibrium phase.

[Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney, [1805.00961](#), [1805.01604](#)]

Predicted by the color glass condensate (CGC).

Longstanding question in the field: do we see initial-state CGC anisotropy?

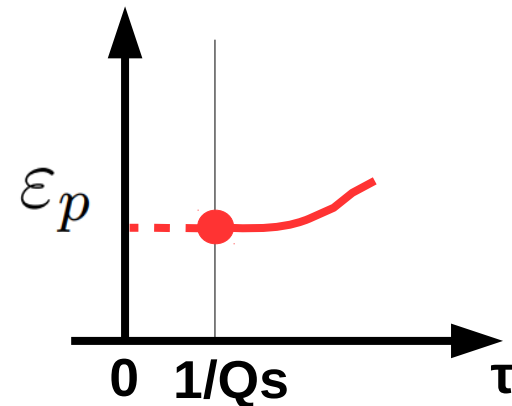
[Altinoluk, Armesto, [2004.08185](#)]

Evaluations in the IP-GLASMA+MUSIC+urQMD framework.

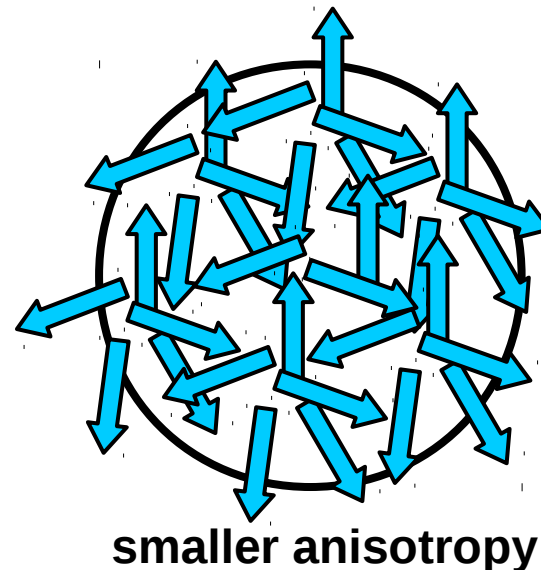
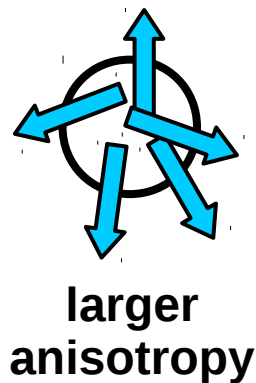
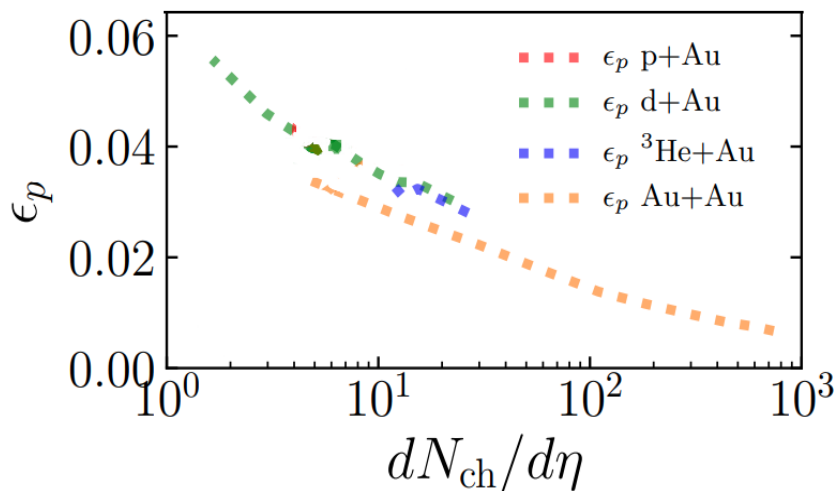
**System is anisotropic (n=2) shortly after the collision.**

$$\mathcal{E}_p \equiv \varepsilon_p e^{i2\Psi_2^p} \equiv \frac{\langle T^{xx} - T^{yy} \rangle + i\langle 2T^{xy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$$

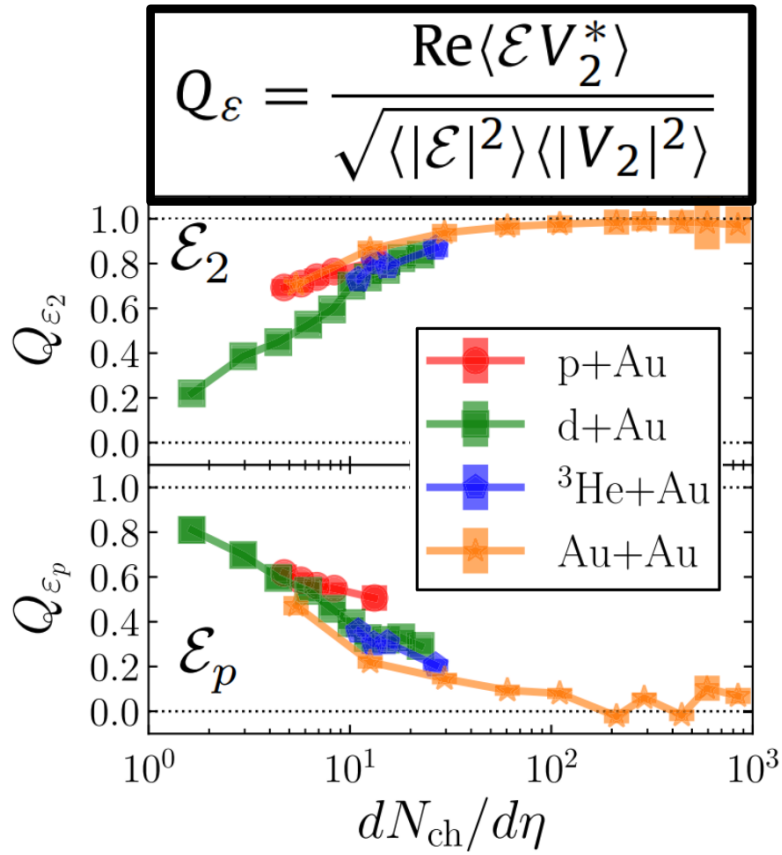
[Schenke, Shen, Tribedy, [1908.06212](#)]



**Salient property: system-size dependence.**



# Does the primordial anisotropy play a role?



– Q coefficient of linear correlation.

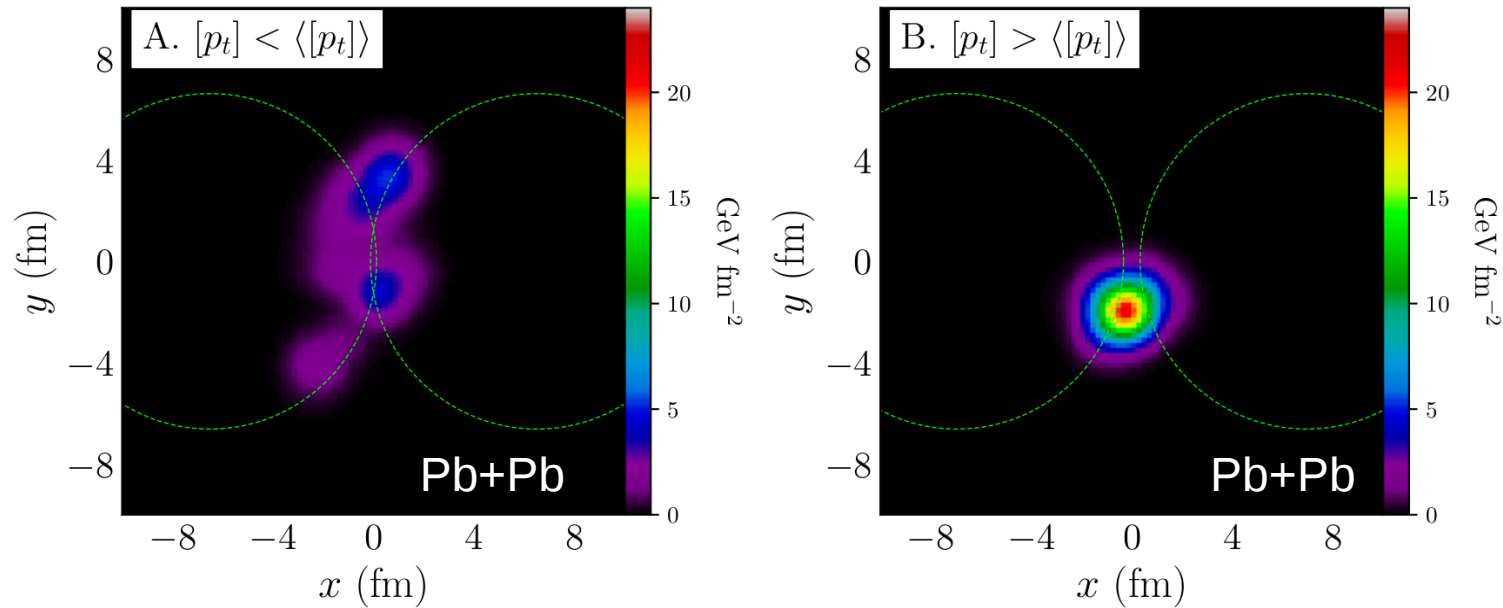
–  $\varepsilon_2$  is the dominant contribution to  $V_2$  for  $dN/d\eta \geq 20$ .

– At low multiplicity,  $V_2$  is instead in a stronger correlation with  $E_p$ .

[Schenke, Shen, Tribedy, [1908.06212](#)]

What observables can reveal this transition and probe  $\varepsilon_p$  ?

Select two **peripheral events** (69-70%) at the **same multiplicity** but **different [pt]**.



| event                         | A     | B     |
|-------------------------------|-------|-------|
| $N_{\text{ch}}$               | 134   | 134   |
| $b$ (fm)                      | 13.0  | 13.9  |
| $[p_t]/\langle [p_t] \rangle$ | 0.907 | 1.143 |
| $R$ (fm)                      | 2.97  | 1.34  |
| $\varepsilon_2$               | 0.675 | 0.133 |
| $\varepsilon_3$               | 0.229 | 0.067 |

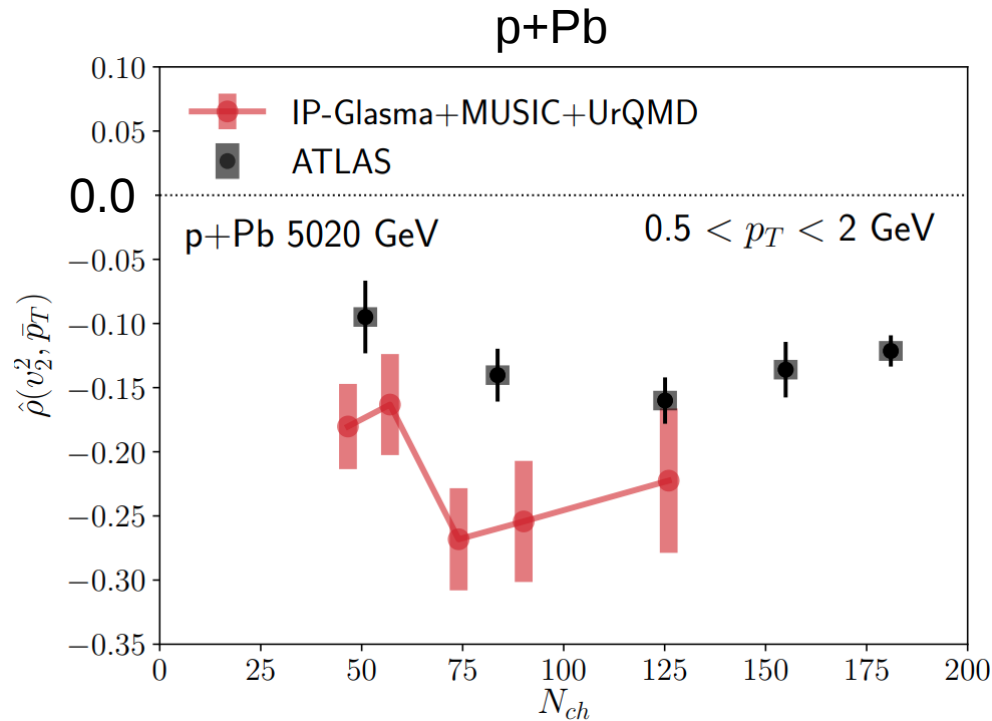
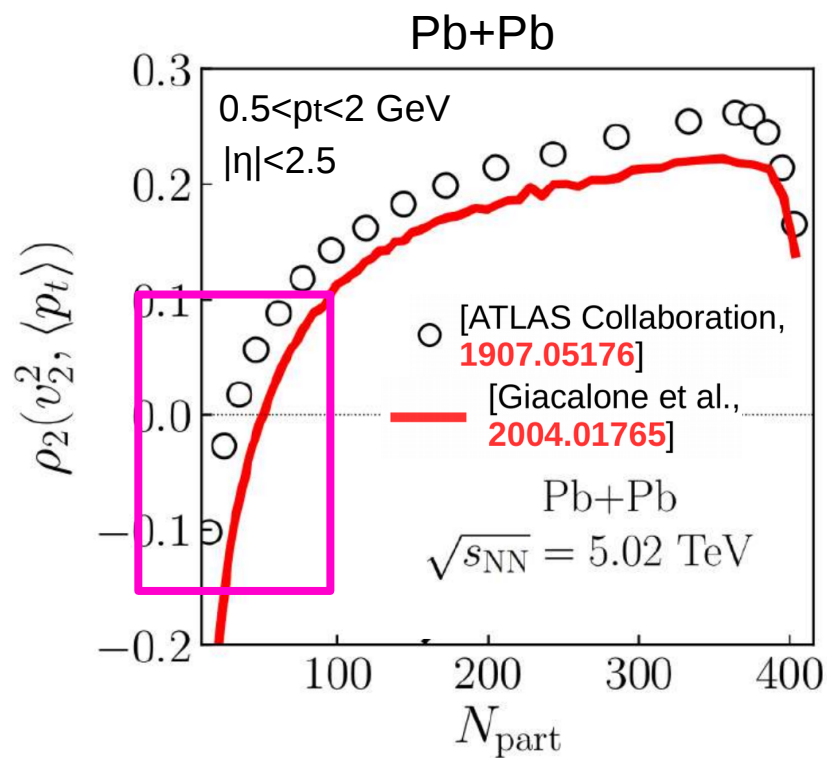
@large [pt]: hot spots clustered around one transverse point. Round system.

**Prediction.** In small systems:  $\rho(v_2^2, [p_t]) < 0$

[Božek, Mehrabpour, [2002.08832](#)]

[Schenke, Shen, Teaney, [2004.00690](#)]

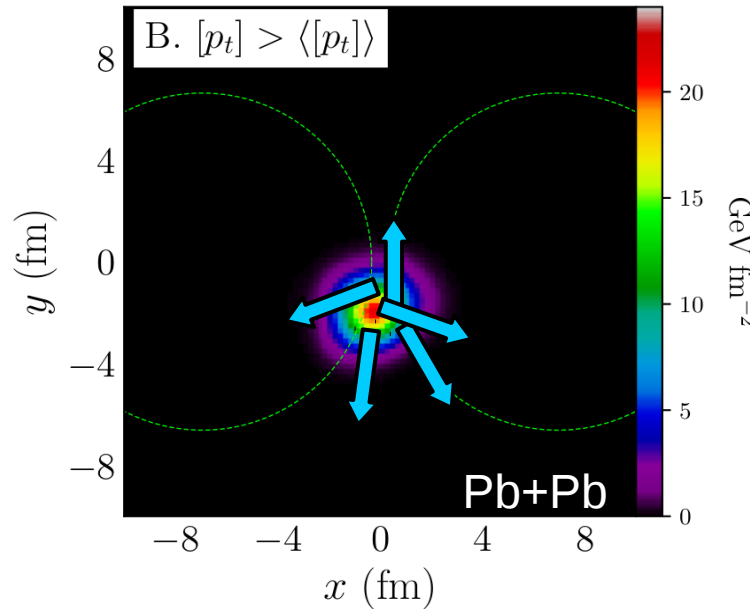
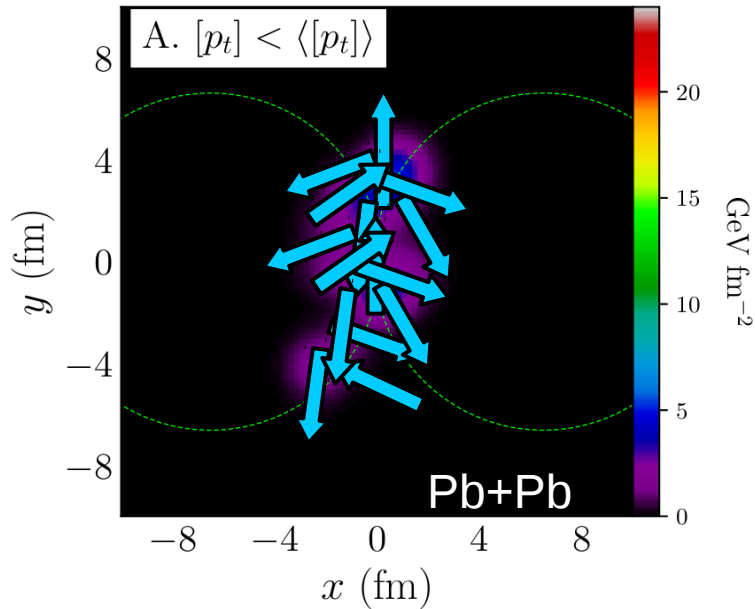
**Verified at LHC.** Correlation is negative. Captured by hydrodynamic models.



[Schenke, Shen, Teaney, [2004.00690](#)]



# What about the initial momentum anisotropy? Intuitive picture.



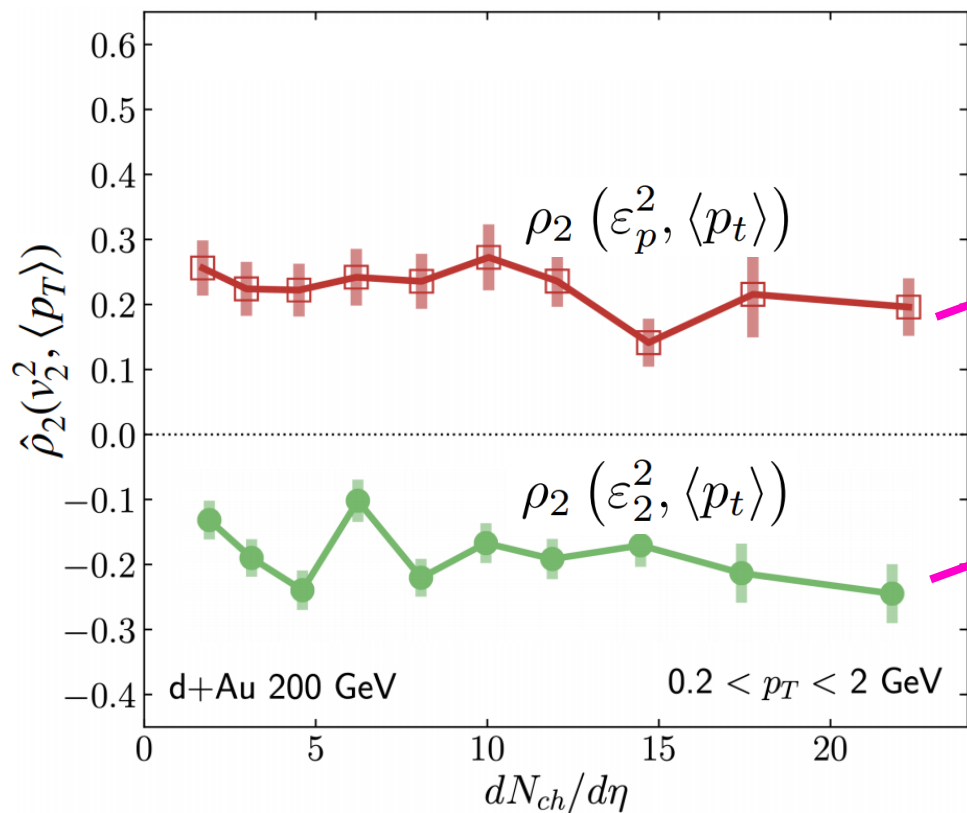
| event                         | A     | B     |
|-------------------------------|-------|-------|
| $N_{\text{ch}}$               | 134   | 134   |
| $b$ (fm)                      | 13.0  | 13.9  |
| $[p_t]/\langle [p_t] \rangle$ | 0.907 | 1.143 |
| $R$ (fm)                      | 2.97  | 1.34  |
| $\varepsilon_2$               | 0.675 | 0.133 |
| $\varepsilon_3$               | 0.229 | 0.067 |

@large  $[p_t]$ : hot spots clustered around one point. Smaller size, more  $\varepsilon_p$ .

**Prediction.** In small systems:  $\rho(\varepsilon_p^2, \langle [p_t] \rangle) > 0$

Consider  $V_2 = \kappa_2 \mathcal{E}_2 + \kappa_p \mathcal{E}_p$ , we expect:

$$\rho(v_2^2, [p_t]) = \kappa_2^2 \rho(\varepsilon_2^2, [p_t]) + \kappa_p^2 \rho(\varepsilon_p^2, [p_t])$$

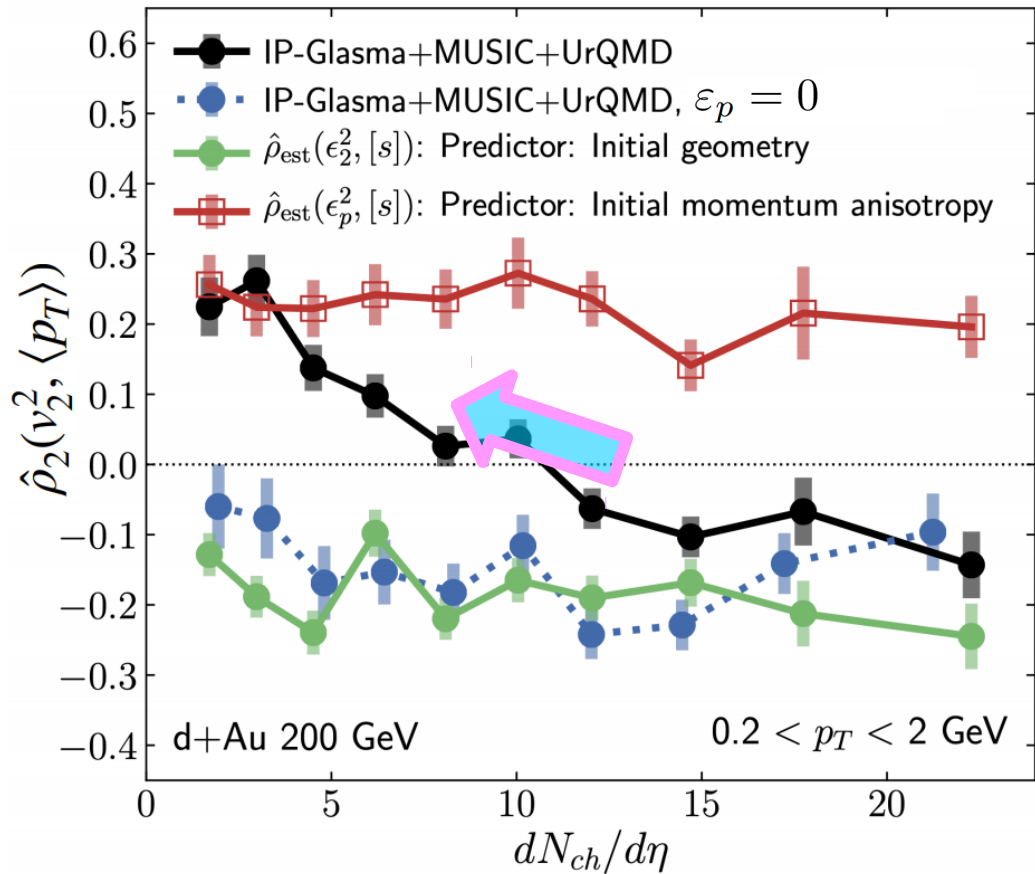


**Positive!**  
**Confirms the intuitive picture.**  
New feature of high-energy QCD.

**Negative.** As expected.

**They are qualitatively different.**

## IP-Glasma+Hydro: full prediction.



– **Sign change** occurs as expected around  $dN/d\eta=10$ .  
**Neat prediction! (AA and pA).**

– **No sign change** if we set  $E_p=0$ .

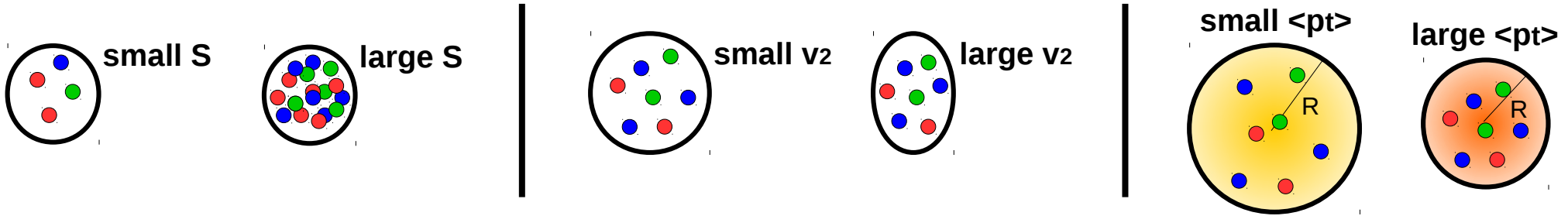
– **Non-flow mimics the signal.**  
**Must be carefully addressed.**

[Behera, Bhatta, Jia, Zhang, 2102.05200]

[Lim, Nagle, 2103.01348]

# SUMMARY

– Established picture of the soft sector in hydrodynamics:



– Nuclear phenomena through [pt]- $v_n$  correlations:

—► **Strong EM fields:**  $\rho^\pm (\langle p_t \rangle, v_1^\pm)$

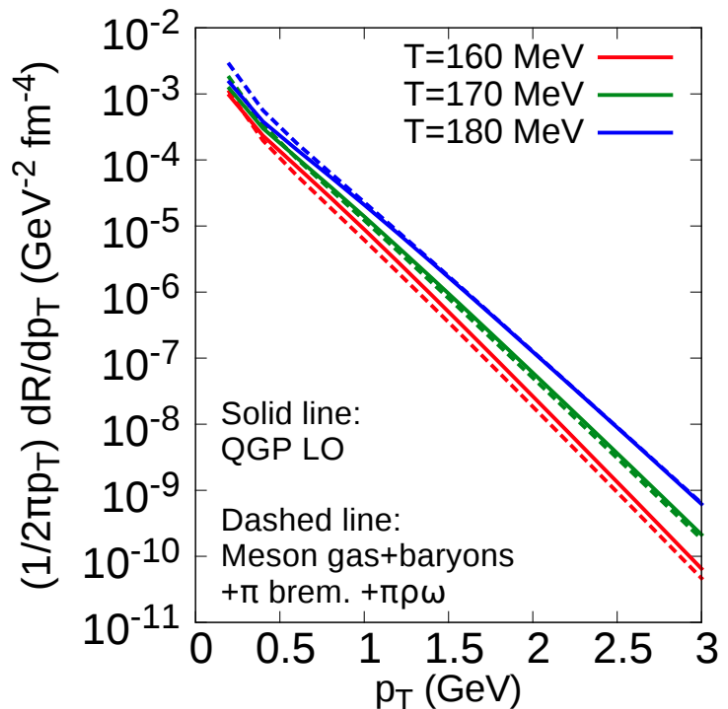
—► **Deformation in nuclei:**  $\rho(v_2^2, [p_t]) < 0$

—► **Primordial momentum anisotropy:**  $\rho(\varepsilon_p^2, [p_t]) > 0$

# OUTLOOK

Thermal yields,  $N$ .  
(photon, di-leptons)

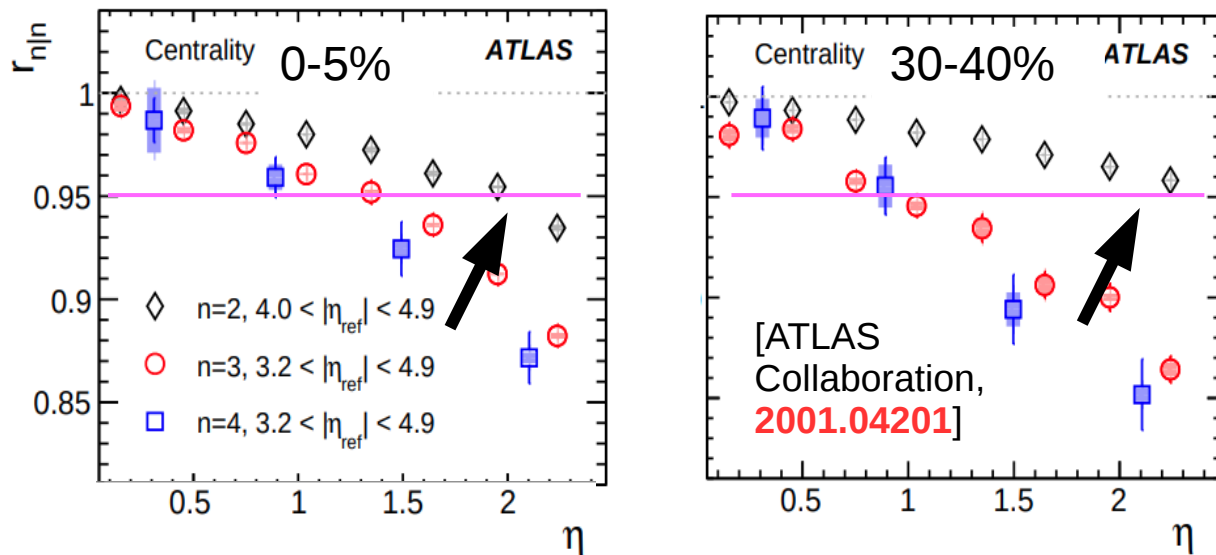
**prediction:**  $\rho(\langle p_T \rangle, N) > 0$ .



[Paquet, Shen, Denicol, Luzum, Schenke, Jeon, Gale, [1509.06738](#)]

## Longitudinal de-correlations.

Smaller de-correlation at large impact parameters.



**prediction:** large- $\langle p_T \rangle$  events de-correlate less.

### More candidates:

- vorticity?
- energy loss?
- ...
- other ideas?

**EXCITING PROSPECTS  
FOR THE NEXT DECADE**

**THANK YOU!**