

Black Holes Decohere Quantum Superpositions

[“Black Holes are Watching you”]

Robert M. Wald

Key References:

- 1) A. Belenchia, R.M. Wald, F. Giacomini, E. Castro–Ruiz, C. Brukner, and M. Aspelmeyer, Phys. Rev. D 98, 126009 (2018) [arXiv: 1807.07015].
- 2) D. Danielson, G. Satishchandran, and R.M. Wald, Phys. Rev. D 105, 086001 (2022) [arXiv:2112.10798].
- 3) D. Danielson, G. Satishchandran, and R.M. Wald, Int. J. Mod. Phys. D 2241003 (2022) [arXiv:2205.06279].
- 4) D. Danielson, G. Satishchandran, and R.M. Wald, arXiv:2301.00026

A Gedankenexperiment

There is both an electromagnetic and gravitation version of this gedankenexperiment; the discussion applies to either/both.

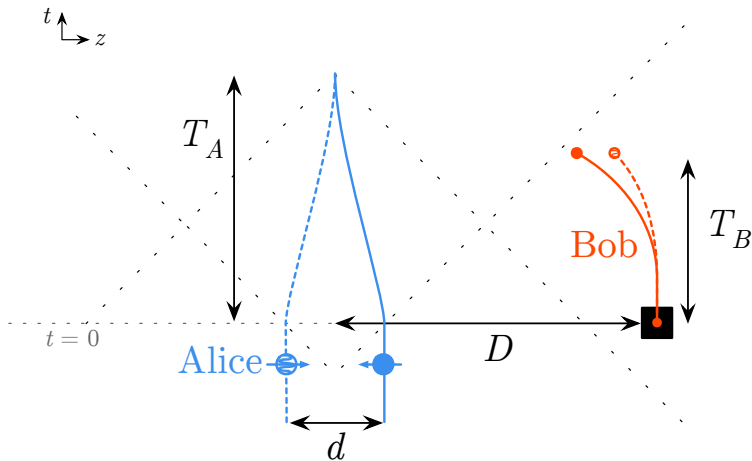
Well prior to time $t = 0$, Alice started with her particle with spin in the x -direction and sent it through a Stern-Gerlach apparatus, thereby putting it into a 50%-50% superposition of spin “up” and spin “down.” Prior to $t = 0$, Bob kept his particle in a trap.

Beginning at time $t = 0$, Alice sends her particle through a “reversing Stern-Gerlach apparatus” and determines its coherence (e.g., by measuring its x -spin).

At time $t = 0$, Bob releases his particle from the trap and attempts to obtain “which path” information on Alice’s particle by measuring the strength of the Coulomb/Newtonian field of Alice’s particle.

If Alice and Bob complete their measurements within a light travel time of each other will Alice’s superposition remain coherent?

Spacetime Diagram of the Gedankenexperiment



Will Alice's particle maintain coherence?

Fundamental Limit on Coherence of Alice's Particle

Emission of entangling quantum electromagnetic/gravitational radiation. Estimate of effect in electromagnetic case: When Alice “closes the superposition” of the components of her particle, the effective dipole \mathcal{D}_A will be reduced to zero in time T_A . The corresponding radiated energy will be

$$\mathcal{E} \sim \left(\frac{\mathcal{D}_A}{T_A} \right)^2 T_A.$$

This energy will appear in the form of photons of frequency $\sim 1/T_A$, so the number of entangling photons will be

$$N \sim \left(\frac{\mathcal{D}_A}{T_A} \right)^2.$$

If $N \gtrsim 1$, the components of Alice's particle will be entangled with emitted photons, and the coherence of her particle will be destroyed, independently of what Bob does.

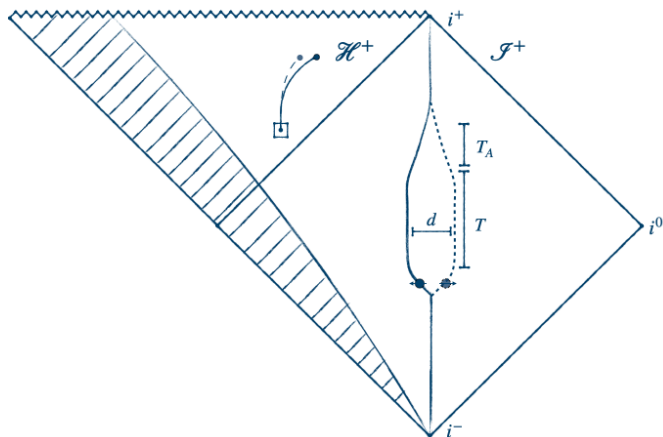
Resolution of the Apparent Paradox

If Alice performs the recombination of her particle “quickly” ($T_A < \mathcal{D}_A$) she will destroy the coherence of her particle by emission of radiation. No paradox will arise if Bob is able to get “which path” information.

By performing her experiment adiabatically ($T_A \gg \mathcal{D}_A$), Alice can ensure that $N \ll 1$ and thus that she doesn't destroy the coherence of her particle. But then Bob will have to be correspondingly far from Alice in order to do his measurement at spacelike separation from Alice's recombination, so his “signal” will be weak. In that case, vacuum fluctuations prevent Bob from acquiring sufficient “which path” information to result in a contradiction. Indeed, we showed that if Alice and Bob follow the protocols of the gedankenexperiment, then Bob can be interpreted as measuring the “on-shell” photons/gravitons emitted by Alice's particle during recombination—even though he is spacelike separated from this recombination!—so Bob can never acquire enough “which path” information to cause a paradox. (But that would be a subject for a different talk!)

A New Twist to the Paradox: Black Holes

Now suppose a black hole is present and Bob performs the measurements from inside the black hole:



New Twist: Black Holes

A black hole is not a particularly good place to set up an experiment, since Bob cannot remain stationary and (presumably) will fall into the singularity inside the black hole within a finite time. Nevertheless, the Coulomb/Newtonian field of Alice's particle will penetrate into the black hole, and if Bob can acquire some (limited) "which path" information if he turns on his measuring apparatus after falling into the black hole. But if T is arbitrarily large, a sequence of Bob's should be able to acquire arbitrarily good "which path" information.

Since Bob's measurements cannot be relevant to Alice's experiment, the black hole itself must be harvesting "which path" information. In the remainder of this talk, I will explain how black holes do this.

Decoherence of Alice's Particle Due to Emission of Radiation

Suppose Alice is in Minkowski spacetime and after recombination, she keeps her recombined particle stationary forever. At asymptotically late times, the state of the total system is of the form

$$\frac{1}{\sqrt{2}} (|\uparrow; A\rangle_{i+} \otimes |\Psi_1\rangle_{\mathcal{I}^+} + |\downarrow; A\rangle_{i+} \otimes |\Psi_2\rangle_{\mathcal{I}^+})$$

where $|\Psi_1\rangle_{\mathcal{I}^+}$ and $|\Psi_2\rangle_{\mathcal{I}^+}$ represent the quantum states of the electromagnetic radiation at future null infinity corresponding to whether Alice's particle followed "path 1" (spin $|\uparrow\rangle$) or "path 2" (spin $|\downarrow\rangle$). The decoherence of Alice's superposition is

$$\mathcal{D} = 1 - |\langle \Psi_1 | \Psi_2 \rangle_{\mathcal{I}^+}| .$$

Decoherence in the Presence of a Black Hole

Now suppose Alice does her experiment in the presence of a black hole. Then the final state is now

$$\frac{1}{\sqrt{2}} \left(|\uparrow; A_1\rangle_{i+} \otimes |\Psi_1\rangle_{\mathcal{I}+} |\Phi_1\rangle_{\mathcal{H}+} + |\downarrow; A_2\rangle_{i+} \otimes |\Psi_2\rangle_{\mathcal{I}+} |\Phi_2\rangle_{\mathcal{H}+} \right)$$

where $|\Phi_1\rangle_{\mathcal{H}+}$ and $|\Phi_2\rangle_{\mathcal{H}+}$ represent the quantum states of the electromagnetic radiation that passes through the event horizon of the black hole. As previously discussed, if $T_A \gg \mathcal{D}_A$, the decohering radiation that reaches infinity will be negligible, i.e., $|\langle \Psi_1 | \Psi_2 \rangle_{\mathcal{I}+}| \approx 1$. The decoherence of Alice's superposition is

$$\mathcal{D}_{\text{BH}} = 1 - |\langle \Phi_1 | \Phi_2 \rangle_{\mathcal{H}+}|.$$

Electromagnetic Radiation Through the Horizon

Let n^a denote the (affinely parametrized) normal to the horizon. Electromagnetic radiation through the horizon is described by the pullback, E_A , of the “electric field” $E_a = F_{ab}n^b$ to the horizon, \mathcal{H}^+ , (where capital latin indices are used for dual vector fields on the horizon that are orthogonal to n^a , i.e., having “purely angular” components). For a static point charge outside the horizon, we have $E_A = 0$, i.e., there is no radiation through the horizon. However, $E_r \neq 0$, on \mathcal{H}^+ , where $E_r = F_{ab}l^an^b$ where l^a is a past directed null vector with $l^an_a = 1$.

Now, suppose that the point charge is moved to a new location. By Maxwell’s equations,

$$D^A E_A = -\partial_V E_r$$

so there will necessarily be some radiation through the horizon. Furthermore, $\int D^A E_A dV$ is constrained by the initial and final values of E_r , and is independent of how slowly the charge is moved.

Radiation of Through the Horizon

If the point charge is moved very slowly, the total energy radiated into the black hole $\propto \int E^A E_A r^2 d\Omega V dV$ can be made arbitrarily small. However, as I shall now explain, if the charge remains in its new position forever, the number of photons radiated into the black hole is infinite!

For an unperturbed black hole formed by gravitational collapse, the state of the electromagnetic field on the horizon of the black hole is described by the Unruh vacuum. However, we will be concerned here only with low frequency phenomena ($\omega \ll c^3/GM$), in which case the Unruh and Hartle-Hawking vacua near the horizon are essentially indistinguishable. In the Fock space associated with the Hartle-Hawking vacuum, a “particle” corresponds to a solution that is purely positive frequency with respect to affine parameter on the horizon.

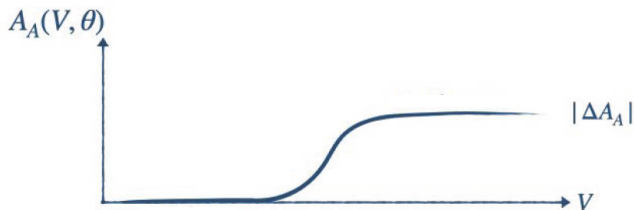
Soft Photons

A classical charge-current source will perturb the state of the quantum field from the Unruh/Hartle-Hawking vacuum to a coherent state associated with the classical retarded solution. In a gauge where $A_a n^a = 0$, the number of photons radiated through the horizon is given by

$$\langle N \rangle_{\mathcal{H}^+} = \frac{1}{\pi \hbar} \int r^2 d\Omega \int_0^\infty \omega d\omega |\hat{A}_A(\omega, x^A)|^2$$

where \hat{A}_A is the Fourier transform of A_A with respect to affine parameter V . In this gauge, $E_A = -\partial_V A_A$, so since $\int E_A dV \neq 0$, it follows that A_A does not return to its initial value at late times. **But this implies that \hat{A}_A diverges as $1/\omega$ as $\omega \rightarrow 0$, which, in turn, implies that $\langle N \rangle_{\mathcal{H}^+} = \infty$.** This is a precise analog of the infrared divergences that occur in scattering theory (for $d = 4$).

Infrared Divergence



$$\Delta A_A \neq 0 \implies \tilde{A}_A(\omega, \theta) \sim \frac{1}{\omega}$$

$$\implies \langle N \rangle = ||A||^2 \sim \int_0^\infty \frac{d\omega}{\omega}$$

Thus moving a point charge to a different location and leaving it there forever results in the radiation of an infinite number of “soft photons” through the horizon!

How Large is ΔA_A ?

Radial electric field of a point charge located a distance b from the black hole:

$$E_r \sim \frac{q}{b^2}$$

If the charge is moved a distance $d \ll b$, we have

$$\Delta E_r \sim \frac{qd}{b^3}$$

Since we have

$$\Delta E_r = - \int dV D^A E_A = \int dV D^A \partial_V A_A = D^A (\Delta A_A)$$

and the horizon is at $r \sim M$, we obtain

$$|\Delta A_A| \sim \frac{M^2 q d}{b^3}$$

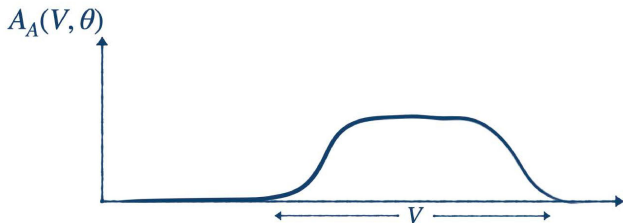
Radiation When Point Charge is Moved Back to Original Position after time T

The infrared divergence is avoided if the charge is moved back to its original position. Now we obtain

$$\langle N \rangle_{\mathcal{H}^+} \sim |\Delta A_A|^2 \ln V \sim \frac{M^4 q^2 d^2}{b^6} \ln V$$

But $V = e^{\kappa T}$ with $\kappa = 1/4M$, so

$$\langle N \rangle_{\mathcal{H}^+} \sim \frac{M^3 q^2 d^2}{b^6} T$$



Alice's experiment

In a stationary lab outside of the black hole, Alice puts her particle in a spatial superposition, keeps it in the superposition for a (proper) time T , and then recombines the particle. The above estimate for $\langle N \rangle_{\mathcal{H}^+}$ holds for the number of “entangling soft photons” emitted into the black hole in this process. If $\langle N \rangle_{\mathcal{H}^+} \gtrsim 1$, then her particle will have decohered. Thus, Alice's particle will decohere in a time

$$\begin{aligned} T_D &\sim \frac{b^6}{M^3 q^2 d^2} \\ &\sim 10^{43} \text{ years} \left(\frac{b}{\text{a.u.}} \right)^6 \cdot \left(\frac{M_\odot}{M} \right)^3 \cdot \left(\frac{e}{q} \right)^2 \cdot \left(\frac{\text{m}}{d} \right)^2 \end{aligned}$$

(But $T_D \sim 5$ minutes if Alice's lab is at $b = 6M$ rather than at 1 a.u.)

Gravitational Case

The analysis of the gravitational case follows in close analogy with the electromagnetic case, with the electric part of the Weyl tensor $E_{ab} = C_{acbd}n^cn^d$ playing the role analogous to E_a . For a static point mass, the only nonvanishing component of E_{ab} on the horizon is $E_{rr} = C_{acbd}l^an^cl^bn^d$. Radiation through the horizon is described by the pullback, E_{AB} , of E_{ab} to the horizon. In parallel with Maxwell's equations, the Bianchi identity yields

$$D^A E_{AB} = -\partial_V E_{rB}, \quad D^A E_{rA} = -\partial_V E_{rr}$$

which implies

$$D^A D^B E_{AB} = \partial_V^2 E_{rr}.$$

Similarly, we now have

$$E_{AB} = -\frac{1}{2}\partial_V^2 h_{AB}.$$

Gravitational Decoherence

An important difference that occurs in the gravitational case is that Alice does not create a “mass dipole” in the superposition—her lab moves oppositely to cancel the mass dipole of the superposition. Thus, it is now the effective mass quadrupole md^2 of the superposition that now enters, rather than the effective electrostatic dipole qd . Otherwise, the analysis proceeds in complete parallel, and decoherence time is now

$$\begin{aligned} T_D^{\text{GR}} &\sim \frac{b^{10}}{M^5 m^2 d^4} \\ &\sim 10 \mu\text{s} \left(\frac{b}{\text{a.u.}}\right)^{10} \cdot \left(\frac{M_\odot}{M}\right)^5 \cdot \left(\frac{M_{\text{Earth}}}{m}\right)^2 \cdot \left(\frac{R_{\text{Earth}}}{d}\right)^4 \end{aligned}$$

Generalizations

The only crucial property of black holes used in the above analysis is that the event horizon is a Killing horizon. A closely parallel analysis (in both the electromagnetic and gravitational cases) holds in other cases where a Killing horizon is present. This includes the case where Alice's lab is accelerating in Minkowski spacetime (Rindler horizon) and the case of de Sitter spacetime (cosmological horizon). For an inertial lab in de Sitter spacetime, in the electromagnetic case, we have

$$T_D \sim \frac{R_H^3}{q^2 d^2}$$

whereas in the gravitational case, we have

$$T_D \sim \frac{R_H^5}{m^2 d^4}.$$

Conclusions

Black holes—and more generally, Killing horizons—harvest information about quantum superpositions of spatially separated components. They do so by absorbing “soft photons/gravitons” associated with the long range fields sourced by the matter comprising these components. Eventually, a black hole will decohere any quantum superposition. Although this is not likely to be of practical importance for experiments, it remains to be explored as to whether this could be of fundamental significance for our understanding of the nature of black holes in a quantum theory of gravity.