From Scattering Amplitudes to Gravitational Waves

Zvi Bern

August 22, 2023

Current Themes in High Energy Physics

Copenhagen, Denmark

ZB, C. Cheung, R. Roiban, C. H. Shen, M. Solon, M. Zeng, arXiv:1901.04424 and arXiv:1908.01493.

ZB, A. Luna, R. Roiban, C. H. Shen, M. Zeng, arXiv:2005.03071

ZB, J. Parra-Martinez, R. Roiban, M. Ruf. C.-H. Shen, M. Solon, M. Zeng, arXiv:2101.07254; arXiv:2112.10750

ZB, Kosmopoulos, Luna, Roiban, Teng, arXiv: 2203.06202.

ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov: arXiv:2305.08981

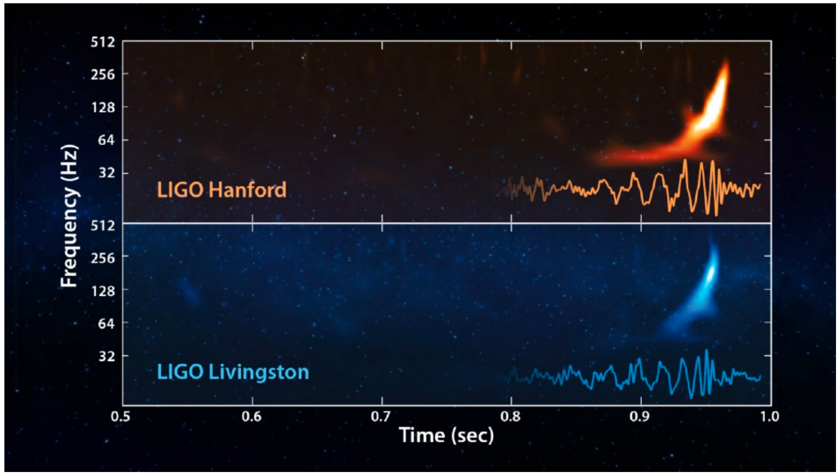
ZB, Kosmopoulos, Luna, Roiban, Sheopner, Teng, Vines, to appear soon.

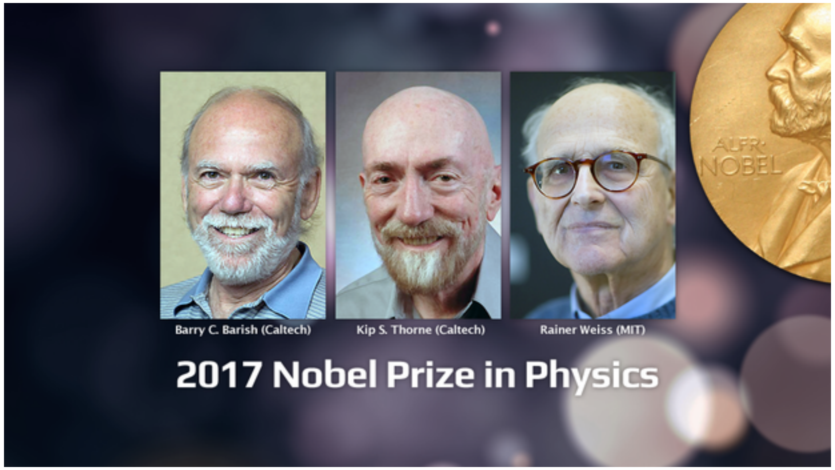
UCLA Mani L. Bhaumik Institute for Theoretical Physics

Outline

Era of gravitational-wave astronomy has begun.

See talks from Campanelli, Cardoso, Chtziioannou, Damour, Di Vecchia, Farr, Vecchio





How can we, as particle theorists, help out?

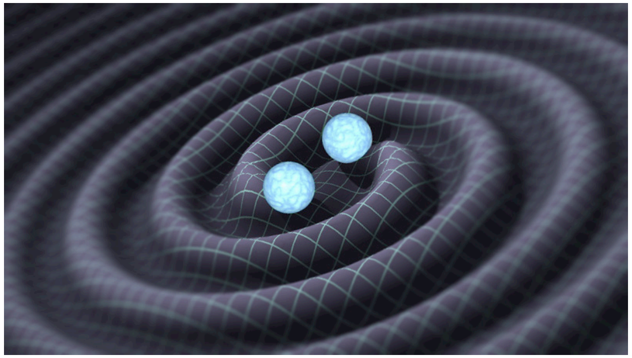
Outline

1. Overview.

2. Brief review of basics.
   — Scattering amplitudes and gravity.

3. Three examples where amplitudes help with grav. wave physics
   — Higher orders. Towards 5th post-Minkowskian order, G^5.
   — Basic issues on defining spin. Puzzle and new results.
   — Compact representations of observables:
     Eikonal formula for spin.

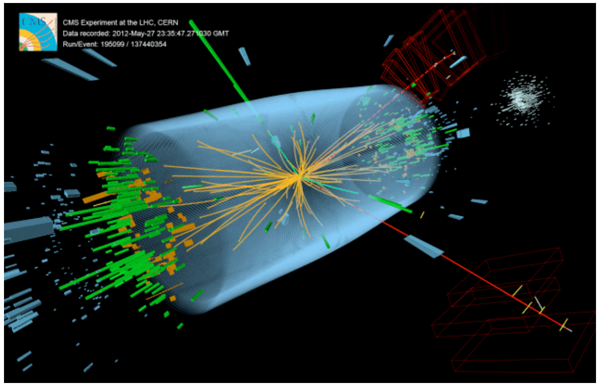
4. Conclusions and Outlook.



Can Particle Theory Help with Gravitational Waves?

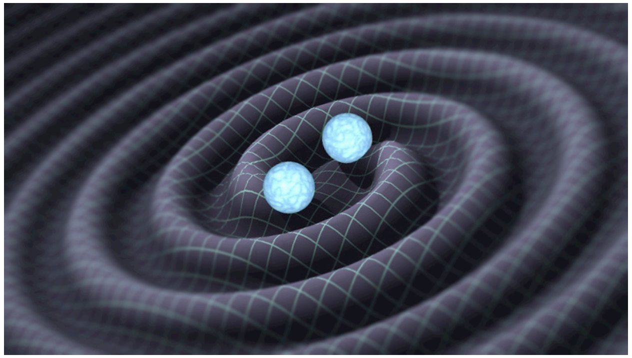
What does particle physics have to do with classical dynamics of astrophysical objects?

unbounded trajectory



gauge theories, QCD, electroweak
quantum field theory

bounded orbit



General Relativity
classical physics

Black holes and neutron stars are point particles as far as long wavelength radiation is concerned.

Iwasaki (1971); **Goldberger, Rothstein (2006)**; Vaydia, Foffa , Porto, Rothstein, Sturani; Kol; Bjerrum-Bohr, Donoghue, Holstein, Plante, Pierre Vanhove; Levi, Steinhoff; Vines etc

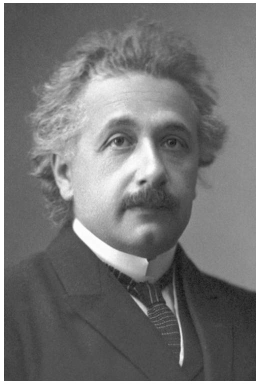
Will explain that QFT scattering amplitudes are well suited for perturbative gravitational wave calculations in post-Minkowskian framework.

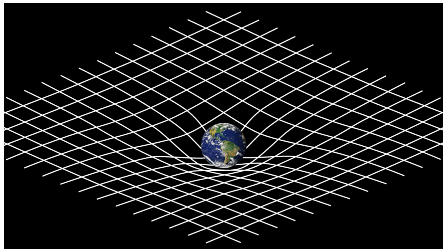
Approach to General Relativity

Our appoach does *not* start from usual Einstein Field equations.

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$$

geometry





Amplitude Approach to General Relativity

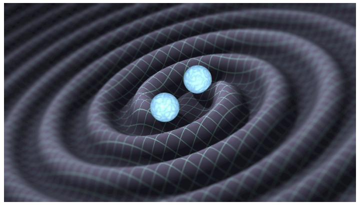
Our appoach does *not* start from usual Einstein Field equations.

$$\cancel{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

~~**geometry**~~

Gravitons are spin 2 particles

- **Not suited for all problems. Works very well for asymptotically flat space-times in context of perturbation theory.**
- **Well suited for gravitational-wave physics from compact astrophysical objects**



Gravity from Gauge Theory

Kawai, Lewellen, Tye; ZB, Carrasco, Johansson

color factor

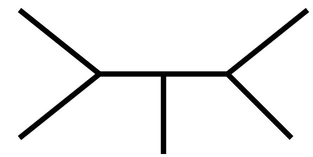
kinematic numerator factor

Feynman propagators

gauge theory (QCD): $$\mathcal{A}_n^{\text{tree}} = i g^{n-2} \sum_i \frac{c_i\, n_i}{D_i}$$

$$c_k = c_i - c_j$$
$$n_k = n_i - n_j$$

$$\boxed{c_i \to n_i}$$



sum over diagrams with only 3 vertices

Einstein gravity: $$\mathcal{M}_n^{\text{tree}} = i \kappa^{n-2} \sum_i \frac{n_i^2}{D_i}$$

$$n_i \sim k_4 \cdot k_5\, k_2 \cdot \varepsilon_1\, \varepsilon_2 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

Gravity and gauge theory kinematic numerators are the same!

Underlying physical reason still unclear.

Cries out for a unified description of gravity with gauge theory, presumably along the lines of string theory.

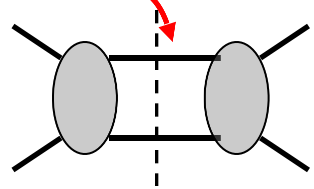
From Tree to Loops: Generalized Unitarity Method

Use tree amplitudes to build higher order (loop) amplitudes.

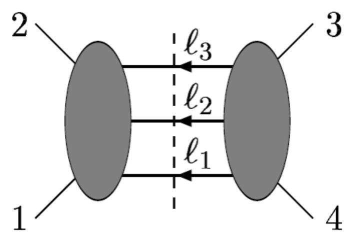
$$E^2 = \vec{p}^{\,2} + m^2$$ **on-shell**

ZB, Dixon, Dunbar and Kosower (1994)

Two-particle cut:



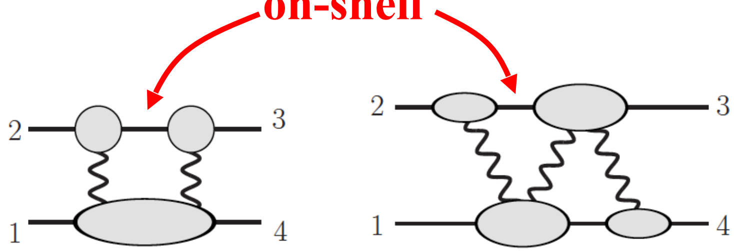
Three-particle cut:



- **Systematic assembly of complete loop amplitudes from tree amplitudes.**
- **Works for any number of particles or loops.**

Generalized unitarity as a practical tool for loops.

on-shell

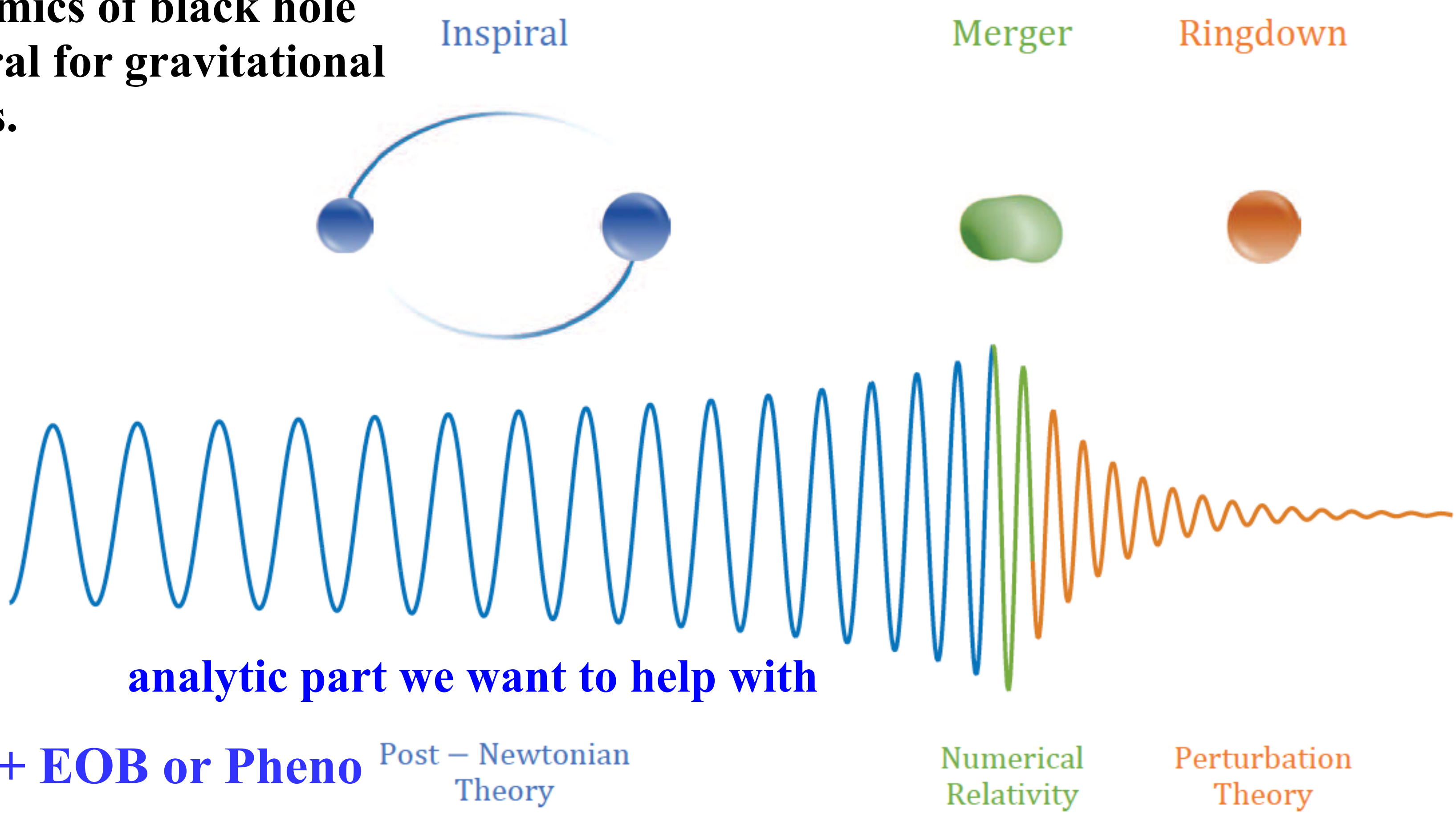


ZB, Dixon and Kosower;
ZB, Morgan;
Britto, Cachazo, Feng;
Ossala,Pittau,Papadopoulos;
Ellis, Kunszt, Melnikov;
Forde; Badger;
ZB, Carrasco, Johansson, Kosower
and many others

Idea used in the "NLO revolution" in QCD collider physics and high loop supergravity calculations.
Are applying it to gravitational wave problem.

Goal: Higher Precision.

Dynamics of black hole inspiral for gravitational waves.



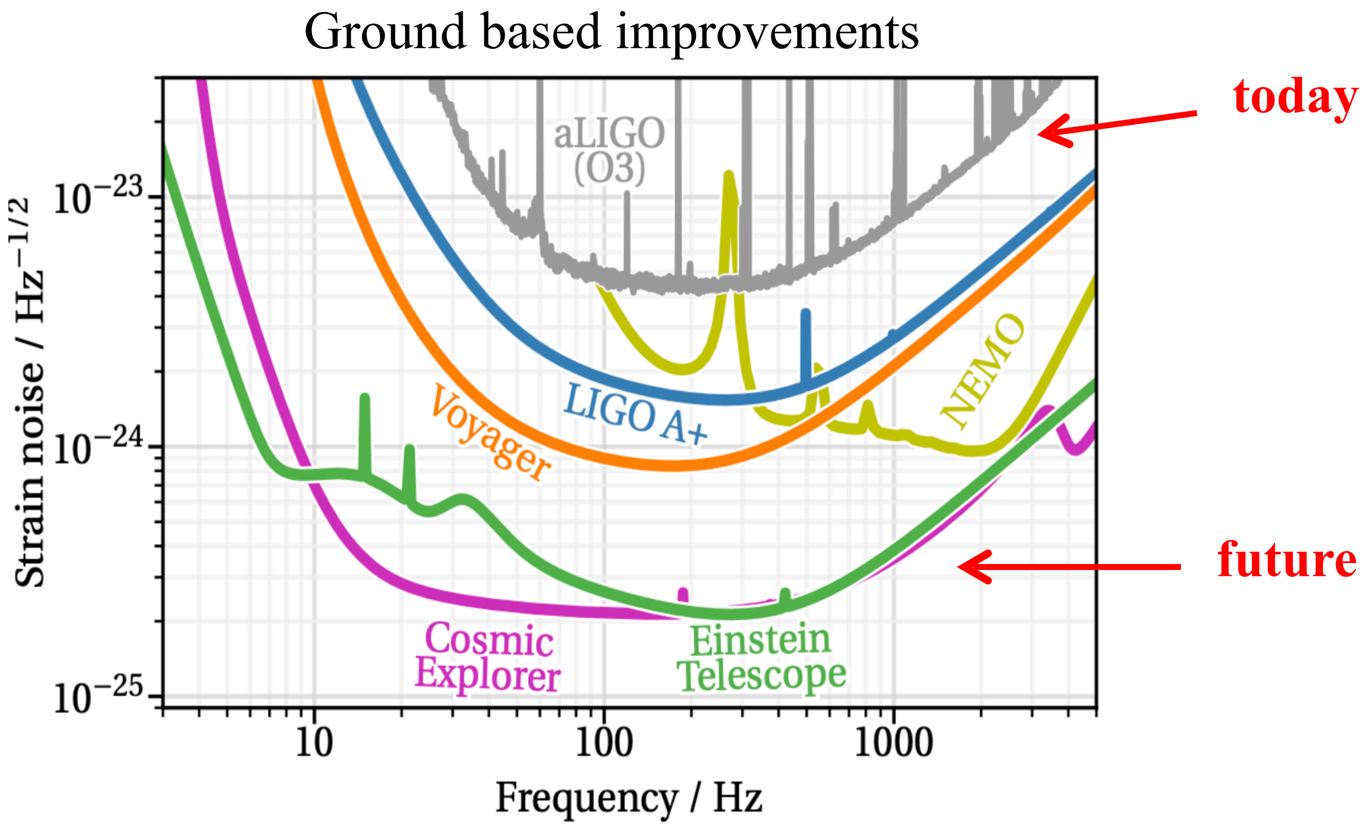
analytic part we want to help with

PN + EOB or Pheno

Small errors accumulate. Need for high precision.

From Antelis and Moreno, arXiv:1610.03567

Future Detectors



https://cosmicexplorer.org/sensitivity.html

- **Depending on parameters, sensitivity improvements up to factor of 100.**
- **Also vast improvements at low frequency from space-based: Extreme mass ratio inspirals (EMRI): LISA, TianQin.**
- **Highly nontrivial theoretical challenge to match upcoming experimental precision.**

High-energy gravitational scattering and the general relativistic two-body problem

Thibault Damour*
Institut des Hautes Etudes Scientifiques, 35 route de Chartres, 91440 Bures-sur-Yvette, France
(Dated: October 31, 2017)

A technique for translating the classical scattering function of two gravitationally interacting bod-

tum grav... amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian

"… and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian."

Hard to resist an invitation with this kind of clarity!

nals from inspiralling and coalescing binary black holes has been significantly helped, from the theoretical side, by the availability of a large bank of waveform templates, defined [5, 6] within the analytical effective one-body (EOB) formalism [7–11]. The EOB formalism combines, in a suitably resummed format, perturbative, analytical results on the motion and radiation of compact binaries, with some non-perturbative information extracted from numerical simulations of coalescing black-hole binaries (see [12] for a review of perturbative results on binary systems, and [13] for a review of the numerical relativity of binary black holes). Until recently, the perturbative results used to define the EOB conservative dynamics were mostly based on the post-Newtonian (PN) approach to the general relativistic two-body interaction. The conservative two-body dynamics was derived, successively, at the second post-Newtonian (2PN) [14, 15], third post-

…ntly intro- … to derive … from the (gauge-invariant) *scattering function* Φ linking (half) the center of mass (c.m.) classical gravitational scattering angle χ to the total energy, $E_{\rm real} \equiv \sqrt{s}$, and the total angular momentum, J, of the system[1]

$$\frac{1}{2}\chi = \Phi(E_{\rm real}, J; m_1, m_2, G)\,. \tag{1.1}$$

The (dimensionless) scattering function can be expressed as a function of dimensionless ratios, say

$$\frac{1}{2}\chi = \Phi(h, j; \nu)\,, \tag{1.2}$$

where we denoted

$$h \equiv \frac{E_{\rm real}}{M}\,;\ j \equiv \frac{J}{G m_1 m_2} = \frac{J}{G \mu M}\,, \tag{1.3}$$

with

$$M \equiv m_1 + m_2;\ \mu \equiv \frac{m_1 m_2}{m_1 + m_2};\ \nu \equiv \frac{\mu}{M} = \frac{m_1 m_2}{(m_1 + m_2)^2}.$$

PN versus PM expansion for conservative two-body dynamics

From Buonanno Amplitudes 2018

$$\mathcal{L} = -Mc^2 + \underbrace{\frac{\mu v^2}{2} + \frac{GM\mu}{r}}_{\downarrow} + \frac{1}{c^2}\left[\ldots\right] + \frac{1}{c^4}\left[\ldots\right] + \ldots$$

$$E(v) = -\frac{\mu}{2}\, v^2 + \cdots$$

non-spinning compact objects

		0PN	1PN	2PN	3PN	4PN	5PN	...
0PM:	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	...
1PM:		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	...
2PM:			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	...
3PM:				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	...
4PM:					$1/r^4$	v^2/r^4	v^4/r^4	...
...						...	...	

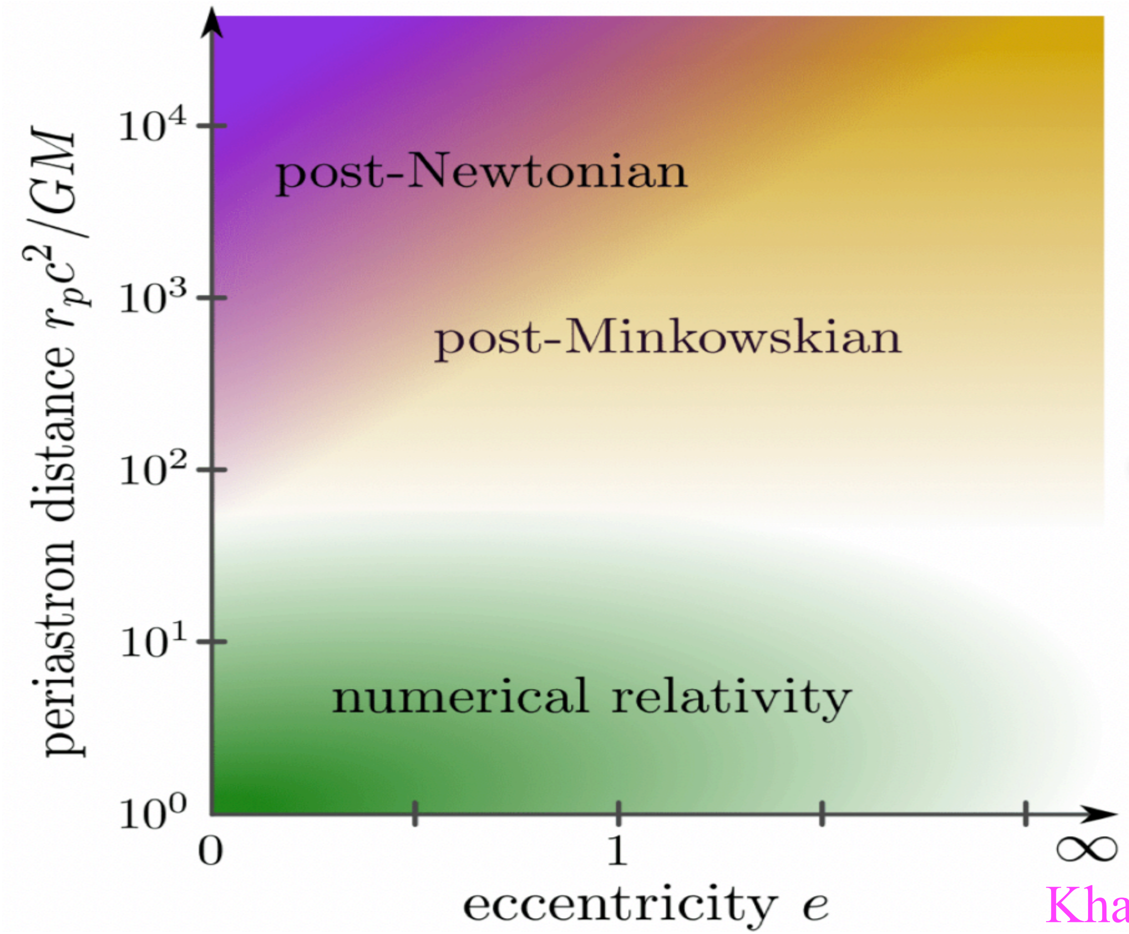
(credit: Justin Vines)

$$1 \to Mc^2, \qquad v^2 \to \frac{v^2}{c^2}, \qquad \frac{1}{r} \to \frac{GM}{rc^2}.$$

current known PN results | current known PM results | overlap between PN & PM results | unknown

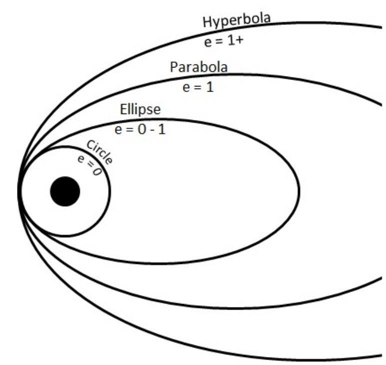
- **PM results** *(Westfahl 79, Westfahl & Goller 80, Portilla 79-80, Bel et al. 81, Ledvinka et al. 10, Damour 16-17, Guevara 17, Vines 17, Bini & Damour 17-18, Vines in prep)*

Post-Minkowskian Approach



Comments:

- **Unbound orbits cleaner theoretical environment.**
- **Asymptotic flat space for scattering processes**



Khalil, Buonanno, Steinhoff, Vines

Different approaches needed for high precision in all regions. EOB.

Buonanno and Damour

Methods for Extracting Classical Physics.

There are now multiple alternative ways to extract classical physics.

- **EFT matching to 2 body Hamiltonian** — Cheng, Solon, Rothstein; ZB, Cheung, Roiban, Shen, Zeng
- **Map to EOB** — Bini, Damour, Geralico
- **Calculate physical observables** — Kosower, Maybee, O'Connell
- **Eikonal phase** — Amati, Ciafaloni, Veneziano; Di Vecchia, Heissenberg, Russo, Veneziano
- **Amplitude radial-action relation** — ZB, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng
- **Exponential representation** — Damgaard, Plante, Vanhove; Bjerrum-Bohr, Plante, Vanhove
- **Heavy mass field theory** — Brandhuber, Chen, Travaglini, Wen; Damgaard, Haddad, Helset
- **World line formalisms** — Goldberger, Rothstein; Levi, Steinhoff; Dlapa, Kälin, Liu, Porto; Jakobson, Mogul, Plefka, Steinhoff; Edison, Levi; etc

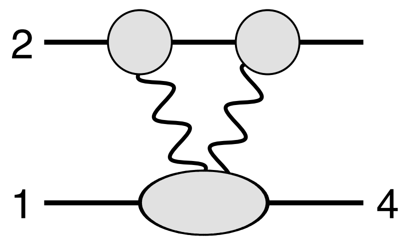
For pushing into new territory we still prefer EFT matching.

General Relativity: Unitarity + Double Copy

- **Long-range force: Two matter lines must be separated by on-shell propagators.**
- **Classical potential: 1 matter line per loop is cut (on-shell).**

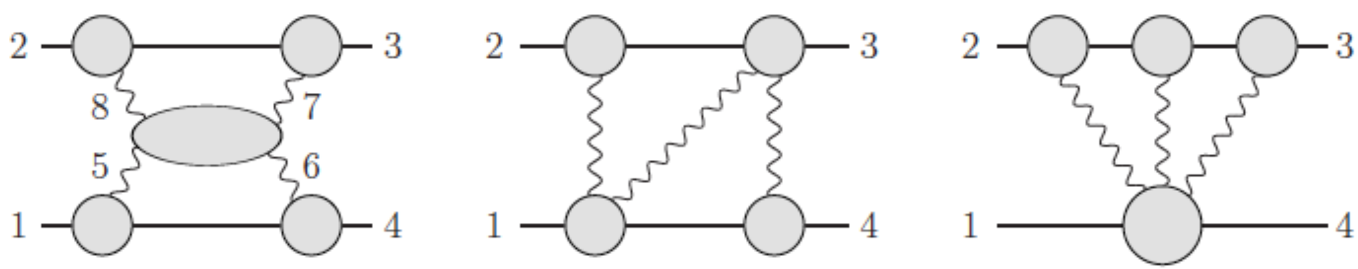
Neill and Rothstein ; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove; Cheung, Rothstein, Solon

Only independent unitarity cut for 2 PM.

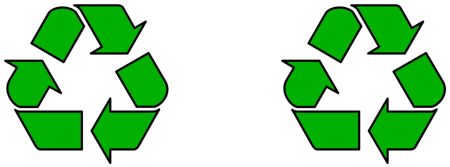


Treat exposed lines on-shell (long range).
Pieces we want are simple!

Independent generalized unitarity cuts for 3 PM.

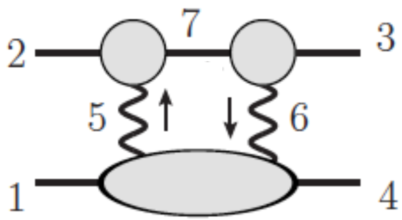


Our amplitude tools fit perfectly with extracting pieces we want.



gravity **loops**

Generalized Unitarity Cuts



2nd post-Minkowkian order

$$
\begin{aligned}
C_{\rm GR} &= \sum_{h_5,h_6=\pm} M_3^{\rm tree}(3^s, 6^{h_6}, -7^s)\, M_3^{\rm tree}(7^s, -5^{h_5}, 2^s)\, M_4^{\rm tree}(1^s, 5^{-h_5}, -6^{-h_6}, 4^s) \\
&= \sum_{h_5,h_6=\pm} it[A_3^{\rm tree}(3^s, 6^{h_6}, -7^s)\, A_3^{\rm tree}(7^s, -5^{h_5}, 2^s)\, A_4^{\rm tree}(1^s, 5^{-h_5}, -6^{-h_6}, 4^s)] \\
&\qquad \times [A_3^{\rm tree}(3^s, 6^{h_6}, -7^s)\, A_3^{\rm tree}(7^s, -5^{h_5}, 2^s)\, A_4^{\rm tree}(4^s, 5^{-h_5}, -6^{-h_6}, 1^s)]
\end{aligned}
$$

Problem of computing the generalized cuts in gravity is reduced to multiplying and summing gauge-theory tree amplitudes.

This is then appropriately integrated and processed into observables

Amplitude in Conservative Classical Potential Limit

ZB, Cheung, Roiban, Shen, Solon, Zeng (BCRSSZ)

The O(G^3) or 3PM conservative terms are:

$$\mathcal{M}_3 = \frac{\pi G^3 \nu^2 m^4 \log \boldsymbol{q}^2}{6\gamma^2 \xi}\Bigg[3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3 - \frac{48\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} - \frac{18\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{(1+\gamma)(1+\sigma)}\Bigg] + \frac{8\pi^3 G^3 \nu^4 m^6}{\gamma^4 \xi}\Big[3\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)F_1 - 32 m^2\nu^2\left(1-2\sigma^2\right)^3 F_2\Big]$$

$$m = m_A + m_B, \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$

$$\xi = E_1 E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1 \cdot p_2/m_1 m_2,$$

- **Amplitude remarkably compact.**
- **Arcsinh and the appearance of a mass singularity is new feature.**

  Di Vecchia, Heissenberg, Russo, Veneziano; Damour
- **Derived conservative scattering angle has simple mass dependence.**

  Antonelli, Buonanno, Steinhoff, van de Meent, Vines

  Comprehensive understanding: Damour

Conservative 3PM Hamiltonian

ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)

The 3PM Hamiltonian:

$$H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2+m_1^2}+\sqrt{\boldsymbol{p}^2+m_2^2}+V(\boldsymbol{p},\boldsymbol{r})$$

Newton in here

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{3} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i,$$

$$c_1 = \frac{\nu^2 m^2}{\gamma^2\xi}\left(1-2\sigma^2\right), \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2\xi}\left[\frac{3}{4}\left(1-5\sigma^2\right) - \frac{4\nu\sigma\left(1-2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1-2\sigma^2\right)^2}{2\gamma^3\xi^2}\right],$$

$$\begin{aligned} c_3 = \frac{\nu^2 m^4}{\gamma^2\xi}\Bigg[&\frac{1}{12}\left(3-6\nu+206\nu\sigma-54\sigma^2+108\nu\sigma^2+4\nu\sigma^3\right) - \frac{4\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} \\ &- \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7-20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3+8\gamma-3\xi-15\sigma^2-80\gamma\sigma^2+15\xi\sigma^2\right)\left(1-2\sigma^2\right)}{4\gamma^3\xi^2} \\ &+ \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\Bigg], \end{aligned}$$

$$m = m_A + m_B, \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$

$$\xi = E_1 E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1\cdot p_2/m_1 m_2,$$

How do we know it is right?

Original checks:

- **Compared to 4PN Hamiltonians after canonical transformation**
- **In test mass limit, $m_1 \ll m_2$, matches Schwarzschild Hamiltonian**

Damour, Jaranowski, Schäfer; Bernard, Blanchet, Bohé, Faye, Marsat

Thibault Damour seriously questioned correctness.
Specific corrections proposed. Damour, arXiv:1912.02139v1

New calculations confirm our 3PM result:

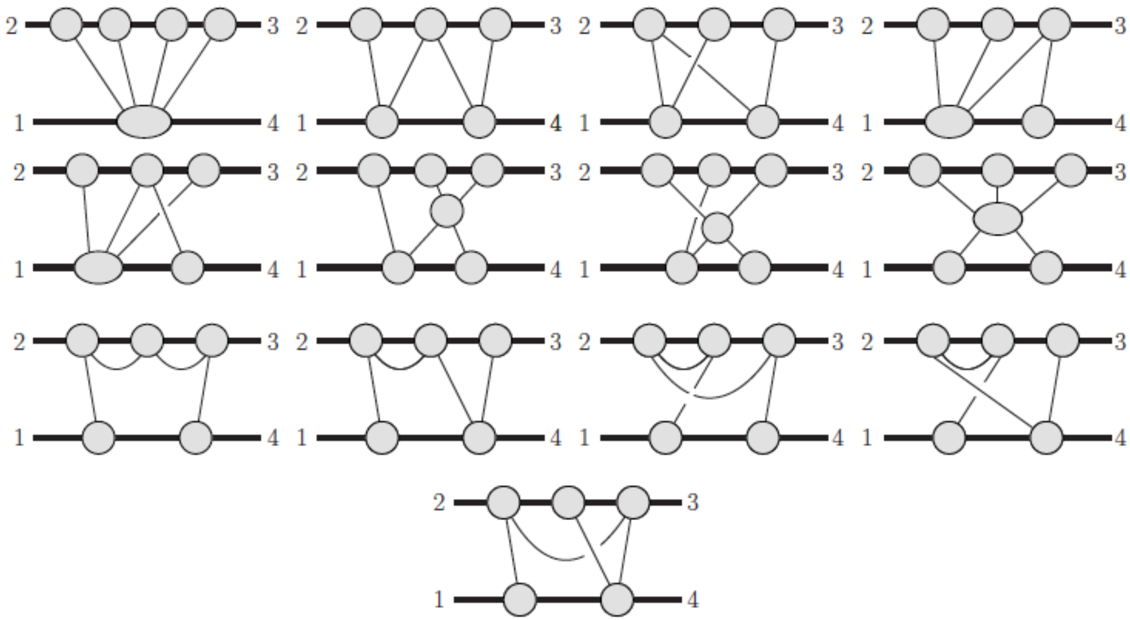
1. **Subsequent papers confirm our result in 6PN overlap.** Blümlein, Maier, Marquard, Schäfer; Bini, Damour, Geralico
2. **Calculations reproducing our 3PM results.** Cheung and Solon; Kälin, Liu, Porto; Bjerrum-Bohr, Damgaard, Planté, Vanhove
3. **Adding real radiation removes mass singularity.** Di Vecchia, Heissenberg, Russo, Veneziano; Damour

3PM results have passed highly nontrivial checks and careful scrutiny.

Higher Order Scalability: $O(G^4)$

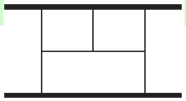
Methods scale well to higher orders

ZB, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng (2022)



New feature: Tail effect. Gravitons bounce off of curved space.
Small and large eccentricity orbits controlled by different local Hamiltonians.

Conservative Contribution O(G^4)



ZB, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng (2022)

O(G^4) amplitude **test particle** **1st self force** **Iteration. No need to compute**

$$\mathcal{M}_4^{\rm cons} = G^4 M^7 \nu^2 |\boldsymbol{q}| \pi^2 \left[\mathcal{M}_4^{\rm p} + \nu\left(4\mathcal{M}_4^{\rm t}\log\left(\tfrac{p_\infty}{2}\right) + \mathcal{M}_4^{\pi^2} + \mathcal{M}_4^{\rm rem}\right)\right] + \int_\ell \frac{\tilde{I}^4_{r,1}}{Z_1 Z_2 Z_3} + \int_\ell \frac{\tilde{I}^2_{r,1}\tilde{I}_{r,2}}{Z_1 Z_2} + \int_\ell \frac{\tilde{I}_{r,1}\tilde{I}_{r,3}}{Z_1} + \int_\ell \frac{\tilde{I}^2_{r,2}}{Z_1}$$

$D = 4 - 2\epsilon$ **tail effect**

$$\mathcal{M}_4^{\rm t} = r_1 + r_2 \log\left(\tfrac{\sigma+1}{2}\right) + r_3 \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}, \qquad \mathcal{M}_4^{\rm p} = -\frac{35\left(1-18\sigma^2+33\sigma^4\right)}{8\left(\sigma^2-1\right)}$$

$$\mathcal{M}_4^{\pi^2} = r_4\pi^2 + r_5\,\mathrm{K}\left(\tfrac{\sigma-1}{\sigma+1}\right)\mathrm{E}\left(\tfrac{\sigma-1}{\sigma+1}\right) + r_6\,\mathrm{K}^2\left(\tfrac{\sigma-1}{\sigma+1}\right) + r_7\,\mathrm{E}^2\left(\tfrac{\sigma-1}{\sigma+1}\right), \quad \longleftarrow \text{ elliptic}$$

$$\mathcal{M}_4^{\rm rem} = r_8 + r_9\log\left(\tfrac{\sigma+1}{2}\right) + r_{10}\frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} + r_{11}\log(\sigma) + r_{12}\log^2\left(\tfrac{\sigma+1}{2}\right) + r_{13}\frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\log\left(\tfrac{\sigma+1}{2}\right) + r_{14}\frac{\operatorname{arccosh}^2(\sigma)}{\sigma^2-1}$$
$$+ r_{15}\,\mathrm{Li}_2\left(\tfrac{1-\sigma}{2}\right) + r_{16}\,\mathrm{Li}_2\left(\tfrac{1-\sigma}{1+\sigma}\right) + r_{17}\frac{1}{\sqrt{\sigma^2-1}}\left[\mathrm{Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - \mathrm{Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right)\right].$$

$\nu = m_1 m_2/(m_1+m_2)^2$ $\quad \sigma = p_1\cdot p_2/m_1 m_2,$ $\quad r_{ij}$ rational coefficients

This is complete conservative contribution.

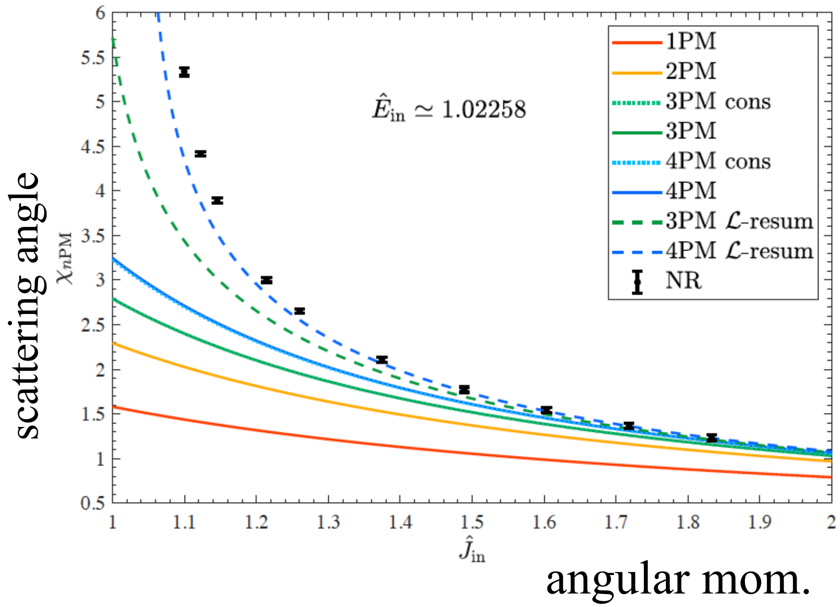
$$\mathcal{M}_4^{\rm radgrav,f} = \frac{12044}{75}p_\infty^2 + \frac{212077}{3675}p_\infty^4 + \frac{115917979}{793800}p_\infty^6 - \frac{9823091209}{76839840}p_\infty^8 + \frac{115240251793703}{1038874636800}p_\infty^{10} + \cdots$$

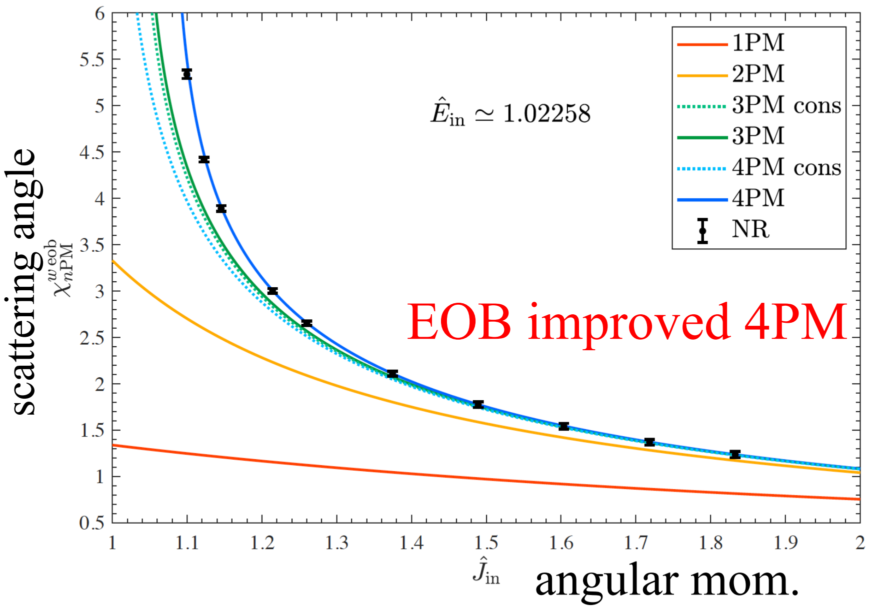
First 3 terms match 6PN results of Bini, Damour, Geralico!

- **Some potential subtlety with Bluemlein, Maier, Marquard, Schafer; Foffa, Sturani. — almost tracked down by Luz Almeida, Muller, Foffa, Sturani,**
- **Analytic continuation to bound case not trivial: tail effect.**

Comparison with Numerical Relativity

Khalil, Buonanno, Vines, Steinhoff; Damour and Rettegno





Plot uses:

4PM Conservative: ZB, Parra-Martinez, Roiban, Ruf. Shen, Solon, Zeng;
Damgaard, Hansen, Planté, Vanhove; Jakobsen, Gustav Mogull, Plefka, Sauer, Xu;
Bjerrum-Bohr, Plante, Vanhove.

4PM Dissipative: Manohar, Shen and Ridgeway; Dlapa, Kalen, Lui, Neef, Porto;
Damgaard, Hansen, Planté, Vanhove.

NR: Damour,. Guercilena, Hinder, Hopper, Nagar, Rezzolla;

- **Surprisingly good agreement with numerical relativity!**
- **Proves we are on a good track!**
- **Motivates us to go on 5 PM order.**

What is next?

There are many problems to work on:

1. **Tail effect causes nontrivial analytic continuation between unbound and bound cases starting at $O(G^4)$.**

   Bini, Damour; Dlapa, Kälin, Liu, R Porto

2. **Radiation.**

3. **Tidal effects.**

4. **Absorption of energy by black holes.**

5. **Push on to 5 PM $O(G^5)$. This is nontrivial. Crucial for future.**

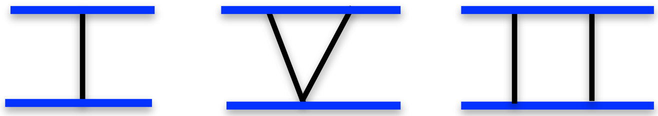
6. **Spin effects.**

Example 1: Towards 5PM

Structure of Higher Orders

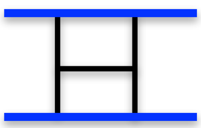
Moving up in orders of PM new effects and features encountered:

1PM and 2PM: **Fixed by geodesic motion, 0SF.**



$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta)\, d\phi^2$$

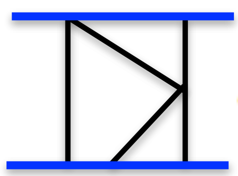
3PM: **Interesting structure in high energy limit. 1SF, m_1/m_2**



$\log(E^2/m_1 m_2)$ Cancels against real radiation

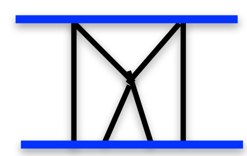
Di Vecchia, Heissenberg, Russo, Veneziano; Damour

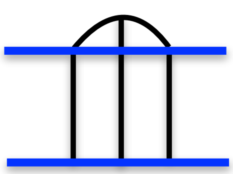
4PM: **Tail effect, nontrivial analytic continuations, elliptic integrals, *non*cancellation of poor high-energy behavior. Nonlocal in time effects.**



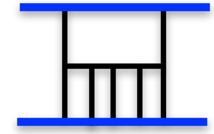
$$\sim K^2\left(\frac{\sigma - 1}{\sigma + 1}\right)$$

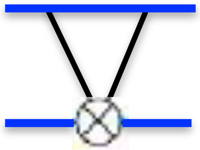
5PM: **2SF, new nontrivial functions, memory effect.**





6PM: **Mixing with tidal operators, UV divergences. Distinguish BHs from neutron stars.**





Example 1: Towards 5PM, $O(G^5)$

ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov

Scattering Amplitudes

Double copy
Generalized unitarity
Expansion in classical limit

Straightforward

Loop Integrand

Reduction to master integrals
DE’s for master integrals
Solutions of DEs.

Hard

Current Bottleneck

Integrated Amplitude

Eikonal, EFT matching computations
Amplitude action relation,
Pick your favorite formalism.

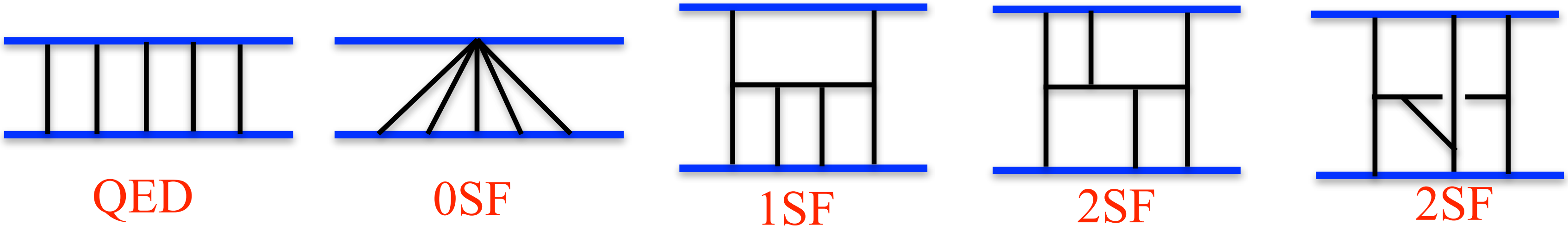
Straightforward

2-Body Hamiltonian or Observables

5PM problem nontrivial, so attack in stages.

Deal With in Stages

ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov



Stages:

1. **QED warmup. Potential mode contributions.** **Done.** ✔
2. **1 SF Conservative.** **Working on it.**
3. **2 SF Conservative.** **Hard, but in reach.**
4. **Radiative effects.** **Similar**

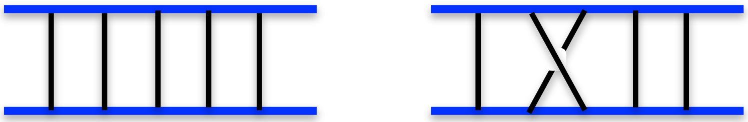
$$M_{5\mathrm{PM}} = M_{5\mathrm{PM}}^{0\mathrm{SF}} + \nu M_{5\mathrm{PM}}^{1\mathrm{SF}} + \nu^2 M_{\mathrm{PM}}^{2\mathrm{SF}} \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

- **Learn from each stage to push forward the next one.**

Electrodynamics Warmup

ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov (2023)

1. Electrodynamics is simpler: 23 families of integrals vs 395.
2. In 23 family overlap have *identical* complexity.
   — soft expansion makesgravity and electrodynamics difficulty similar.
3. Diagrams shared across families: 25 percent of electrodynamics integrals GR integrals.



Integral reduction: 10^6 integrals → 1107 master integrals in 23 families.

Standard tools work!

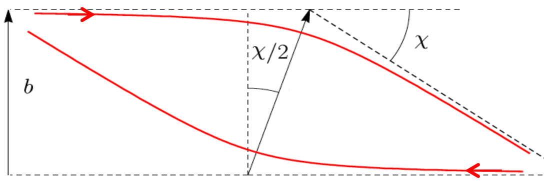
- FIRE6 + LiteRed, but tune code, algorithms and choice of master integrals.

A. Smirnov and Chuharev; Lee
V. Smirnov, Usovitsch

- Already have factor of ~ 1000 improvement compared to 4PM.

5th Order Scattering Angle (Potential)

ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov (2023)



$$\chi^{\rm 5PL}_{\rm pot} = \frac{\alpha^5 (m_1+m_2)^4}{30 J^5 E^4 (\sigma^2-1)^{5/2}} \times \left[r_0^{(0)} + \sum_{k=1}^{12} \left(\nu r_k^{(1)} + \nu^2 r_k^{(2)} \right) f_k \right]$$

Mass polynomiality, GSF expansion

Only potential modes.

$$r_1^{(1)} = \frac{15}{\sigma^2} - 208\sigma^6 + 128\sigma^5 - 625\sigma^4 - 320\sigma^3 + 705\sigma^2 + 240\sigma + 65\,,$$

$$r_2^{(1)} = \sqrt{\sigma^2-1}\left[-\frac{60\,(5\sigma^2-1)}{\sigma^3} - 80\sigma\,(16\sigma^2+23) \right],$$

$$r_3^{(1)} = \frac{90\,(6\sigma^2-1)}{\sigma^4} - 10\,(350\sigma^2+319)\,,$$

$$r_1^{(2)} = \frac{405\sigma\,(15-44\sigma^2)}{16\,(1-4\sigma^2)^2} - \frac{15\,(10\sigma^2+2\sigma-3)}{\sigma^3} + \frac{-2048\sigma^7+6656\sigma^6+17872\sigma^5+20000\sigma^4}{16} + \frac{-7740\sigma^3-22560\sigma^2-6635\sigma-2080}{16}\,,$$

$$r_2^{(2)} = \sqrt{\sigma^2-1}\left[\frac{45\,(1232\sigma^4-1168\sigma^2+287)}{16\,(4\sigma^2-1)^3} + \frac{30\,(20\sigma^3-9\sigma^2-4\sigma+3)}{\sigma^4} + \frac{5}{16}\,(1776\sigma^4+8192\sigma^3+10820\sigma^2+11776\sigma+3223) \right],$$

etc.

$$f_1 = 1\,,\quad f_2 = C_0^0(x)\,,\quad f_3 = C_{0,0}^{0,0}(x)\,,\quad f_4 = C_{0,0,0}^{0,0,0}(x)\,,$$

$$f_5 = -C_{1,0}^{0,0}(x) + C_{2,0}^{0,0}(x) + \frac{\pi^2}{4}\,,$$

$$f_6 = -C_{2,0}^{0,0}(x) + C_{4,0}^{1,0}(x) - \frac{\pi^2}{16}\,,$$

$$f_7 = C_{3,0}^{0,0}(x) + 2C_{3,0}^{1,0}(x) + C_{6,0}^{0,0}(x) - 2C_{6,0}^{1,0}(x) + \frac{\pi^2}{6}\,,$$

$$f_8 = -C_{0,1,0}^{0,0,0}(x) + C_{0,2,0}^{0,0,0}(x) + \frac{\pi^2}{4} C_0^0(x) + \frac{7\zeta_3}{2}\,,$$

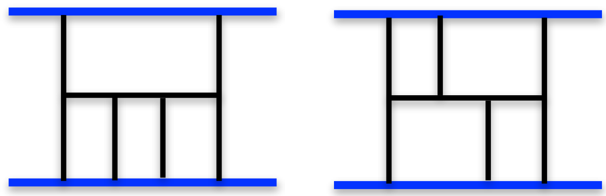
etc.

Cyclotomic polylogs are natural functions to use. Ablinger, Bluemlein, Schneider.

We can carry out 4-loop calculations using our setup.

Status of Gravity 5PM Conservative

ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov

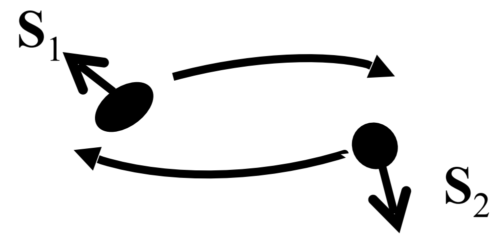


- **Key Message from QED: Standard tools work, with some care.**
- **Now marching on 5PM gravity.**
- **1SF terms in 5PM gravity being calculated, 2SF harder but doable.**

Example 2: Description of Spinning Objects in GR

Example 2: Spinning Objects

ZB, Luna, Roiban Shen, Zeng



Many hundreds of papers

Two spinning black holes or neutron stars.

- **Orbital angular momentum not conserved.**
- **Orbital motion complicated, not in a plane.**

Suppose we toss two spinning black holes or neutron stars at each other.

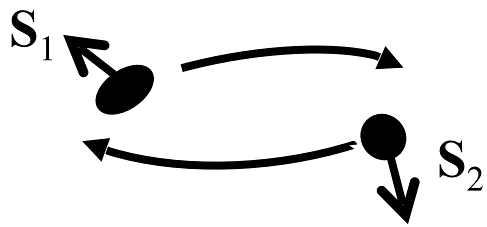
1. **What are the scattering angles?**
2. **How do spins and momenta change?**

Various approaches to spin

- **Worldline approaches** — Porto, Rothstein; Levi, Steinhoff.
- **Massive spinor helicity** — Arkani-Hamed, Huang; Chen, Chung, Huang, Kim
- **Heavy Mass EFT** — Damgaard, Haddad, Helset; Aoude, Haddad, Helset
- **Amplitudes for low spins.** — Vaydia; Febres Cordero,Lin, Ruf, Zeng
- **Amplitudes for generic spins.** — ZB, Luna, Roiban Shen, Zeng; ZB, Kosmopoulos, Luna, Roiban, Teng

Field Theory Formalism for General Spin

ZB, Luna, Roiban Shen, Zeng (2019)



Back in 2019 we set up a field theory formalism for dealing with problem of general spins.

Wilson coeffs

$$\mathcal{L} = \frac{1}{2}(-1)^s \phi_s(-\nabla^2 - m^2)\phi^s + \frac{H_2}{8} R_{abcd}\phi_s M^{ab} M^{cd} \phi^s - \frac{C_{ES^2}}{2m} R_{a f_1 b f_2} \nabla^a \phi_2 \mathbb{S}^{(f_1} \mathbb{S}^{f_2)} \nabla^b \phi_s + \ldots$$

$$\mathbb{S}^a \equiv \frac{-i}{2m} \epsilon^{abcd} M_{cd} \nabla(\omega)_b$$

Lorentz generators

Higher dimension operators

Rules for taking classical limit:

$$\varepsilon(\boldsymbol{s}, p_1) M^{ab} \varepsilon(\boldsymbol{s}, p_2) = S(p_1, \boldsymbol{S})^{ab}\, \varepsilon(\boldsymbol{s}, p_1) \cdot \varepsilon(\boldsymbol{s}, p_2) + \mathcal{O}(q^0)$$

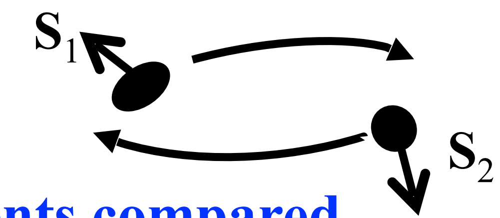
Some features:

- **Arbitrary spin field ϕ_s.**
- **For simplicity field not an irreducible representation.**

Puzzle at High Spin Order

Last year carried out calculations through $G^2 S^5$

ZB, Kosmopoulos, Luna, Roiban, Teng; Aoude, Haddad, Helset.



Puzzle: We have more independent Wilson coefficients compared to more standard worldline formulation.

$$\mathcal{L} = \frac{1}{2}(-1)^s \phi_s(-\nabla^2 - m^2)\phi^s + \frac{H_2}{8} R_{abcd}\phi_s M^{ab} M^{cd}\phi^s - \frac{C_{ES^2}}{2m} R_{af_1bf_2}\nabla^a \phi_2 \mathbb{S}^{(f_1}\mathbb{S}^{f_2)}\nabla^b \phi_s + \dots$$

Match to worldline terms in $G^2 S^3$ scattering:

$$\boxed{H_2 = 1 \qquad C_{ES^2} = C_2}$$

Why do we have more Wilson coefficients?

- **Physical effects on observables starting only at $G^2 S^3$.**
- **For black holes standard WL description looks complete.**
- **For generic objects such as neutron stars, extra coefficients.**
- **Only a single PN calculation has reached the order where this is relevant.**

Levi and Yin

Questions

ZB, Kosmopoulos, Luna, Roiban, Sheopner, Teng, Vines, to appear next week

1. **What is a complete description of a spinning body in GR?**
2. **Can one construct a worldline theory matching our field-theory?**
3. **Should a classical spin be modeled as a definite-spin field or as superposition of fields with different spins?**
4. **What two-body Hamiltonian matches theories with extra Wilson coefficients?**

Here we answer these questions

Electrodynamics as a Toy Model

ZB, Kosmopoulos, Luna, Roiban, Sheopner, Teng, Vines, to appear next week

Key simplification: **Similar to GR except extra Wilson coefficients affect observables linear in spin, not cubic.**

To track the origin compare multiple formalisms:

1) 2 types field theories.
- **Similar construction as earlier for GR, with reducible representation.**
- **Single transverse traceless quantum field in (*s*, *s*) Lorentz representation.**

2) Effect of spin supplementary condition (SSC) on worldlines. $\mathsf{S}^{\mu\nu} p_\nu = 0$

3) Extra Wilson coefficient and degrees of freedom for two-body Hamiltonians.

Kinetic Term for Field Theories

FT1:

$$\mathcal{L}_{\rm EM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad \mathcal{L}_{\rm min} = -(-1)^s\phi_s(D^2+m^2)\bar{\phi}_s$$

1. **Original construction (except E&M here, not gravity)**
2. **Has extra states. Not irreducible representation of spin.**
3. **Only meant to be interpreted in classical limit.**

FT2:

$$\mathcal{L}^s_{\rm min} = -(-1)^s\Big[\phi_s(D^2+m^2)\bar{\phi}_s + s(D\phi_s)(D\bar{\phi}_s) + \ldots\Big]$$

e.g.

$$\begin{aligned}\mathcal{L}_{s=3} = {} & \phi^{\mu_1\mu_2\mu_3}(D^2+m^2)\bar{\phi}_{\mu_1\mu_2\mu_3} + 3(D_\mu\phi^{\mu\mu_2\mu_3})(D^\nu\bar{\phi}_{\nu\mu_2\mu_3}) \\ & - 3\phi_\mu{}^{\mu\mu_3}(D^2+m^2)\bar{\phi}^\nu{}_{\nu\mu_3} + 3\phi_\mu{}^{\mu\mu_3}D^\rho D^\lambda\bar{\phi}_{\rho\lambda\mu_3} + 3\bar{\phi}^\mu{}_{\mu\mu_3}D_\rho D_\lambda\phi^{\rho\lambda\mu_3} \\ & + \frac{3}{2}(D_\mu\phi^{\mu\rho}{}_\rho)(D_\nu\bar{\phi}^{\nu\lambda}{}_\lambda) + 2\varphi(D^2+4m^2)\bar{\varphi} + m(\varphi D_\mu\bar{\phi}^{\mu\lambda}{}_\lambda + \bar{\varphi}D_\mu\phi^{\mu\lambda}{}_\lambda)\end{aligned}$$

FT1 ~ $\mathcal{L}^s_{\rm min} + \mathcal{L}^{s-1}_{\rm min} + \cdots$

Different field theories to probe different effects.

Standard World Line for Electrodynamics

ZB, Kosmopoulos, Luna, Roiban, Sheopner, Teng, Vines, to appear

Steinhoff's dynamical mass worldline formalism:

$$S[\mathsf{e},\xi,\chi,z,p,e,\mathsf{S}] = \int_{-\infty}^{\infty}\Big(-(p_\mu - QA_\mu)\dot z^\mu + \frac{1}{2}\mathsf{S}_{\mu\nu}\Omega^{\mu\nu} + \mathsf{e}(|p| - \mathcal{M}(z,\hat p,\mathsf{S}))$$

$$+\,\xi_\mu \mathsf{S}^{\mu\nu}p_\nu + \chi_\mu({e^\mu}_0 - \hat p^\mu)\Big)\,d\lambda\,.$$

single Wilson coefficient *O*(*S*)

Lagrange multiplier

$$\mathcal{M} = m - \frac{C_1}{2m}\mathsf{S}^{\mu\nu}F_{\mu\nu}$$

Other potential operator vanishes $\quad \frac{D_1}{m}\hat p^\mu \mathsf{S}^{\mu\nu}F_{\nu\rho}\hat p^\rho = 0$ by SSC

SSC: $\quad \mathsf{S}^{\mu\nu}\hat p_\nu = 0$

$\quad K^\mu = 0$

$$\hat p_\nu \equiv p_\nu/\sqrt{p^\mu p_\mu} \qquad \mathsf{S}^{\mu\nu} = \hat p^\mu K^\nu - K^\mu \hat p^\nu + \epsilon^{\mu\nu\rho\sigma}\hat p_\rho S_\sigma$$

Compare to field theory:

Two Wilson coefficient

$$(-1)^s\mathcal{L}_{\text{non-min}} = C_1 F_{\mu\nu}\phi_s M^{\mu\nu}\bar\phi_s + \frac{D_1}{m^2}F_{\mu\nu}(D_\rho\phi_s M^{\rho\mu}D^\nu\bar\phi_s + \text{c.c.})$$

At *O*(S) one more independent Wilson coefficient than worldline

Modified World Line for Electrodynamics

ZB, Kosmopoulos, Luna, Roiban, Sheopner, Teng, Vines, to appear

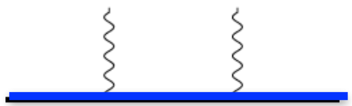
$$S[\mathsf{e},\xi,\chi,z,p,e,\mathsf{S}] = \int_{-\infty}^{\infty}\left(-(p_\mu - QA_\mu)\dot{z}^\mu + \frac{1}{2}\mathsf{S}_{\mu\nu}\Omega^{\mu\nu} + \mathsf{e}(|p| - \mathcal{M}(z,\hat{p},\mathsf{S}))\right)d\lambda$$

$$\mathcal{M} = m - \frac{C_1}{2m}\mathsf{S}^{\mu\nu}F_{\mu\nu} - \frac{D_1}{m}\hat{p}^\mu\mathsf{S}^{\mu\nu}F_{\nu\rho}\hat{p}^\rho$$

$$\hat{p}_\nu \equiv p_\nu/\sqrt{p^\mu p_\mu}$$

$$\mathsf{S}^{\mu\nu} = \hat{p}^\mu K^\nu - K^\mu\hat{p}^\nu + \epsilon^{\mu\nu\rho\sigma}\hat{p}_\rho S_\sigma$$

- **No SSC**
- **Matches FT in the number of Wilson coefficients.**
- **Compton amplitude matches field theory**



$$\begin{aligned}\mathcal{A}^{\mathrm{FT1g}}_{4,\,\mathrm{cl.}}\Big|_{S^1} = (-1)^s\mathcal{E}_1\cdot\bar{\mathcal{E}}_4\Bigg\{S(p_1)_{\mu\nu}\Bigg[&\frac{iC_1}{(p_1\cdot q_2)^2}(f_2^{\mu\nu}q_{2\rho}f_3^{\rho\lambda} + f_3^{\mu\nu}q_{3\rho}f_2^{\rho\lambda})p_{1\lambda} + \frac{2iC_1^2}{p_1\cdot q_2}f_2^{\nu\rho}f_{3\rho}{}^{\mu} \\ &+ \frac{2iD_1(2C_1 - D_1 - 2)}{(p_1\cdot q_2)m^2}p_{1\rho}f_2^{\rho\mu}f_3^{\nu\lambda}p_{1\lambda}\Bigg] + \frac{2K(p_1)_\mu p_{1\nu}}{m}\Bigg[\frac{D_1 - C_1}{(p_1\cdot q_2)^2}(q_2^\mu + q_3^\mu)f_2^{\nu\rho}f_{3\rho\lambda}p_1^\lambda \\ &- \frac{C_1(1 - C_1 + D_1)}{p_1\cdot q_2}(f_2^{\nu\rho}f_{3\rho}{}^{\mu} - f_3^{\nu\rho}f_{2\rho}{}^{\mu})\Bigg]\Bigg\}.\end{aligned}$$

$$f_i^{\mu\nu} \equiv \varepsilon_i^\mu q_i^\nu - \varepsilon_i^\nu q_i^\mu$$

2 Body Hamiltonian

usual

Extra

$$\mathcal{H}_2 = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V^{(0)}(\boldsymbol{r}^2, \boldsymbol{p}^2) + V^{(1)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\frac{\boldsymbol{L}\cdot\mathbf{S}_1}{\boldsymbol{r}^2} + V^{(2)}(\boldsymbol{r}^2, \boldsymbol{p}^2)\frac{\boldsymbol{r}\cdot\mathbf{K}_1}{\boldsymbol{r}^2}$$

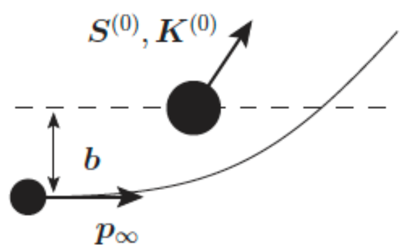
magnetic dipole

electric dipole
(For GR need K² and KS)

$$[S_{1,i}, S_{1,j}] = i\epsilon_{ijk}S_{1,k}\,, \qquad [S_{1,i}, K_{1,j}] = i\epsilon_{ijk}K_{1,k}\,,$$

$$[r_i, S_{1,j}] = [p_i, S_{1,j}] = 0 \qquad [r_i, K_{1,j}] = [p_i, K_{1,j}] = 0 \qquad [K_{1,i}, K_{1,j}] = -i\epsilon_{ijk}S_{1,k}\,,$$

To interpret Hamiltonian as classical:
replace commutators with Poisson bracket



Compton amplitudes from $\mathcal{H}_2$ match Compton from WL with no SSC & FT1. Also two boys scattering matches.

Results in Electrodynamics O(α^2 S)

1. Single quantum spin *s* state propagating, extra Wilson coefficients decouple.

Steinhoff and Kim; Haddad

2. If 2 or more states propagate with transitions:
   — Extra Wilson coefficients allowed.
   — Spin vector magnitude not preserved.

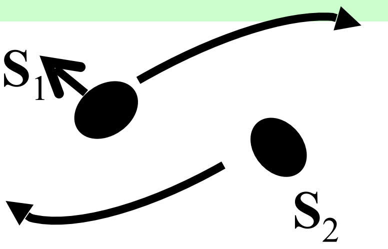
3. By introducing new worldline degrees of freedom by removing SSC match field theory with extra Wilson coefficients. Identical results.

4. EFT Hamiltonian that matches has extra degrees of freedom. S and K. Magnetic and electric dipoles.

Consistent picture from FT, WL and 2 body Hamiltonians:

At sufficiently high orders in perturbation theory (conservative) description of spin requires extra Wilson coefficients.

Return to Gravity



Now that we have tracked the origin in electrodynamics two problems:

1. **Need to redo analysis for gravity.**
2. **Identify physical systems via matching calculations where these effects are measurable.**

Example 3: Eikonal Phase for Spin

Example 3: Eikonal-Phase Insight

Recall eikonal phase, geometric optics. Scattering amplitudes:

slowly varying

See Di Vecchia's talk

$$\mathcal{M}(\boldsymbol{q}) \sim \int d^{2-2\epsilon}\boldsymbol{b}\, C(s, \boldsymbol{b}, \ldots) e^{i\chi(\boldsymbol{b}, s, \ldots)} e^{i\boldsymbol{b}\cdot\boldsymbol{q}}$$

eikonal phase

Dominant part: stationary phase approximation.

$$\Delta p = q = -\nabla_b \chi(b)$$

Change in momentum given by derivative of eikonal phase from stationary phase.

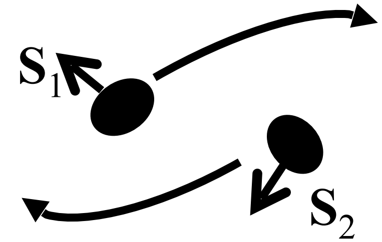
- **A completely natural object to study in classical limit of amplitudes**
- **Directly extracted from gauge-invariant amplitude.**

Expect there to be a scalar function—the eikonal phase—from which all classical scattering observables *with spin* can be extracted.

May seem bold, but we can explicitly check.

Example 3: Eikonal phase and spin.

ZB, Luna, Roiban, Shen, Zeng; ZB, Luna, Roiban, Sheopner, Teng, Vines



Observables in scattering with spin.

Poisson bracket

$$\Delta\mathbb{O} = \mathbb{O}(t=+\infty) - \mathbb{O}(t=-\infty)$$
$$= \{\chi, \mathbb{O}\} + \frac{1}{2}\{\chi, \{\chi, \mathbb{O}\}\} + \mathcal{D}_L(\chi, \{\chi, \mathbb{O}\}) - \frac{1}{2}\{\mathcal{D}_L(\chi, \chi), \mathbb{O}\} + \mathcal{O}(\chi^3)$$

$$\chi\mathcal{D}g = \chi g + \mathcal{D}_L(\chi, g)$$

$$\mathbb{O} \in \{\mathbf{p}_\perp, \mathbf{S}, \mathbf{K}\}$$

$$\mathcal{D}_L(f, g) = -\sum_{a=1}^{2} \epsilon_{ijk}\left(S_{ai}\frac{\partial f}{\partial S_{aj}} + K_{ai}\frac{\partial f}{\partial K_{aj}}\right)\frac{\partial g}{\partial L_k}$$

$$\{S_{1i}, S_{1j}\} = \epsilon_{ijk}S_{1k}\,, \quad \{S_{1i}, K_{1j}\} = \epsilon_{ijk}K_{1k}\,, \quad \{K_{1i}, K_{1j}\} = -\epsilon_{ijk}S_{1k}$$

eikonal phase

$$\chi = \frac{1}{4E|\boldsymbol{p}|}\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}(\mathbb{M}^{\text{1PL}} + \mathbb{M}_\triangle^{\text{2PL}}) + \mathcal{O}(\alpha^3)$$

Eikonal phase indeed encodes the motion (at least to checked order). At this order same as radial action. Might be more natural.

- **Remarkable that a single function encoded complicated dynamics.**
- **Can we find all order proof?**

Conjecture:

$$\Delta\mathbb{O} = e^{-\chi\mathcal{D}}\{\mathbb{O}, e^{\chi\mathcal{D}}\}$$

Outlook

Amplitude methods have a lot of promise and their use has already been tested for a variety of problems.

- **Pushing state of the art for high orders in *G*.** ZB, Cheung, Roiban, Parra-Martinez, Ruf, Shen, Solon, Zeng; Di Vecchia, Heissenberg, Russo, Veneziano; Damgaard, Hansen, Plante, Vanhove
- **Radiation.** Cristofoli, Gonzo, Kosower, O'Connell; Herrmann, Parra-Martinez, Ruf, Zeng; Di Vecchia, Heissenberg, Russo, Veneziano; Herderschee, Roiban, Teng; Georgoudis, Heissenberg , Vazquez-Holm; Brandhuber, Brown, Chen, De Angelis, Gowdy , Travaglini
- **Finite-size effects.** Cheung and Solon; Haddad and Helset; Kälin, Liu, Porto; Cheung, Shah, Solon; ZB, Parra-Martinez, Roiban, Sawyer, Shen
- **Spin.** Vaidya; Geuvara, O'Connell, Vines; Chung, Huang, Kim, Lee; ZB, Luna, Roiban, Shen, Zeng; Kosmopoulos, Luna; Febres Cordero, Kraus, Lin, Ruf, Zeng; Aoude Haddad, Helset; ZN, Kosmopoulos, Luna, Roiban, Teng; etc.
- **Dissipation.** Goldberger and Rothstein; Aoude, Ochirov

The standard quantities of interest for the inspiral phase can all be computed via amplitudes. Formalism far from exhausted.

Conclusions

Scattering amplitudes offer new perspective on gravitational-wave physics.

1. **Novel way to look at perturbative gravity.**
   — **Everything flows from graviton being a massless spin-2 particle.**
   — **Double copy shows gravity follows from gauge theory.**

2. **Example 1: High orders, marching on 5PM.**
   — **Electrodynamics warmup complete.**
   — **5PM integrands easy.**
   — **Marching on 5PM integrals. 1 GSF part under control. 2 GSF harder.**

3. **Example 2: Description of spin appears incomplete at higher orders.**
   — **Extra Wilson coefficients and degrees of freedom. Found in GR.**
   — **Electrodynamics studied in great detail.**
   — **Similar studies to be completed in gravity.**

4. **Example 3: Compact eikonal phase encodes physical scattering observables for spinning case.**

Amplitudes gives us new ways to think about gravitational waves.

Extra

Extra

Compton Amplitudes

$$
\begin{aligned}
\mathcal{A}^{\mathrm{FT1g}}_{4,\text{ cl.}}\Big|_{S^1} = (-1)^s \mathcal{E}_1 \cdot \bar{\mathcal{E}}_4 \Big\{ S(p_1)_{\mu\nu} \Big[& \frac{iC_1}{(p_1 \cdot q_2)^2} (f_2^{\mu\nu} q_{2\rho} f_3^{\rho\lambda} + f_3^{\mu\nu} q_{3\rho} f_2^{\rho\lambda}) p_{1\lambda} + \frac{2iC_1^2}{p_1 \cdot q_2} f_2^{\nu\rho} f_{3\rho}{}^{\mu} \\
& + \frac{2iD_1(2C_1 - D_1 - 2)}{(p_1 \cdot q_2) m^2} p_{1\rho} f_2^{\rho\mu} f_3^{\nu\lambda} p_{1\lambda} \Big] + \frac{2K(p_1)_\mu p_{1\nu}}{m} \Big[\frac{D_1 - C_1}{(p_1 \cdot q_2)^2} (q_2^\mu + q_3^\mu) f_2^{\nu\rho} f_{3\rho\lambda} p_1^\lambda \\
& - \frac{C_1(1 - C_1 + D_1)}{p_1 \cdot q_2} (f_2^{\nu\rho} f_{3\rho}{}^{\mu} - f_3^{\nu\rho} f_{2\rho}{}^{\mu}) \Big] \Big\} .
\end{aligned}
$$

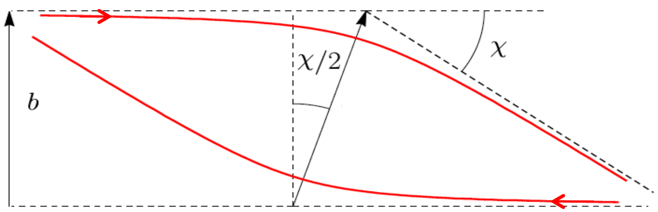
$$f_i^{\mu\nu} \equiv \varepsilon_i^\mu q_i^\nu - \varepsilon_i^\nu q_i^\mu$$

FT1 matches FT3 (positive norm) and matches modified WL2.

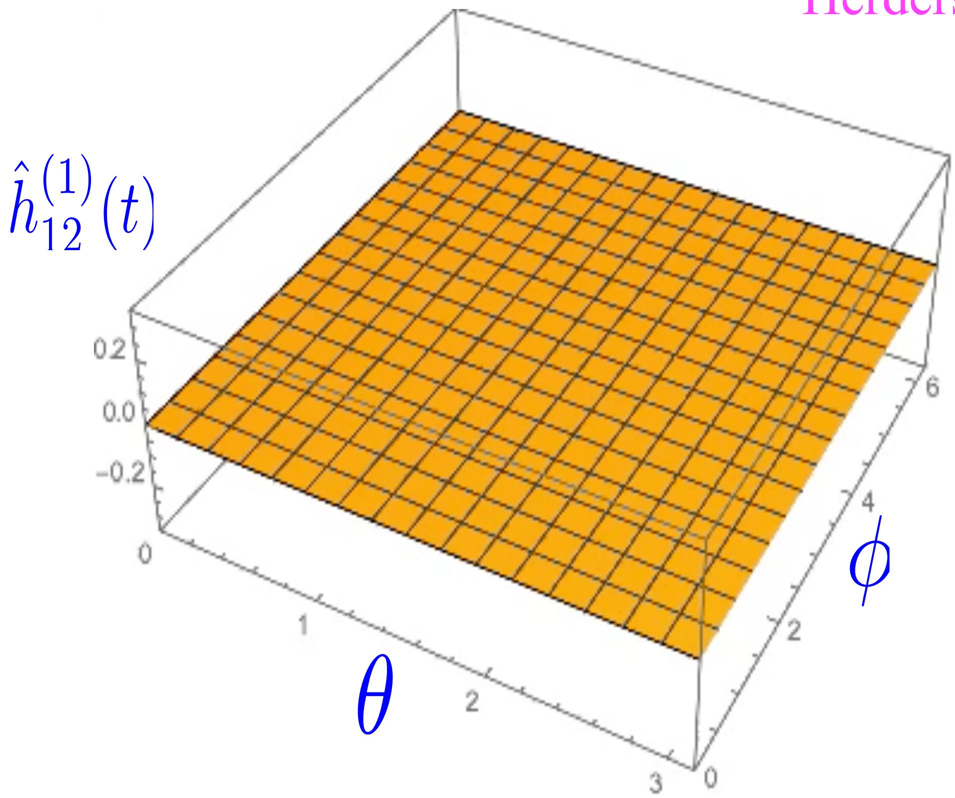
- **Multiple quantum spin states with transitions needed.**
- **Negative norm states not relevant in classical limit.**
- **Extra degrees of freedom needed on WL.**

Gravitational radiation from scattering two black holes

$$g_{\mu\nu}\Big|_{|\boldsymbol{x}|\to\infty} = \eta_{\mu\nu} + \frac{\kappa^2 \mathrm{M}}{8\pi|\boldsymbol{x}|}\left[\frac{\kappa^2\mathrm{M}}{\sqrt{-b^2}}\hat{h}^{(1)}_{\mu\nu} + \left(\frac{\kappa^2\mathrm{M}}{\sqrt{-b^2}}\right)^2 \hat{h}^{(2)}_{\mu\nu} + \ldots\right]$$



Herderschee, Roiban, Teng



Gravitational radiation at infinity.

Scattering amplitudes directly useful for determining radiation

Questions

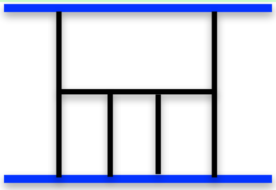
ZB, Kosmopoulos, Luna, Roiban, Sheopner, Teng, Vines, to appear next week

1. **What is a complete description of a spinning body in GR?**
2. **Can one construct a worldline theory matching our field-theory?**
3. **Our field-theory construction uses reducible representations of rotation group (some with negative norm). What happen if only a single (positive norm) quantum state propagates?**
4. **Can one build a field theory based on only positive-norm irreducible representation displaying extra Wilson coefficients?**
5. **Should a classical spin be modeled as a definite-spin field or as superposition of fields with different spins?**
6. **What two-body Hamiltonian matches theories with extra Wilson coefficients?**

Here we will answer these questions

High Loop Integration

ZB, Parra-Martinez, Roiban, Ruf. Shen, Solon, Zeng



Developed a hybrid approach:

1. **Method of regions to separate potential and radiation.**
   Beneke and Smirnov
2. **Nonrelativistic integration. Velocity expand and then mechanically integrate. Get first few orders in velocity. Boundary conditions.**
   Cheung, Rothstein, Solon
3. **Integration by parts and differential equations. Imported from QCD. Single scale integrals!** Chetyrkin, Tkachov; Laporta; Kotikov, Bern, Dixon and Kosower; Gehrmann, Remiddi.
   Parra-Martinez, Ruf, Zeng

IBP: $$0 = \int \prod_i^L \frac{d^D \ell_i}{(2\pi)^D} \frac{\partial}{\partial \ell_i^\mu} \frac{N^\mu(\ell_k, p_M)}{Z_1 \dots Z_n}$$

Solve linear relations between integrals in terms of master integrals.

Solve DEs either as series or basis of functions.

DEs: $$\frac{\partial}{\partial s_i} I_j^{\text{master}} = \text{simplified via IBP}$$

Many tools available: We use FIRE6 + LiteRed Smirnov, Chuharev; Lee

Spin Magnitude

- **When extra degrees of freedom present (and do not decouple).**
- **Spin vector magnitude can change (conservative).**

$$\frac{d}{dt}(S_{1\mu\nu}S_1^{\mu\nu}) = 0 \qquad \frac{d}{dt}\left(\mathbf{S}_1^2 - \mathbf{K}_1^2\right) = 0 \qquad c_1^{(2)} = \frac{m_2\gamma\,(-C_1 + D_1 + 1)}{4E_1E_2}$$

1PL (tree):

$$\Delta\mathbf{S}_1^2 = \Delta\mathbf{K}_1^2 = \frac{4\alpha E_1 E_2\left(K_{1z}^{(0)}S_{1y}^{(0)} - K_{1y}^{(0)}S_{1z}^{(0)}\right)c_1^{(2)}(p_\infty^2)}{b\,p_\infty(E_1+E_2)} + \mathcal{O}(\alpha^2)$$

Spin vector magnitude conserved at 1PL order if K(t=0) = 0

2PL:

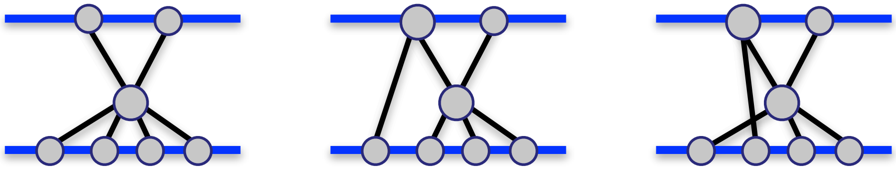
$$\Delta\mathbf{S}_1^2\Big|_{\mathbf{K}_1^{(0)}\to 0} = \Delta\mathbf{K}_1^2\Big|_{\mathbf{K}_1^{(0)}\to 0} = \frac{4\alpha^2E_1^2E_2^2\left(\left(S_{1y}^{(0)}\right)^2 + \left(S_{1z}^{(0)}\right)^2\right)\left(c_1^{(2)}(p_\infty^2)\right)^2}{b^2\,p_\infty^2(E_1+E_2)^2} + \mathcal{O}(\alpha^3)$$

Spin vector magnitude *not* conserved at 2PL order

Extra Wilson coefficients linked to changes in spin vector magnitude

Towards 5PM

ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov



+ 48 other independent generalize unitarity cuts

- **Integrand construction straightforward using generalized unitarity even at 6PM (five loops).**
  — generalized unitarity
  — double copy

Done

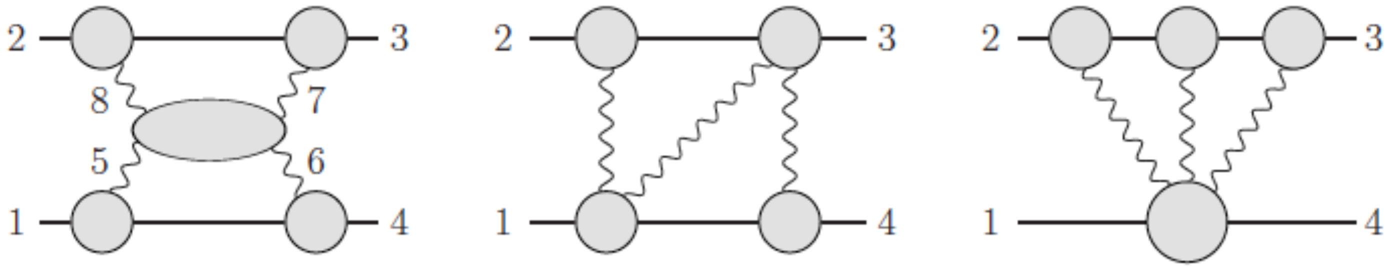
- **Construction of master integrals and DEs**
  — 5PM master integrals and DEs all known except for a few
  — Series solutions straightforward, analytic results harder.

Mostly done

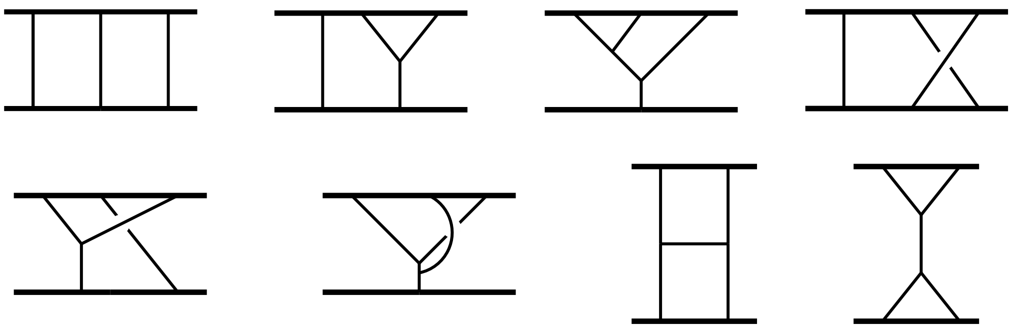
- **Integral reduction of amplitude master integrals.**
  — only difficult step.
  — Key realization: with some care standard methods work.
  — improve coding, choices of masters, and ibp system chosen carefully.

5PM problem nontrivial, so attack in stages.

Two Loops and 3 PM



- More complicated than one loop, but no problem.
- To interface easily with integration, rearrange unitarity cuts into conventional-looking diagrams.



Results

$$(-1)^s \mathcal{L}_{\text{non-min}} = QC_1 F_{\mu\nu}\phi_s M^{\mu\nu}\bar{\phi}_s + \frac{QD_1}{m^2} F_{\mu\nu}(D_\rho\phi_s M^{\rho\mu} D^\nu\bar{\phi}_s + \text{c.c}) - \frac{iQC_2}{2m^2}\partial_{(\mu}F_{\nu)\rho}(D^\rho\phi_s \mathbb{S}^\mu\mathbb{S}^\nu\bar{\phi}_s - \text{c.c}) - \frac{iQD_2}{2m^2}\partial_\mu F_{\nu\rho}(D_\alpha\phi_s M^{\alpha\mu}M^{\nu\rho}\bar{\phi}_s - \text{c.c})$$

Field theory	Lagrangian	Amplitude	External state
FT1	$\mathcal{L}_{\text{EM}} + \mathcal{L}_{\text{min}} + \mathcal{L}_{\text{non-min}}$	$\mathcal{A}^{\text{FT1s}}$	spin-s
		$\mathcal{A}^{\text{FT1g}}$	generic
FT2	$\mathcal{L}_{\text{EM}} + \mathcal{L}^s_{\text{min}} + \mathcal{L}_{\text{non-min}}$	$\mathcal{A}^{\text{FT2}}$	spin-s
FT3	$\mathcal{L}_{\text{EM}} + \mathcal{L}^{s,s-1}_{\text{min}} + \mathcal{L}^{s,s-1}_{\text{non-min}}$	$\mathcal{A}^{\text{FT3s}}$	spin-s
		$\mathcal{A}^{\text{FT3g}}$	indefinite spin

4 Loop Integrals

IBPs used to create differential equations for master integrals:

$$\partial_x \vec{\mathcal{I}}(x,\epsilon) = \epsilon \sum_{w\in\mathbb{W}} w(x)\, A_w\, \vec{\mathcal{I}}(x,\epsilon)$$

$$\mathbb{W} = \left\{ \frac{1}{x}, \frac{1}{1+x}, \frac{1}{x-1}, \frac{2x}{1+x^2}, \frac{1+2x}{1+x+x^2}, \frac{2x-1}{1-x+x^2} \right\}$$

$$\sigma = \frac{p_1 \cdot p_2}{m_1 m_2} = \frac{1}{(1-v^2)^{1/2}}$$

$$\sigma = \frac{1+x^2}{2x} + O(q^2)$$

Cyclotomic polylogs are natural functions to use.

Ablinger, Bluemlein, Schneider.

cyclotomic polylog

$$f_1 = 1\,,\ f_2 = C_0^0(x)\,,\ f_3 = C_{0,0}^{0,0}(x)\,,\ f_4 = C_{0,0,0}^{0,0,0}(x)\,,$$

$$f_5 = -C_{1,0}^{0,0}(x) + C_{2,0}^{0,0}(x) + \frac{\pi^2}{4}\,,$$

$$f_6 = -C_{2,0}^{0,0}(x) + C_{4,0}^{1,0}(x) - \frac{\pi^2}{16}\,,$$

$$f_7 = C_{3,0}^{0,0}(x) + 2C_{3,0}^{1,0}(x) + C_{6,0}^{0,0}(x) - 2C_{6,0}^{1,0}(x) + \frac{\pi^2}{6}\,,$$

$$f_8 = -C_{0,1,0}^{0,0,0}(x) + C_{0,2,0}^{0,0,0}(x) + \frac{\pi^2}{4}C_0^0(x) + \frac{7\zeta_3}{2}\,,$$

$$f_9 = -C_{0,2,0}^{0,0,0}(x) + C_{0,4,0}^{0,1,0}(x) - \frac{\pi^2}{16}C_0^0(x) - \frac{21\zeta_3}{16}\,,$$

$$f_{10} = C_{0,3,0}^{0,0,0}(x) + 2C_{0,3,0}^{0,1,0}(x) + C_{0,6,0}^{0,0,0}(x) - 2C_{0,6,0}^{0,1,0}(x) + \frac{1}{6}\pi^2 C_0^0(x) + \frac{28\zeta_3}{9}\,,$$

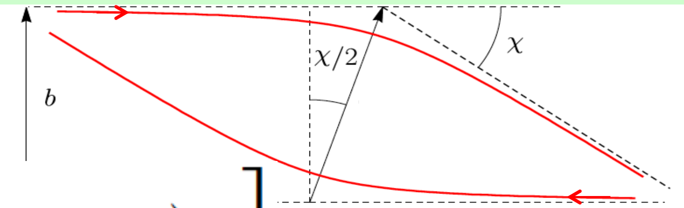
$$f_{11} = -C_{1,0,0}^{0,0,0}(x) + C_{2,0,0}^{0,0,0}(x) - \frac{7\zeta_3}{4}\,,$$

$$f_{12} = -C_{2,0,0}^{0,0,0}(x) + C_{4,0,0}^{1,0,0}(x) + \frac{21\zeta_3}{32}\,.$$

All functions in amplitude can be rewritten in terms of ordinary polylogs

5 PL Scattering Angle (Potential)

5th post-Lorentzian



$$\chi^{5\mathrm{PL}}_{\mathrm{pot}} = \frac{\alpha^5 (m_1+m_2)^4}{30 J^5 E^4 (\sigma^2-1)^{5/2}} \times \left[r_0^{(0)} + \sum_{k=1}^{12} \left(\nu r_k^{(1)} + \nu^2 r_k^{(2)} \right) f_k \right]$$

Only potential modes.

Mass polynomiality, GSF expansion

$$\begin{aligned}
r_1^{(1)} &= \frac{15}{\sigma^2} - 208\sigma^6 + 128\sigma^5 - 625\sigma^4 - 320\sigma^3 + 705\sigma^2 \\
&\quad + 240\sigma + 65\,, \\
r_2^{(1)} &= \sqrt{\sigma^2-1}\left[-\frac{60\,(5\sigma^2-1)}{\sigma^3} - 80\sigma\,(16\sigma^2+23) \right], \\
r_3^{(1)} &= \frac{90\,(6\sigma^2-1)}{\sigma^4} - 10\,(350\sigma^2+319)\,, \\
r_4^{(1)} &= -\frac{5760\sigma}{\sqrt{\sigma^2-1}}\,, \\
r_5^{(1)} &= 120\,(\sigma^2-1)^{3/2}\,(2\sigma^2-1)\,, \\
r_8^{(1)} &= 120\,(\sigma^2-1)\,(\sigma^2+\sigma-1)\,,
\end{aligned}$$

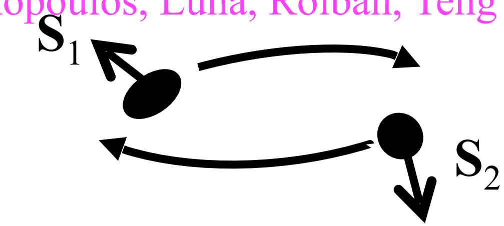
$$\begin{aligned}
r_9^{(1)} &= r_{12}^{(1)} = 240(\sigma^2-1)^2\,, \\
r_{11}^{(1)} &= 120(\sigma^2-1)(\sigma^2+2\sigma-1)\,, \\
r_6^{(1)} &= r_7^{(1)} = r_{10}^{(1)} = 0\,,
\end{aligned}$$

$$\begin{aligned}
r_1^{(2)} &= \frac{405\sigma\,(15-44\sigma^2)}{16\,(1-4\sigma^2)^2} - \frac{15\,(10\sigma^2+2\sigma-3)}{\sigma^3} \\
&\quad + \frac{-2048\sigma^7+6656\sigma^6+17872\sigma^5+20000\sigma^4}{16} \\
&\quad + \frac{-7740\sigma^3-22560\sigma^2-6635\sigma-2080}{16}\,, \\
r_2^{(2)} &= \sqrt{\sigma^2-1}\Bigg[\frac{45\,(1232\sigma^4-1168\sigma^2+287)}{16\,(4\sigma^2-1)^3} \\
&\quad + \frac{30\,(20\sigma^3-9\sigma^2-4\sigma+3)}{\sigma^4} \\
&\quad + \frac{5}{16}\,(1776\sigma^4+8192\sigma^3+10820\sigma^2+11776\sigma+3223) \Bigg], \\
r_3^{(2)} &= -\frac{30\,(16\sigma^4+36\sigma^3-11\sigma^2-6\sigma+3)}{\sigma^5} \\
&\quad + 20\,(212\sigma^3+350\sigma^2+328\sigma+319)\,, \\
r_4^{(2)} &= \frac{2880(\sigma+1)(3\sigma+1)}{\sqrt{\sigma^2-1}}\,, \\
r_6^{(2)} &= 480\,(\sigma^2-1)^{3/2}\,(2\sigma^2-1)\,, \\
r_7^{(2)} &= 45\sigma\,(\sigma^2-1)^{5/2}\,, \\
r_9^{(2)} &= -480\,(\sigma^2-1)\,(\sigma^2-\sigma-1)\,, \\
r_{10}^{(2)} &= -135\,(\sigma^2-1)^2\,, \\
r_{12}^{(2)} &= -480\,(\sigma^2-1)\,(\sigma^2-2\sigma-1)\,, \\
r_5^{(2)} &= r_8^{(2)} = r_{11}^{(2)} = 0\,.
\end{aligned}$$

We can get to end of a 4 loop calculation. Integrals doable!

Basic Puzzle

ZB, Kosmopoulos, Luna, Roiban, Teng



How can it be that the basic WL discription of conservative dynamics of spinning objects has missing interactions when compared to the field theory?

It seems surprising:

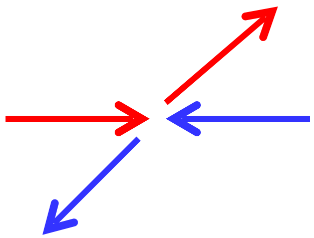
No evidence in the many dozens of PN spin studies that anything is missing.
Origin of the SSC and spin gauge symmetry seems completely reasonable.
You might worry that negative norm states are a problem in classical limit.
You might worry that high spin states in our constructio are not transverse is a problem.

On the other hand it evades checks:

Effects disappears for blackholes. Extra Wilson coefficients play no role (to the order where we work).
So far, only a single WL paper has reached high enough order to even have a chance to find anything.

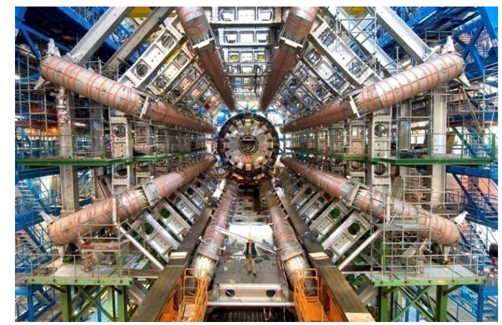
Quantum Field Theory and Scattering Amplitudes

Scattering amplitudes give us quantum mechanical description of events at particle colliders.

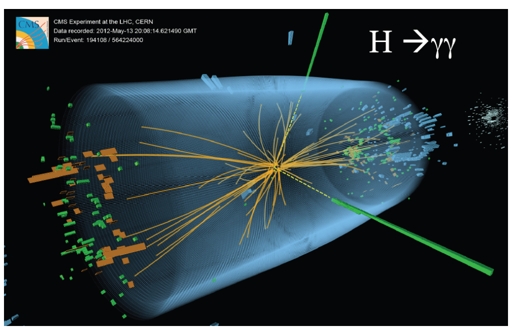


particle scattering

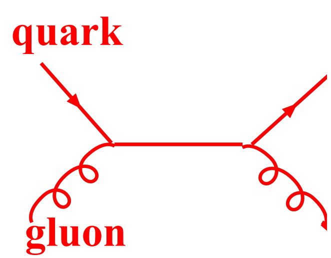
Large Hadron Collider



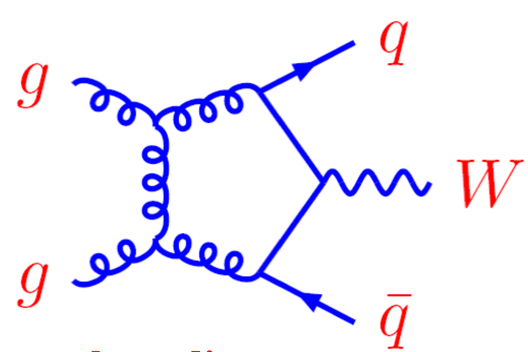
ATLAS Detector



Higgs boson event



Tree Feynman diagram



loop diagram
higher order

At first sight, does not seem to have much to do with gravitational waves

What are we after?

- Replace scattering in General Relativity with a two body potential that is easy to use in bound-state problem.
- Extract physics juice, leaving behind complexity of

  general relativity.

$$V(\mathbf{r}, \mathbf{p}) = -\frac{G m_1 m_2}{r} + \cdots$$

Just like Newton's potential, except:

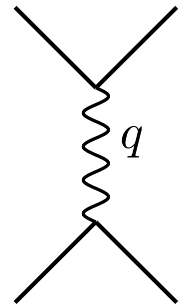
- **Compatible with special relativity (all orders in velocity)**
- **Valid through O(G^5).**

2 Body Potentials and Amplitudes

Iwasaki; Gupta, Radford; Donoghue; Holstein, Donoghue; Holstein and A. Ross; Bjerrum-Bohr, Donoghue, Vanhove; Neill, Rothstein; Bjerrum-Bohr, Damgaard, Festuccia, Planté. Vanhove; Chueng, Rothstein, Solon; Chung, Huang, Kim, Lee; etc.

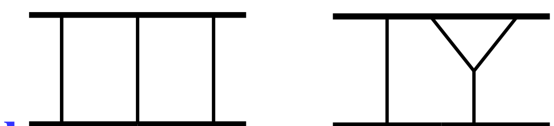
Tree-level: Fourier transform gives classical potential.

$$V(r) \sim \int \frac{d^3 q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} A^{\text{tree}}(\mathbf{q})$$



Newtonian potential follows from Feynman diagrams

Beyond 1 loop things quickly become much less obvious:



- **What I learned in grad school on $\hbar$ and cl**
  Loops have classical pieces.
- $1/\hbar^L$ **scaling of at L loop.**
- **Double counting and iteration.**
- **Cross terms between $1/\hbar$ and $\hbar$.**

$$e^{iS_{\text{classical}}/\hbar}$$

Piece of loops are classical: Our task is to efficiently extract these pieces.

We harness EFT to clean up confusion

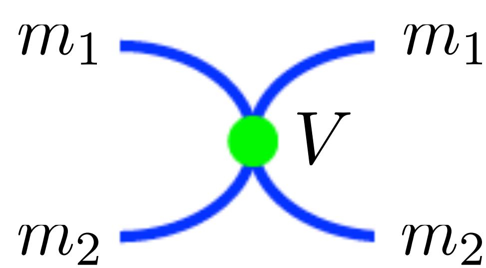
Effective Field Theory is a Clean Approach

Build EFT from which we can read off potential.
Want a Newtonian-like potential,
with GR corrections

Goldberger and Rothstein
Neill, Rothstein
Cheung, Rothstein, Solon (2018)

$$L_{\rm kin} = \int_{\boldsymbol{k}} A^\dagger(-\boldsymbol{k})\left(i\partial_t + \sqrt{\boldsymbol{k}^2 + m_A^2}\right)A(\boldsymbol{k}) + \int_{\boldsymbol{k}} B^\dagger(-\boldsymbol{k})\left(i\partial_t + \sqrt{\boldsymbol{k}^2 + m_B^2}\right)B(\boldsymbol{k})$$

A, B scalars represents spinless black holes



$$L_{\rm int} = -\int_{\boldsymbol{k},\boldsymbol{k}'} V(\boldsymbol{k},\boldsymbol{k}')\, A^\dagger(\boldsymbol{k}')A(\boldsymbol{k})B^\dagger(-\boldsymbol{k}')B(-\boldsymbol{k})$$

potential we want to obtain

$$H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p},\boldsymbol{r})$$

2 body Hamiltonian in c.o.m. frame.

Match amplitudes of this theory to the full theory in classical limit to extract a classical potential of the type Newton would like.

Our gravitational-wave theory friends want Hamiltonians.

4 Loop Integrals

ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov

IBPs used to create differential equations for master integrals:

cyclotomic polylog

$$\partial_x \vec{\mathcal{I}}(x,\epsilon) = \epsilon \sum_{w \in \mathbb{W}} w(x)\, A_w\, \vec{\mathcal{I}}(x,\epsilon)$$

$$\mathbb{W} = \left\{ \frac{1}{x}, \frac{1}{1+x}, \frac{1}{x-1}, \frac{2x}{1+x^2}, \frac{1+2x}{1+x+x^2}, \frac{2x-1}{1-x+x^2} \right\}$$

$$\sigma = \frac{p_1 \cdot p_2}{m_1 m_2} = \frac{1}{(1-v^2)^{1/2}}$$

$$\sigma = \frac{1+x^2}{2x} + O(q^2)$$

$$f_1 = 1\,, \quad f_2 = C_0^0(x)\,, \quad f_3 = C_{0,0}^{0,0}(x)\,, \quad f_4 = C_{0,0,0}^{0,0,0}(x)\,,$$

$$f_5 = -C_{1,0}^{0,0}(x) + C_{2,0}^{0,0}(x) + \frac{\pi^2}{4}\,,$$

$$f_6 = -C_{2,0}^{0,0}(x) + C_{4,0}^{1,0}(x) - \frac{\pi^2}{16}\,,$$

$$f_7 = C_{3,0}^{0,0}(x) + 2C_{3,0}^{1,0}(x) + C_{6,0}^{0,0}(x) - 2C_{6,0}^{1,0}(x) + \frac{\pi^2}{6}\,,$$

$$f_8 = -C_{0,1,0}^{0,0,0}(x) + C_{0,2,0}^{0,0,0}(x) + \frac{\pi^2}{4} C_0^0(x) + \frac{7\zeta_3}{2}\,,$$

Cyclotomic polylogs are natural functions to use.

Ablinger, Bluemlein, Schneider.

All functions in amplitude can be rewritten in terms of ordinary polylogs

EFT Matching

Cheung, Rothstein, Solon

full general relativity
(complicated)

Amplitude methods
double copy

tree amplitude

generalized
unitarity

$\hbar \to 0$

loop integrand

loop
integration

GR loop amplitude

identical physics

$=$

effective theory
(simpler)

build
ansatz

Potential ***V(r)***

Feynman
diagrams

loop integrand

loop
integration

EFT loop amplitude

Roundabout, but robust mean to extracts potential.
New methods bypass this, directly giving radial action.

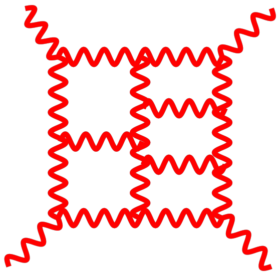
Summary

In a very precise sense:

Gravity ~ (gauge theory) x (gauge theory)

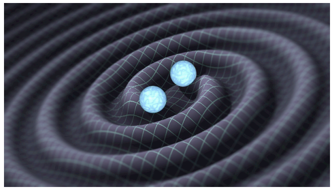
- **Gives us a good way to carry out calculations.**
- **Use it to do difficult calculations, to answer questions of physical interest.**

Examples:

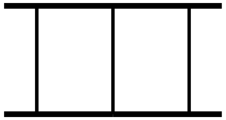


- **5 loop supergravity to study nonrenormalizability of gravity theories.**

ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng (2018)



- **G^4 corrections to Newton's potential from GR.**



ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)
ZB, Parra-Martinez, Roiban, Ruf. Shen, Solon, Zeng (2021)

KLT Relation Between Gravity and Gauge Theory

KLT (1985)

Kawai-Lewellen-Tye string relations in low-energy limit:

gravity

gauge-theory color ordered

$$M_4^{\rm tree}(1,2,3,4) = -i s_{12} A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3)\,,$$

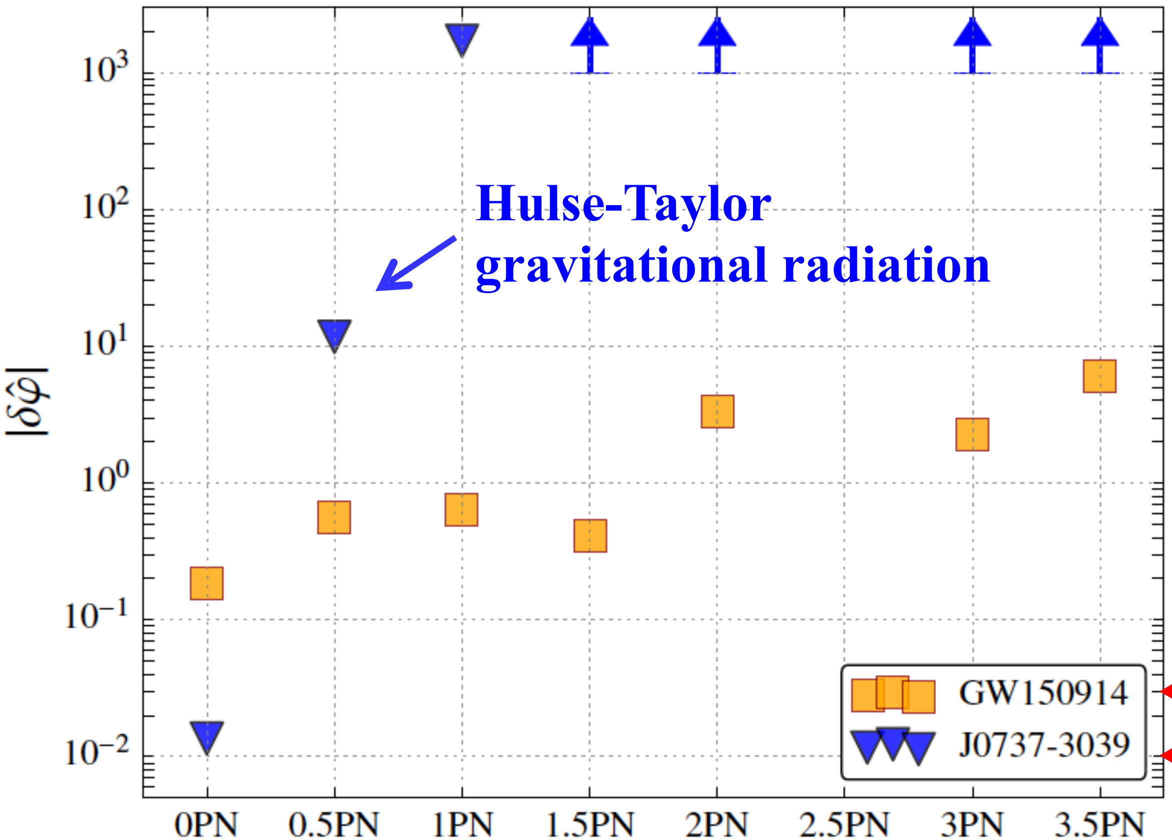
$$M_5^{\rm tree}(1,2,3,4,5) = i s_{12} s_{34} A_5^{\rm tree}(1,2,3,4,5)\, A_5^{\rm tree}(2,1,4,3,5) + i s_{13} s_{24} A_5^{\rm tree}(1,3,2,4,5)\, A_5^{\rm tree}(3,1,4,2,5)$$

Generalizes to explicit all-leg form. ZB, Dixon, Perelstein, Rozowsky

1. **Gravity is derivable from gauge theory.**
2. **Standard Lagrangian methods offer no hint why this is possible.**
3. **It is very general property of gravity.**

Importance of higher orders for LIGO/Virgo

LIGO/Virgo Collaboration arXiv:1602.03841



LIGO

Binary pulsar

Newton (1686)	Einstein (1938)	Ohta (1974) Damour (1981)	Blanchet (2000) Damour (2000)	Blanchet (2015) Damour (2014)

Binary pulsar confirms quadrupole radiation formula and not much else.

LIGO/Virgo tests PN terms from GR

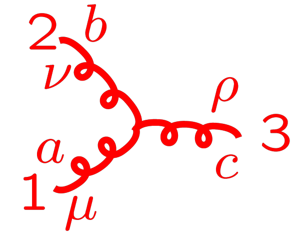
LIGO/Virgo sensitive to high PN orders.

Duality Between Color and Kinematics

ZB, Carrasco, Johansson (2007)

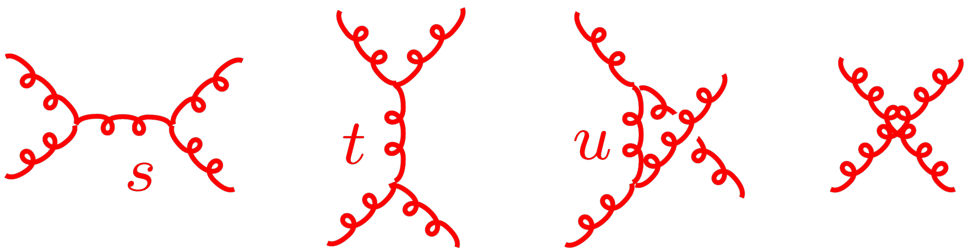
coupling constant **color factor** **momentum dependent kinematic factor**

$$-g f^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$



Color factors based on a Lie algebra: $[T^a, T^b] = i f^{abc} T^c$

Jacobi Identity $f^{a_1 a_2 b} f^{b a_4 a_3} + f^{a_4 a_2 b} f^{b a_3 a_1} + f^{a_4 a_1 b} f^{b a_2 a_3} = 0$



Use 1 = *s/s* = *t/t* = *u/u* to assign 4-point diagram to others.

$s = (k_1 + k_2)^2 \quad t = (k_1 + k_4)^2$

$u = (k_1 + k_3)^2$

$$\mathcal{A}_4^{\text{tree}} = g^2 \left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u} \right)$$

Color factors satisfy Jacobi identity: $c_u = c_s - c_t$

Numerator factors satisfy similar identity: $n_u = n_s - n_t$

Proven at tree level and extends to higher points

ZB, Carrasco, Johansson; Kiermaier; Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove; Cachazo, etc

Post Newtonian Approximation

For orbital mechanics:

$$v^2 \sim \frac{GM}{R} \ll 1$$

Expand in *G* and v^2

virial theorem

In center of mass frame:

$$m = m_A + m_B, \quad \nu = \mu/M,$$

$$\mu = m_A m_B/m, \quad P_R = P \cdot \hat{R}$$

$$\frac{H}{\mu} = \frac{P^2}{2} - \frac{Gm}{R}$$

Newton

$$+ \frac{1}{c^2}\left\{ -\frac{P^4}{8} + \frac{3\,\nu\,P^4}{8} + \frac{Gm}{R}\left(-\frac{{P_R}^2\,\nu}{2} - \frac{3\,P^2}{2} - \frac{\nu\,P^2}{2}\right) + \frac{G^2m^2}{2R^2}\right\}$$

$$+ \ldots$$

1PN: Einstein, Infeld, Hoffmann; Droste, Lorentz

Hamiltonian known to 4PN order.

2PN: Ohta, Okamura, Kimura and Hiida.

3PN: Damour, Jaranowski and Schaefer; L. Blanchet and G. Faye.

4PN: Damour, Jaranowski and Schaefer; Foffa (2017), Porto, Rothstein, Sturani (2019).