A minimal SM/LCDM cosmology

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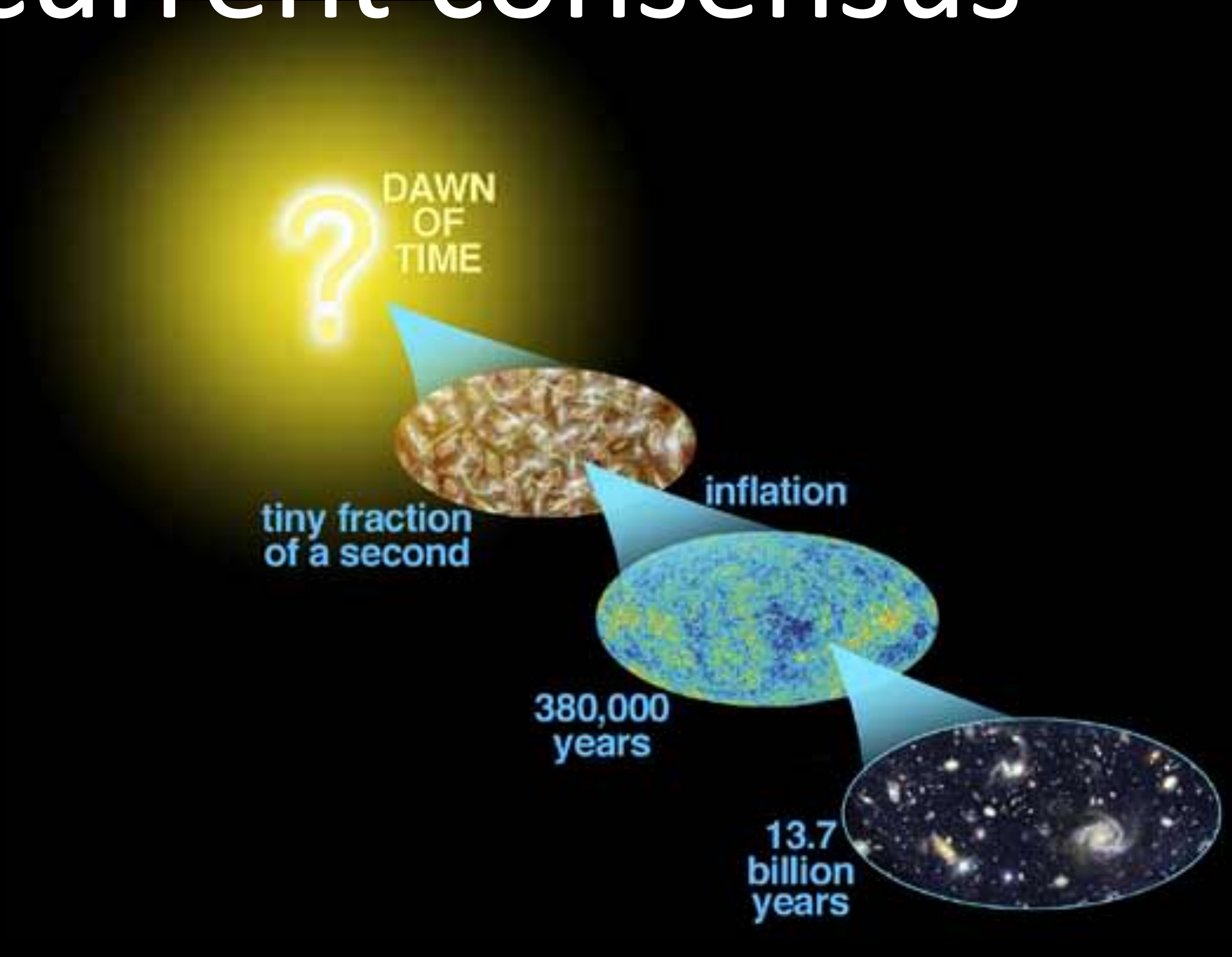
with Latham Boyle

current consensus



Inflation was a groundbreaking idea but

1. Observations are very well fit by vanilla LCDM
   cf. overabundance of inflationary models

2. No sign of inflation's "smoking gun" signal: long wavelength gravitational waves
   (Current bound is r <0.03; CMB experimenters project r<0.003 by 2027)

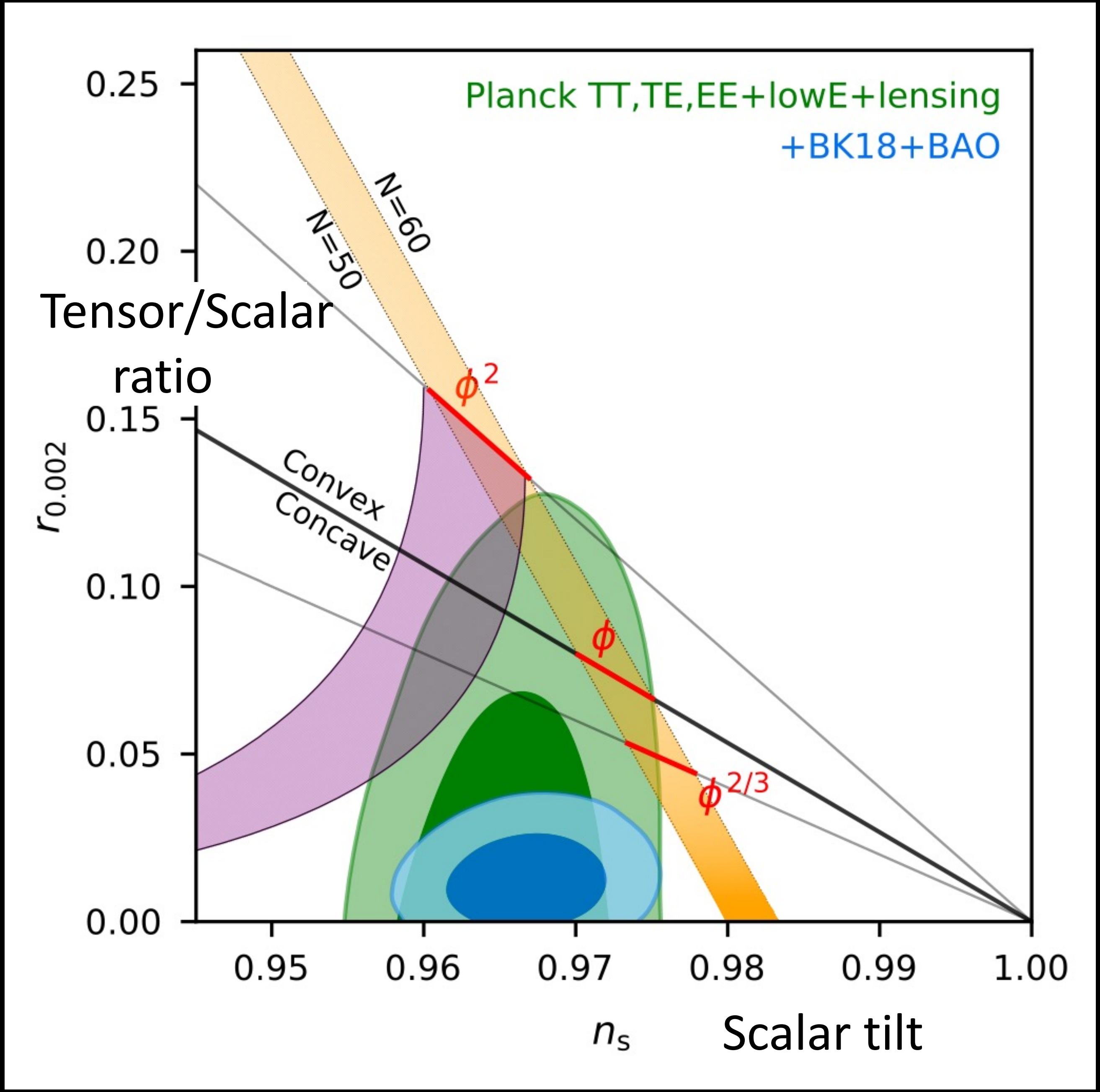
3. No satisfactory measure on inflationary universes

In this talk (and in the spirit of Bohr's atomic model)
I will attempt to outline a comprehensive alternative

no sign of
inflationary
tensors

BICEP/Keck
Collaboration
2203.16556
[astro-ph]

anticipated limit
r<.003
using SPT for
“delensing”
(2027)



Tensor/Scalar ratio

Scalar tilt

vanilla LCDM:

just 5 fundamental physics parameters

matter/energy content

1. ρ_Λ cosmological constant
2. ρ_{DM}/ρ_B DM/baryon density
3. n_B/n_γ baryons per photon

fluctuations

Newtonian potential

$$\langle \Phi^2 \rangle = \int \frac{dk}{k} A \left(\frac{k}{k_*}\right)^{n_s - 1} \quad (k_* \equiv 0.05\mathrm{Mpc}^{-1})$$

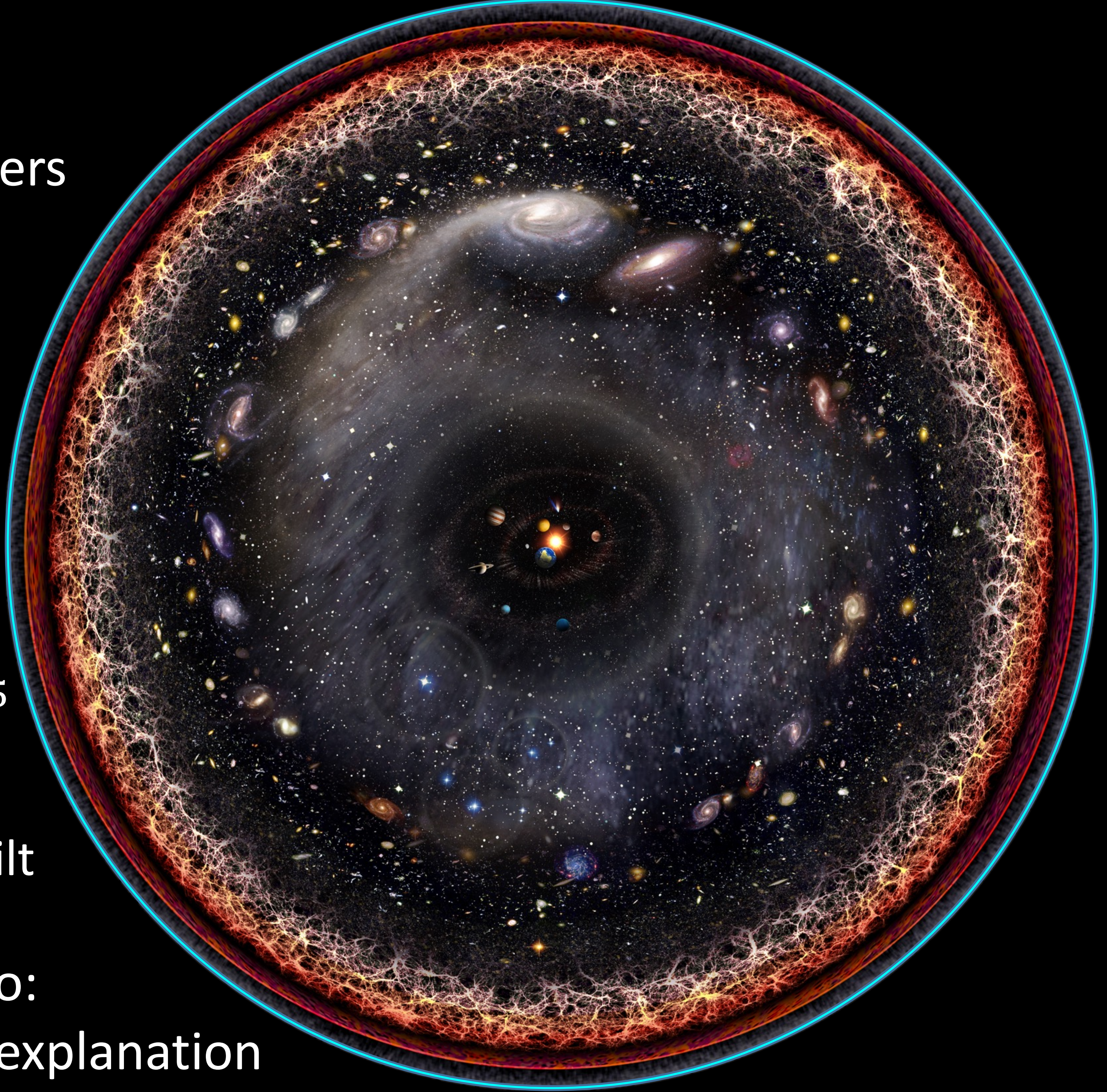
4. large scale $\Phi_{rms} \approx \sqrt{A} \approx 3\times10^{-5}$

Sachs-Wolfe $\delta T/T \approx \frac{1}{3}\Phi \approx 10^{-5}$

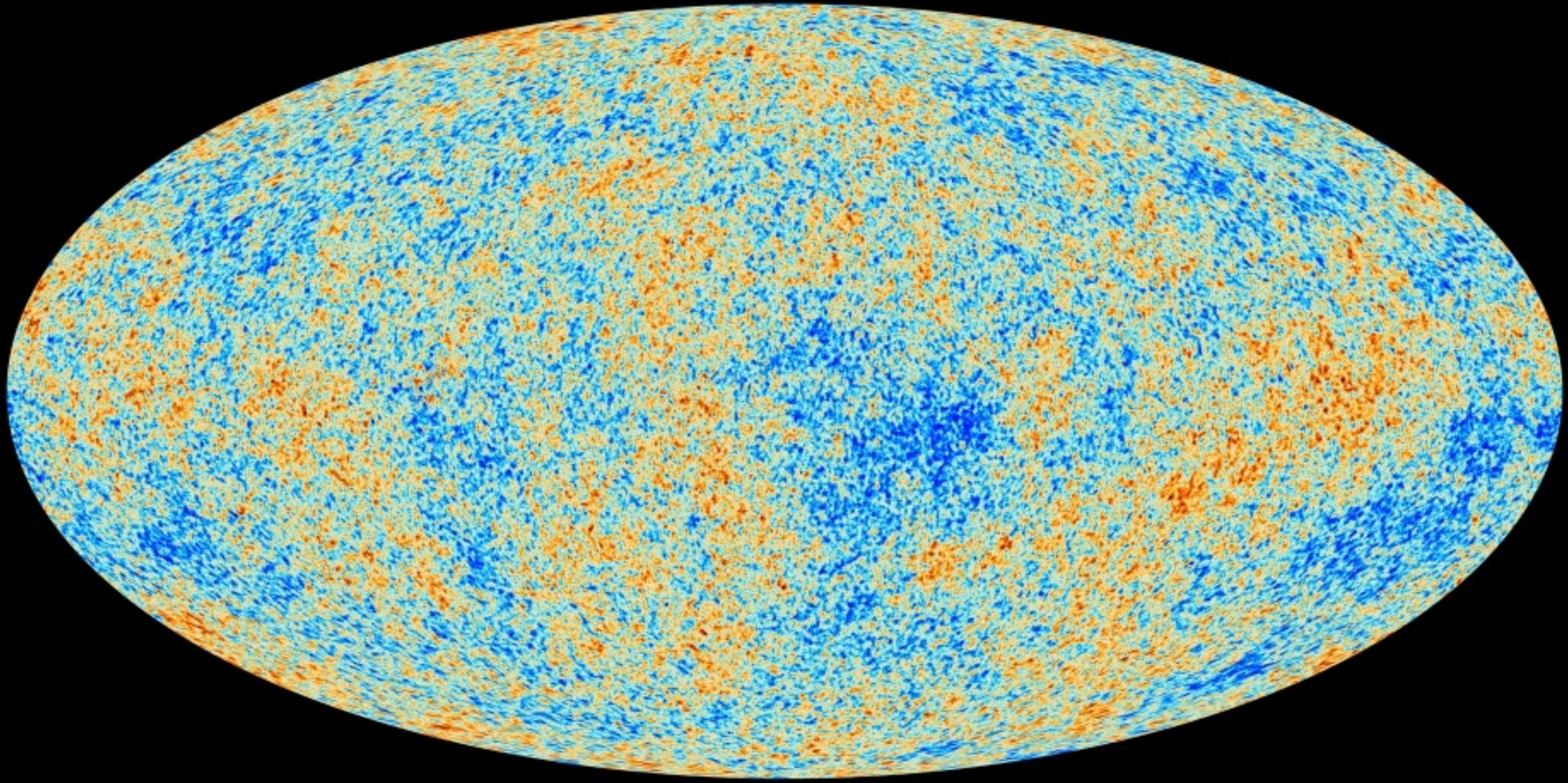
5. $n_s - 1 \approx -0.04 \pm .006$ small red tilt

many quantities consistent with zero:

suggests looking for an economical explanation

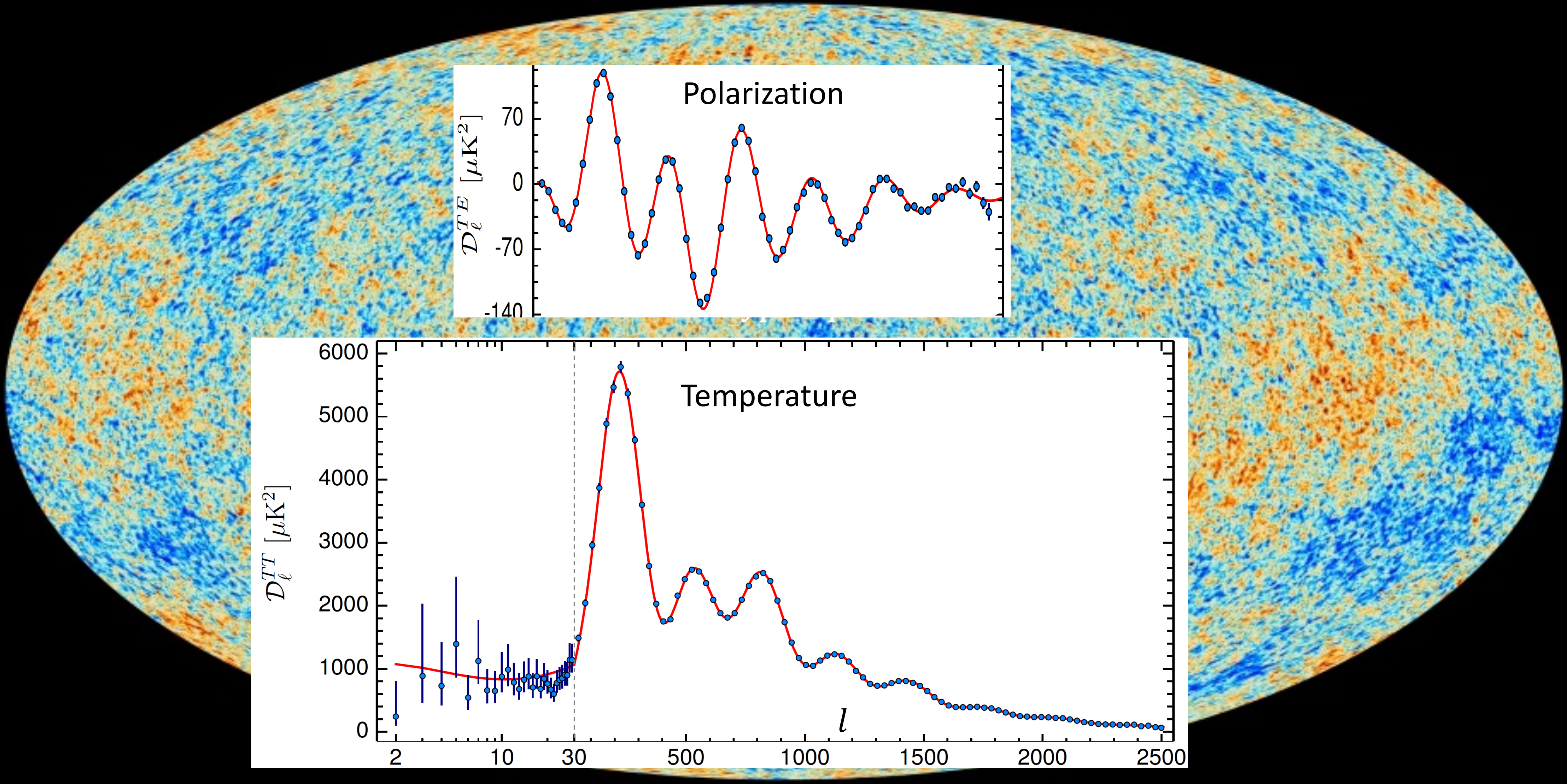


Looking back to the bang



ESA Planck satellite

Large scale perturbations

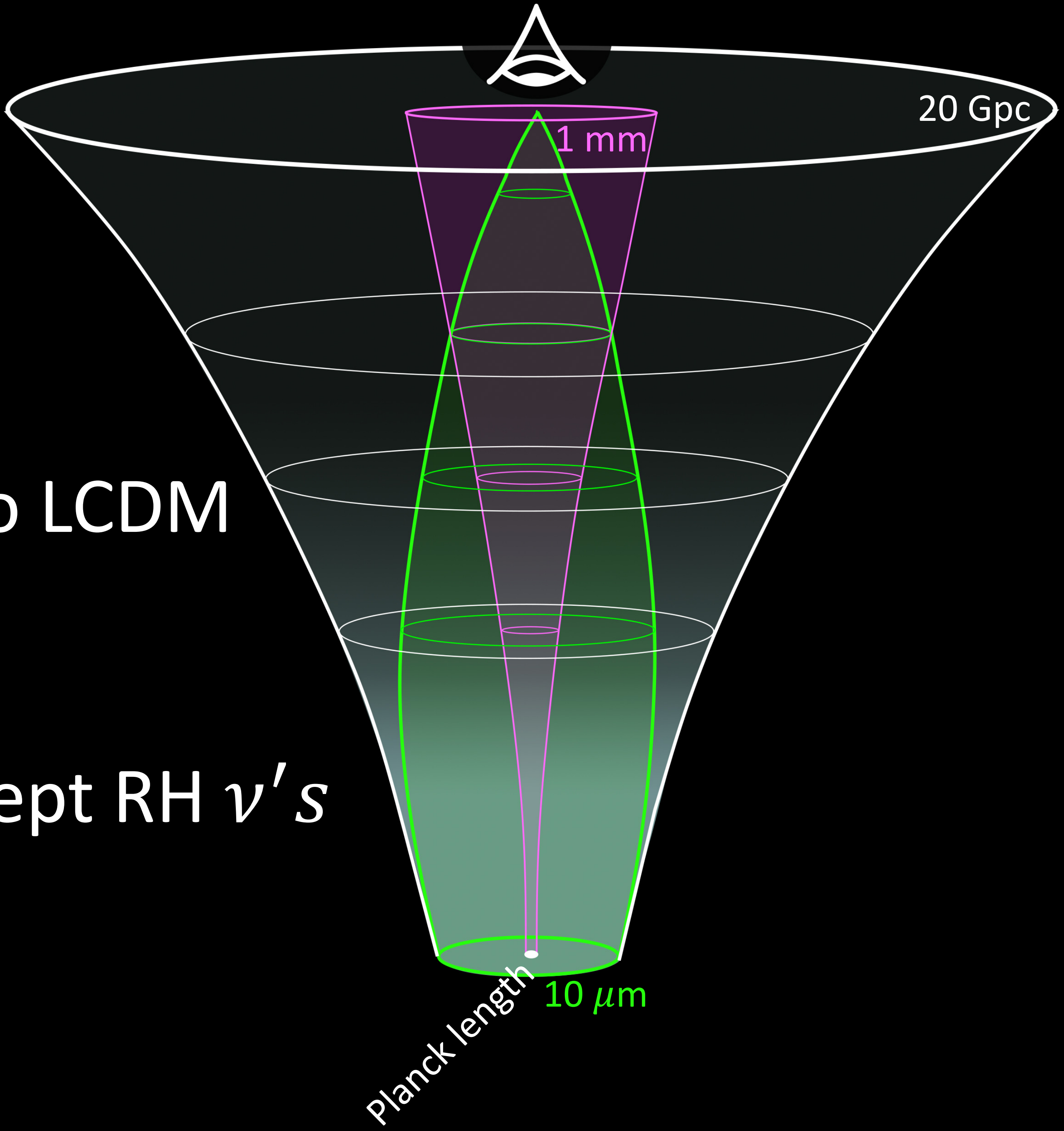


ESA Planck satellite

this talk:

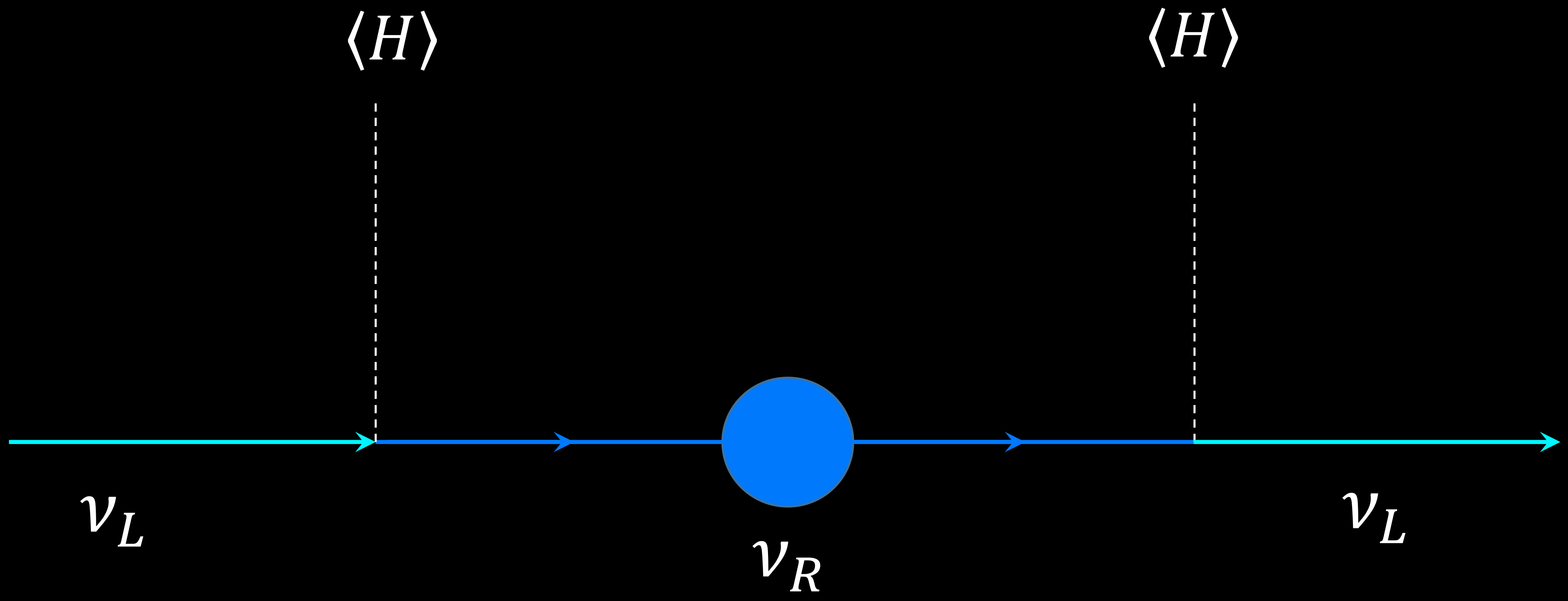
a new framework
connecting the SM to LCDM
(all 5 parameters)

no new particles except RH $\nu' s$
no need for inflation



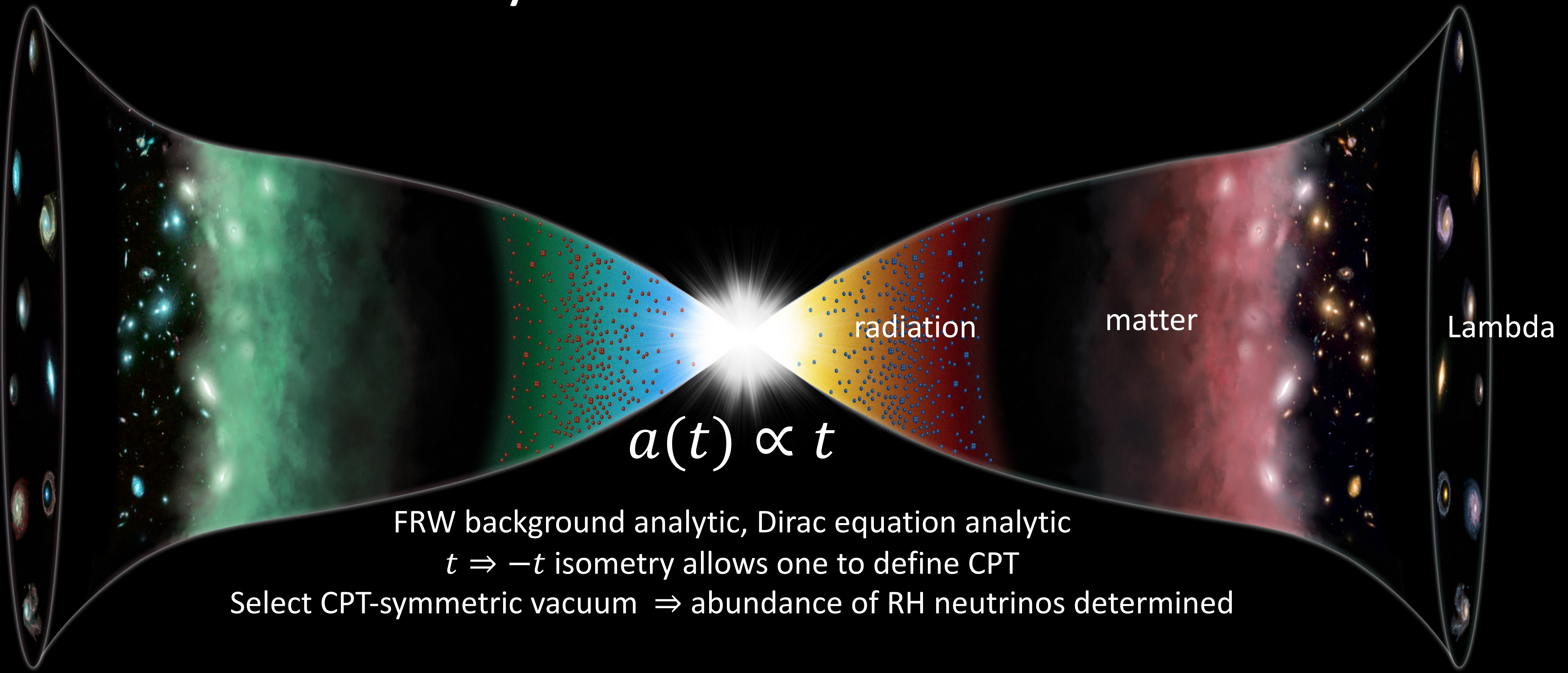
dark matter

Right-handed neutrinos:



naturally explain observed neutrino masses

CPT symmetric universe



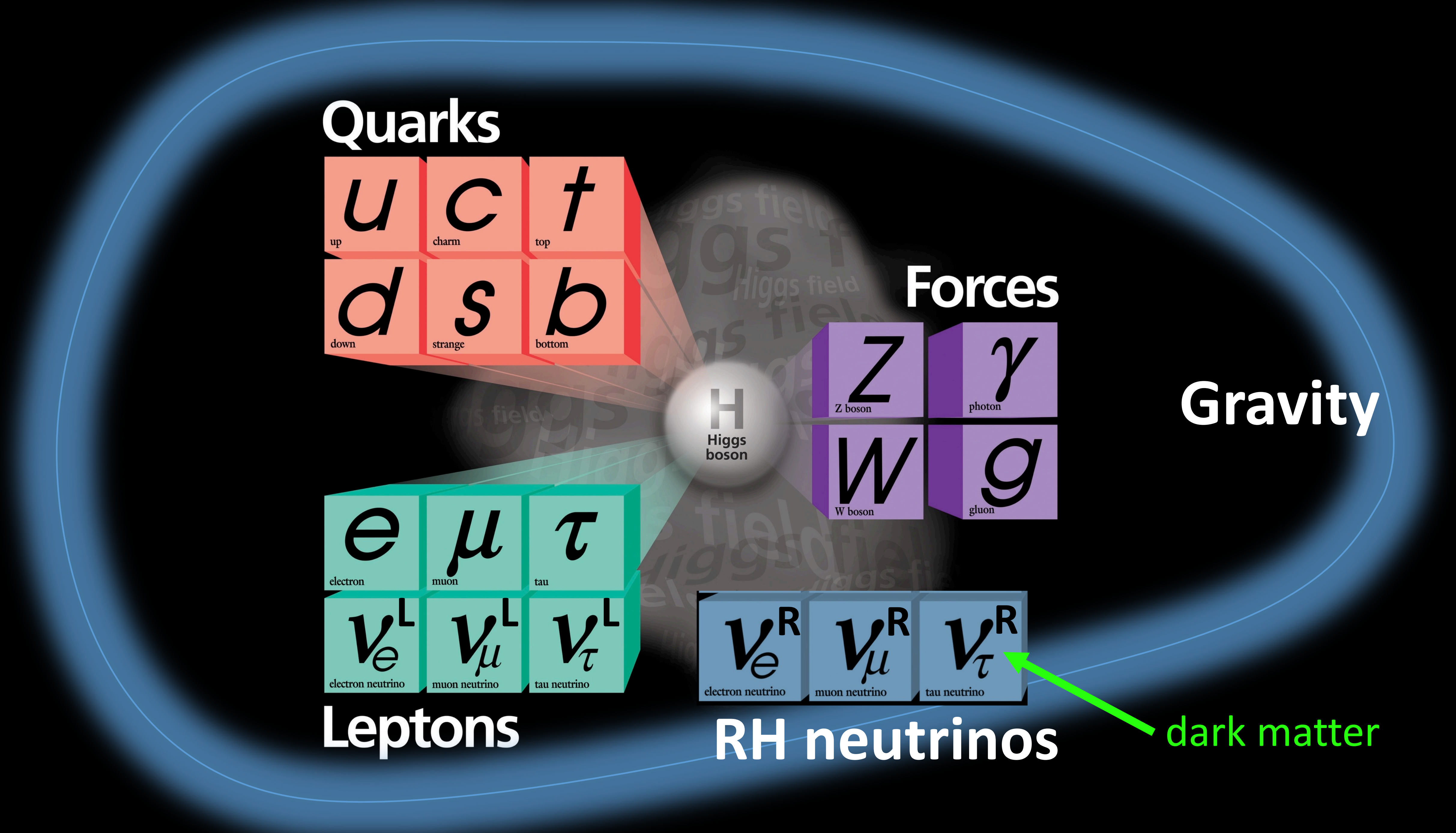
$$a(t) \propto t$$

FRW background analytic, Dirac equation analytic

$t \Rightarrow -t$ isometry allows one to define CPT

Select CPT-symmetric vacuum $\Rightarrow$ abundance of RH neutrinos determined

RH neutrinos produced as Hawking radiation



Quarks

Forces

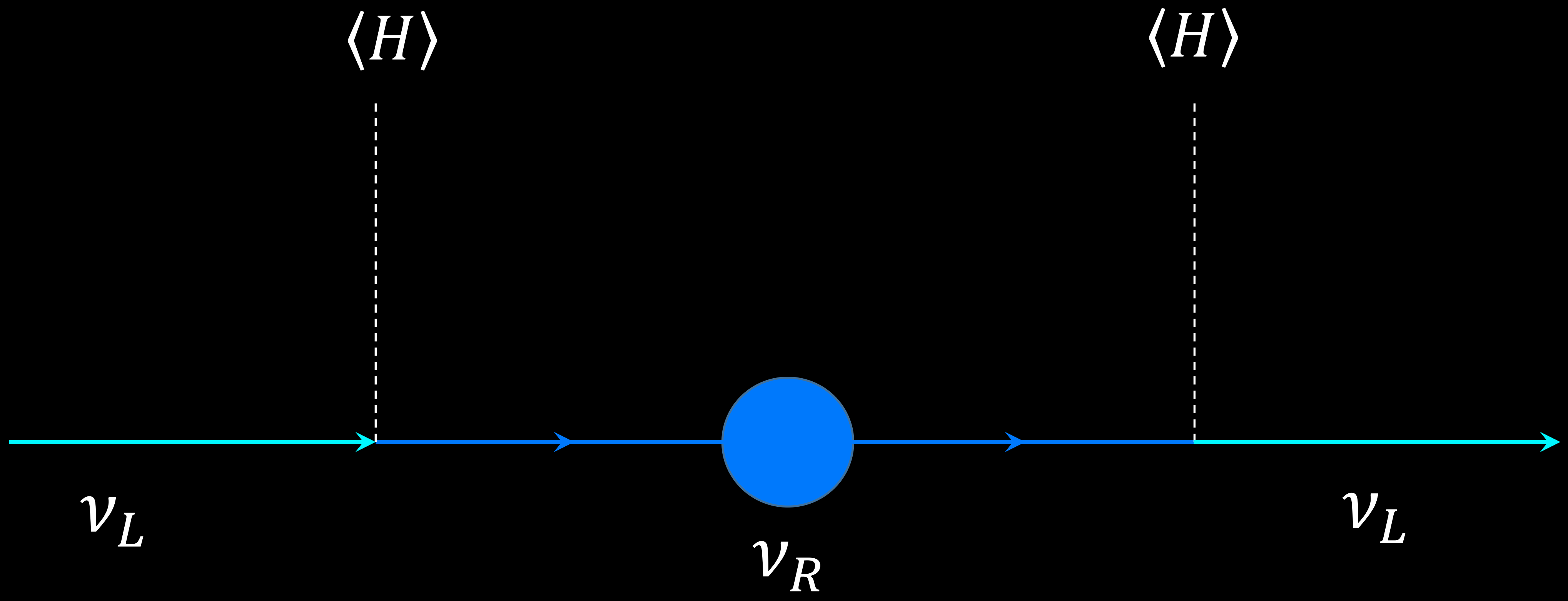
Gravity

Leptons

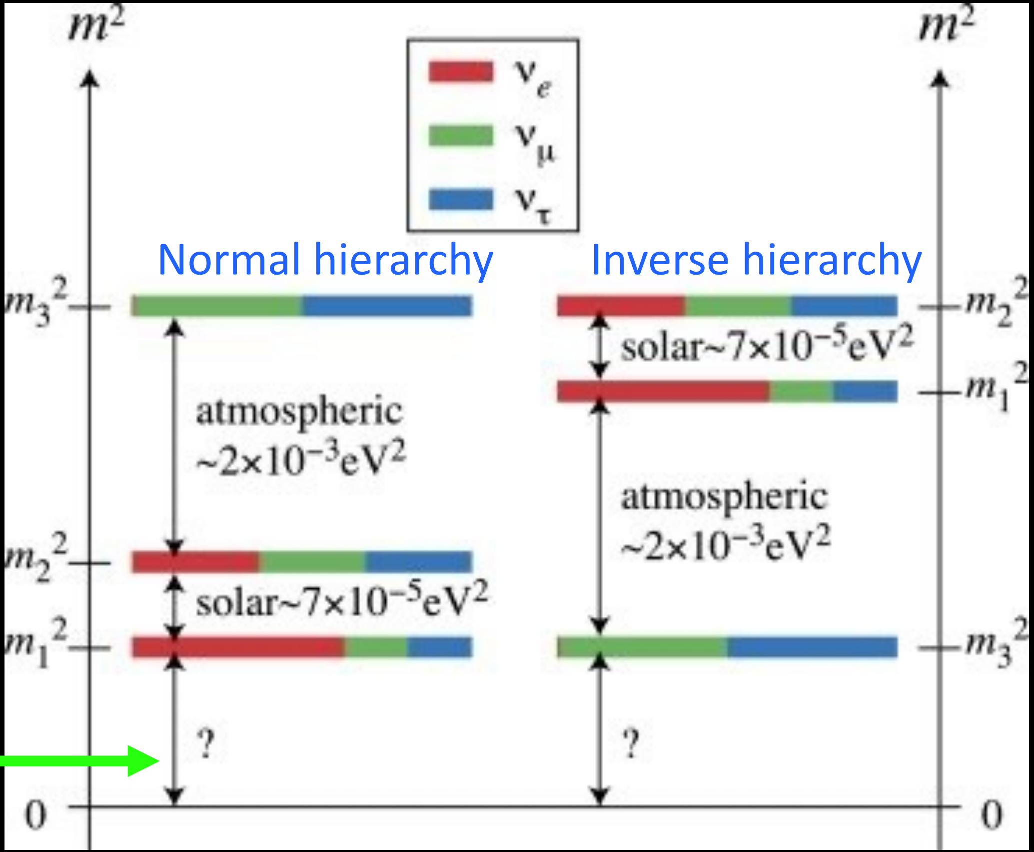
RH neutrinos

dark matter

Stability of one right-handed neutrino ⇒ lightest ν is massless



Light neutrinos: observations



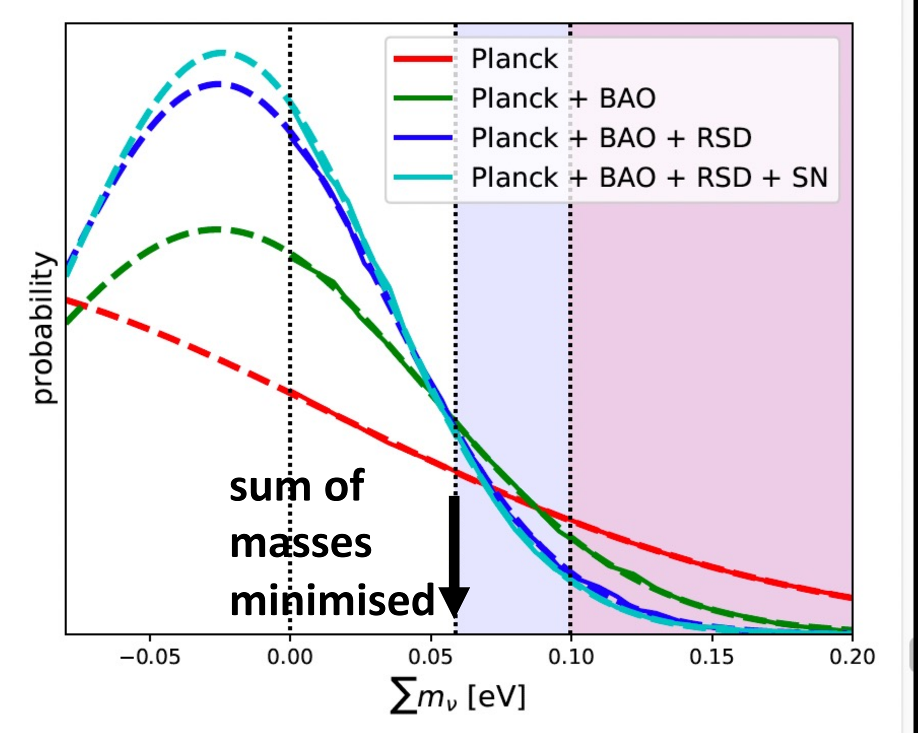
we
predict
zero →

Normal hierarchy: $M_\nu \equiv \sum m_\nu \approx 0.06\ eV$

Inverted hierarchy: $M_\nu \approx 0.1\ eV$

current data

eBOSS 2007.08991

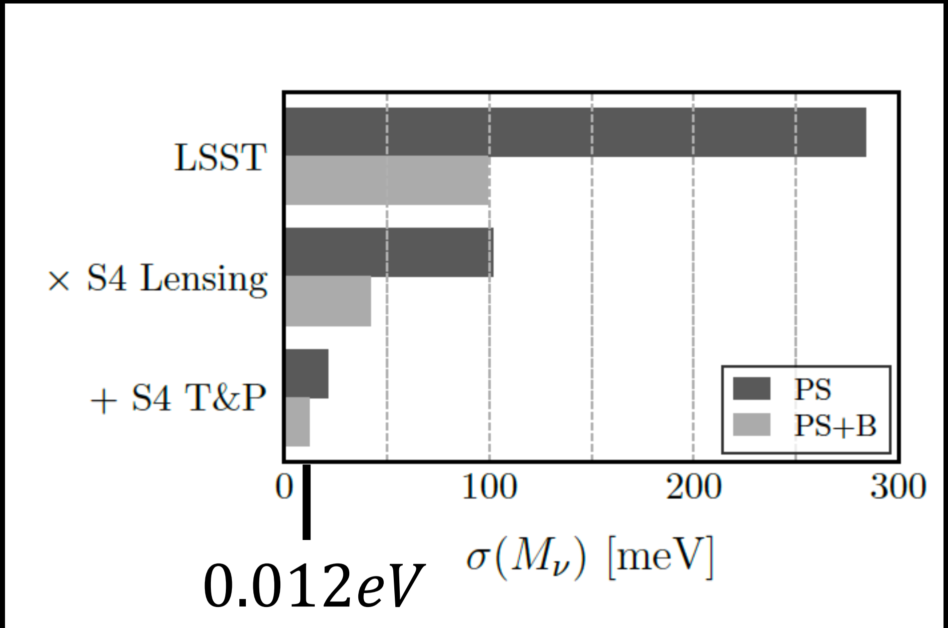


sum of masses minimised

FIG. 13.— Posterior for sum of neutrino masses for selected con binations of data with a $\nu\Lambda$CDM cosmology. Dashed curves sho the implied Gaussian fits. Shaded regions correspond to lower lin its on normal and inverted hiearchies. Likelihood curves are no malized to have the same area under the curve for $\sum m_\nu > 0$.

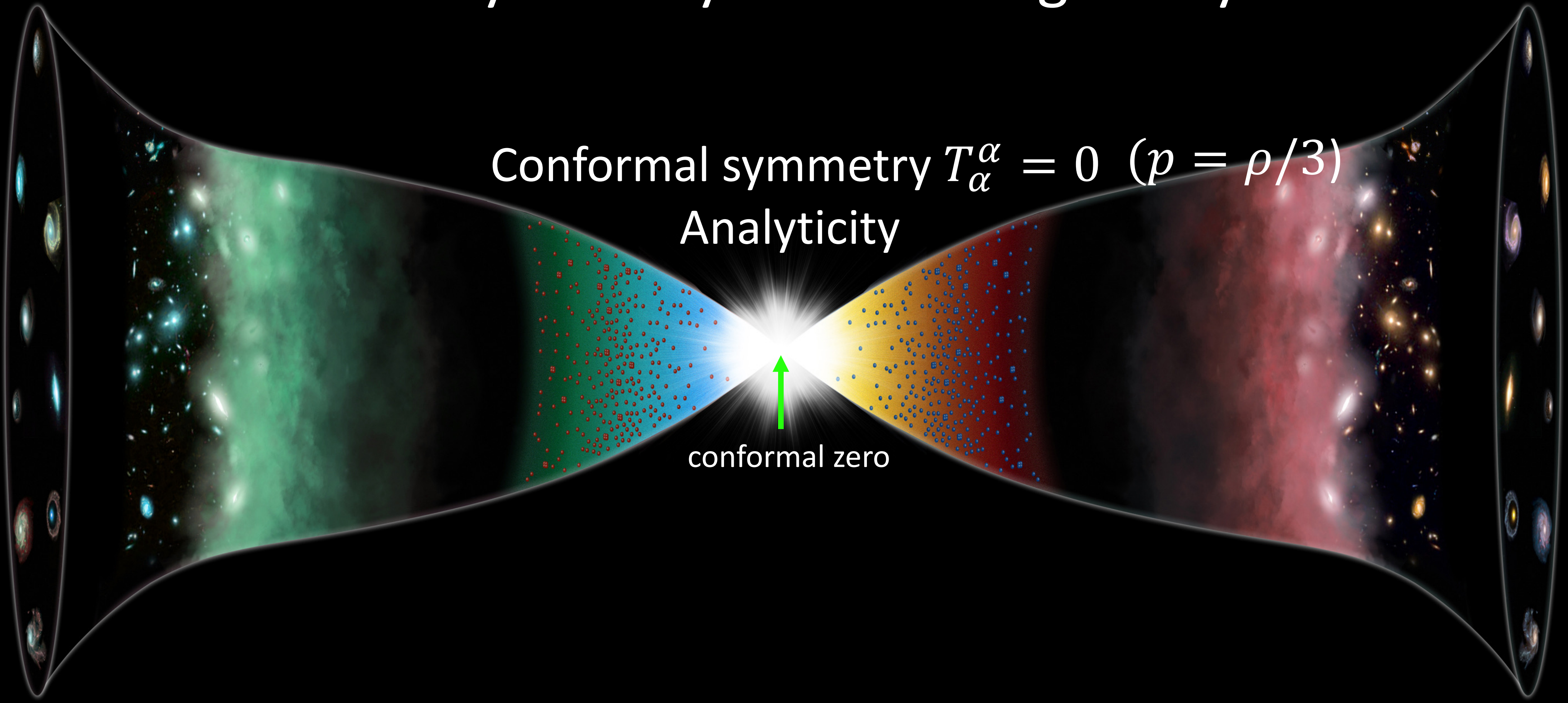
Chen et al. 2103.01229

Vera C. Rubin Observatory
Legacy Survey of Space and Time



(similar bounds hoped for from EUCLID telescope)

CPT symmetry and the singularity

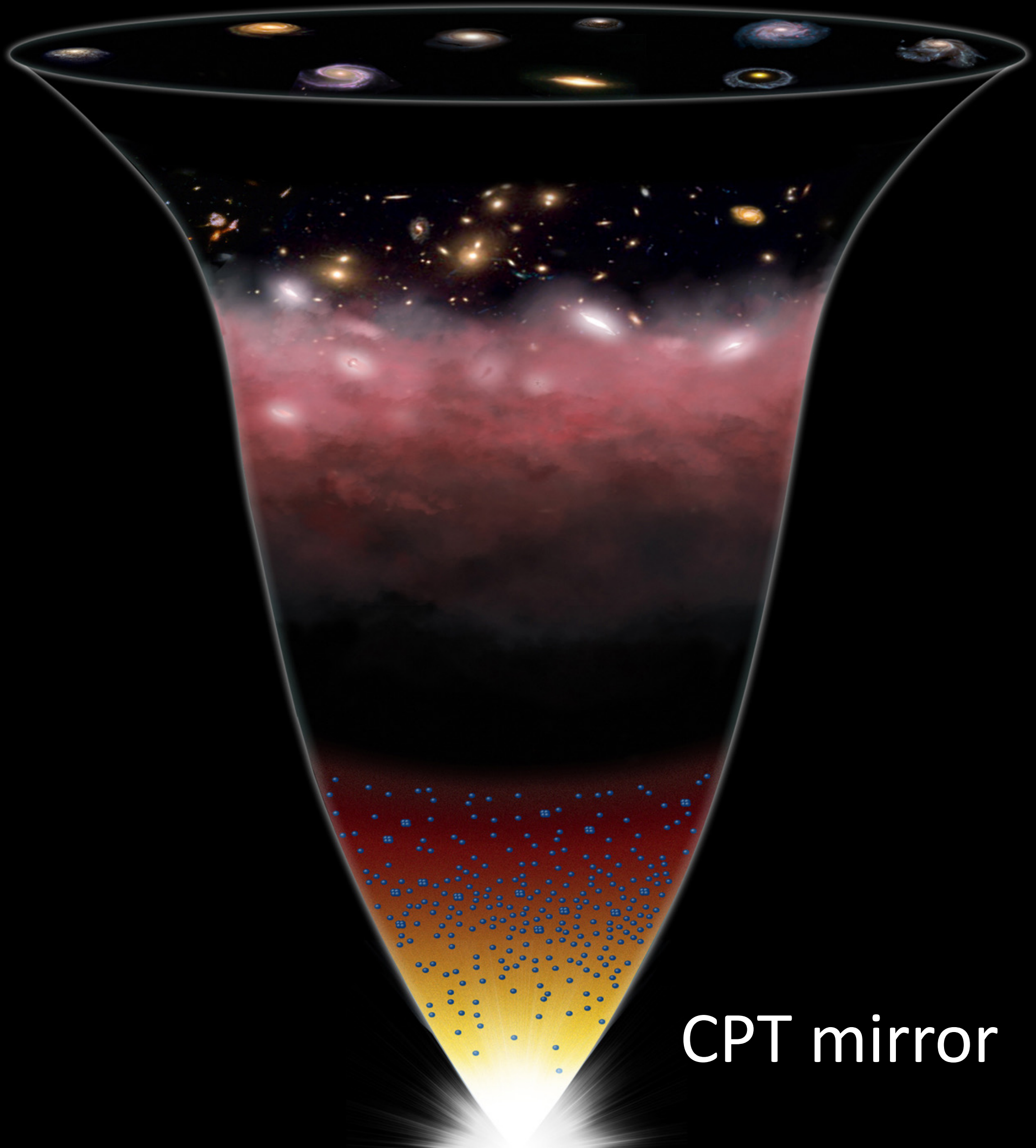


Conformal symmetry $T^{\alpha}_{\alpha} = 0 \;\; (p = \rho/3)$

Analyticity

CPT-symmetry imposed via the “method of images”

classically, the big bang is an “analytic mirror”



CPT mirror

Striking fact: for perfect fluid with $T^{\mu}{}_{\mu} = 0$, *i.e.*, local conformal symmetry,
$\exists\ \infty^3$ solutions to the Einstein equations which are analytic at $t = 0$:

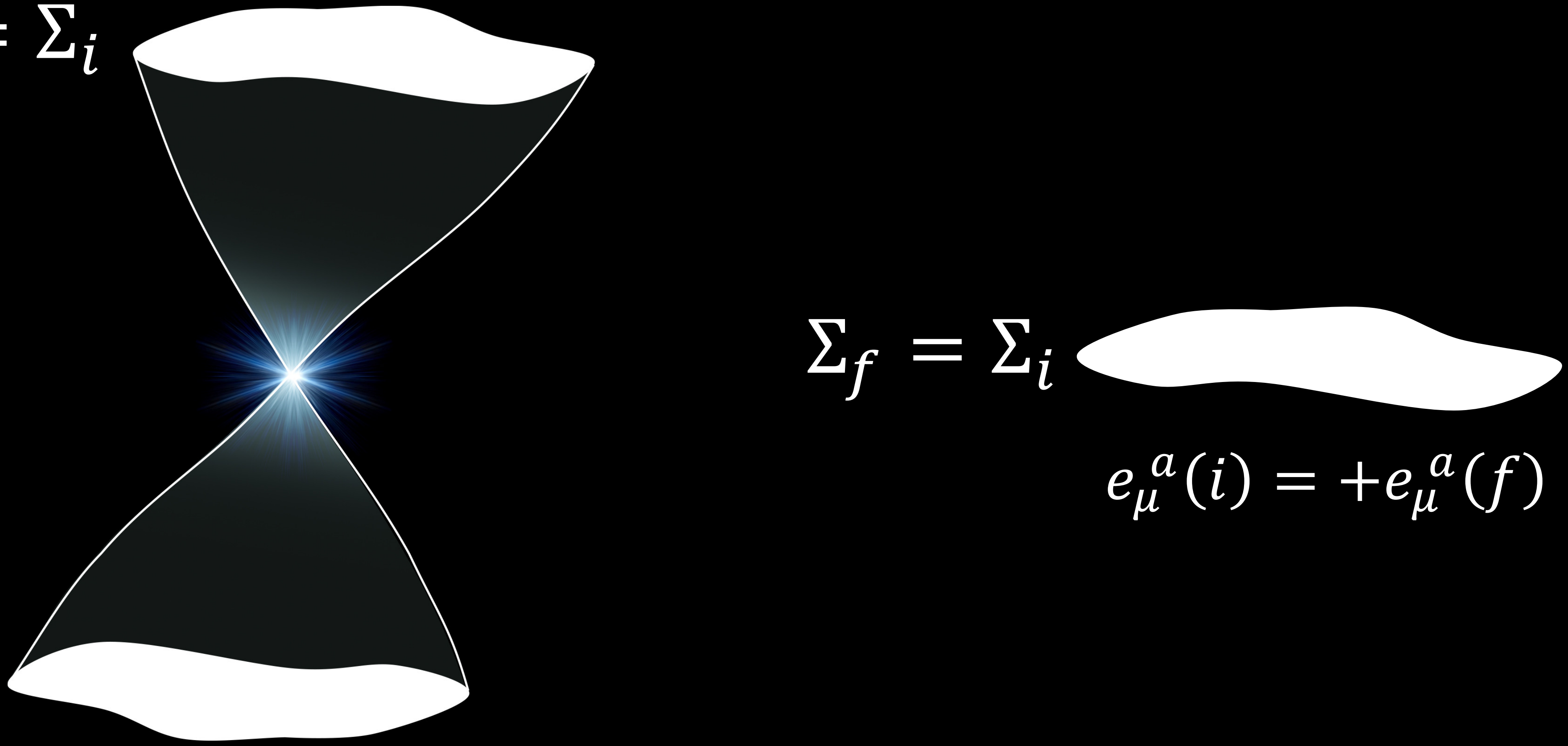
$$ds^2 = t^2\underbrace{(-dt^2 + h_{ij}(t,\boldsymbol{x})dx^i\,dx^j)}_{\text{regular 4-metric}};\; h_{ij}(t,\boldsymbol{x}) = \underbrace{h^0_{ij}(\boldsymbol{x})}_{\text{regular 3-metric}} + t^2\,\underbrace{h^2_{ij}(\boldsymbol{x})}_{\text{determined by Einstein eqns}}+..$$

The extended spacetime is symmetric under $t \Rightarrow -t$; provides a saddle to the path integral for gravity with CPT-symmetric boundary conditions

The big bang singularity is purely conformal: the Weyl tensor vanishes there
Penrose conjecture is a consequence of the CPT-symmetric path integral

BKL or Mixmaster metrics are excluded because they are singular and hence not genuine saddles

Lorentzian path integral for gravity with CPT-symmetric boundary conditions



$\Sigma_f = \Sigma_i$

Σ_i

$e_\mu{}^a(i) = -e_\mu{}^a(f)$

Past

$\Sigma_f = \Sigma_i$

$e_\mu{}^a(i) = +e_\mu{}^a(f)$

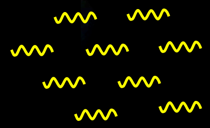
Present

the puzzling large-scale geometry of the cosmos

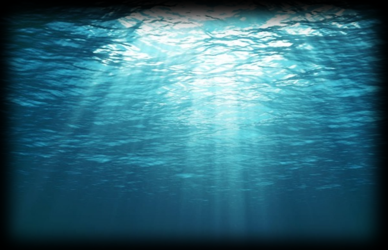
Penrose

Path integrals and gravity

gauge fields

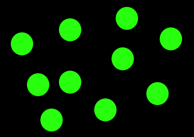


Higgs



$$\int e^{\frac{i}{\hbar}\int\left(\frac{R}{16\pi G} - \frac{1}{4}F^2 + \overline{\psi} i \not{D}\psi - \lambda H\overline{\psi}\psi + |DH|^2 - V(H) \right)}$$

gravity

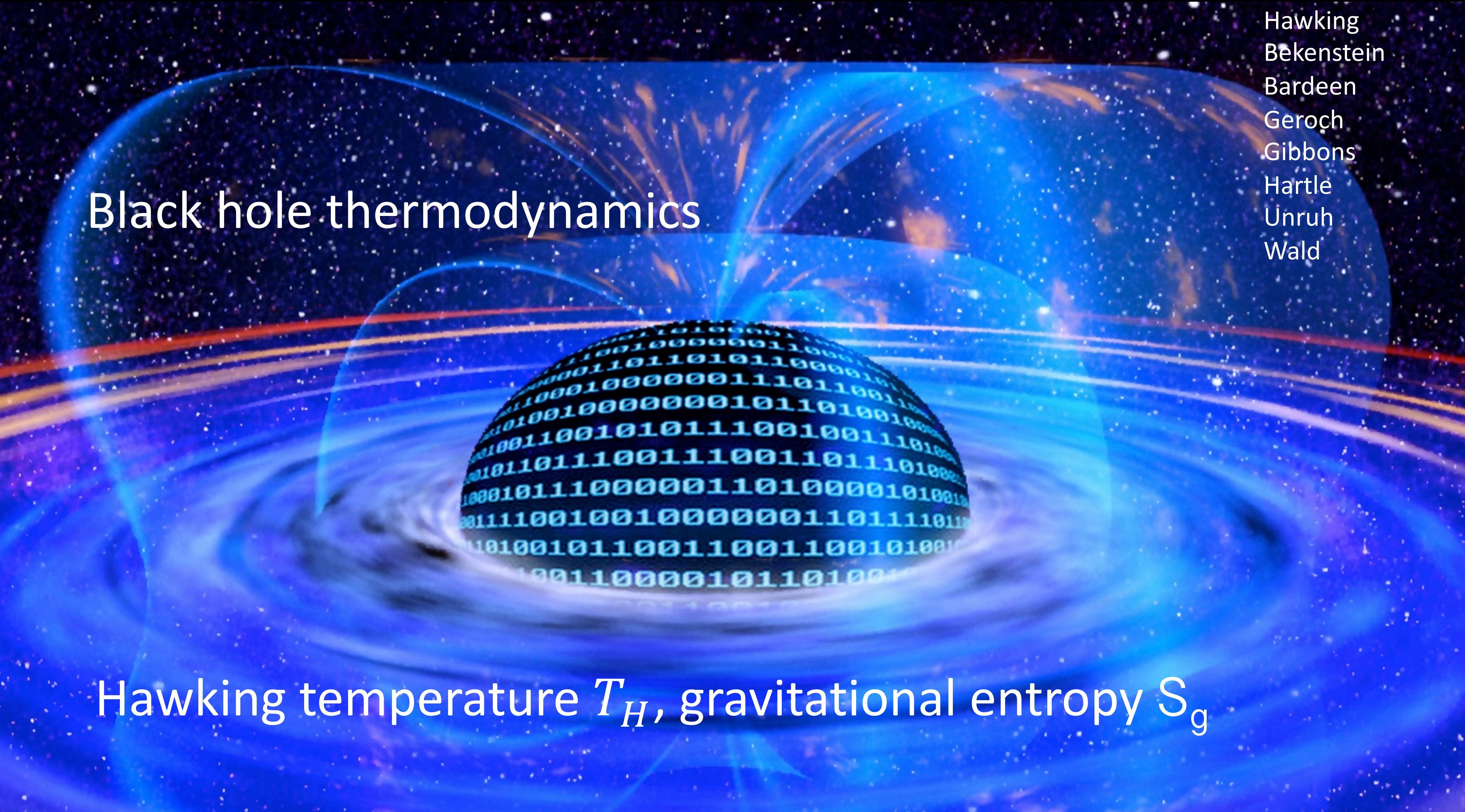


particles

gravitational entropy

With pbcs in imaginary time, $Z = e^{S_g}$

partition function

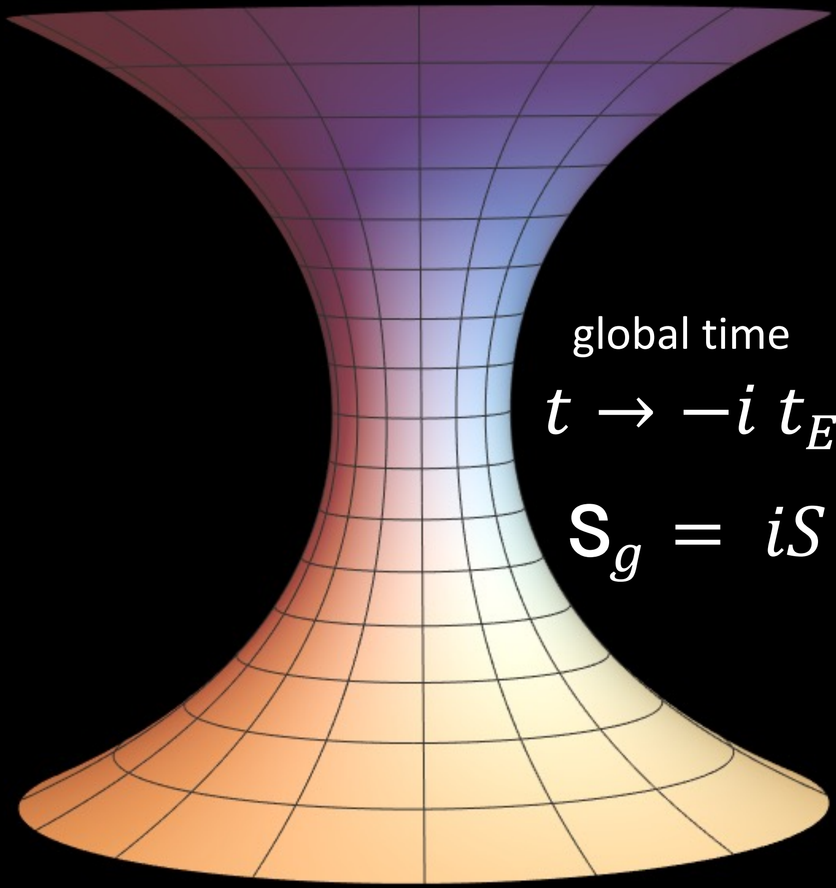


Black hole thermodynamics

Hawking
Bekenstein
Bardeen
Geroch
Gibbons
Hartle
Unruh
Wald

Hawking temperature T_H, gravitational entropy S_g

de Sitter

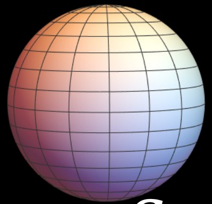


gravitational entropy from the Euclidean path integral

global time

$t \to -i\, t_E$

S^4



trace of Einstein

$$R = \frac{4\rho_\Lambda}{M_P^2}$$

$$\mathbf{S}_g = iS = -S_E = \int\left(\tfrac{1}{2}M_P^2 R - \rho_\Lambda\right) = \rho_\Lambda\,\mathrm{Vol} = \frac{24\pi^2 M_P^4}{\rho_\Lambda}$$

$\equiv \mathbf{S}_\lambda \approx 3.26\times10^{122}$ for measured ρ_Λ

de Sitter Entropy

cf. entropy of radiation in our Hubble volume $\sim 10^{90}$

realistic cosmology:

scale factor

symmetric space $R^{(3)} = 6\kappa$

$$ds^2 = a(t)^2\left(-dt^2 + \gamma_{ij}dx^i dx^j\right)$$

conformal time

comoving space (assume compact)

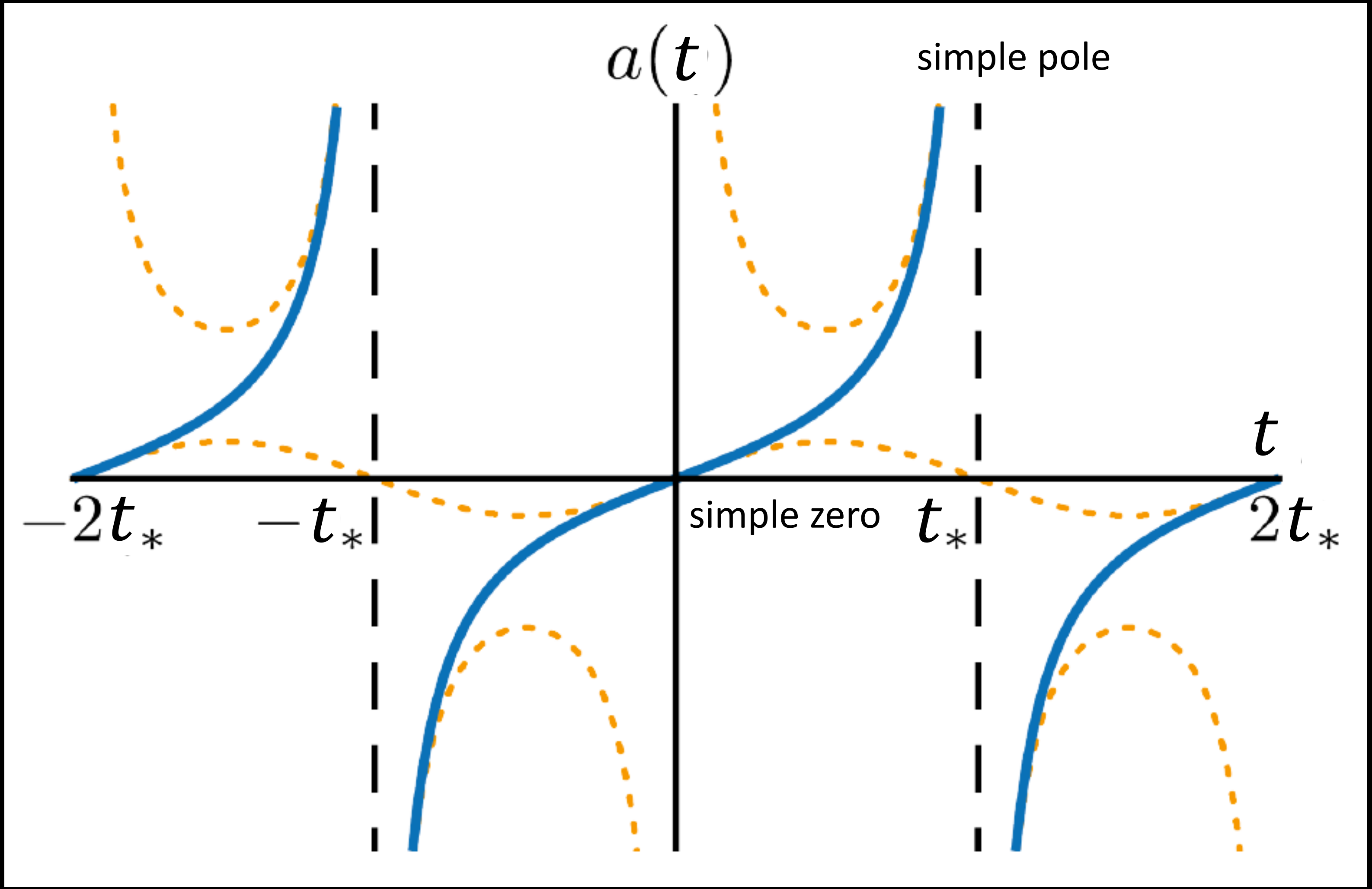
In suitable units

radiation matter space curvature Lambda

Friedmann $$3\dot{a}^2 = r + \mu a - 3\kappa a^2 + \lambda a^4$$

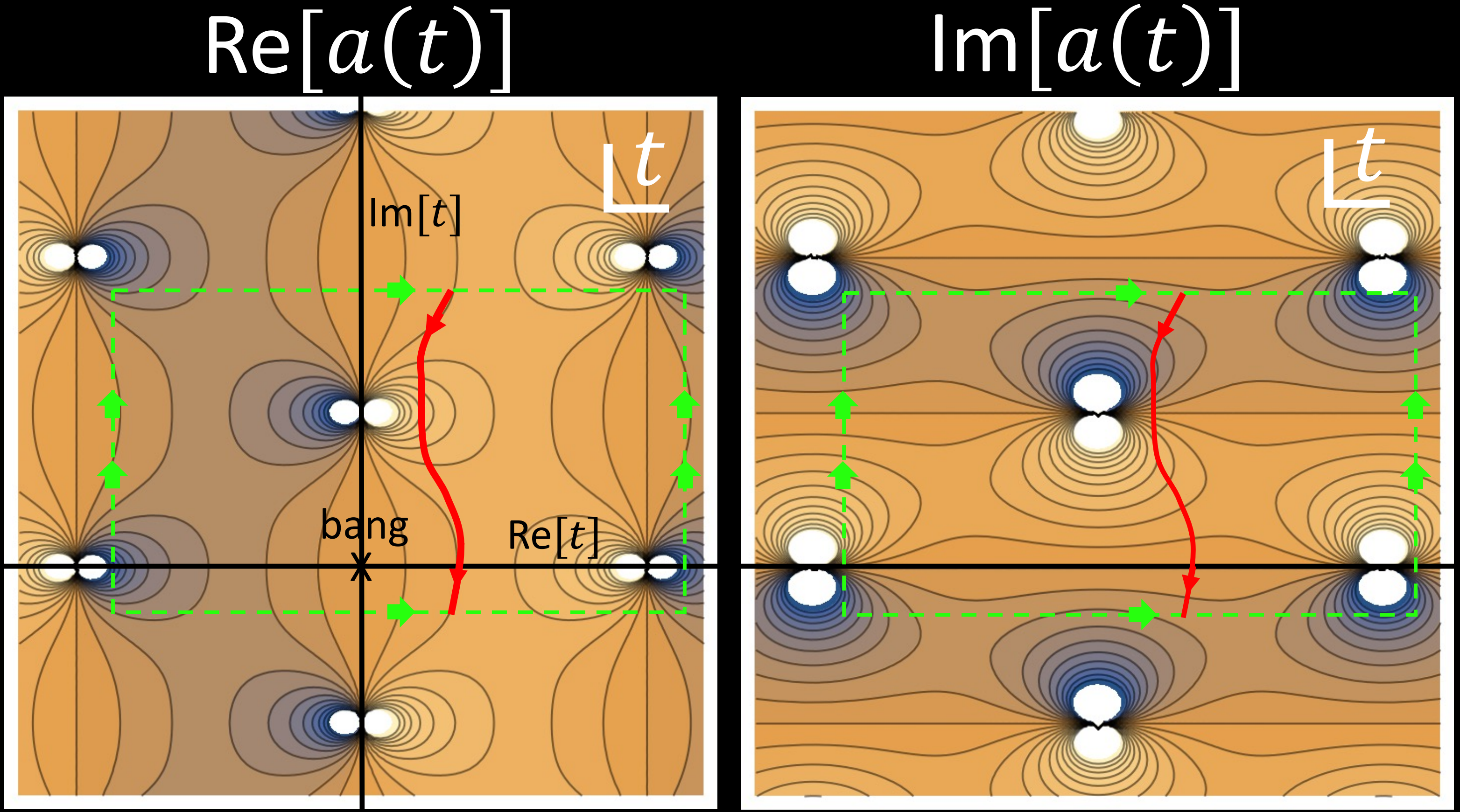
$$T^{\mu}{}_{\mu} = 0 \overset{\text{Einstein}}{\Longrightarrow} R = 0 \Longrightarrow a(t) \text{ analytic at } t = 0$$

general solution has remarkable analytical properties real time

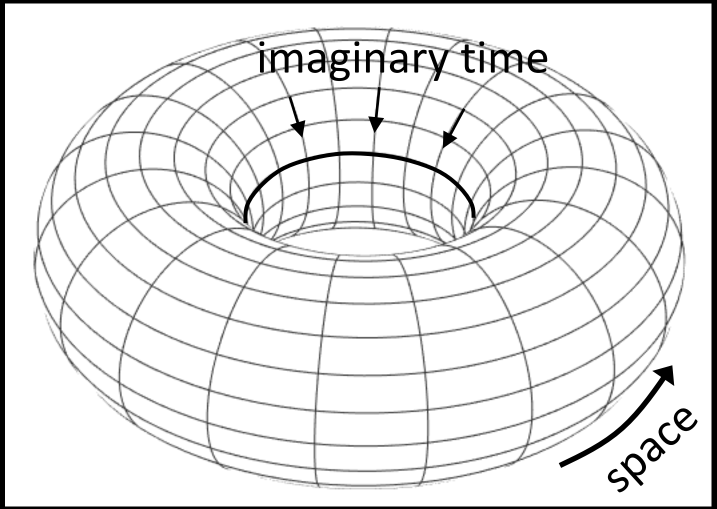


Jacobi

$a(t)$ is single-valued and doubly periodic in the complex t-plane: its only singularities are simple poles. The imaginary time period and the action computed over a period determine T_H and the gravitational entropy S$_g$


Re$[a(t)]$

Im$[a(t)]$



Euclidean instanton for a universe w/radiation, matter, curvature, Lambda

We recently computed S_g analytically for a general cosmology with radiation, matter, space curvature and a cosmological constant (*i.e.*, all conserved quantities).

Inhomogeneities and anisotropies treated in cosmological perturbation theory.

We found that S_g is greatest for:

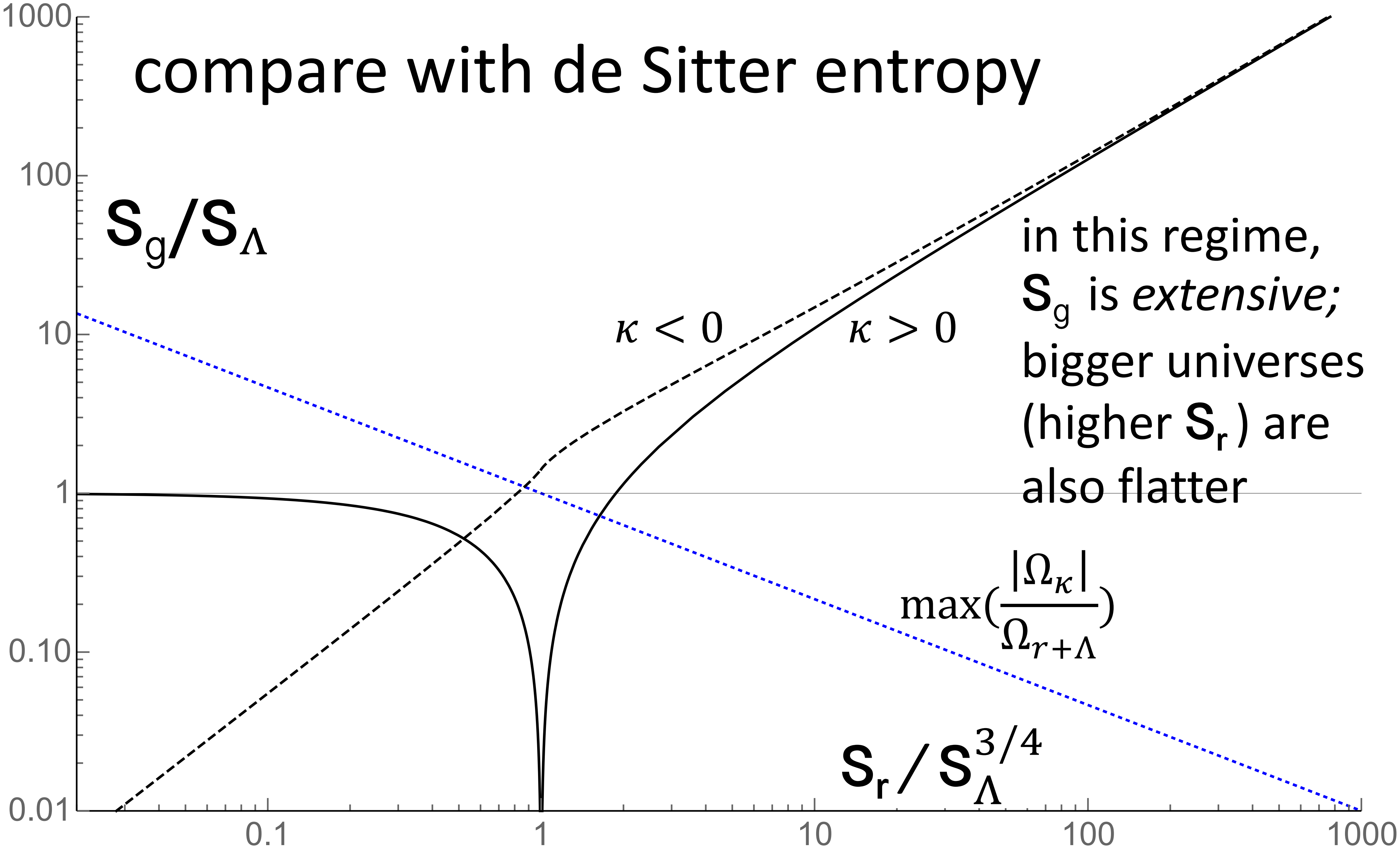
1. a spatially flat, homogeneous, isotropic universe
2. a small, positive cosmological constant

(echoing earlier arguments of Baum, Hawking, Coleman…)

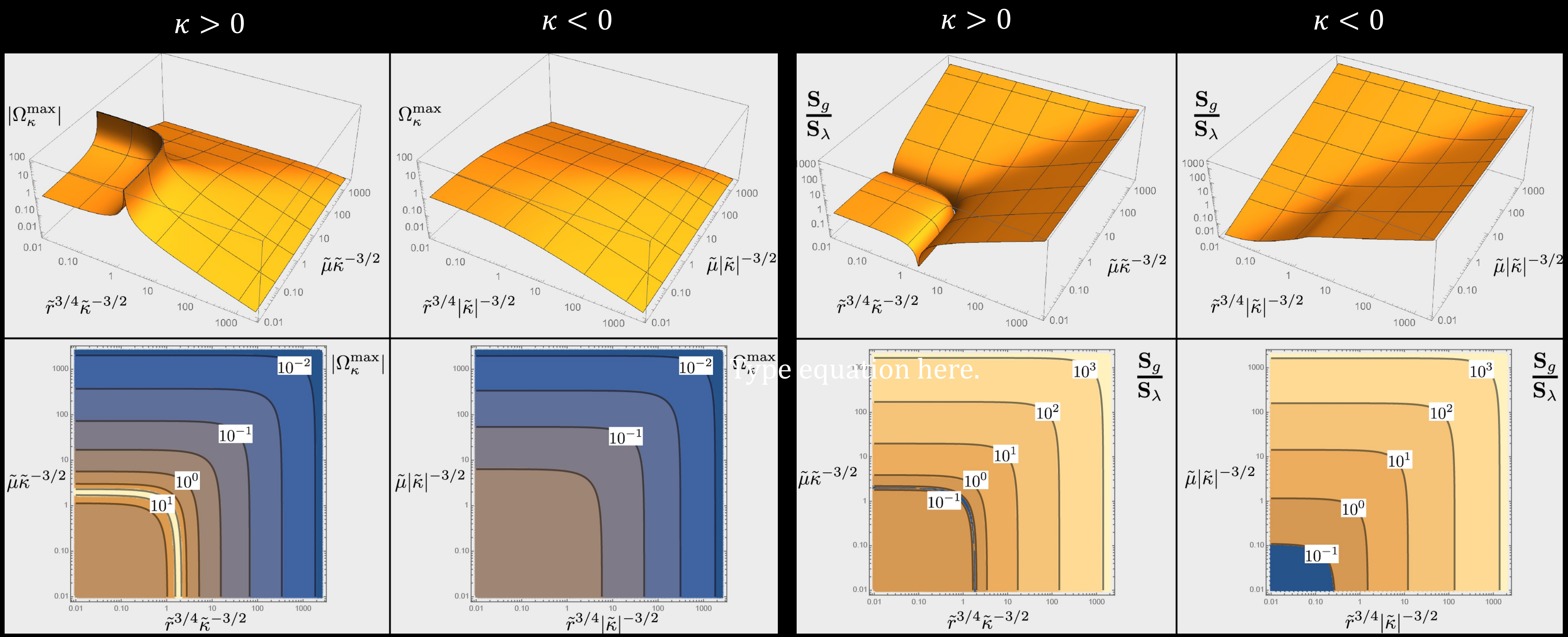
Note:

S_g is the *global* entropy for the entire spacetime. It is a fixed number, independent of Lorentzian time (via Cauchy theorem), depending on the cosmological parameters.

compare with de Sitter entropy



in this regime,
S_g is *extensive;*
bigger universes
(higher **S_r**) are
also flatter

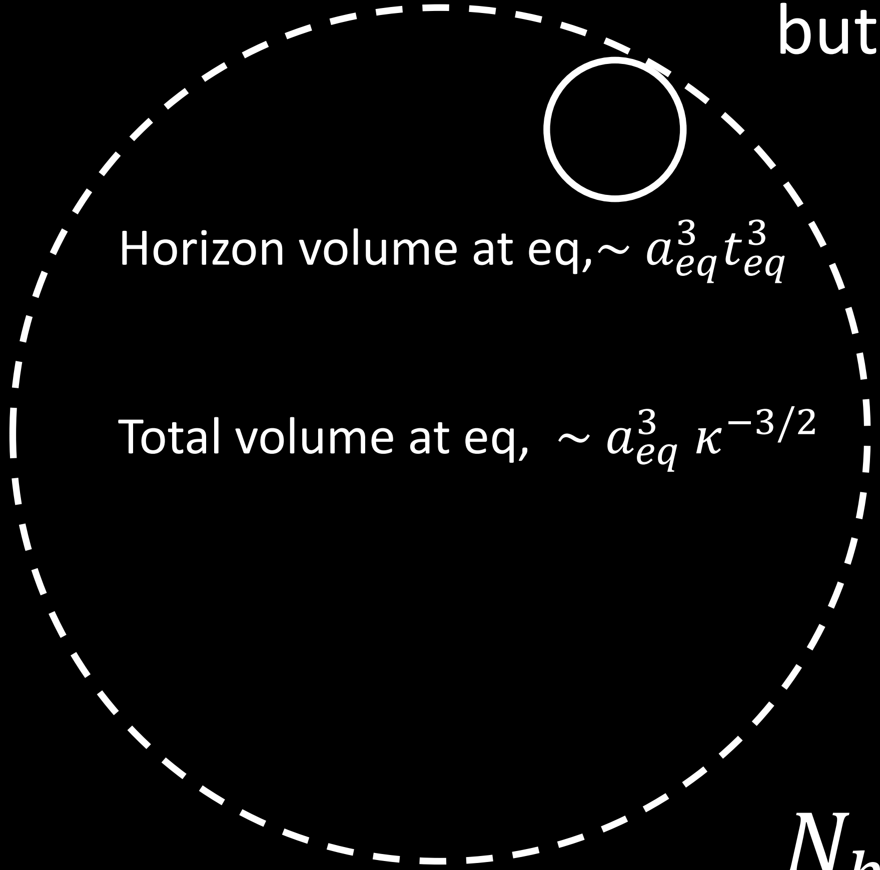


$$\tilde{r} \equiv \frac{r}{\lambda};\ \tilde{\mu} \equiv \frac{\mu}{\lambda}; \tilde{\kappa} \equiv \frac{\kappa}{\lambda};\ \mathrm{S}_\lambda = \frac{24\pi^2}{L_{Pl}^2\lambda}$$

understanding in terms of horizons

$3\dot{a}^2 = -3\kappa a^2 + r + \lambda a^4;$ equal Λ, radiation density at $a_{eq} = (r/\lambda)^{1/4}$

but for $a < a_{eq},\ a \sim r^{1/2} t$ so $t_{eq} \sim 1/(r\lambda)^{1/4}$



Horizon volume at eq,$\sim a_{eq}^3 t_{eq}^3$

Total volume at eq, $\sim a_{eq}^3\ \kappa^{-3/2}$

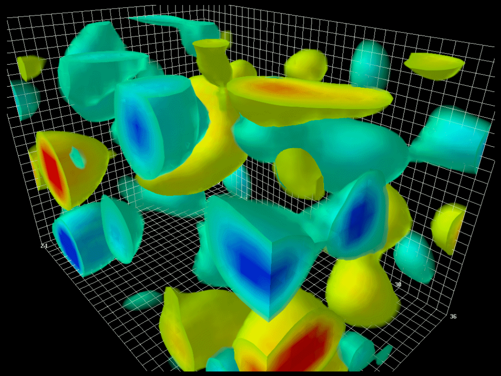
number of horizon volumes at equality

$N_{hor} \sim (\lambda r/\kappa^2)^{3/4}$

multiply by de Sitter entropy $\sim \lambda^{-1}$

$$N_{hor}\lambda^{-1} \sim (r/\kappa^2)^{3/4}\ \lambda^{-1/4} \sim \mathrm{S}_r \mathrm{S}_\Lambda^{1/4}$$

Quantum fields and gravity



vacuum energy and pressure are divergent,
simple physical regularizations such as point splitting give (for, *e.g.*, Maxwell):

B.S.DeWitt, Phys. Rep. 19 (1975) 295

$$\Longrightarrow \langle T^{\mu\nu}\rangle_{vac} \sim \frac{3}{\pi^2 \Delta t^4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \end{pmatrix},$$

where $\Delta t^2 =$ invart time-like separation

Breaks Lorentz invariance! Can be renormalized away but leaves us with little physical understanding of the QFT vacuum.

Worse still are Weyl anomalies where quantum divergences spoil the local scale invariance of Maxwell and Dirac fields: violations cannot be renormalized away

Dimension zero scalars

A four-derivative, Weyl-invariant (*i.e.,* locally scale-invariant) action

$$S_4 = -\frac{1}{2}\int d^4x\sqrt{-g}\varphi\Delta_4\varphi; \quad \Delta_4 = \Box^2 + \ldots$$

Heisenberg (1957), Pauli, Thirring, Nakanishi, …
Flato, Fronsdal (‘70s , ‘80s) forerunner of AdS/CFT

φ is Heisenberg’s “dipole ghost” or Dirac’s “singleton”; a very interesting theory

It has an infinite dimensional symmetry: $\varphi(x) \longrightarrow \varphi(x) + \alpha(x)$ with $\Box\alpha = 0$

The only physical state is the vacuum: there are no excited states

Bogoliubov et al (1987); Rivelles (2003)

The vacuum fluctuations are scale-invariant

$$\langle\varphi(0,\boldsymbol{x})\varphi(0,\boldsymbol{y})\rangle = \int \frac{d^3\boldsymbol{k}}{(2\,\pi)^3}\,\frac{e^{i\boldsymbol{k}.(\boldsymbol{x}-\boldsymbol{y})}}{4\,k^3}$$

cf. observed Newtonian potential
in cosmology

SM + dim-zero scalar numerology: the vacuum energy and conformal anomalies (*at lowest order*)

$$E_{\boldsymbol{k}} = \frac{1}{2}\hbar k\left(\overset{\text{dim-one scalars}}{n_{s,1}} - \overset{\text{chiral fermions}}{2n_F} + \overset{\text{gauge fields}}{2n_A} + \overset{\text{dim-zero scalars}}{2n_{s,0}}\right)$$ per mode $\boldsymbol{k}$

$$\langle T^{\mu}{}_{\mu}\rangle = -a\,E + c\,C^2 \quad E = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2; \quad C^2 = C^{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}$$

$$a = \frac{1}{360(4\,\pi)^2}\left[n_{s,1} + \frac{11}{2}\,n_F + 62\,n_A - 28\,n_{s,0}\right]$$

$$c = \frac{1}{120(4\,\pi)^2}\left[n_{s,1} + 3\,n_F + 12\,n_A - 8\,n_{s,0}\right]$$

Cancellation of all three implies $n_F = 4n_A$; $n_{s,0} = 3n_A$; $n_{s,1} = 0$.

Given SU3×SU2×U1, n_A=12, predicts 3 generations of fermions, each with a ν_R

Also requires no fundamental dim-1 scalars so the Higgs must be composite (exponentiating a dim 0 scalar gives an operator with nontrivial scaling dimension)

Promising development in self-dual gravity – twistor formulation of (exact) path integral for gravity also has anomalies but these can be removed *to all orders* using dimension zero scalars

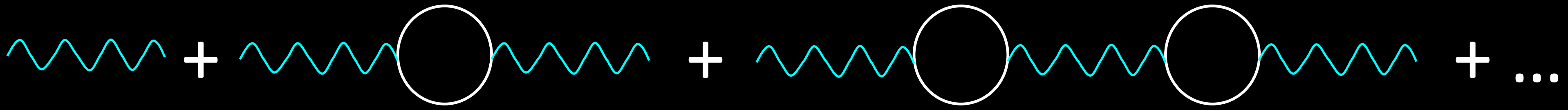
K. Costello 2111.08879, R. Bittleston *in prep*. 2023

Tomboulis 70's

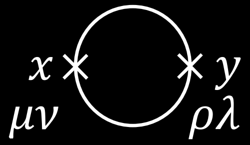
Han Willenbrook

Donoghue Menezes

Graviton propagator with 1 loop SM corrections



Loop is given by the Fourier transform of the stress-energy correlator: for a CFT,



$$= \langle T^{\mu\nu}(x)T^{\rho\lambda}(y)\rangle = C^T \frac{1}{4\,\pi^4 x^8}\, I^{\mu\nu,\rho\lambda}(x-y)$$

where $I^{\mu\nu,\rho\lambda}(x) = \frac{1}{2}\left(I^{\mu\rho}(x)I^{\nu\lambda}(x) + I^{\mu\lambda}(x)I^{\rho\nu}(x)\right) - \frac{1}{4}\eta^{\mu\nu}\eta^{\rho\lambda}$ and $I^{\mu\nu}(x) = \eta^{\mu\nu} - 2\frac{x^\mu x^\nu}{x^2}$

$C^T = \frac{4}{3}[n_{s,1} + 3\,n_F + 12\,n_A - 8\,n_{s,0}] \equiv \frac{4}{3} n_{eff}$ (∝ coefft of Weyl squared in the trace anomaly)

Projector onto spin 2 component
- gauge invariant

Dim reg and min subt $\Rightarrow$ $$D^{\alpha\beta,\mu\nu}(k) = \frac{P^{\alpha\beta,\mu\nu}(k)}{k^2\left((1-\frac{n_{eff}}{240\pi}Gk^2\ln(-\frac{k^2}{\mu^2})\right)}$$

SM corrections to the graviton propagator are problematic:

1. Inconsistent with Källén-Lehmann repn. $D(k) = \int_0^\infty dm^2\, \rho(m^2) \frac{1}{k^2 - m^2 + i\varepsilon}$

   (follows from Poincare invariance and positivity of the physical Hilbert space)

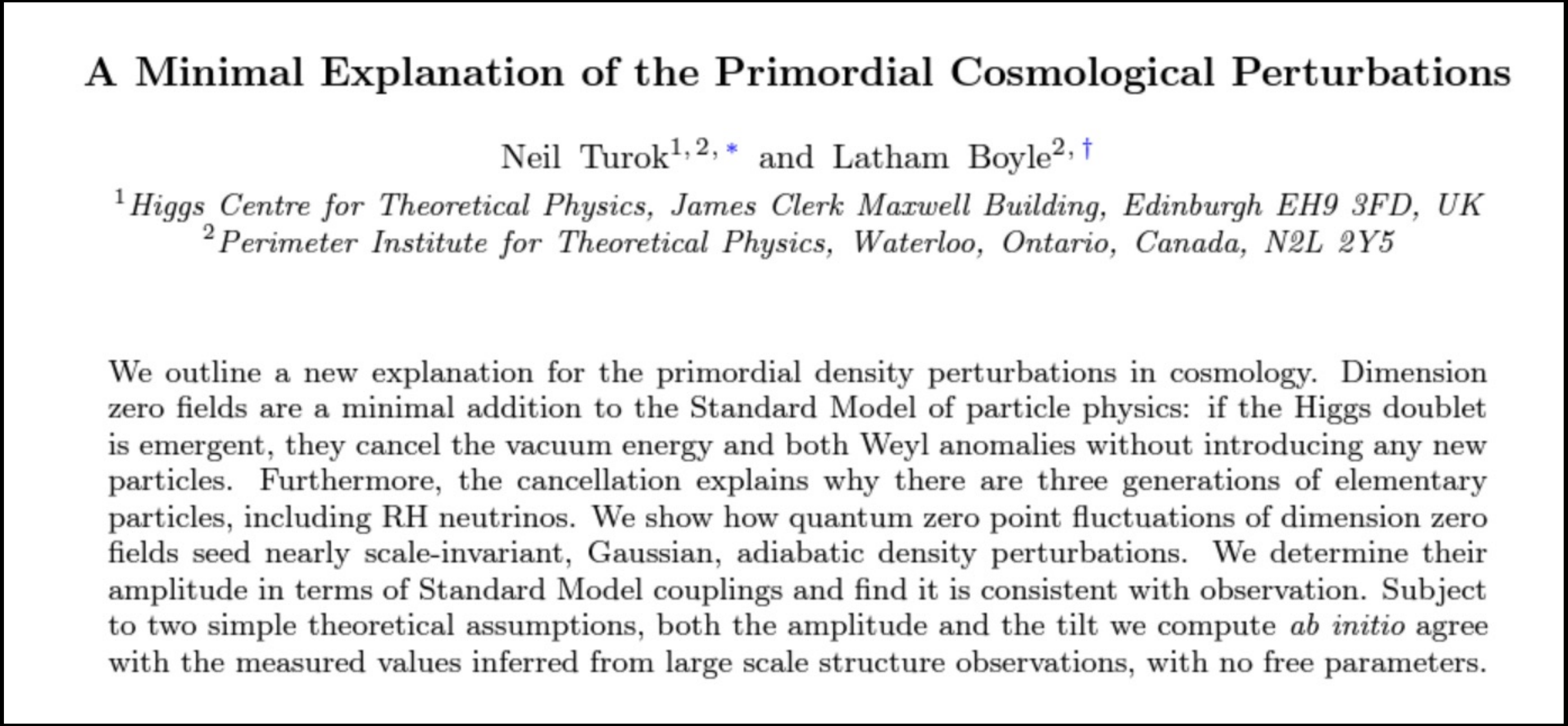
2. Specifically, resummed $D(k)$ (i) falls off as $|k|^{-4}$ at large $|k|$

   (ii) has complex (acausal) poles on physical sheet

Similarly, Dim-0 scalar loops alone violate K-L: (i) $|k|^{-4}$ fall off; (ii) a tachyonic pole

BUT:

SM + Dim-0 combination is consistent with Poincaré, positivity and microcausality (at one loop in SM gauge+fermion fields: we are examining higher orders)



A Minimal Explanation of the Primordial Cosmological Perturbations

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We outline a new explanation for the primordial density perturbations in cosmology. Dimension zero fields are a minimal addition to the Standard Model of particle physics: if the Higgs doublet is emergent, they cancel the vacuum energy and both Weyl anomalies without introducing any new particles. Furthermore, the cancellation explains why there are three generations of elementary particles, including RH neutrinos. We show how quantum zero point fluctuations of dimension zero fields seed nearly scale-invariant, Gaussian, adiabatic density perturbations. We determine their amplitude in terms of Standard Model couplings and find it is consistent with observation. Subject to two simple theoretical assumptions, both the amplitude and the tilt we compute *ab initio* agree with the measured values inferred from large scale structure observations, with no free parameters.

arXiv:2302.00344[hep-ph]

primordial perturbations from dim-0 fields and the SM

Running couplings violate scale symmetry: at high temperature,

$$T_\beta^{SM} \equiv \langle T^{SM\mu}{}_{\mu} \rangle_\beta = 3P - \rho \approx \sum c_i \alpha_i^2 T^4 \equiv c_\beta^{SM} T^4$$

This anomalous trace can be cancelled by introducing a linear coupling in the effective action,

$$\Gamma^\varphi = \sum_{j=1}^{n_{s,0}} \frac{1}{2} \int -a\varphi_j \Delta_4 \varphi_j + \left[a\left(E - \tfrac{2}{3}\boxdot R\right) + cC^2 - n_{s,0}^{-1} T_\beta^{SM}\right]\varphi_j$$

↑ non-Weyl invariant term used to cancel anomalies

(generalizing sigma models in string theory)

The final linear term is chosen to cancel the trace anomaly due to running couplings at high T
It corrects the Einstein-fluid equations, converting quantum correlations inthe dim-0 fields into large scale curvature fluctuations: Friedmann equation becomes

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_r a^4 (1 + c_\varphi \bar{\varphi}(x)) \text{ with } \bar{\varphi}(x) = n_{s,0}^{-1} \sum \varphi_j(x),\ c_\varphi = c_\beta^{SM} / \left(\tfrac{\pi^2}{30}\mathcal{N}_{eff}\right), \mathcal{N}_{eff} \approx 106\tfrac{1}{4}$$

Conformal factor translates directly into “comoving curvature perturbation” $\mathcal{R}(x) = \frac{1}{4} c_\varphi \bar{\varphi}(x)$

(adiabatic, Gaussian, scalar: no primordial long-wavelength gravitational waves)

Spectral tilt

Dominated by QCD: asymptotic freedom $\Longrightarrow$ red tilt!

To understand quantitatively, consider the trace anomaly (for QCD)

$$S = -\int \tfrac{1}{4} F^2 \Rightarrow -\int \tfrac{1}{4g^2} F^2. \quad \alpha \equiv \tfrac{g^2}{4\pi};\ \mu\partial_\mu \alpha \equiv \beta_\alpha; \Longrightarrow \mu\partial_\mu S = \int \tfrac{\beta\alpha}{\alpha}\tfrac{1}{4g^2} F^2 \Rightarrow \int \tfrac{\beta\alpha}{4\ \alpha} F^2$$

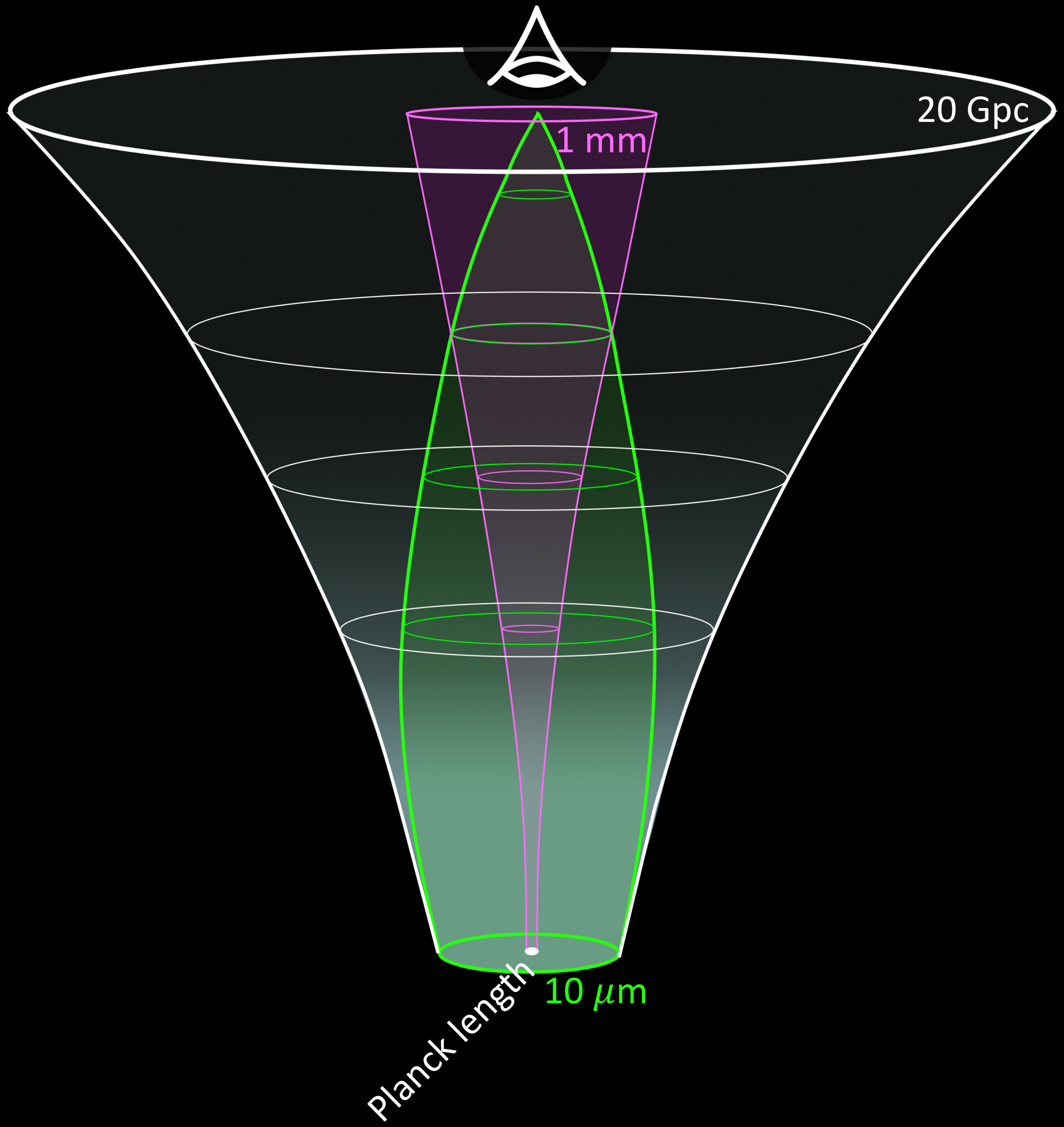
$$\Longrightarrow T_\lambda^\lambda = \frac{\beta\alpha}{4\ \alpha} F^2; \quad \frac{\beta\alpha}{\alpha} = -(11 - \tfrac{2}{3} n_f)\tfrac{\alpha}{2\pi} T^4; \quad \langle F^2 \rangle_\beta = \tfrac{2\pi\alpha}{9}(12+5n_f)T^4$$

running coupling: energy scale of φ plasma interactions: energy scale of T

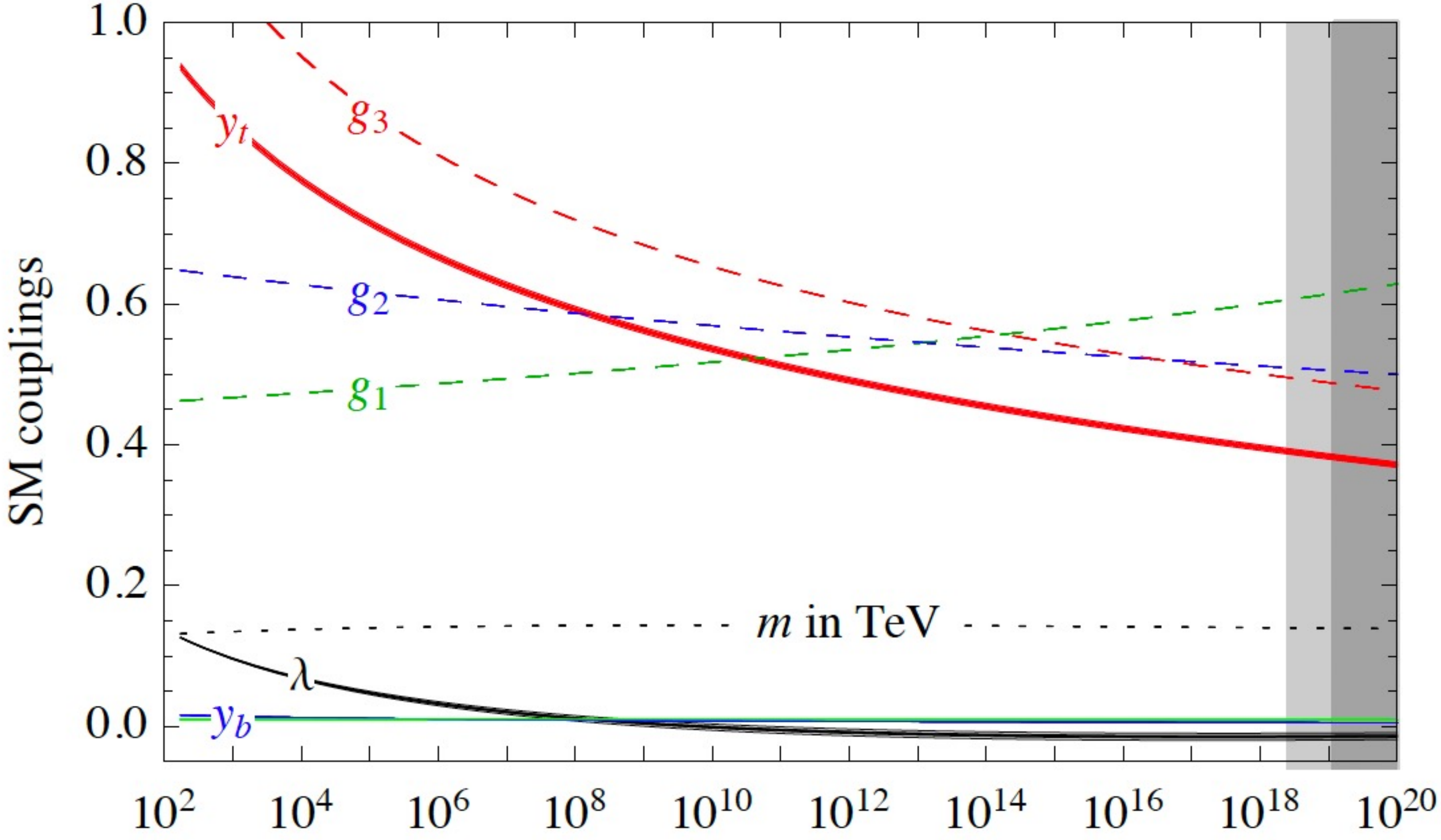
Thus, $\mathcal{P}_\mathcal{R}(k)$ scales with k as $\alpha^2(k)$; $n_s - 1 \equiv \frac{d \ln \mathcal{P}_\mathcal{R}(k)}{\mathrm{d} \ln k} = 2\frac{\beta\alpha}{\alpha} = -\frac{7}{\pi}\alpha_{QCD}(M_P)$

The red tilt is a critical exponent which can be computed perturbatively

If so, we can extrapolate over 30 orders of magnitude in length scale…


20 Gpc
1 mm
10 μm
Planck length

Buttazzo et al
1307.3536
[hep-ph]



Planck energy 10^{19}

Energy scale in GeV

Comparison with observation

$\mathcal{P}_{\mathcal{R}}(k) = \frac{3^2 5^2}{7(2\,\pi)^4}\left(\frac{c_\beta^{SM}}{\mathcal{N}_{eff}}\right)^2\left(\frac{k}{k_P}\right)^{-\frac{7\alpha_3}{\pi}}$; k_P= comoving Planck wavenumber

with $c_\beta^{SM} \equiv \frac{125}{108}\alpha_Y^2 - \frac{95}{72}\alpha_2^2 - \frac{49}{6}\alpha_3^2$ and $\mathcal{N}_{eff} = 106\frac{1}{4}$ (to lowest order, neglect Higgs)

Now use $(k_P/k_*)^{1-n_s} = 14.8 \pm 5.1,$ $k_* \equiv$0.05 Mpc^{-1}

Thus, we find $\mathcal{P}_{\mathcal{R}} = A\left(\frac{k}{k_*}\right)^{n_s-1}$, $A = (13 \pm 5)\times 10^{-10}$; $n_s = 0.958$

cf. Planck satellite: $A = (21 \pm 0.3)\times 10^{-10}$; $n_s = 0.959 \pm 0.006$

Comparison with observation

$11-\frac{2}{3}n_f$

$$\mathcal{P}_{\mathcal{R}}(k)=\frac{3^2 5^2}{7(2\,\pi)^4}\left(\frac{c_\beta^{SM}}{\mathcal{N}_{eff}}\right)^2\left(\frac{k}{k_P}\right)^{-\frac{7\alpha_3}{\pi}};$$

k_P= comoving Planck wavenumber

with $c_\beta^{SM}\equiv\frac{125}{108}\alpha_Y^2-\frac{95}{72}\alpha_2^2-\frac{49}{6}\alpha_3^2$ and $\mathcal{N}_{eff}=106\frac{1}{4}$ (to lowest order, neglect Higgs)

Now use $(k_*/k_P)^{n_s-1}=14.8\pm5.1,$ $k_*\equiv$0.05 Mpc^{-1}

Thus, we predict $\mathcal{P}_{\mathcal{R}}=A\left(\frac{k}{k_*}\right)^{n_s-1},$ $A=(13\pm5)\times10^{-10};\ n_s=0.958$

cf. Planck satellite: $A=(21\pm0.3)\times10^{-10};\ n_s=0.959\pm0.006$

prediction will be tested further as observations and theory improve

summary

analytic extension of cosmological solutions of the Einstein equations lead to

- a new picture of the big bang singularity as a CPT “mirror”
- a calculation of the gravitational entropy for cosmologies

new explanations and predictions for

- the large-scale homogeneity, isotropy and flatness of the cosmos (and a hint about Lambda)
- the dark matter
- the arrow of time and the strong CP problem

Dimension zero scalars

- cancel the vacuum energy and both Weyl anomalies at leading (free field) order
- explain why there are 3 generations of SM fermions, including RH neutrinos
- explain the amplitude, tilt and character of the primordial perturbations
- require the Higgs to be emergent/composite, a new approach to the gauge-gravity hierarchy?

All without adding any new propagating degrees of freedom to the SM and Einstein gravity

These are encouraging signs but much remains to be understood

Thank You!

Boyle, Finn, NT Phys. Rev. Lett. 121 (2018) 251301; Annals of Physics 438 (2022) 168767
arXiv: 2109.06204, 2110.06258, 2201.07279, 2208.10396, 2210.01142, 2302.00344