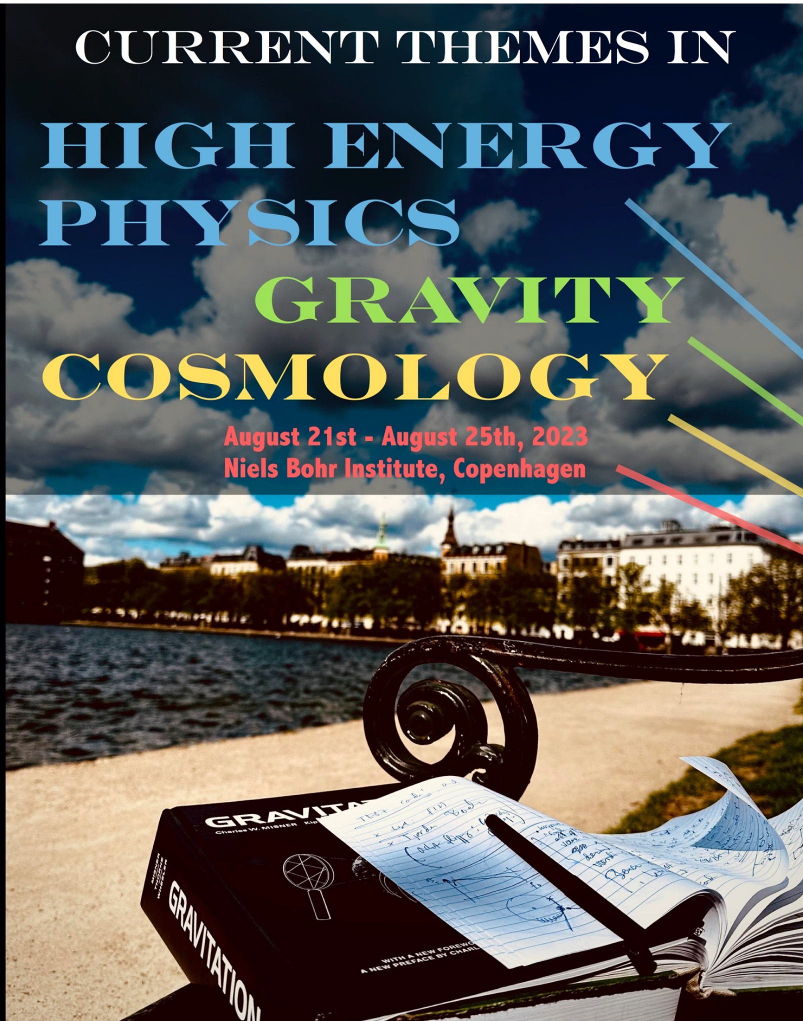
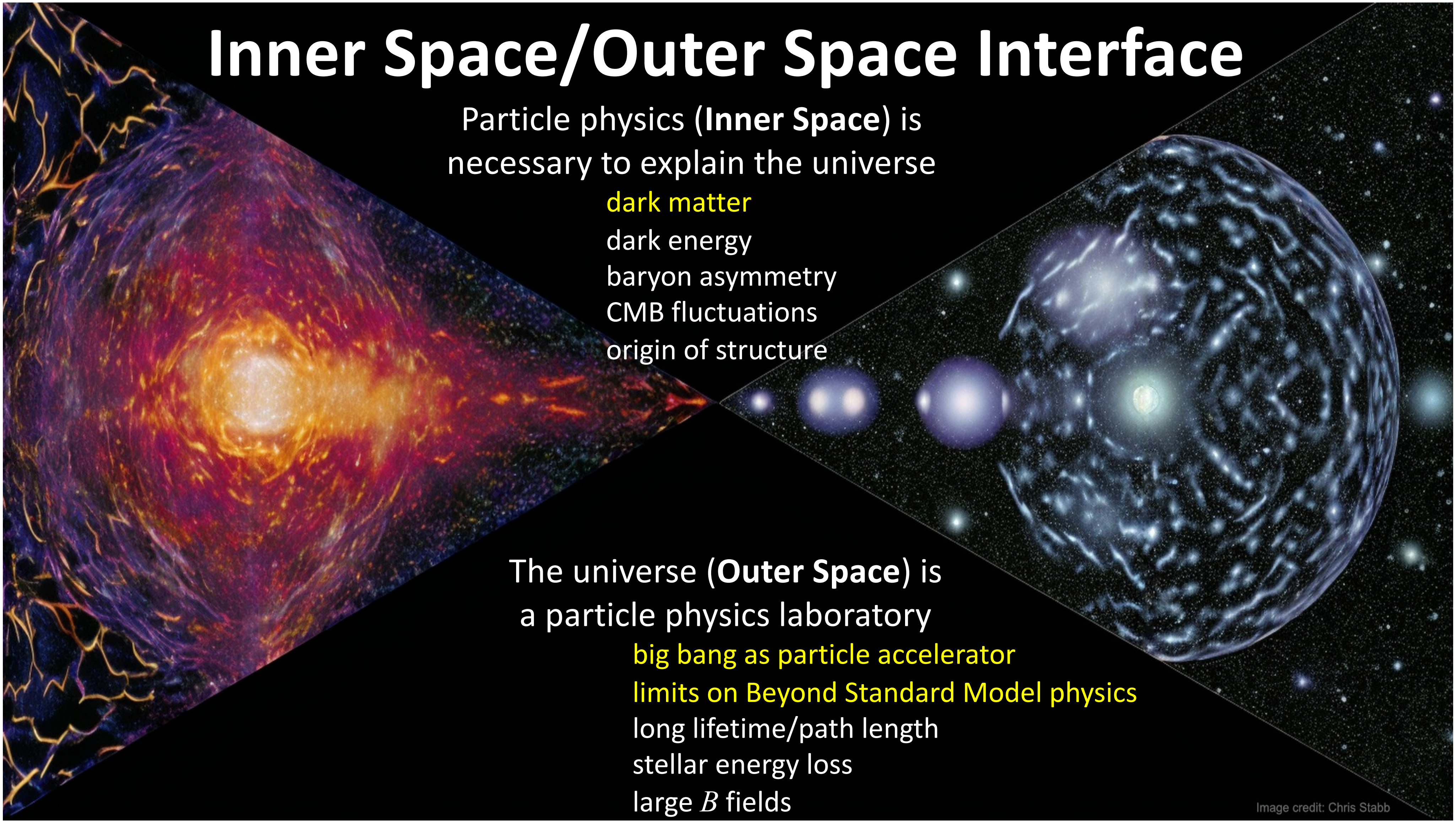


Cosmological Gravitational Particle Production (CGPP) involving High Energy Physics, Gravity, & Cosmology

Rocky Kolb
University of Chicago

August 2023

The Niels Bohr International Academy


Inner Space/Outer Space Interface
Particle physics (**Inner Space**) is
necessary to explain the universe
dark matter
dark energy
baryon asymmetry
CMB fluctuations
origin of structure
The universe (**Outer Space**) is
a particle physics laboratory
big bang as particle accelerator
limits on Beyond Standard Model physics
long lifetime/path length
stellar energy loss
large B fields
Image credit: Chris Stabb

Inner Space/Outer Space Interface

Assumption: particle of interest (e.g., dark matter) was a component of the primordial soup with present abundance determined by, e.g., freeze-out/freeze-in.

Requires: $\left\{ \begin{array}{l} \text{1. at some point } T > m \\ \text{2. particle has SM interactions} \end{array} \right.$

BUT

Maximum temperature of the radiation-dominated universe is the "reheat" temperature after inflation, $T_{\rm RH}$

$T_{\rm RH}$ may be as low as $8\ \mathrm{MeV}$ (to set stage for BBN)!

What about particles with no SM interactions (or) too weak to be populated in the primordial soup?

(No evidence that dark matter interacts with SM particles)

Image credit: Chris Stabb


The big question: origin of dark matter?
JWST image

For 40 Years, Leading DM Candidate: “Weak”-Scale Cold Thermal Relic

- Mass: GeV – TeV
- “Weak-scale” interaction strength with SM (WIMP miracle)
- No self-interactions
- Produced by “freeze-out” from primordial plasma. COLD dark matter. CDM.
- “Detectable” by direct detection, indirect detection, decay products, production at colliders
- Just BSM, e.g., low-energy SUSY!

The WIMP "Miracle"

Merriam-Webster

Merriam-Webster OnLine

mir·a·cle
\ˈmir-i-kəl \
noun

1 : an extraordinary event manifesting divine intervention in human affairs

Miracle

From Wikipedia, the free encyclopedia

WIKIPEDIA
The Free Encyclopedia

… often used to give an impression of great and unusual value in a trivial context …

For 40 Years, Leading DM Candidate: “Weak”-Scale Cold Thermal Relic

- Mass: GeV – TeV
- “Weak-scale” interaction strength with SM (WIMP miracle)
- No self-interactions
- Produced by “freeze-out” from primordial plasma. COLD dark matter. CDM.
- “Detectable” by direct detection, indirect detection, decay products, production at colliders
- Just BSM, e.g., low-energy SUSY!

But WIMPs have stubbornly evaded detection!

What if DM interacts only gravitationally with SM?

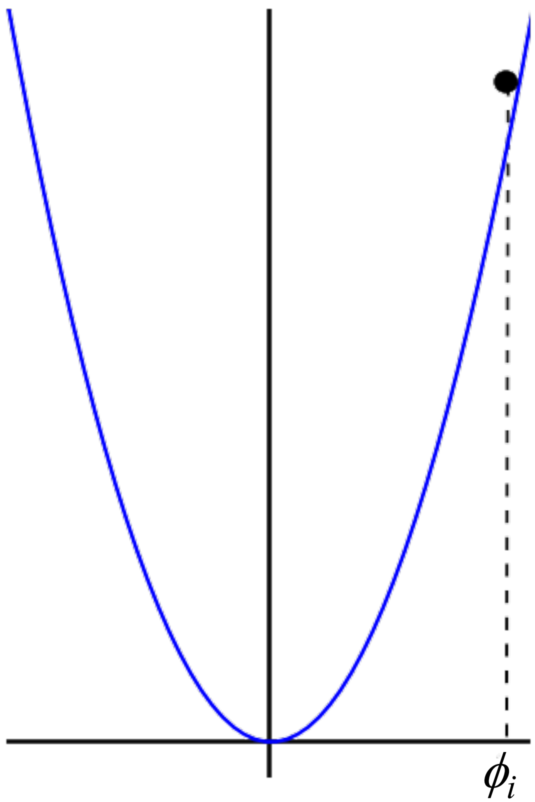
- Gravity must play a role in its cosmological production
- But gravity weak!

Cosmological Gravitational Particle Production (CGPP) can be the origin of DM!

- CGPP is not optional! Can’t hide from gravity.

Ideas for gravitational particle production

Produce particles through misalignment mechanism



- EOM of scalar field

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

- Scalar field has quantum fluctuations during inflation

$$\Delta\phi = \frac{H}{2\pi}$$

- After inflation field frozen by "Hubble drag" until

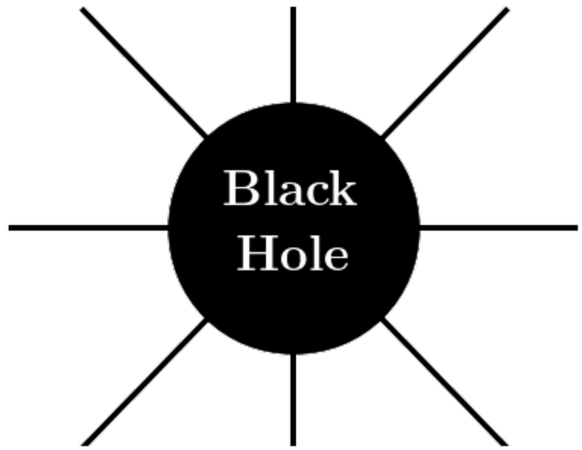
$$H \simeq m_\phi$$

- After which it oscillates with energy density in oscillating field

- E.g., axion

Ideas for gravitational particle production

Produce particles via Hawking radiation from primordial black holes
(Hooper, Krnjaic, & McDermott)



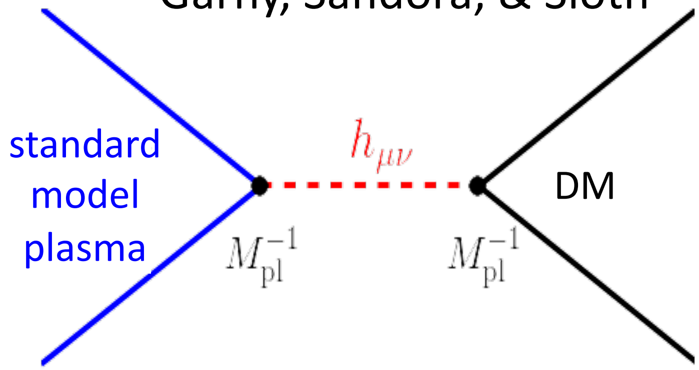
$$\frac{\Omega h^2}{0.12} \approx \left(\frac{10^{11}\mathrm{GeV}}{m}\right)\left(\frac{10^{12}\mathrm{GeV}}{T_i}\right)^3\left(\frac{\epsilon_{\mathrm{BH}}}{10^{-16}}\right)$$

- PBHs of current interest (after first LIGO event)
- Seeds for PBHs from inflation
- Assumes DM mass about $10^{11}\,\mathrm{GeV}$ (WIMPzilla)

Ideas for gravitational particle production

$$\mathcal{L} = M_{\rm Pl}^{-1} h_{\mu\nu} T^{\mu\nu}$$

Produce particles from SM plasma via
graviton exchange
Garny, Sandora, & Sloth



$$\frac{\Omega h^2}{0.12} \approx \left(\frac{\langle \sigma v \rangle}{T^2/M_{\rm Pl}^4}\right)\left(\frac{m}{10^{13}{\rm GeV}}\right)\left(\frac{T_{\rm RH}}{10^{14}{\rm GeV}}\right)^3$$

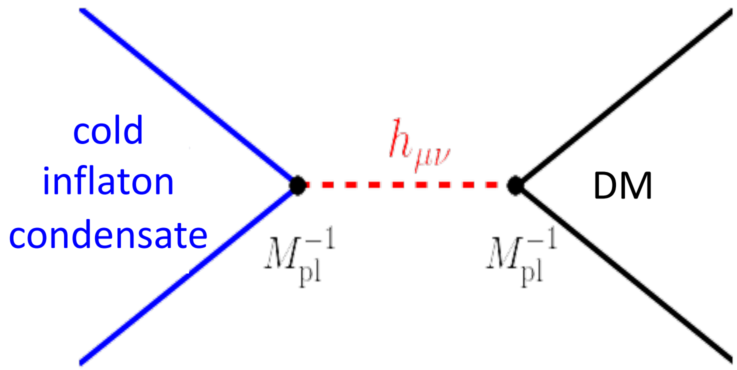
- Freeze-in
- For DM mass about 10^{13} GeV (WIMPzilla)
- Assumes $m < T_{\rm RH}$

Ideas for gravitational particle production

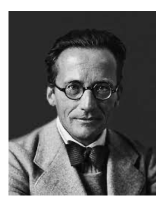
$$\mathcal{L} = M_{\rm Pl}^{-1} h_{\mu\nu} T^{\mu\nu}$$

Produce particles from inflaton field after quasi-de Sitter era via graviton exchange

Ema, Nakayama, Tang; Mambrini & Olive



- Only works for DM mass < inflaton mass
- DM mass for correct Ωh^2 involved function of several parameters
- "Boltzmann" approach not complete treatment (Kaneta, Lee, Oda; Basso, Chung, EWK, Long)

Boltzmann  $\subset$  Schrödinger $+$  Bolgolubov

Particle creation through expansion of the universe

Physica VI, no 9 October 1939

THE PROPER VIBRATIONS OF THE EXPANDING UNIVERSE

by ERWIN SCHRÖDINGER

§ 1. *Introduction and summary.* Wave mechanics imposes an a priori reason for assuming space to be closed; for then and only then are its proper modes discontinuous and provide an adequate description of the observed atomicity of matter and light. — Einsteins theory of gravitation imposes an a priori reason for assuming space to be, if closed, expanding or contracting; for this theory does not admit of a stable static solution. — The observed facts are, to say the least, not contrary to these assumptions.

This makes it imperative to generalize to expanding (or contracting) universes the investigation of proper vibrations, started for the the static cases (Einstein- and De Sitter-universe) by the present writer and two of his collaborators [1]). The task is an easy one. The broad results are largely (in part even entirely) independent of the time-law of expansion. In the cases of main practical interest, i.e. with the present slow time rate of expansion and with wave lengths small compared with the radius of curvature of space (R), they are the following.

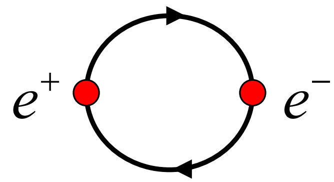
These are the broad results. A finer and particularly interesting phenomenon is the following.

The decomposition of an arbitrary wave function into proper vibrations is rigorous, as far as the functions of space (amplitude-functions) are concerned, which, by the way, are exactly the same as in the static universe. But it is known, that, with the latter, two frequencies, equal but of opposite sign, belong to every space function. *These two* proper vibrations cannot be rigorously separated in the expanding universe. That means to say, that if in a certain moment only one of them is present, the other one can turn up in the course of time.

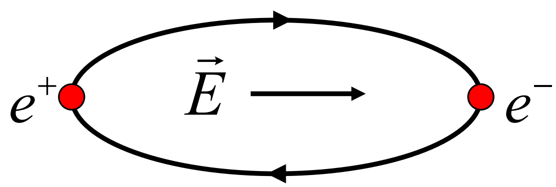
Generally speaking this is a phenomenon of outstanding importance. With particles it would mean production or anihilation of matter, merely by the expansion, whereas with light there would be a production of light travelling in the opposite direction, thus a sort of reflexion of light in homogeneous space. Alarmed by these prospects, I have investigated the question in more detail. Fortunately the equations admit of a solution by familiar functions, if R is a *linear* function of time. It turns out, that in this case the alarming phenomena do not occur, even within arbitrarily long periods of time.

Disturbing the Quantum Vacuum

Electric Field $\longrightarrow$ Particle creation

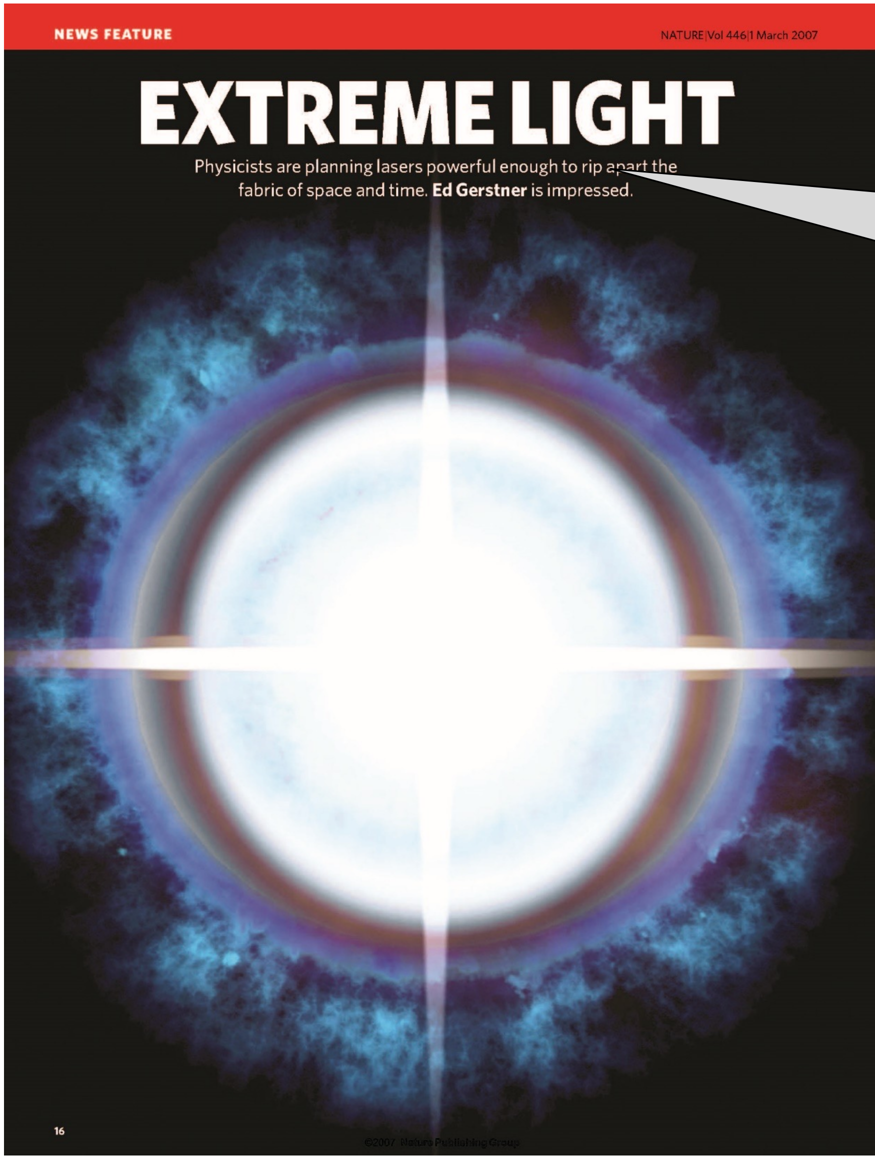


Particle creation if energy gained in acceleration from E-field over a Compton wavelength exceeds the particle's rest mass.



$$\left|\vec{E}_{\text{crit}}\right| = \frac{m_e^2 c^3}{e\hbar} \approx 10^{16}\ \text{V cm}^{-1}$$

Sauter (1931); Heisenberg & Euler (1935); Weisskopf (1936); Schwinger (1951)



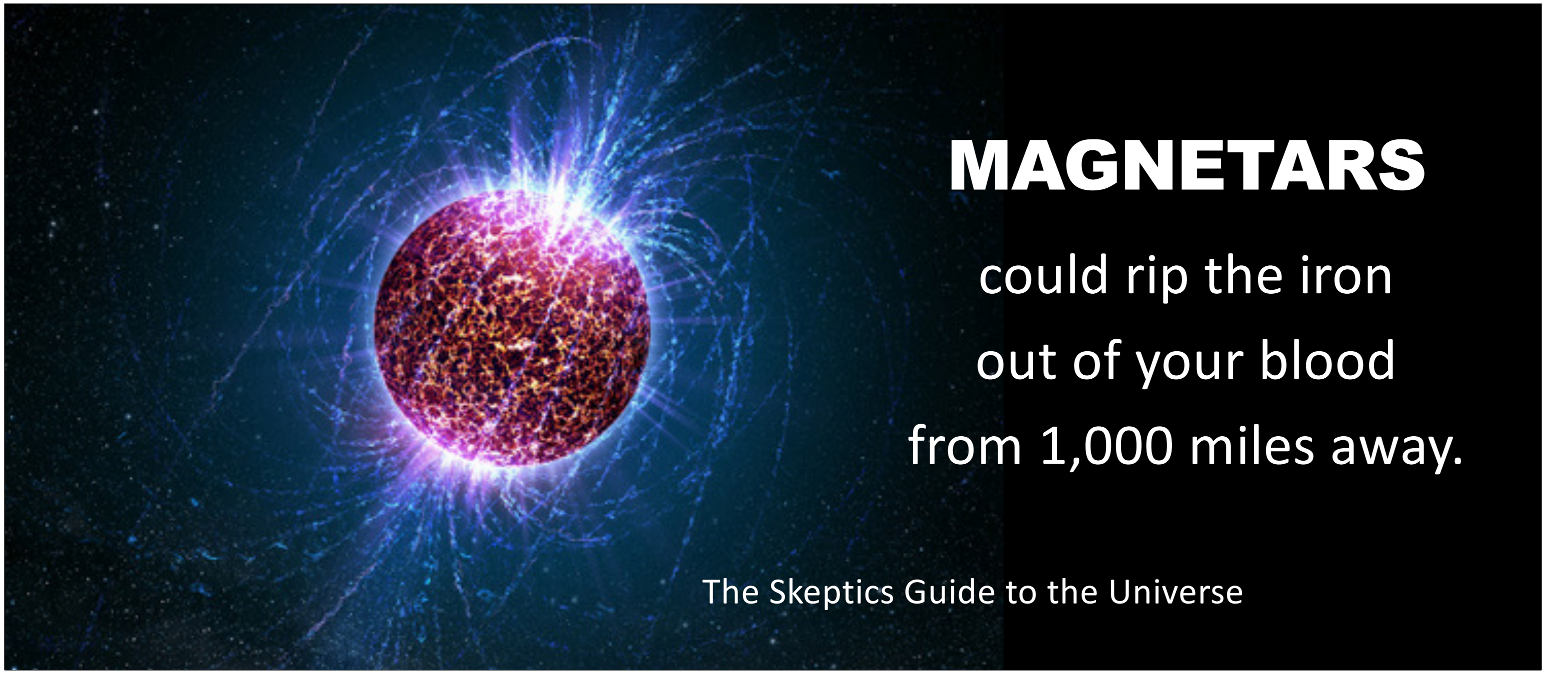
NATURE, Vol 446/1 March 2007

Physicists are planning lasers powerful enough to rip apart the fabric of space and time.

"We're going to change the index of refraction of the vacuum and produce new particles."

Gérard Mourou

$$I_C \approx \frac{c}{8\pi}\left|\vec{E}_{\mathrm{crit}}\right|^2 \approx 10^{30}\ \mathrm{W\ cm^{-2}}$$



$$\left|\vec{B}_{\rm crit}\right| = \frac{m_e^2}{e} \approx 5 \times 10^{13}\,\mathrm{G}$$

Crab pulsar	3×10^{13} G
Magnetars	$10^{14} - 10^{15}$ G

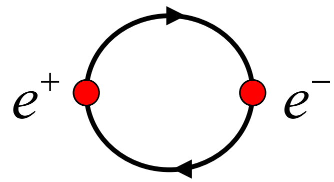
Strong magnetic fields imply existence of strong electric fields.

Many unexplained phenomena associated with pulsars, magnetars, etc.

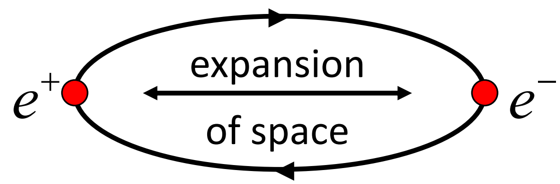
Damour & Ruffini

Disturbing the Quantum Vacuum

Expanding universe $\longrightarrow$ Particle creation



Particle creation if energy gained in acceleration from expansion over a Compton wavelength exceeds the particle's rest mass.



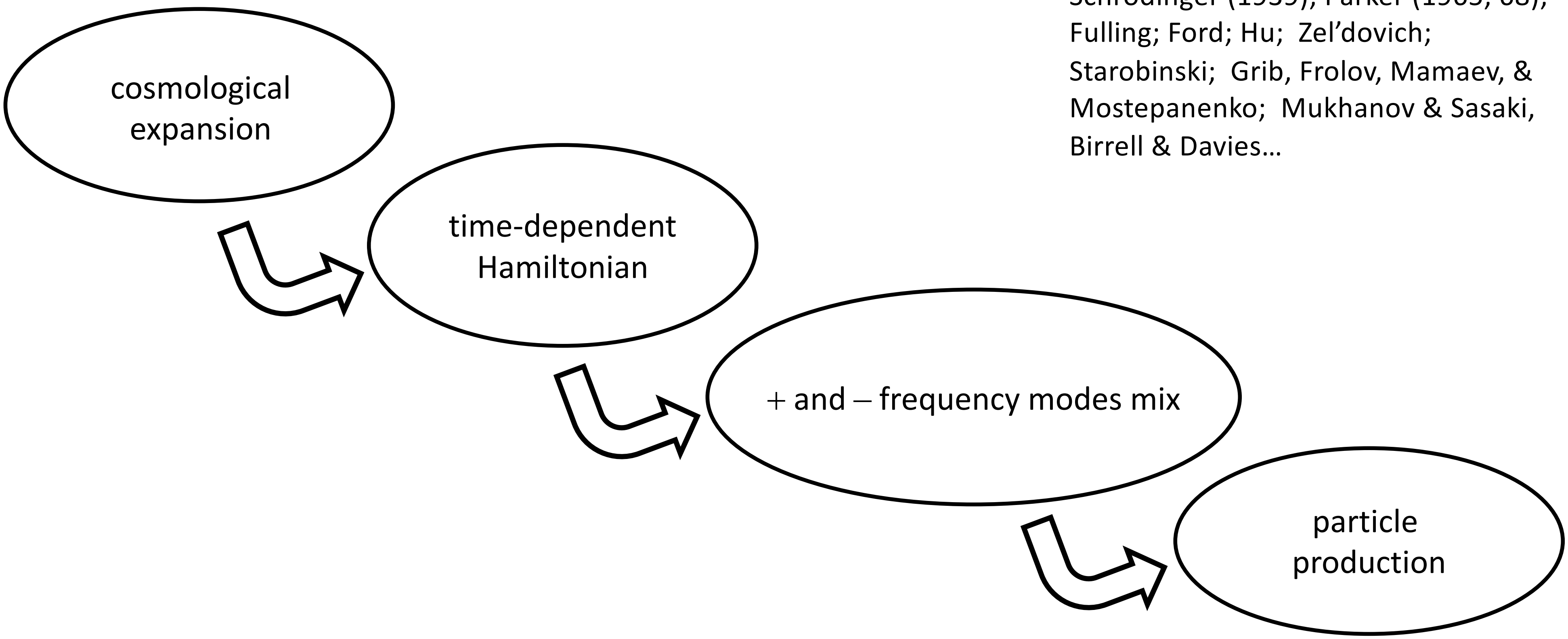
$\mathrm{v} = c$ at Hubble radius

$$H_{\text{crit}} = m$$

CGPP Through Expansion of the Universe

- In Minkowskian QFT, a particle is an IR of the Poincaré group.
- But, expanding universe not Poincaré invariant.
- Notion of a “particle” is approximate.

In the early days:
Schrodinger (1939); Parker (1965, 68); Fulling; Ford; Hu; Zel’dovich; Starobinski; Grib, Frolov, Mamaev, & Mostepanenko; Mukhanov & Sasaki, Birrell & Davies…



Representation	**Particle**	**1-point function Dark Matter**	**2-point function CMB Isocurvature**	**3-point function CMB Nongaussian**
(0,0)	Conformally Coupled Scalar $\xi = 1/6$ (use as template)	Kuzmin & Tkachev (99)	Expected to be very small (blue)	Chung & Yoo (13)
(0,0)	Minimally Coupled Scalar $\xi = 0$ (e.g., inflaton)	Kuzmin & Tkachev (99)	Chung, EWK, Riotto, & Senatore (05)	
(1/2,0) $\oplus$ (0,1/2)	"Dirac" Fermion	Chung, EWK, & Riotto (98)	Expected to be very small (blue)	
(1/2,1/2)	de Broglie-Proca Vector	Graham & Mardon (16); Ahmed, Grzadkowski,& Socha (20); EWK & Long (21)		
(1,0) $\oplus$ (0,1)	2-Form (Pseudo) Vector (e.g., Kalb-Ramond)	Capanelli, Jenks, EWK, & McDonough (next week)		
(1/2,1) $\oplus$ (1,1/2)	Rarita-Schwinger Fermion (e.g., gravitino)	EWK, Long, & McDonough (21)		
(1,1)	Fierz-Pauli (massive graviton)	EWK, Liang, Long, Rosen (23)		
Higher-spin bosons		Jenks, Koutrolikos, McDonough, Alexander, Gates (23)		

Scalar field in FLRW background

Covariant action for spectator scalar field (not the inflaton)

$$S[\varphi(x), g_{\mu\nu}(x)] = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2 + \frac{1}{2}\xi R\varphi^2\right]$$

Gravity enters the picture

ξ is a dimensionless constant: $\xi = 0$ minimal coupling; $\xi = 1/6$ conformal coupling.

In principle, ξ could be anything (and presumably there is RGE).

In spatially-flat FLRW background in conformal time: $dt = a\, d\eta$; rescaled field $\phi = a\, \varphi$

$$S[\phi(\eta, \boldsymbol{x})] = \int_{-\infty}^{\infty} d\eta \int d^3\mathbf{x}\left[\frac{1}{2}(\partial_\eta\phi)^2 - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}m_{\text{eff}}^2\phi^2\right]$$

Time-dependent effective mass

$$m_{\text{eff}}^2(\eta) = a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right)R(\eta)\right]$$

cosmological expansion ⇒
time-dependent background field ⇒
time-dependent Hamiltonian for spectator field

CGPP Through Expansion of the Universe

Expansion of the universe causes explicit time dependence in action for "spectator" fields.

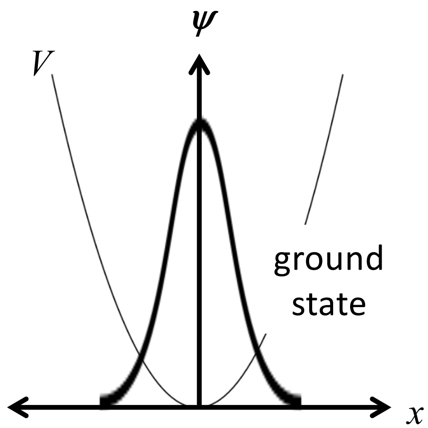
Initial State ~ Minkowski (early-time) vacuum may not evolve to

Final State ~ Minkowski (late-time) vacuum, but to an excited state populated by particles.

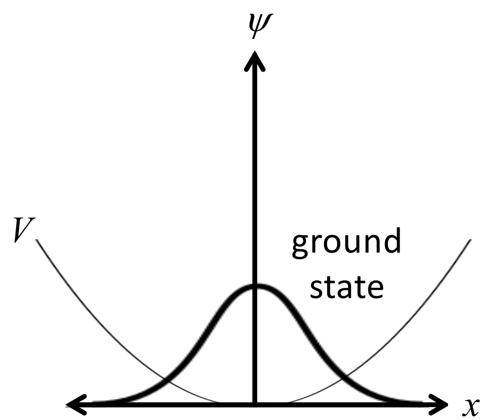
Think of a harmonic oscillator

$$\ddot{x}(t) + \omega^2(t)\ x(t) = 0$$

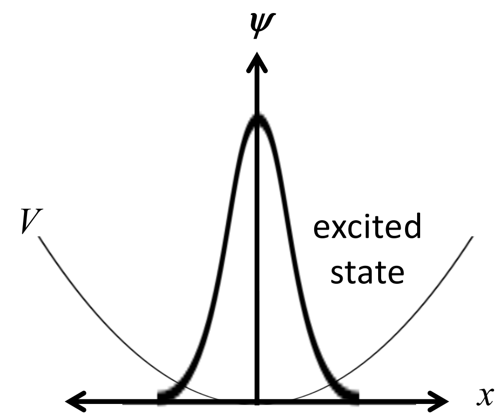
Initial State



Spring constant varied slowly (adiabatically)



Spring constant varied abruptly (nonadiabatically)



Scalar field in FLRW background

Fourier modes of ϕ obey wave equation: $\partial_\eta^2 \chi_k(\eta) + \omega_k^2 \chi_k(\eta) = 0$

Solutions to wave equation for mode functions include both + and – frequency terms

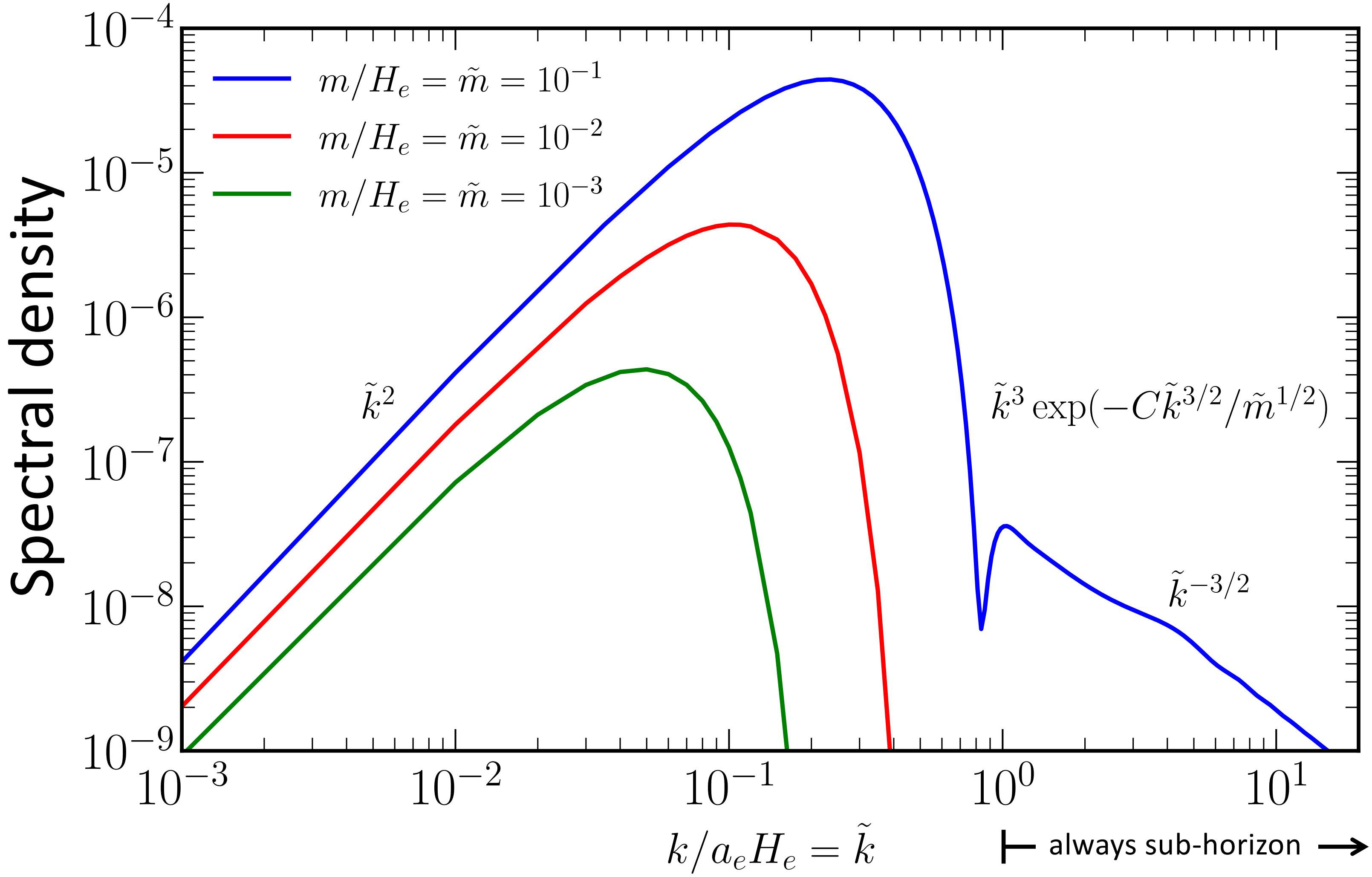
$$\chi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{-i\int \omega_k(\eta) d\eta} - \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{+i\int \omega_k(\eta) d\eta} \qquad |\alpha_k|^2 - |\beta_k|^2 = 1$$

If start with only positive frequency modes, $|\alpha_k| = 1$ & $|\beta_k| = 0$,
Expansion of the universe will generate negative frequency modes (particles), $\beta_k \neq 0$.

<u>Comoving</u> number density of particles at late time is

$$a^3 n = \int \frac{dk}{k} \underbrace{\frac{k^3}{2\pi^2}\, |\beta_k|^2}_{n_k = \text{ spectral density}}$$

Quadratic Inflaton Potential for Conformally-Coupled Scalar: $\xi = 1/6$

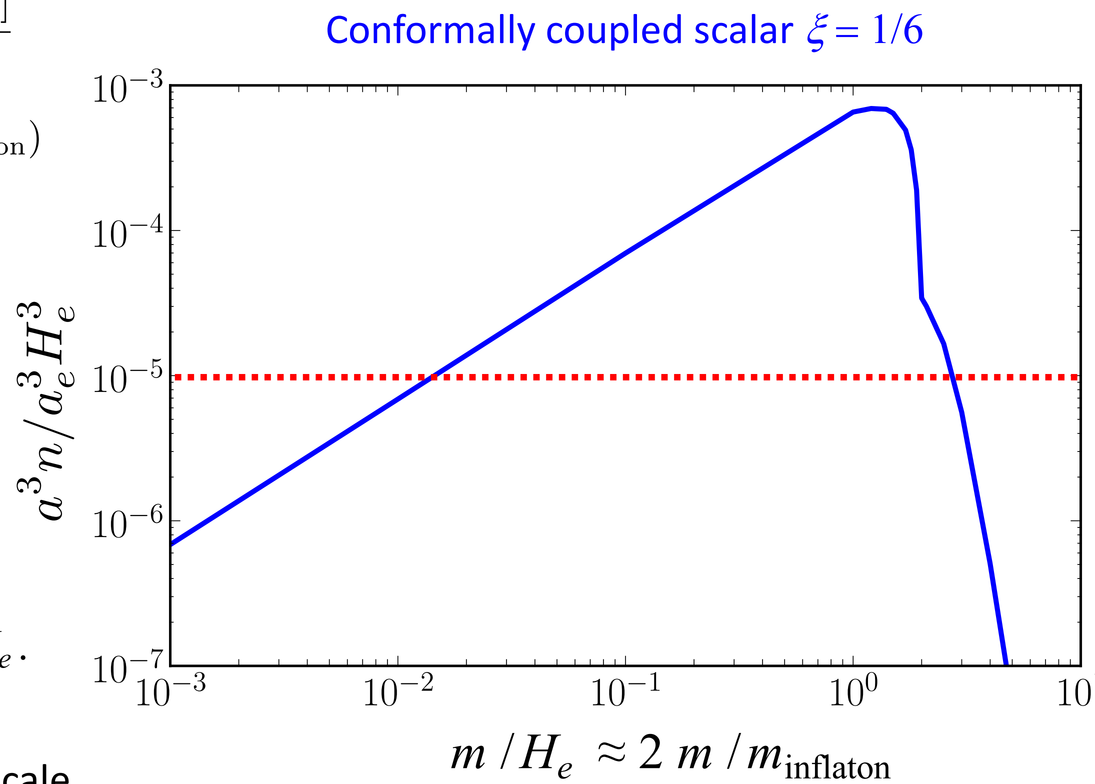


Quadratic Inflaton Potential for Conformally-Coupled Scalar: $\xi = 1/6$

$$\frac{\Omega h^2}{0.12} = \frac{m}{H_e}\left(\frac{H_e}{10^{12}\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\mathrm{GeV}}\right)\ \frac{[na^3/\,a_e^3H_e^3]}{10^{-5}}$$

$$\sim \left(\frac{m}{10^{11}\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\mathrm{GeV}}\right) \qquad (m \lesssim m_{\mathrm{inflaton}})$$

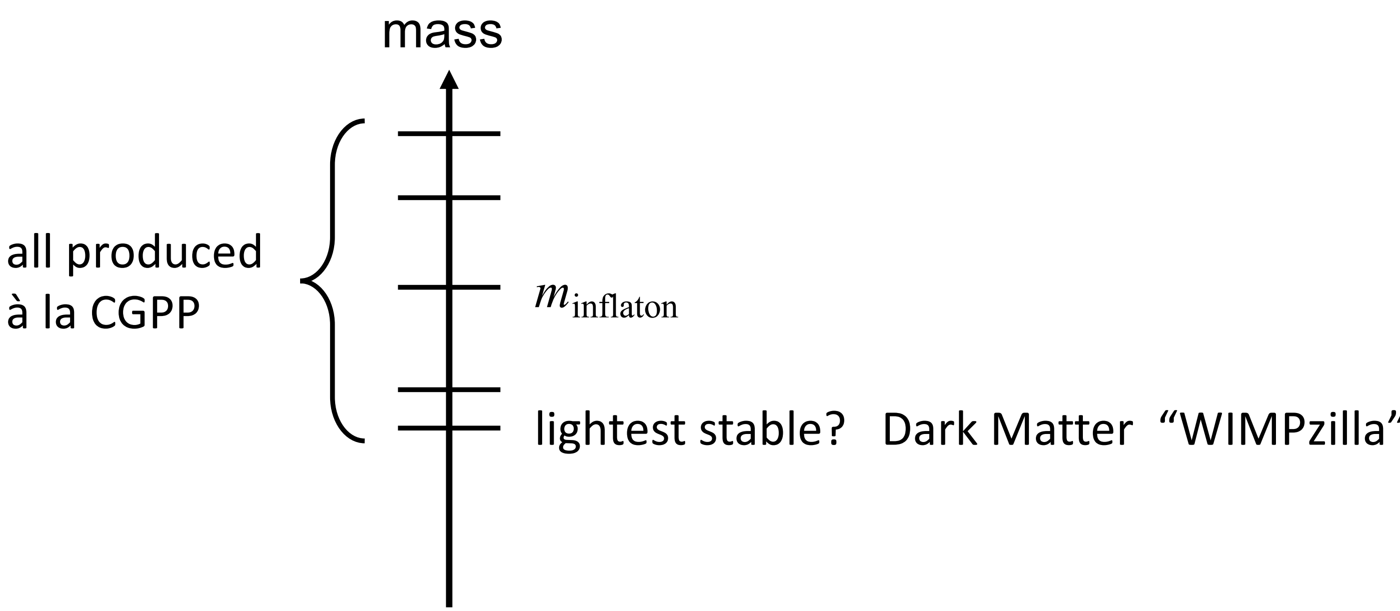
- Calculation assumes inflationary model (quadratic, which is ruled out).
- But general picture holds in other models since action occurs around end of inflation.
- Don't know, but $H_e \approx 10^{11}$ GeV and $T_{\mathrm{RH}} \approx 10^9$ GeV are "common."
- If stable and dark matter, $\Omega h^2 = 0.12 \implies m \approx H_e$.
  Could have been anything! WIMPZILLA miracle!
- Perhaps inflation scale represents new physics scale, stable particle at that mass scale natural DM candidate.



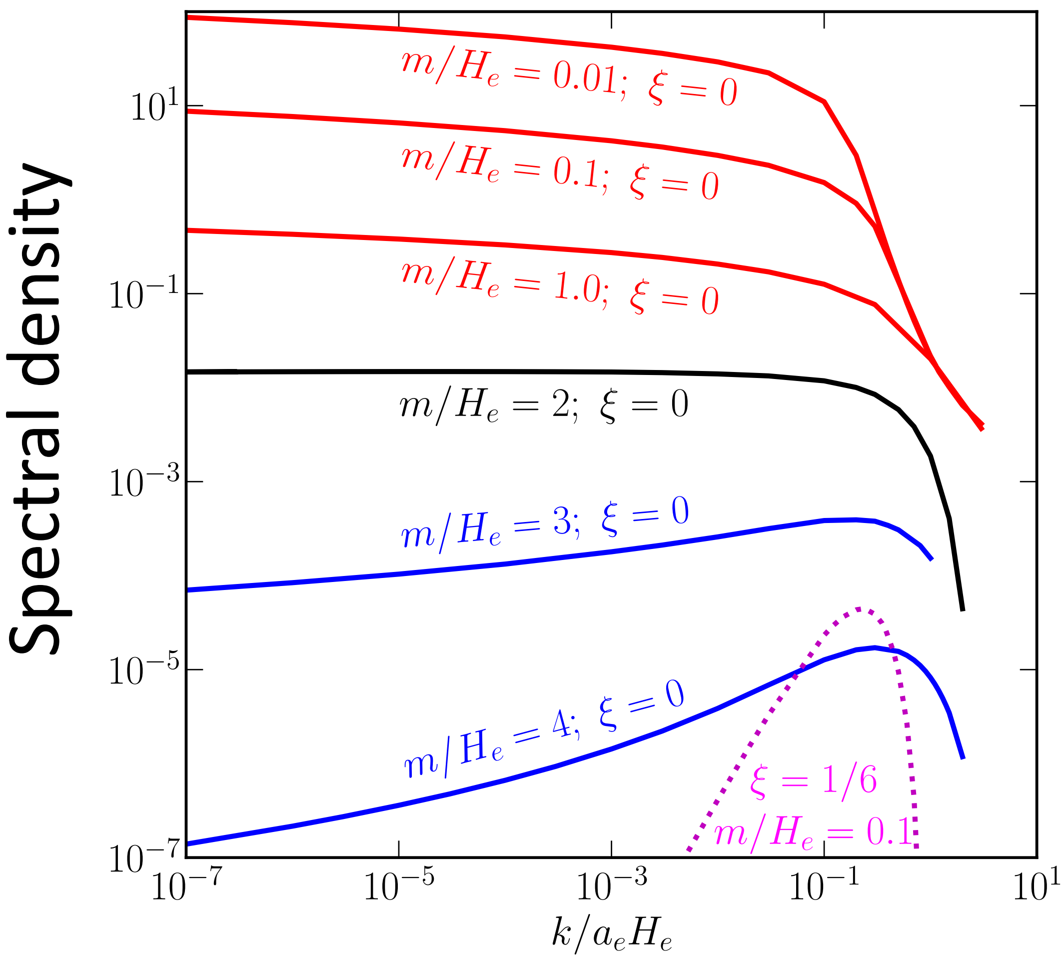
Conformally-coupled scalar WIMPZILLA DM candidate
if $m_\chi = \mathcal{O}(m_{\mathrm{inflaton}})$

GPP & Dark Matter

- Inflation indicates a new mass scale
- In most models, $m_{\text{inflaton}} \approx H_{\text{inflation}} \approx 10^{12} - 10^{14}$ GeV?
- $H_{\text{inflation}}$ detectable via primordial gravitational waves in CMB
- (I, at least) expect other particles with mass $\approx m_{\text{inflaton}}$



Quadratic Inflaton Potential for Minimally-Coupled Scalar: $\xi = 0$



Red Spectrum leads to dangerous isocurvature fluctuations

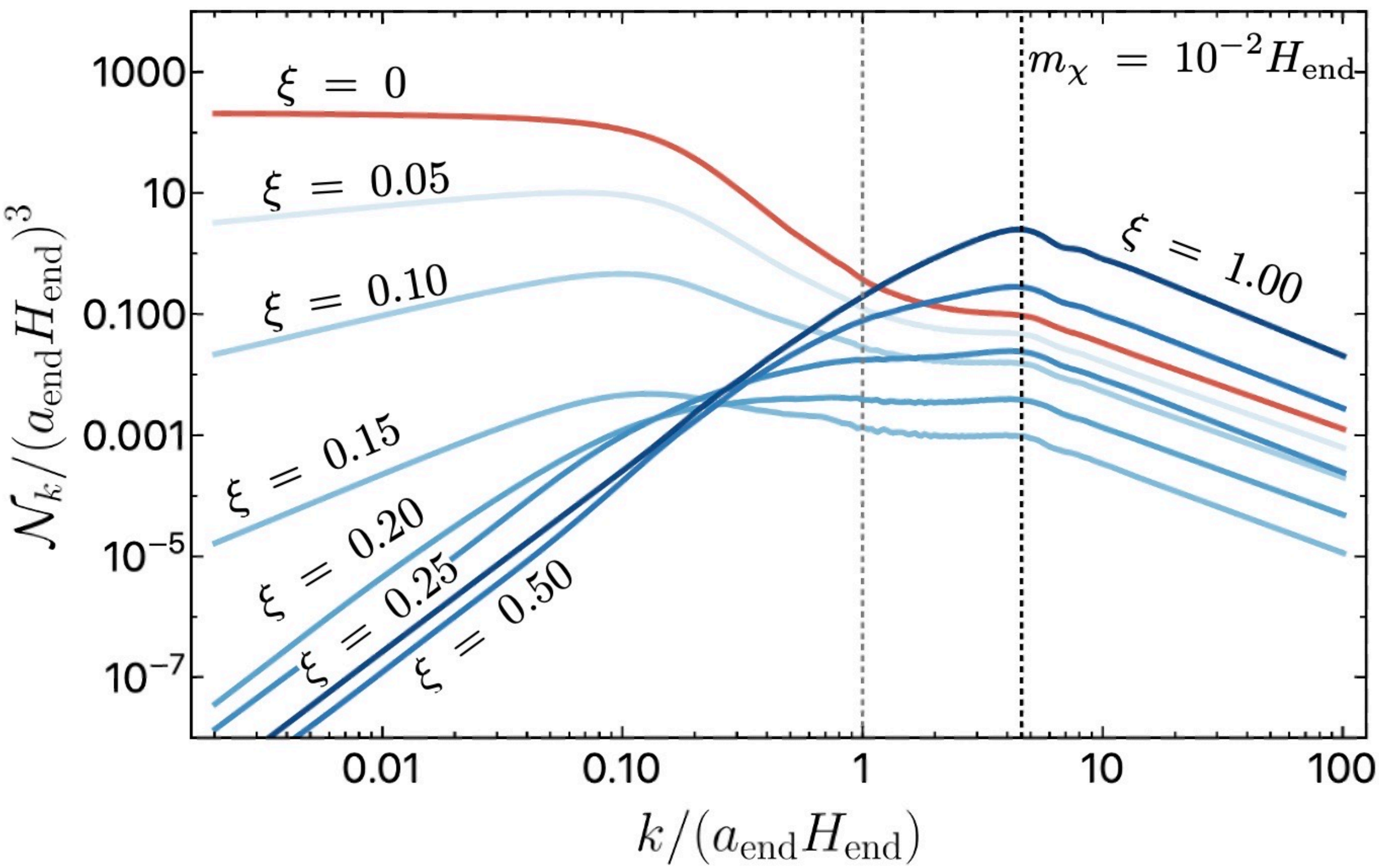
CMB limits

Chung, EWK, Riotto, Senatore (05)

Stable, minimally-coupled scalars are disallowed if $m \lesssim$ few H_e

Model-T inflation model (Kallosh & Linde): $V(\varphi) = 10^{-10} M_{\rm Pl}^4 \tanh^2(\varphi/\sqrt{6}M_{\rm Pl})$

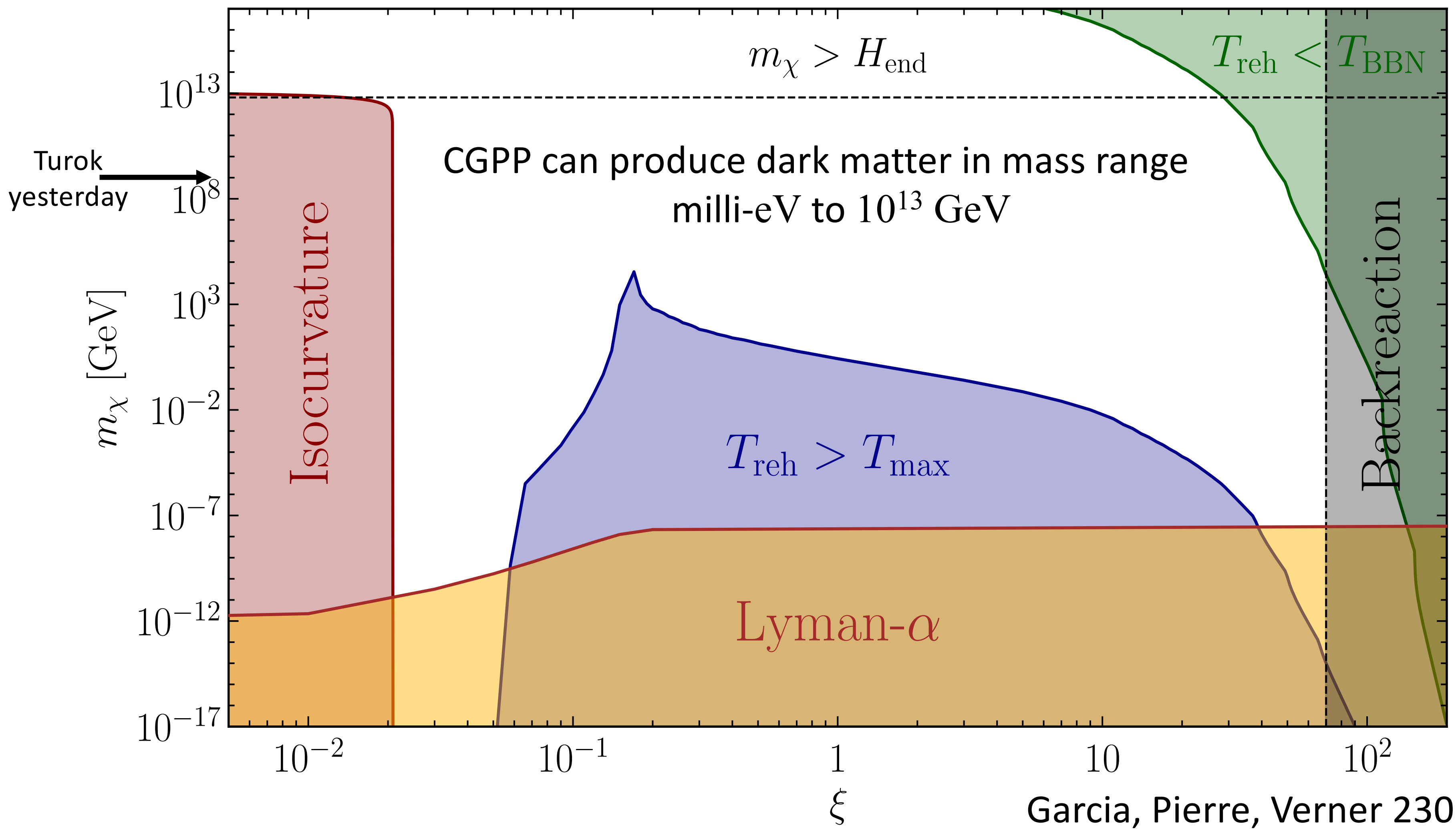
$$m_{\rm eff}^2 = m^2 + \frac{1}{6}(1-6\xi)R$$



Garcia, Pierre, Verner 2305.14446

also EWK, Long, McDonough, Payeur 2211.14323

Model-T inflation model (Kallosh & Linde): $V(\varphi) = 10^{-10} M_{\rm Pl}^4 \tanh^2(\varphi/\sqrt{6} M_{\rm Pl})$



Garcia, Pierre, Verner 2305.14446

Dirac field ψ in FRW background

$$\frac{\Omega h^2}{0.12} = \frac{m}{H_e}\left(\frac{H_e}{10^{12}\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\mathrm{GeV}}\right)\ \frac{\left[na^3/\,a_e^3H_e^3\right]}{10^{-5}}$$

$$\sim\left(\frac{m}{10^{11}\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\mathrm{GeV}}\right)\qquad (m\lesssim m_{\mathrm{inflaton}})$$

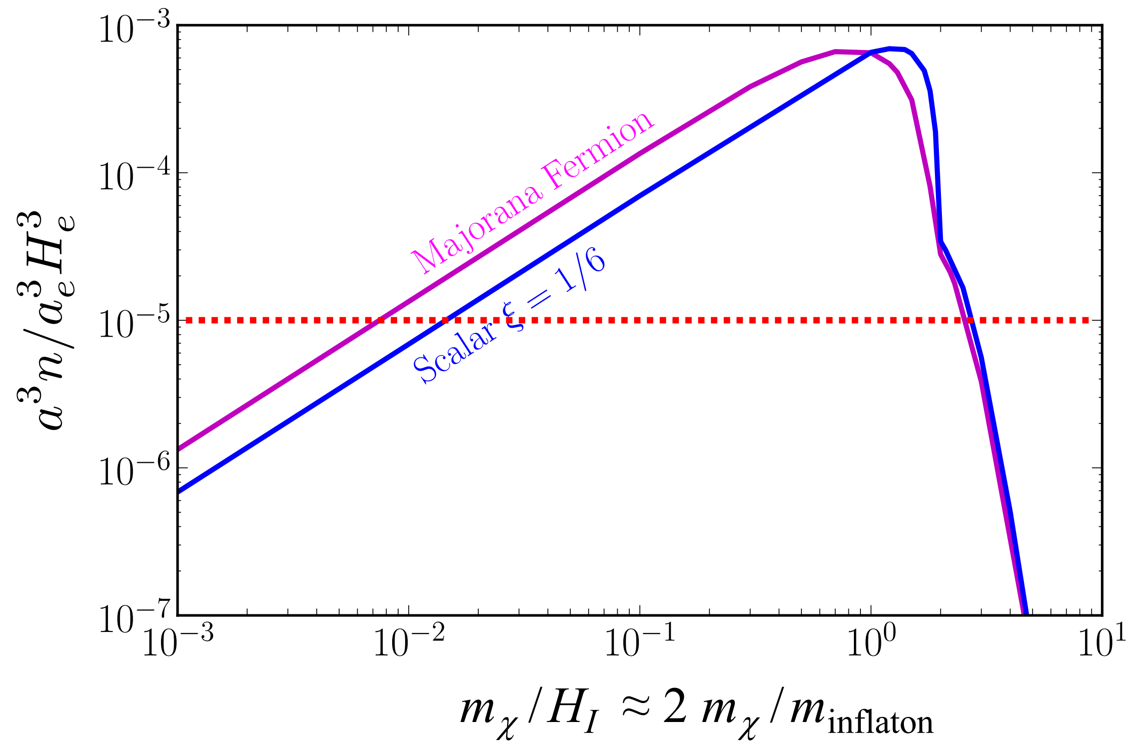
Dirac Equation in FRW:

$$i\partial_\eta\begin{pmatrix}u_A(\eta)\\ u_B(\eta)\end{pmatrix} = \begin{pmatrix}a(\eta)m & k\\ k & -a(\eta)m\end{pmatrix}\begin{pmatrix}u_A(\eta)\\ u_B(\eta)\end{pmatrix}$$

Dispersion relation same as conformally-coupled scalar

Blue spectrum: no isocurvature issues

Dirac WIMPZILLA DM candidate for $m = \mathcal{O}(m_{\mathrm{inflaton}})$



Fields with Spin $>$ 1/2

For bosons, $\omega_k(\eta)$ tells all:

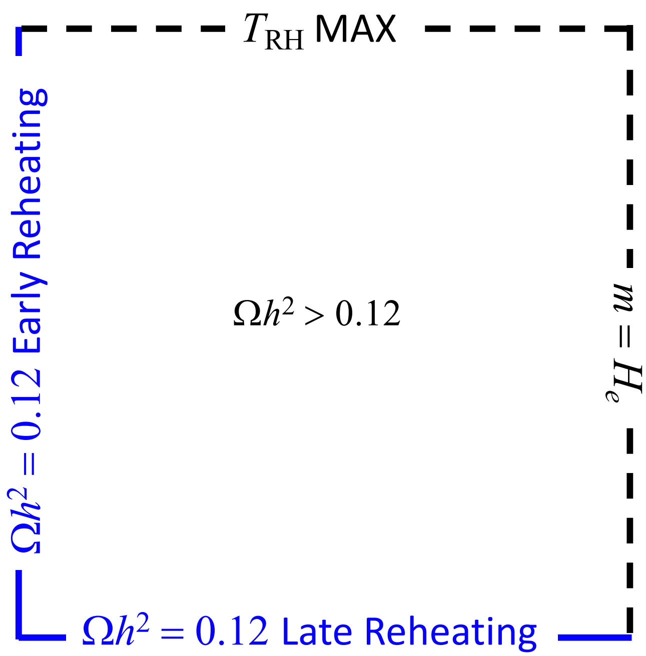
$$\omega_k^2(\eta) = \begin{cases} k^2 + a^2(\eta)m^2 + (\frac{1}{6} - \xi)a^2(\eta)R(\eta) & s=0 \\ k^2 + a^2(\eta)m^2 \quad \text{Like conformally-coupled scalar: in massless limit no production} & s=1 \quad \lambda = \pm 1 \\ k^2 + a^2(\eta)m^2 + \dfrac{1}{6}\dfrac{k^2 a^2(\eta)R(\eta)}{k^2 + a^2(\eta)m^2} + 3\dfrac{k^2 a^4(\eta)H^2(\eta)m^2}{(k^2 + a^2(\eta)m^2)^2} \quad \text{Interesting (i.e., complicated)} & s=1 \quad \lambda = 0 \\ k^2 + a^2(\eta)m^2 + \frac{1}{6}a^2(\eta)R(\eta) \quad \text{Like minimally-coupled scalar; graviton in massless limit} & s=2 \quad \lambda = \pm 2 \\ k^2 + a^2(\eta)m^2 + \frac{1}{6}\dfrac{a^2(\eta)(2k^2 + a^2(\eta)m^2)R(\eta)}{k^2 + a^2(\eta)m^2} - \dfrac{a^2(\eta)k^2(2k^2 - a^2(\eta)m^2)H^2(\eta)}{(k^2 + a^2(\eta)m^2)^2} & s=2 \quad \lambda = \pm 1 \\ \text{way, way too long to show} & s=2 \quad \lambda = 0 \end{cases}$$

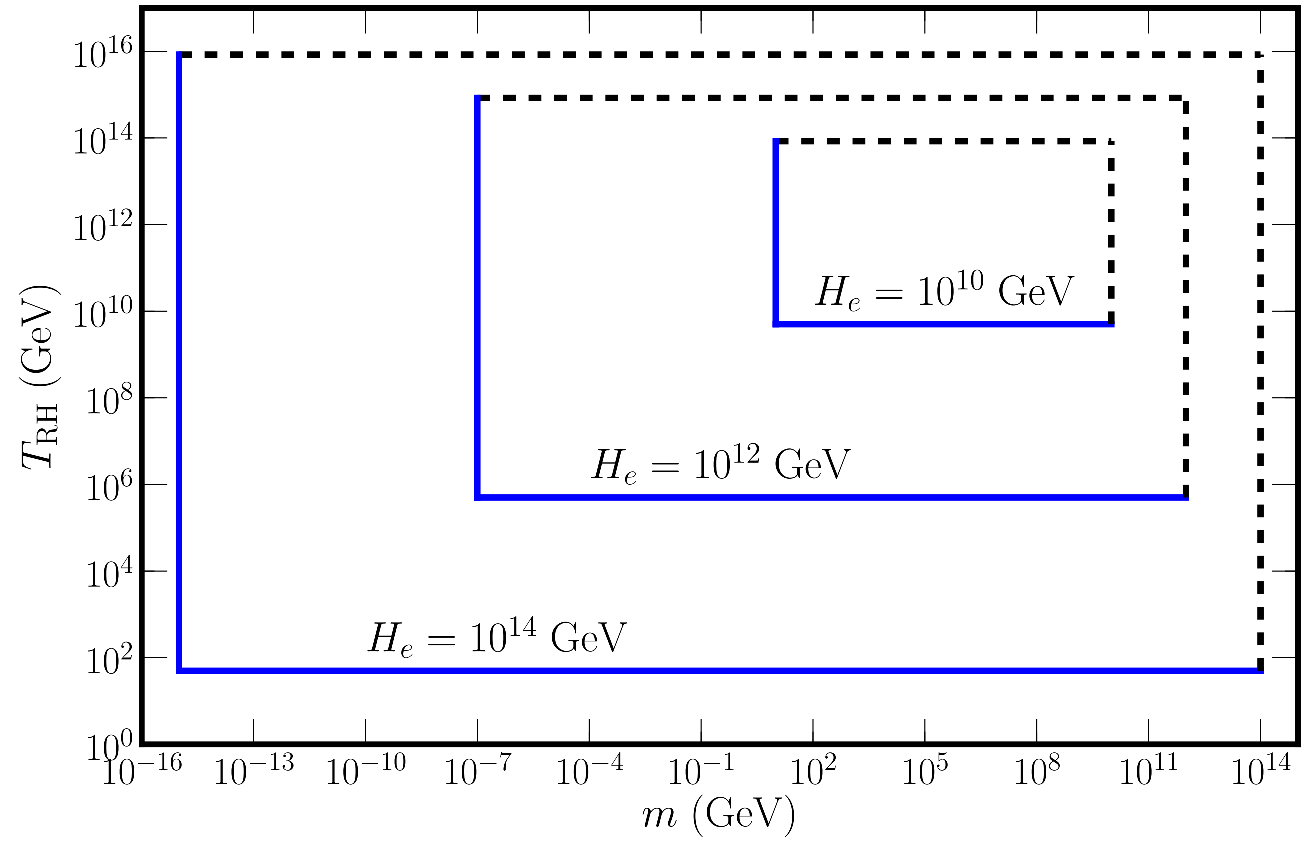
de Broglie—Proca field in FLRW background

$$S[A_\mu(x), g_{\mu\nu}(x)] = \int d^4x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} + \frac{1}{2} m^2 g^{\mu\nu} A_\mu A_\nu - \frac{1}{2}\xi_1 R g^{\mu\nu} A_\mu A_\nu - \frac{1}{2}\xi_2 R^{\mu\nu} A_\mu A_\nu \right]$$

- Two possible nonminimal terms
- Transverse mode looks like conformally-coupled scalar
- Longitudinal mode more complicated
- For some choices of (ξ_1, ξ_2) kinetic term can be negative leading to ghost-like action
- CGPP of longitudinal mode dominates transverse mode

de Broglie—Proca field in FLRW background





Early/Late:

$$T_{\mathrm{RH}} = 8.4 \times 10^{8} \left(\frac{m}{\mathrm{GeV}}\right)^{1/2} \mathrm{GeV}$$

Very light (μeV) DM from GPP

or

Very massive (10^{14} GeV) DM from GPP

Graham, Mardon, & Rajendran

Ahmed, Grzadkowski, & Socha

EWK & Long

Rarita—Schwinger field ψ_μ in FRW background

EWK, Long, McDonough PRD **104**, 075015 (2021);
PRL **127** 13, 131603 (2021)

"Dirac" Equation in FRW:

$$i\partial_\eta \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix} = \begin{pmatrix} a(\eta)m & k \\ k & -a(\eta)m \end{pmatrix} \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix}$$

$s = 3/2$; $\lambda = \pm\, 3/2$ (same as $s = 1/2$)

$$i\partial_\eta \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix} = \begin{pmatrix} a(\eta)m & (C_A + iC_B)k \\ (C_A - iC_B)k & -a(\eta)m \end{pmatrix} \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix}$$

nonzero for gravitino

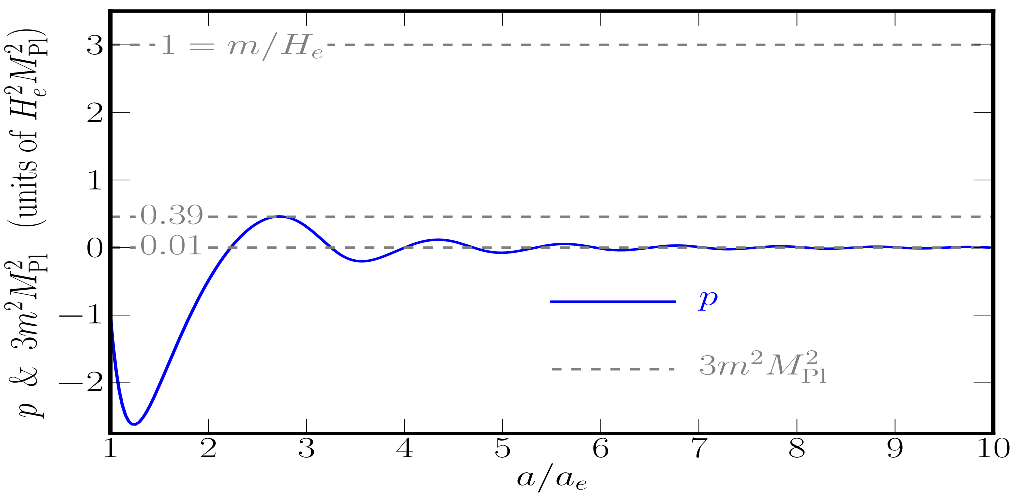
$s = 3/2$; $\lambda = \pm\, 1/2$
C_A & C_B functions of (H, m, R, $\partial_\eta m$)
$C_A{}^2 + C_B{}^2 = c_s{}^2 =$ sound speed

New feature: $c_s = \dfrac{\left|p(\eta) - 3m^2 M_{\rm Pl}^2\right|}{\rho(\eta) + 3m^2 M_{\rm Pl}^2}$ time-dependent effective sound speed!

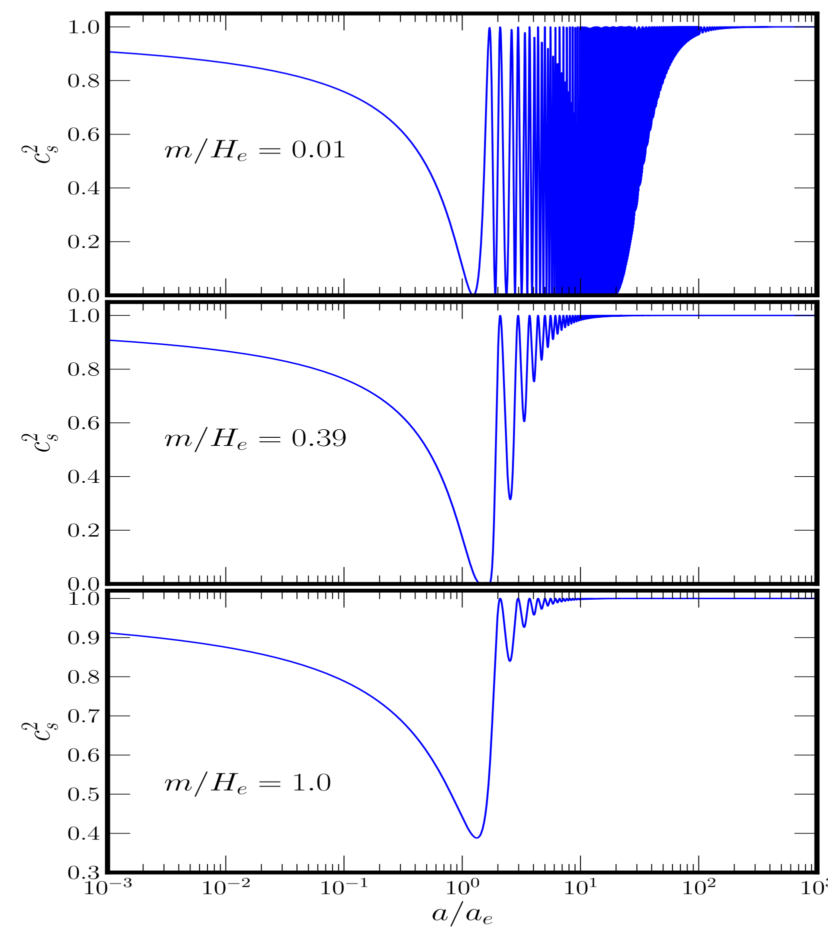
Can vanish when $p = 3m^2 M_{\rm Pl}^2$!!

Rarita—Schwinger field ψ_μ in FRW background

$$c_s = \frac{\left|p(\eta) - 3m^2 M_{\rm Pl}^2\right|}{\rho(\eta) + 3m^2 M_{\rm Pl}^2}$$



vanishing sound speed



Sound speed will vanish (perhaps many times) if $m < 0.39\ H_e$
(assumes harmonic potential after inflation)

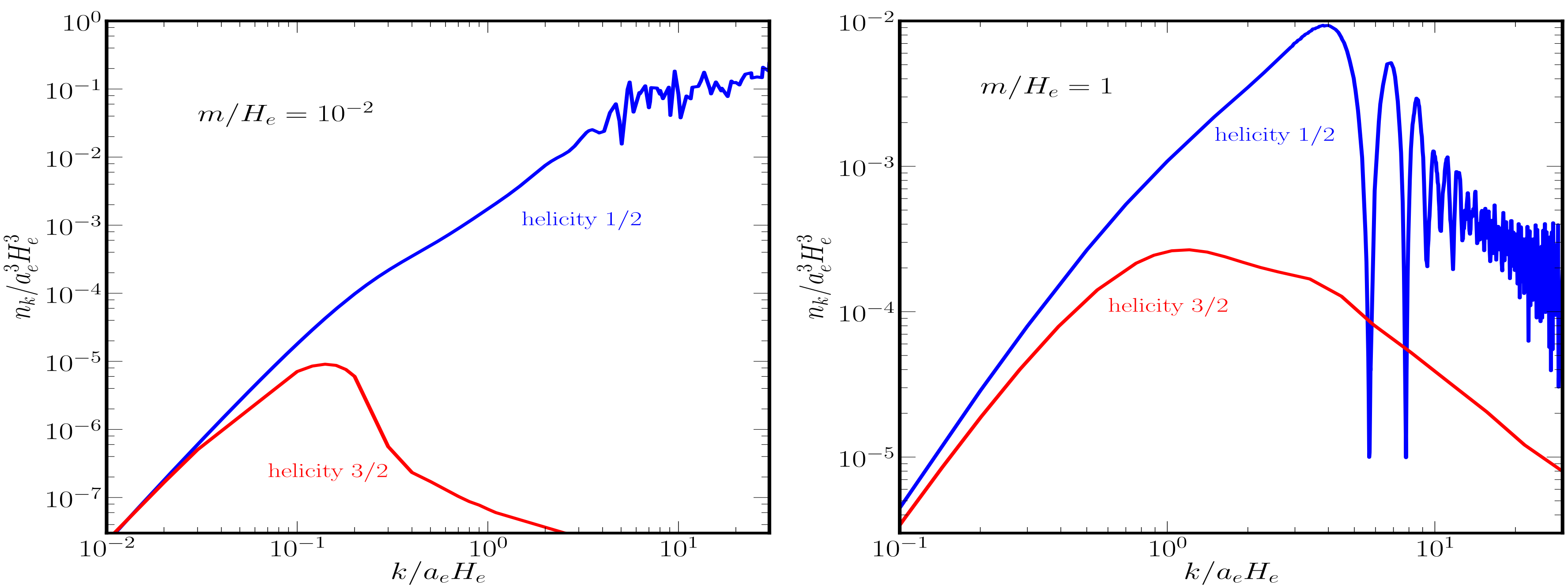
Rarita—Schwinger field ψ_μ in FRW background

Dispersion relation is $\omega_k^2(\eta) = c_s^2 k^2 + a^2(\eta) m^2$

Usual case: $c_s^2 = 1 \Rightarrow \omega_k(\eta) = k$ and constant for $k \Rightarrow \infty$

GPP depends on changing $\omega_k(\eta)$, so no production of high-k modes!

If $c_s^2 = 0$: as $k \Rightarrow \infty$, $\omega_k(\eta)$ is independent of k, production of high-k modes unsuppressed!



Rarita—Schwinger field ψ_μ in FRW background

Supergravity employs spin-3/2 field (gravitino, inflation, …), the superpartner to graviton.

Catastrophic production of gravitinos dependent on model.

For models with a single chiral superfield gravitino mass is time dependent ($\partial_\eta m \neq 0$).

$c_s = 1$ at all times $\Longrightarrow$ no catastrophic production

For models with multiple chiral superfields (most modern models)

c_s depends on relative orientation of inflaton direction & susy breaking

$c_s = 0$ in models with a nilpotent superfield and orthogonal constraint KKLT

mixing between the goldstino & inflatino may avoid the catastrophe (explicit calculation needed)

Dudas, Garcia, Mambrini, Olive, Peloso, & Verner (2021); Antoniadis, Benakli, & Ke (2021)

Models with $c_s = 0$ are in a SWAMPLAND! Kolb, Long, & McDonough (2021)

GGP may provide constraints on SUGRA model building.

Fierz-Pauli field $f_{\mu\nu}$ in FRW background

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Collaborators

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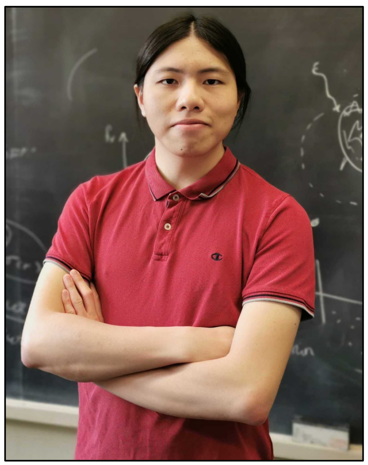

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Metric Perturbations About Minkowski Spacetime

Start with EH action: $S[g_{\mu\nu}] = \int d^4x \sqrt{-g}\ \frac{M_P^2}{2}\ R[g]$

Linearize about Minkowski spacetime: $g_{\mu\nu} \to \eta_{\mu\nu} + \frac{2}{M_P} h_{\mu\nu}$ $\qquad h = \eta^{\mu\nu} h_{\mu\nu}$

$$S[h_{\mu\nu}] = \int d^4x \left[-\tfrac{1}{2} \nabla_\lambda h_{\mu\nu} \nabla^\lambda h^{\mu\nu} + \nabla_\mu h^{\nu\lambda} \nabla_\nu h^\mu{}_\lambda - \nabla_\mu h^{\mu\nu} \nabla_\nu h + \tfrac{1}{2} \nabla_\mu h \nabla^\mu h \right]$$

We will be careful about counting degrees of freedom:

$h_{\mu\nu}$:	16	– 6	– 4	– 4	=	2
		symmetric	gauge	transverse/traceless		helicity modes of massless graviton ±2, or $\times$, $+$

Now Add Fierz-Pauli (1939) Mass Term(s)

(see reviews by Hinterbichler 1105.3735; de Rahm 1401.4173)

$$\delta S[h_{\mu\nu}] = \int d^4x \left[-\tfrac{1}{2}m_1^2 h_{\mu\nu}h^{\mu\nu} - \tfrac{1}{2}m_2^2 h^2\right]$$

Introduces unwanted 6th degree of freedom (a ghost) of mass $m^2_{\text{ghost}} = \dfrac{m_1^2}{4}\dfrac{m_1^2 + 4m_2^2}{m_1^2 + m_2^2}$

So, choose $m_2^2 = -m_1^2$ to banish ghost to ∞ (but no symmetry enforces this!)

$$\delta S[h_{\mu\nu}] = \int d^4x \left[-\tfrac{1}{2}m^2 \left(h_{\mu\nu}h^{\mu\nu} - h^2\right)\right]$$

Fierz-Pauli mass term

Spin-2 theories will be haunted by the spectre of ghosts

Degrees of freedom as expected

$h_{\mu\nu}$:	16	$-$ 6	$-$ 1	$-$ 4	= 5
		symmetric	gauge	transverse/traceless	polarization modes ±2, ±1, 0

Boulware—Deser Ghost

Boulware and Deser (1972) pointed out that Fierz-Pauli tuning breaks down with generic nonlinear extensions of Fierz-Pauli, and a sixth ghostly degree of freedom arises (zombie ghost?).

Once thought that all Lorentz-invariant massive gravity theories were ghostly, until …

… de Rahm-Gabadadze-Tolley (dRGT) developed a ghost-free massive gravity theory in 2010.

dRGT introduced second “reference” metric, taken to be Minkowski. Metrics interact via potential $V(\mathbb{X};\beta_n)$.

Extended/completed to general metric by Hassan & Rosen → ghost-free bigravity (2011).

This is our starting point. Field content: two metric fields, $g_{\mu\nu}$ and $f_{\mu\nu}$, coupling to two scalar fields, ϕ_g and ϕ_f.

Bigravity With Minimal Coupling To Matter (Minimal Model)

$$S = \int \mathrm{d}^4x \left[\frac{M_g^2}{2}\sqrt{-g}\,R[g] + \frac{M_f^2}{2}\sqrt{-f}\,R[f] - m^2 M_*^2 \sqrt{-g}\,V(\mathbb{X};\beta_n) + \sqrt{-g}\,\mathcal{L}_g(g,\phi_g) + \sqrt{-f}\,\mathcal{L}_f(f,\phi_f) \right]$$

Kinetic terms for f and g + dRGT potential + Matter Lagrangians

dRGT Potential: $\mathbb{X}^\mu{}_\nu = (\sqrt{g^{-1}f})^\mu{}_\nu$

$$V(\mathbb{X};\beta_n) \equiv \sum_{n=0}^{4} \beta_n S_n(\mathbb{X})\,, \qquad S_n(\mathbb{X}) \equiv \mathbb{X}^{\mu_1}_{[\mu_1} \ldots \mathbb{X}^{\mu_n}_{\mu_n]}$$

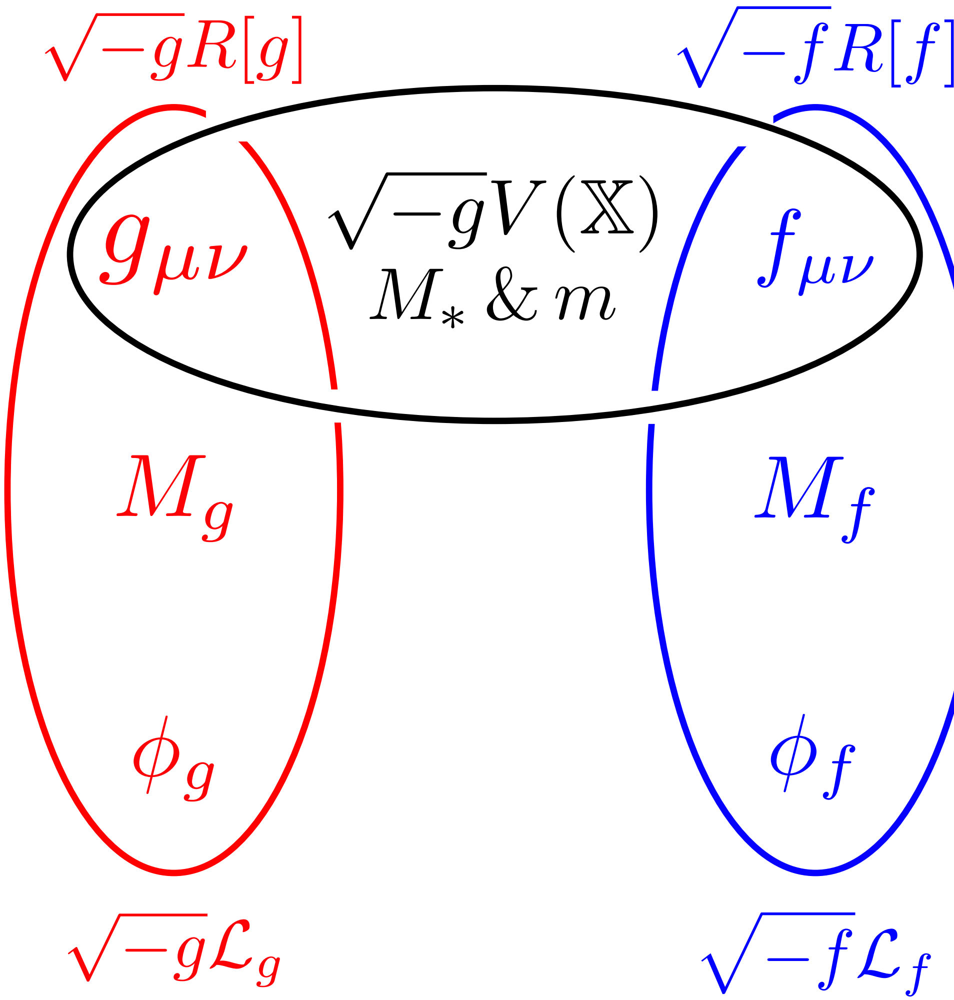
Matter Lagrangians:

$$\left.\begin{aligned} \mathcal{L}_g(g,\phi_g) &= -\tfrac{1}{2} g^{\mu\nu}\nabla_\mu\phi_g\nabla_\nu\phi_g - V_g(\phi_g) \\ \mathcal{L}_f(f,\phi_f) &= -\tfrac{1}{2} f^{\mu\nu}\nabla_\mu\phi_f\nabla_\nu\phi_f - V_f(\phi_f) \end{aligned}\right\} \text{Source FRW background}$$

After sausage making, want to end with: massless spin–2, massive spin–2, two scalar fields

DOFs: $2 + 5 + 2 = 9$

Inflationary Bigravity



Perturbations, Backgrounds, Mirroring (Bar Denotes Background)

Tensor sector

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{2}{M_g} h_{\mu\nu}$$

$$f_{\mu\nu} = \bar{f}_{\mu\nu} + \frac{2}{M_f} k_{\mu\nu}$$

$$\bar{g}_{\mu\nu} = \bar{f}_{\mu\nu} = \mathrm{FRW}$$

Scalar sector

$$\phi_g = \bar{\phi}_g + \varphi_g$$

$$\phi_f = \bar{\phi}_f + \varphi_f$$

$$\frac{1}{M_g}\,\bar{\phi}_g = \frac{1}{M_f}\,\bar{\phi}_f \equiv \frac{1}{M_P}\,\bar{\phi}$$

$$\frac{1}{M_g^2}\,V_g\left(\tfrac{M_g}{M_P}\phi\right) = \frac{1}{M_f^2}\,V_f\left(\tfrac{M_f}{M_P}\phi\right) \equiv \frac{1}{M_P}\,V(\phi)$$

Background EoMs:

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = \frac{1}{M_P^2}\,\bar{T}_{\mu\nu}$$

$$\Box\bar{\phi} - V'(\bar{\phi}) = 0$$

$$\bar{T}_{\mu\nu} = \nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi} + \bar{g}_{\mu\nu}\bar{\mathcal{L}}(\bar{g},\bar{\phi})$$

$$\bar{\mathcal{L}}(\bar{g},\bar{\phi}) = -\tfrac{1}{2}\bar{g}^{\mu\nu}\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi} - V(\bar{\phi})$$

Change Perturbation Variables: Massive and Massless Modes Decouple

$$\{h_{\mu\nu},\ k_{\mu\nu}\} \to \{u_{\mu\nu},\ v_{\mu\nu}\}$$

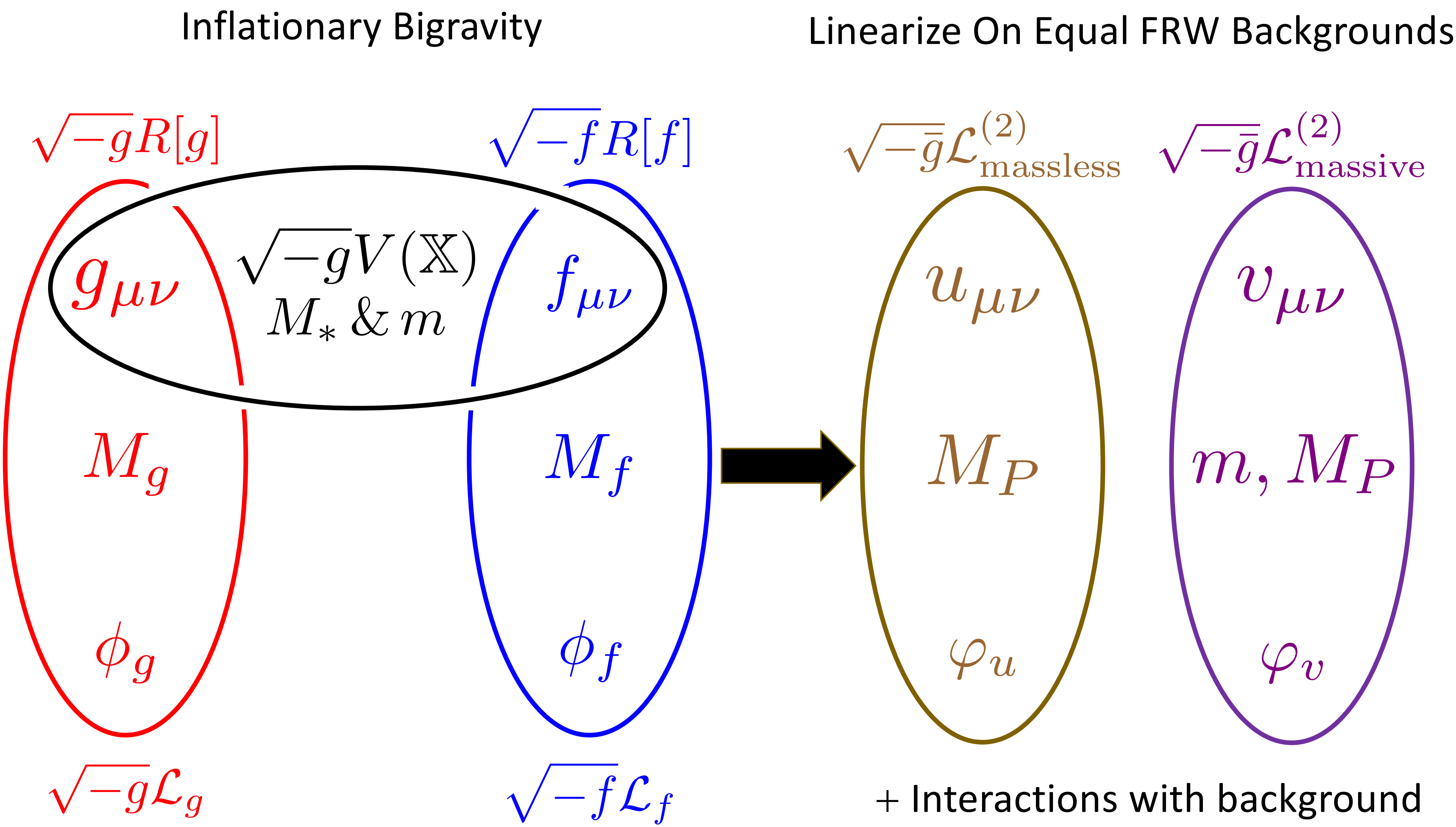
$$\frac{u_{\mu\nu}}{M_*} = \frac{h_{\mu\nu}}{M_f} + \frac{k_{\mu\nu}}{M_g}$$

$$\frac{v_{\mu\nu}}{M_*} = \frac{h_{\mu\nu}}{M_g} - \frac{k_{\mu\nu}}{M_f}$$

$$\{\varphi_g,\ \varphi_f\} \to \{\varphi_u,\ \varphi_v\}$$

$$\frac{\varphi_u}{M_*} = \frac{\varphi_g}{M_f} + \frac{\varphi_f}{M_g}$$

$$\frac{\varphi_v}{M_*} = \frac{\varphi_g}{M_g} - \frac{\varphi_f}{M_f}$$



Inflationary Bigravity
Linearize On Equal FRW Backgrounds
$\sqrt{-g}R[g]$
$\sqrt{-f}R[f]$
$\sqrt{-\bar{g}}\mathcal{L}^{(2)}_{\text{massless}}$
$\sqrt{-\bar{g}}\mathcal{L}^{(2)}_{\text{massive}}$
$\sqrt{-g}V(\mathbb{X})$
$M_* \,\&\, m$
$g_{\mu\nu}$
$f_{\mu\nu}$
$u_{\mu\nu}$
$v_{\mu\nu}$
M_g
M_f
M_P
m, M_P
ϕ_g
ϕ_f
φ_u
φ_v
$\sqrt{-g}\mathcal{L}_g$
$\sqrt{-f}\mathcal{L}_f$
+ Interactions with background

Change Perturbation Variables: Massive and Massless Modes Decouple

$$\mathcal{L}^{(2)}_{\mathrm{massless}} = \mathcal{L}^{(2)}_{uu} + \mathcal{L}^{(2)}_{u\varphi_u} + \mathcal{L}^{(2)}_{\varphi_u\varphi_u}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{uu} =& -\tfrac{1}{2}\nabla_\lambda u_{\mu\nu}\nabla^\lambda u^{\mu\nu} + \nabla_\mu u^{\nu\lambda}\nabla_\nu u^\mu{}_\lambda \\ & - \nabla_\mu u^{\mu\nu}\nabla_\nu u + \tfrac{1}{2}\nabla_\mu u \nabla^\mu u \\ & + \left(\bar{R}_{\mu\nu} - M_P^{-2}\,\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi}\right) \\ & \times \left(u^{\mu\lambda}u_\lambda{}^\nu - \tfrac{1}{2}u^{\mu\nu}u\right)\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{u\,\varphi_u} =& M_P^{-1}\Big[\left(\nabla_\mu\bar{\phi}\nabla_\nu\varphi_u + \nabla_\nu\bar{\phi}\nabla_\mu\varphi_u\right) \\ & \times \left(u^{\mu\nu} - \tfrac{1}{2}\bar{g}^{\mu\nu}u\right) - V'(\bar{\phi})\varphi_u u\Big]\end{aligned}$$

$$\mathcal{L}^{(2)}_{\varphi_u\varphi_u} = -\tfrac{1}{2}\nabla_\mu\varphi_u\nabla^\mu\varphi_u - \tfrac{1}{2}V''(\bar{\phi})\varphi_u^2$$

$$\mathcal{L}^{(2)}_{\mathrm{massive}} = \mathcal{L}^{(2)}_{vv} + \mathcal{L}^{(2)}_{v\varphi_v} + \mathcal{L}^{(2)}_{\varphi_v\varphi_v}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{vv} =& -\tfrac{1}{2}\nabla_\lambda v_{\mu\nu}\nabla^\lambda v^{\mu\nu} + \nabla_\mu v^{\nu\lambda}\nabla_\nu v^\mu{}_\lambda \\ & - \nabla_\mu v^{\mu\nu}\nabla_\nu v + \tfrac{1}{2}\nabla_\mu v \nabla^\mu v \\ & + \left(\bar{R}_{\mu\nu} - M_P^{-2}\,\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi}\right) \\ & \times \left(v^{\mu\lambda}v_\lambda{}^\nu - \tfrac{1}{2}v^{\mu\nu}v\right) \\ & \boxed{-\tfrac{1}{2}m^2\left(v^{\mu\nu}v_{\mu\nu} - v^2\right)}\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{v\,\varphi_v} =& M_P^{-1}\Big[\left(\nabla_\mu\bar{\phi}\nabla_\nu\varphi_v + \nabla_\nu\bar{\phi}\nabla_\mu\varphi_v\right) \\ & \times \left(v^{\mu\nu} - \tfrac{1}{2}\bar{g}^{\mu\nu}v\right) - V'(\bar{\phi})\varphi_v v\Big]\end{aligned}$$

$$\mathcal{L}^{(2)}_{\varphi_v\varphi_v} = -\tfrac{1}{2}\nabla_\mu\varphi_v\nabla^\mu\varphi_v - \tfrac{1}{2}V''(\bar{\phi})\varphi_v^2$$

Scalar/Vector/Tensor (SVT) Decomposition Of Massive Spin-2 Field

Represent 4-tensor by variables that transform under spatial rotations as 3–scalars/3–vectors/3–tensors

$$v_{00} = a^2 E \qquad v_{0i} = a^2 (\partial_i F + G_i) \qquad v_{ij} = a^2 (\delta_{ij} A + \partial_i \partial_j B + \partial_i C_j + \partial_j C_i + D_{ij})$$

Subject to transverse/traceless constraints (repeated indices summed):

$$\partial_i C_i = 0, \quad \partial_i G_i = 0, \quad \partial_i D_{ij} = 0, \quad \text{and} \quad D_{ii} = 0$$

At quadratic order S/V/T decouple: $d\eta = dt/a(t)$

$$S = \int d\eta\, d^3x \left(L_S + L_V + L_T\right) + \mathcal{O}^3$$

(Set your watch to CST—Conformal Standard Time)

For S/V/T

1. Remove nondynamical DoFs.
2. Express in terms of Fourier modes.
3. Canonically normalize kinetic term.
4. Check for ghosts, gradient instabilities.
5. Find mode equation and ω_k: $\tilde{\psi}'' + \omega_k^2 \tilde{\psi} = 0$.
6. Solve with appropriate boundary conditions.
7. Integrate over k.
8. Write paper.

Tensor Sector (Prime Denotes ∂_η)

$$L_T = \frac{1}{2}a^2\Big[D'_{ij}D'_{ij} - \partial_k D_{ij}\partial_k D_{ij} - a^2m^2 D_{ij}D_{ij}\Big]$$

Canonically normalized kinetic term: $\chi_{ij} = aD_{ij}$

Fourier modes of $\chi_{ij}(\eta,\mathbf{x}) \equiv \tilde{\chi}_{ij}(\eta,\mathbf{k})$; can take $\mathbf{k}=(0,0,k)$

$$[\tilde{\chi}_{ij}] = \begin{bmatrix} \tilde{\chi}_+ & \tilde{\chi}_\times & 0 \\ \tilde{\chi}_\times & -\tilde{\chi}_+ & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\chi}''_{\substack{+\\ \times}}(\eta,k) + \omega_k^2(\eta)\,\tilde{\chi}_{\substack{+\\ \times}}(\eta,k) = 0$$

$$\omega_k^2(\eta) = k^2 + a^2m^2 - a''/a$$

If $m=0$, mode equation for gravitational wave propagating on an FRW background, familiar from studies of tensor perturbations in inflation

Vector Sector (Prime Denotes ∂_η)

$$L_V = a^2\Big[\partial_j\big(G_i - C_i'\big)\partial_j\big(G_i - C_i'\big) + a^2m^2\big(G_iG_j - \partial_jC_i\partial_jC_i\big)\Big]$$

G_i not dynamical, can be Integrated out

In Fourier space: $$L_{V,k} = \frac{a^4k^2m^2}{k^2+a^2m^2}|\tilde{C}_i'|^2 - a^4k^2m^2|\tilde{C}_i|^2$$

If $m = 0$, Lagrangian vanishes trivially since massless theory does not propagate vector modes.

Canonically normalize, again taking $\mathbf{k} = (0,0,k)$, and defining $\tilde{\chi}_\pm(\eta,k) = (\tilde{\chi}_1 \mp i\tilde{\chi}_2)/\sqrt{2}$:

$$\begin{aligned}
\tilde{\chi}_\pm''(\eta,k) + \omega_k^2(\eta)\,\tilde{\chi}_\pm(\eta,k) &= 0 \\
\omega_k^2(\eta) &= k^2 + a^2m^2 - f''/f \\
f &= a^2/\sqrt{k^2+a^2m^2}
\end{aligned}$$

Scalar Sector (Prime Denotes ∂_η)

$L_S = L_S(A, B, E, F, \varphi_v)$ (and φ_u decoupled).

After removing non-propagating DoFs, and defining $\hat{\varphi}_v = \varphi_v - \frac{a^{-1}\bar{\phi}'}{M_P H} A$

$$L_{S,k} = K_\varphi \, |\tilde{\hat{\varphi}}_v'|^2 - M_\varphi \, |\tilde{\hat{\varphi}}_v|^2 + K_B \, |\tilde{B}'|^2 - M_B \, |\tilde{B}|^2 + L_2 \, \tilde{\hat{\varphi}}_v^{*\prime} \tilde{B}' + L_1 \, \tilde{\hat{\varphi}}_v^{*} \tilde{B}' - L_0 \, \tilde{\hat{\varphi}}_v^{*} \tilde{B}$$

$$L_{S,k} = K_\varphi \, |\tilde{\hat{\varphi}}_v'|^2 - M_\varphi \, |\tilde{\hat{\varphi}}_v|^2 + K_B \, |\tilde{B}'|^2 - M_B \, |\tilde{B}|^2 + L_2 \, \tilde{\hat{\varphi}}_v^{*\prime} \tilde{B}' + L_1 \, \tilde{\hat{\varphi}}_v^{*} \tilde{B}' - L_0 \, \tilde{\hat{\varphi}}_v^{*} \tilde{B}$$

$$K_B = \frac{a^6 m^2}{8} \frac{\left(8m^2H^2 - 6H^2m_H^2 - m^2m_H^2\right)k^4}{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)} \tag{3.17c}$$

$$M_B = \frac{a^6 m^2}{8} \frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4}{\left[H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)\right]^2} \tag{3.17d}$$

$$\begin{aligned}
c_{10} &= H^2\left(8m^2H^2 - 8H^4 - 2H^2m_H^2 - m^2m_H^2\right) \\
c_8 &= a^2H^2\Big[\left(30m^4H^2 + 32m^2H^4 - 96H^6 - 3m^4m_H^2 - 56m^2H^2m_H^2\right. \\
&\qquad \left. + 48H^4m_H^2 + 5m^2m_H^4 + 6H^2m_H^4\right) \\
&\qquad + \left(4m^2 - 24H^2\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big] \\
c_6 &= \tfrac{3}{8}a^4m^2\Big[\left(96m^4H^4 + 144m^2H^6 - 6m^4H^2m_H^2 - 252m^2H^4m_H^2 - 192H^6m_H^2\right. \\
&\qquad \left. + 8m^2H^2m_H^4 + 200H^4m_H^4 - 10H^2m_H^6 - m^2m_H^6\right) \\
&\qquad + \left(8m^2m_H^2 - 16H^2m_H^2\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big] \\
c_4 &= \tfrac{3}{8}a^6m^4\Big[\left(36m^4H^4 - 48m^2H^6 + 64H^8 - 12m^2H^4m_H^2 - 32H^6m_H^2\right. \\
&\qquad \left. - 12m^2H^2m_H^4 + 4H^4m_H^4 + 12H^2m_H^6 - 3m^2m_H^6 + 2m_H^8\right) \\
&\qquad - \left(24m^2H^2 - 16H^4 - 12m^2m_H^2 - 8H^2m_H^2 + 8m_H^4\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big]
\end{aligned}$$

$$L_2 = \frac{a^3m^2\bar\phi'}{2M_PH} \frac{H^2k^4 + \frac{3}{2}a^2\left(m^2 - m_H^2\right)H^2k^2}{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)} \tag{3.17e}$$

$$L_1 = -\frac{a^4m^2\bar\phi'}{M_P} \frac{\left(H^2 - \frac{1}{4}m_H^2 - \frac{1}{2}\frac{aHV'(\bar\phi)}{\bar\phi'}\right)k^4 - \frac{3}{2}a^2\left(m^2 - m_H^2\right)\left(H^2 + \frac{1}{4}m_H^2 + \frac{1}{2}\frac{aHV'(\bar\phi)}{\bar\phi'}\right)k^2}{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)} \tag{3.17f}$$

$$L_0 = \frac{a^3m^2\bar\phi'}{2M_PH} \frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4 + c_2k^2}{\left[H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)\right]^2} \tag{3.17g}$$

$$\begin{aligned}
c_{10} &= H^4 \qquad\qquad (3.17\text{h}) \\
c_8 &= \tfrac{1}{2}a^2H^4\Big[\left(9m^2 + 12H^2 - 13m_H^2\right) - 4\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_6 &= \tfrac{3}{8}a^4H^2\Big[\left(18m^4H^2 + 32m^2H^4 + 64H^6 - 48m^2H^2m_H^2 - 64H^4m_H^2\right. \\
&\qquad \left. + m^2m_H^4 + 28H^2m_H^4\right) \\
&\qquad + 8\left(-4m^2H^2 + 4H^4 + m^2m_H^2\right)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_4 &= \tfrac{3}{16}a^6m^2H^2\Big[\left(18m^4H^2 - 24m^2H^4 + 256H^6 - 54m^2H^2m_H^2 - 160H^4m_H^2\right. \\
&\qquad \left. + 9m^2m_H^4 + 60H^2m_H^4 - 7m_H^6\right) \\
&\qquad + 4\left(-30m^2H^2 + 32H^4 + 12m^2m_H^2 + 4H^2m_H^2 - 7m_H^4\right)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_2 &= \tfrac{9}{16}a^8m^4H^2\left(2H^2 - m_H^2\right)\Big[-\left(4H^2 + m_H^2\right)\left(3m^2 - 4H^2 - m_H^2\right) \\
&\qquad + 4\left(-3m^2 + 2H^2 + 2m_H^2\right)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big]
\end{aligned}$$

$$K_\varphi = \frac{a^2}{2} \frac{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{9}{4}a^4m^2\left(m^2 - m_H^2\right)H^2}{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)} \tag{3.17a}$$

$$M_\varphi = \frac{a^2}{2} \frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4 + c_2k^2 + c_0}{\left[H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)\right]^2} \tag{3.17b}$$

$$\begin{aligned}
c_{10} &= H^4 \\
c_8 &= \tfrac{1}{2}a^2H^2\Big[\left(12m^2H^2 + 8H^4 - 14H^2m_H^2 - m_H^4\right) + 4\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} + 2H^2V''(\bar\phi)\Big] \\
c_6 &= \tfrac{3}{8}a^4H^2\Big[\left(36m^4H^2 + 72m^2H^4 - 82m^2H^2m_H^2 - 64H^4m_H^2\right. \\
&\qquad \left. - 7m^2m_H^4 + 40H^2m_H^4 + 8m_H^6\right) \\
&\qquad + 8\left(3m^2 - 4m_H^2\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + 16\left(m^2 - m_H^2\right)H^2V''(\bar\phi)\Big] \\
c_4 &= \tfrac{3}{8}a^6\Big[4H^2\left(9m^6H^2 + 36m^4H^4 + 16m^2H^6 - 30m^4H^2m_H^2 - 76m^2H^4m_H^2\right. \\
&\qquad \left. - 3m^4m_H^4 + 31m^2H^2m_H^4 + 24H^4m_H^4 + 6m^2m_H^6 - 6H^2m_H^6 - 3m_H^8\right) \\
&\qquad - 4m^2H^2\left(H^2 - m_H^2\right)\tfrac{V'(\bar\phi)^2}{M_P^2} \\
&\qquad + \left(36m^4H^2 + 8m^2H^4 - 94m^2H^2m_H^2 + m^2m_H^4 + 48H^2m_H^4\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + \left(36m^4H^2 - 58m^2H^2m_H^2 - m^2m_H^4 + 24H^2m_H^4\right)H^2V''(\bar\phi)\Big] \\
c_2 &= \tfrac{9}{32}a^8m^2\Big[H^2\left(18m^6H^2 + 120m^4H^4 + 128m^2H^6 - 78m^4H^2m_H^2 - 384m^2H^4m_H^2\right. \\
&\qquad \left. - 9m^4m_H^4 + 132m^2H^2m_H^4 + 128H^4m_H^4 + 23m^2m_H^6 - 32H^2m_H^6 - 16m_H^8\right) \\
&\qquad - 8H^2\left(2m^2H^2 - 2m^2m_H^2 + m_H^4\right)\tfrac{V'(\bar\phi)^2}{M_P^2} \\
&\qquad + 4\left(6m^4H^2 - 22m^2H^2m_H^2 + m^2m_H^4 + 14H^2m_H^4\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + 4\left(m^2 - m_H^2\right)\left(12m^2H^2 - 10H^2m_H^2 - m_H^4\right)H^2V''(\bar\phi)\Big] \\
c_0 &= \tfrac{27}{32}a^{10}m^4\Big[-2H^2\left(2m^2H^2 - 2m^2m_H^2 + m_H^4\right)\tfrac{V'(\bar\phi)^2}{M_P^2} \\
&\qquad - m^2\left(2H^2 - m_H^2\right)\left(4H^2 + m_H^2\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + \left(m^2 - m_H^2\right)\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)H^2V''(\bar\phi)\Big]
\end{aligned}$$

$\boxed{\text{Why you might not wish to do this!}}$

Scalar Sector (Prime Denotes ∂_η)

$L_S = L_S(A, B, E, F, \varphi_v)$ (and φ_u decoupled).

After removing non-propagating DoFs, and defining $\hat{\varphi}_v = \varphi_v - \frac{a^{-1}\bar{\phi}'}{M_P H} A$

$$L_{S,k} = K_\varphi\,|\tilde{\hat{\varphi}}_v'|^2 - M_\varphi\,|\tilde{\hat{\varphi}}_v|^2 + K_B\,|\tilde{B}'|^2 - M_B\,|\tilde{B}|^2 + L_2\,\tilde{\hat{\varphi}}_v^{*\prime}\tilde{B}' + L_1\,\tilde{\hat{\varphi}}_v^{*}\tilde{B}' - L_0\,\tilde{\hat{\varphi}}_v^{*}\tilde{B}$$

Yet another Field redefinition to diagonalize kinetic terms: $\{\tilde{\hat{\varphi}}_v, \tilde{B}\} \Rightarrow \{\tilde{\Pi}, \tilde{\mathcal{B}}\}$

$$L_{S,k} = K_\Pi\,|\tilde{\Pi}'|^2 - M_\Pi\,|\tilde{\Pi}|^2 + K_{\mathcal{B}}\,|\tilde{\mathcal{B}}'|^2 - M_{\mathcal{B}}\,|\tilde{\mathcal{B}}|^2 + \lambda_1\,\tilde{\Pi}^*\tilde{\mathcal{B}}' - \lambda_0\,\tilde{\Pi}^*\tilde{\mathcal{B}}$$

Scalar Sector (Prime Denotes ∂_η)

$$K_\Pi = \frac{a^2}{2}\frac{H^2k^4 + 3a^2\big(m^2 - m_H^2\big)H^2k^2 + \frac{9}{4}a^4m^2\big(m^2 - m_H^2\big)H^2}{H^2k^4 + 3a^2\big(m^2 - m_H^2\big)H^2k^2 + \frac{3}{8}a^4m^2\big(6m^2H^2 - 4H^2m_H^2 - m_H^4\big)}$$

$$K_{\mathcal{B}} = \frac{3a^6m^2(m^2 - m_H^2)}{4k^4 + 12a^2(m^2 - m_H^2)k^2 + 9a^4m^2(m^2 - m_H^2)}$$

Where we have defined $m_H^2(\eta) = 2H^2(\eta)\big[1 - \epsilon(\eta)\big]$ $\qquad$ $\epsilon(\eta) = -H'/(aH^2)$

ϵ is the first inflationary slow-roll parameter.

$\boxed{\text{If } m < m_H(\eta)\text{, theory propagates a ghost in } \tilde{\mathcal{B}} \text{ (spin–2) sector!}}$

Generalized Higuchi Bound

In 1986 Higuchi studied perturbations of massive gravity on a de Sitter background and found a ghost if $m^2 < 2H^2$.

$m^2 = 2\,H^2$ is a "partially massless" point: mass term also vanishes.

We find a ghost in a general FRW background if $m^2 < 2H^2(\eta)\,[\,1 - \epsilon(\eta)\,]$. (In dS $\epsilon = 0$.)

FRW ghost is not generally a "partially massless" point.

Question For My Wise Colleagues

How should one regard a theory, perfectly healthy in Minkowski spacetime, but ghostly in a non-pathological, classical gravitational background?

In FRW, $m > \sqrt{2}\, H$ to avoid ghosts.

In principle, H could be anything!

Scalar Sector (Prime Denotes ∂_η)

$$L_{S,k} = K_\Pi \, |\tilde{\Pi}'|^2 - M_\Pi \, |\tilde{\Pi}|^2 + K_{\mathcal{B}} \, |\tilde{\mathcal{B}}'|^2 - M_{\mathcal{B}} \, |\tilde{\mathcal{B}}|^2 + \lambda_1 \, \tilde{\Pi}^* \tilde{\mathcal{B}}' - \lambda_0 \, \tilde{\Pi}^* \tilde{\mathcal{B}}$$

<u>At late times:</u>

$$L_{S,k} = \frac{1}{2}\Big[|\tilde{\chi}'_\Pi|^2 - \big(k^2 + a^2 V''(\bar{\phi})\big)|\tilde{\chi}_\Pi|^2\Big] + \frac{1}{2}\Big[|\tilde{\chi}'_{\mathcal{B}}|^2 - \big(k^2 + a^2 m^2\big)|\tilde{\chi}_{\mathcal{B}}|^2\Big] + \mathcal{O}(H/m)$$

Inflaton DoF

Massive spin-2 DoF

CGPP (Finally!)

Have mode equations for $u_{\mu\nu}$(tensor) , $\boldsymbol{\varphi}_u$, $v_{\mu\nu}$(tensor, vector) , Π , $\mathcal{B}$

No DoF left behind: $2 \quad + \; 1 \; + \qquad 2 \quad + \;\; 2 \qquad +1 + 1 \; = \; 9$

1. Mode equation: $\tilde{\psi}''(\eta, k) + \omega_k^2(\eta)\tilde{\psi}(\eta, k) = 0$
2. Bunch-Davies (Minkowski) initial conditions: $\lim\limits_{\eta \to -\infty} \tilde{\psi}(\eta, k) = \dfrac{1}{\sqrt{2k}}\, e^{-ik\eta}$
3. Calculate $\tilde{\psi}$ at late time when mode is
   - nonrelativistic
   - subhorizon
   - evolution approximately adiabatic
4. Calculate Bogolubov coefficient for modes with wavenumber k

$$|\beta_k|^2 = \lim_{\eta \to \infty} \left(\frac{\omega_k}{2} |\tilde{\psi}|^2 + \frac{1}{2\omega_k} |\partial_\eta \tilde{\psi}|^2 - \frac{1}{2} \right)$$

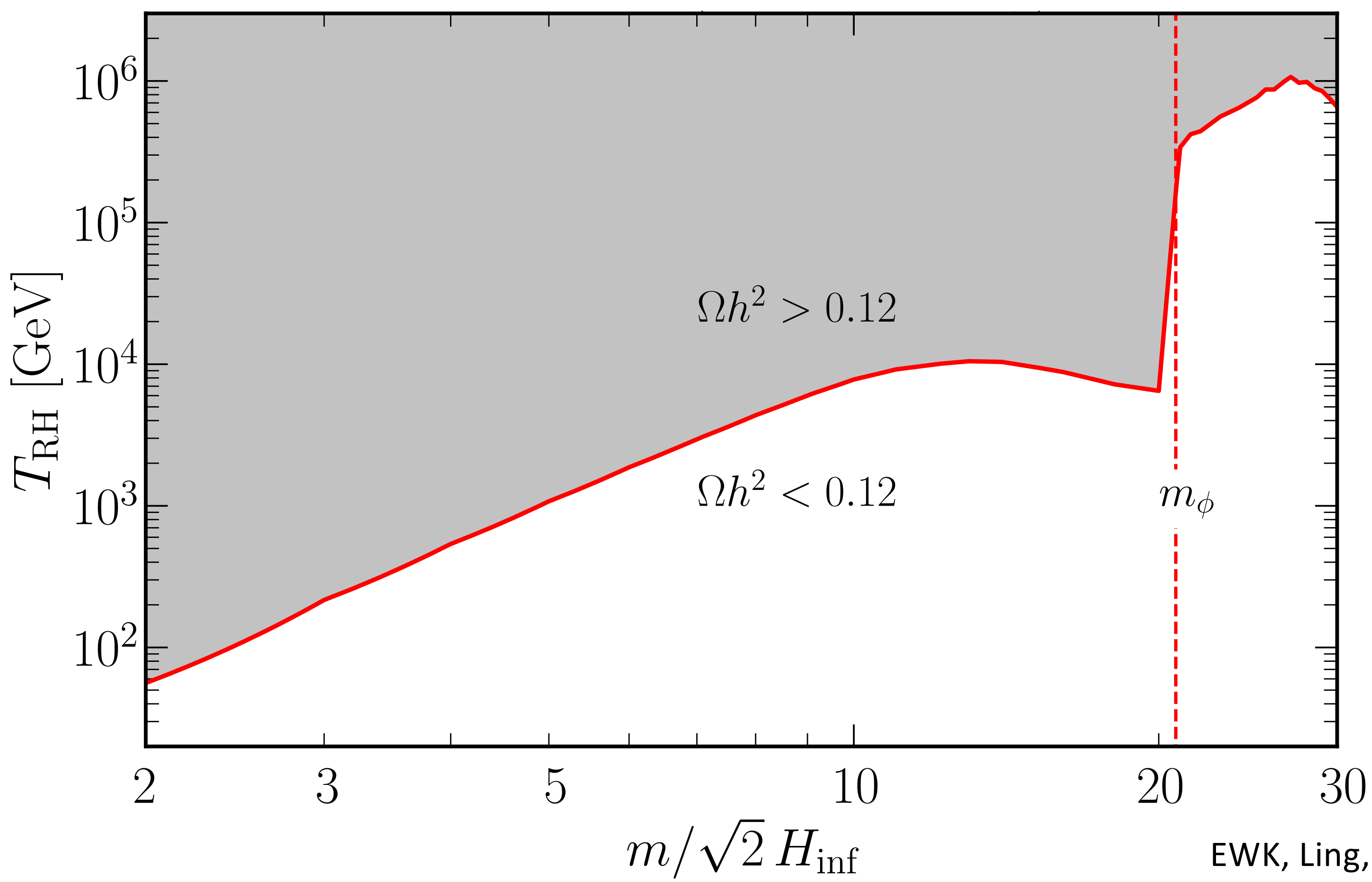
5. Physical number density of particles with comoving momentum $p = k$

$$n_k(\eta) = a^{-3}(\eta) \frac{k^3}{2\pi^2} |\beta_k|^2$$

Total number density: $n(\eta) = \int \frac{dk}{k} n_k(\eta)$

Massive Bigravity

Theory (ghost-free in Minkowski) propagates ghost in FLRW for low-mass, $m^2 < 2H_e^2\,(1-\varepsilon)$



EWK, Ling, Long, Rosen (23)

Finally, Summary: CGPP can produce DM & constrain BSM physics!

Dark matter might have only gravitational interactions (that's all we really "know")

If so, dark matter must have a gravitational origin.

Cosmological Gravitational Particle Production promising.

Scalars:

Conformally-coupled: promising DM candidate if $m \approx H_e$ (WIMPZILLA miracle).

Minimally-coupled: not promising DM candidate, exclude stable particles with $m \lesssim$ few H_e.

If allow $2\times10^{-2} \lesssim \xi \lesssim 10^2$ DM candidate in mass range milli-eV to $10^{13}\,\mathrm{GeV}$.

Dirac fermions:

Like conformally-coupled scalars; promising DM candidate if $m \approx H_e$ (WIMPZILLA miracle).

de Broglie—Proca vectors:

DM candidate could be very light ($\mu\mathrm{eV}$) or very massive (H_e).

Rarita-Schwinger fermions:

Catastrophic production if c_s vanishes. Implications for models of supergravity.

Gravitinos: EWK, Long, McDonough (2021); Dudas, Garcia, Mambrini, Olive, Peloso, Verner (2021)

Fierz-Pauli tensors:

FRW-generalization of the Higuchi bound; DM relic abundance.

Spin greater than 2: Jenks, Koutrolikos, McDonough, Alexander, Gates

Much Recent Work … Many Open Roads

- Complete CGPP for higher-spin fields
- Fully explore Rarita-Schwinger = Gravitino
- Massive particles from K-K reduction in SUGRA/Strings
- Understand what it means to have ghosts
- Develop CMB implications
- Dark matter as Kalb-Ramond-Like-Particle (KRLP)?
- Long-lived massive particles from CGPP
  - Baryo/leptogenesis?
  - ….
- Direct detection?

Coming soon-ish, to a *Reviews of Modern Physics* Near You

Cosmological gravitational particle production and its implications for cosmological relics

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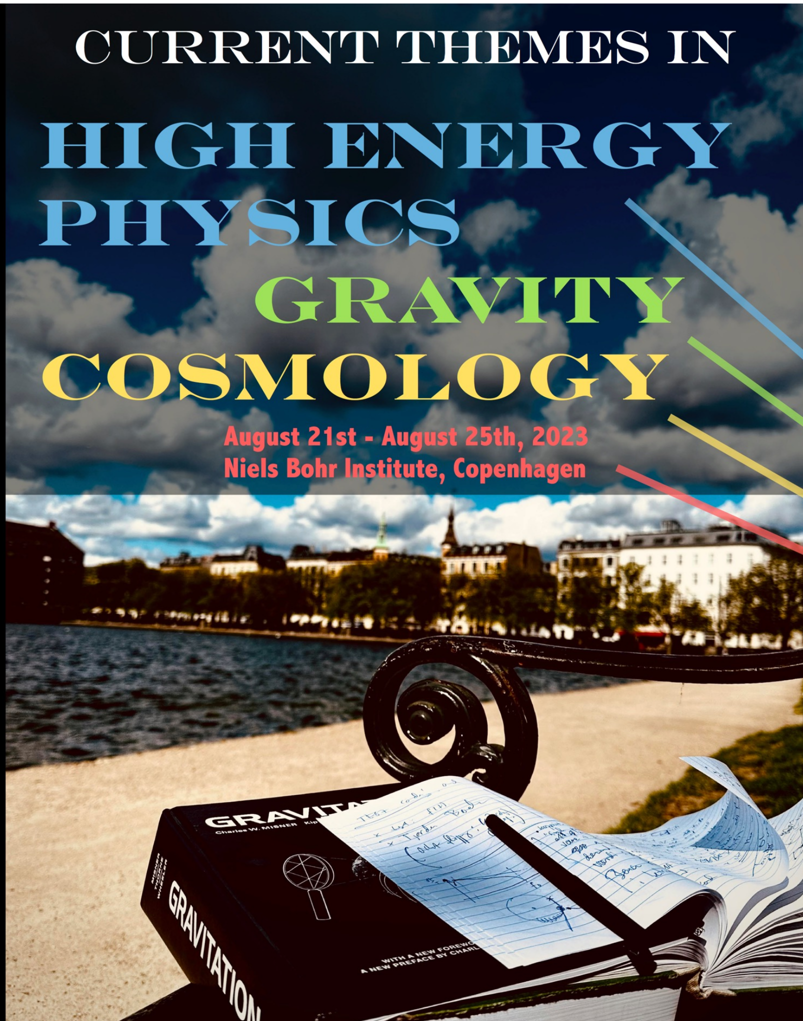
The focus of this review is the phenomenon of particle production in the early universe solely by the expansion of the universe, with particular attention to the possibility that the created particle species could be the dark matter. We will treat particle production by cosmological expansion for particles of spin 0, 1/2, 1, 3/2, and 2, and comment on the possibility of larger spins. For the early-universe evolution of the background spacetime we assume an initial inflationary phase, followed by a transition to a matter-dominated phase, eventually transiting to a radiation-dominated phase. We review the two basic requirements for particle production by the expansion of the universe: 1) the contribution to the matter action from the particle must violate conformal invariance (the trace of the matter stress-energy tensor involving the new field must be nonzero), and 2) the mass of the particle must not be too much in excess of the expansion rate of the universe during inflation. In this review we specialize to a Friedman-Lemaître-Robertson-Walker cosmological model, and calculate the spectrum of particles resulting from the expansion of the universe. We summarize the criteria for the resulting density of particles to be sufficient to account for the dark matter, as well as discuss several other cosmological implications. We then mention other mechanisms for cosmological particle production through gravity: particle production from the standard-model plasma through graviton exchange, particle production through black-hole evaporation, and particle production through a misalignment mechanism.

Thanks to my collaborators in 25 years of CGPP: (Chung, EWK, Riotto, *PRD* **59** (1998) 023501)

Ivone Albuquerque, Edward Basso, Christian Capanelli, Daniel Chung, Patrick Crotty, Michael Fedderke, Gian Giudice, Lam Hui, Leah Jenks, Siyang Ling, Andrew Long, Evan McDonough, Toni Riotto, Rachel Rosen, Leo Senatore, Alexi Starobinski, Igor Tkachev, Mark Wyman



Cosmological Gravitational Particle Production (CGPP) involving High Energy Physics, Gravity, & Cosmology

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August 2023

The Niels Bohr International Academy