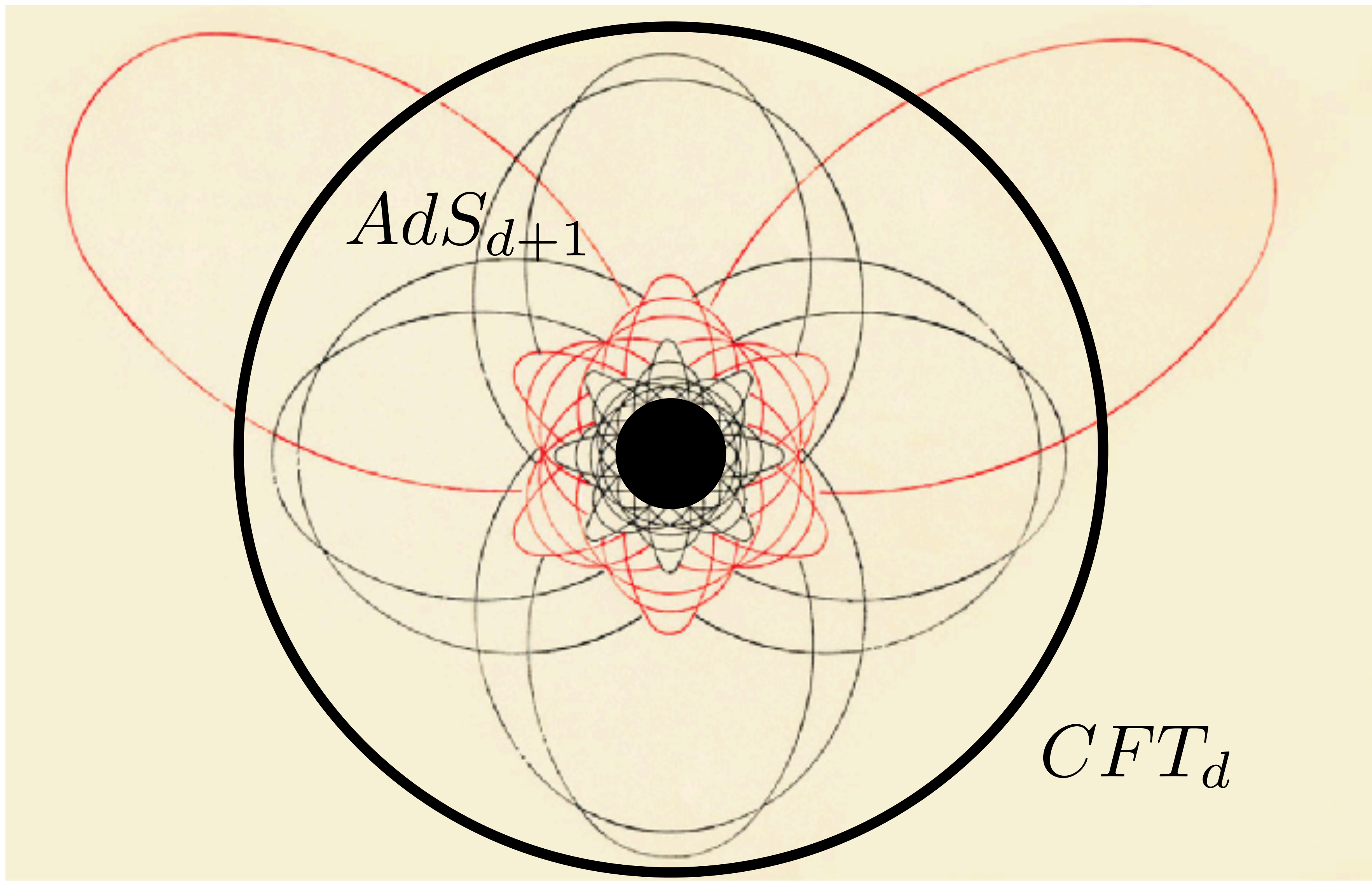
Multi-stress tensor dynamics in four-dimensional CFTs

A. Zhiboedov (CERN)
Bohr-100: Current themes in theoretical physics, Copenhagen

[with M. Dodelson, A. Grassi, C. Iossa, D. Panea Lichtig]

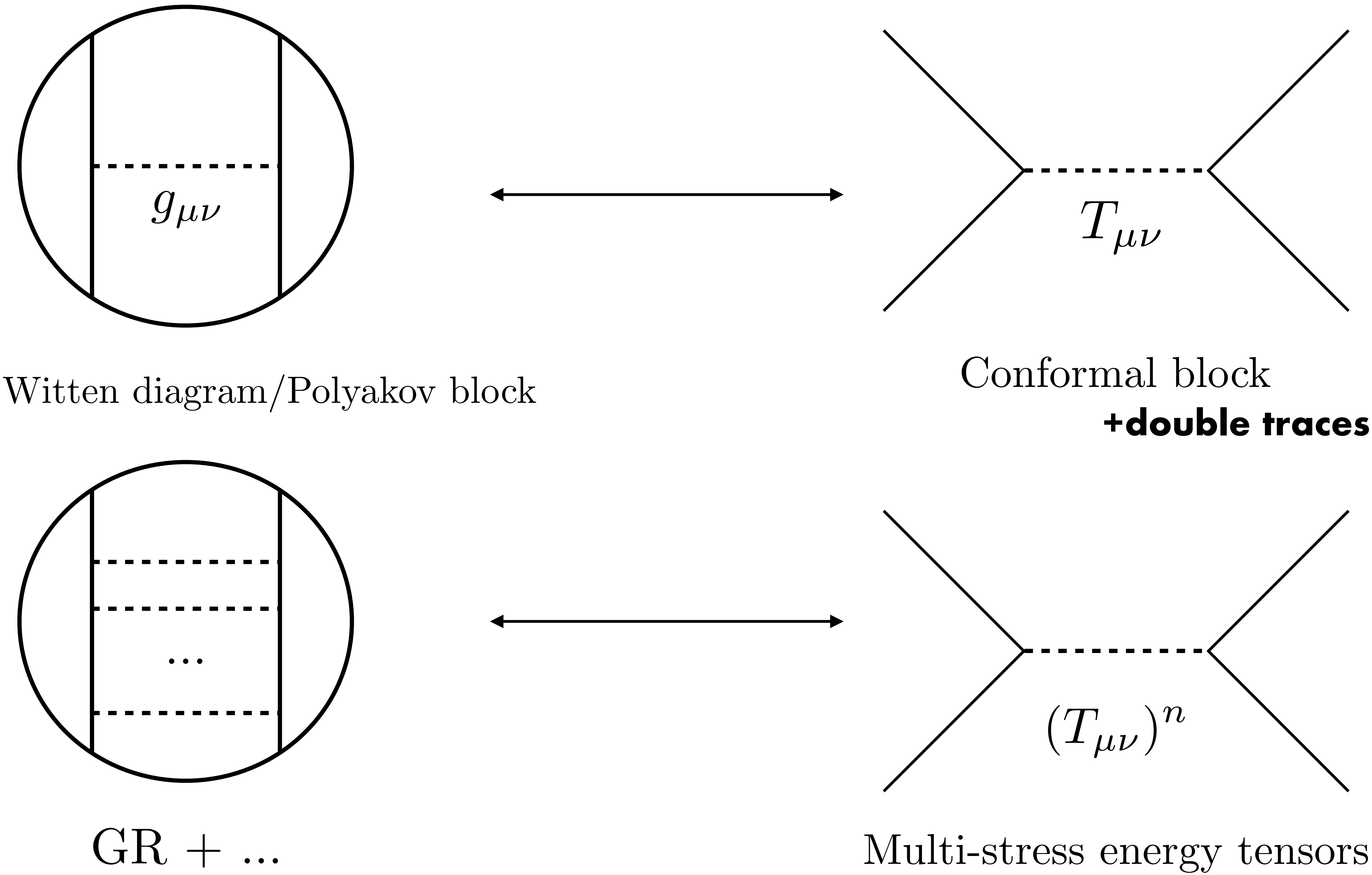
BH in AdS



$g_{\mu\nu}$ ⟷ $T_{\mu\nu}$

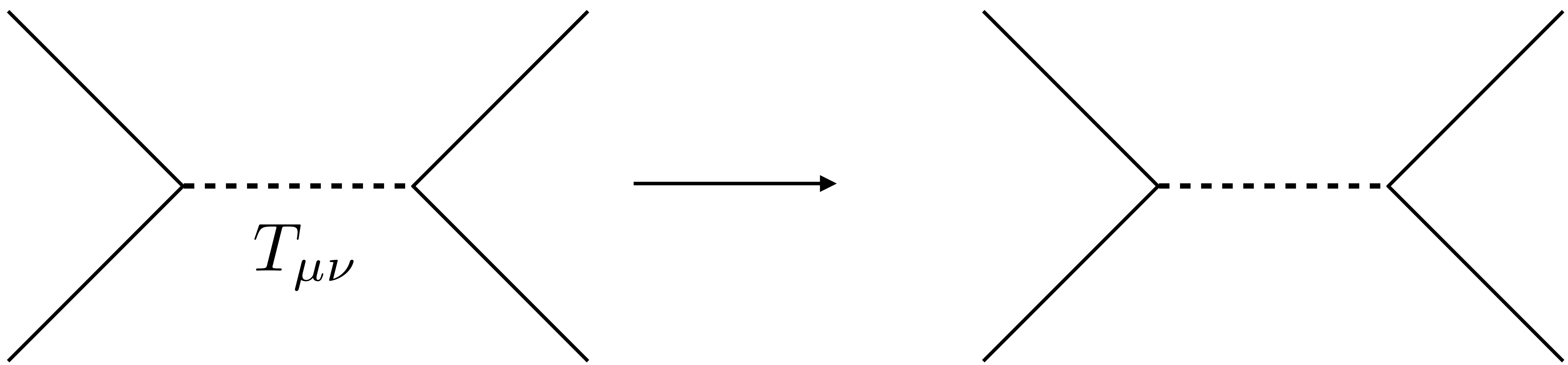
BH ⟷ Heavy operator

Multi-stress tensor dynamics



Multi-stress tensor dynamics

In 2d the multi-stress tensor dynamics is controlled by the Virasoro symmetry. [Belavin, Polyakov, Zamolodchikov '81]
(no gravity waves, geometry is locally AdS_3)

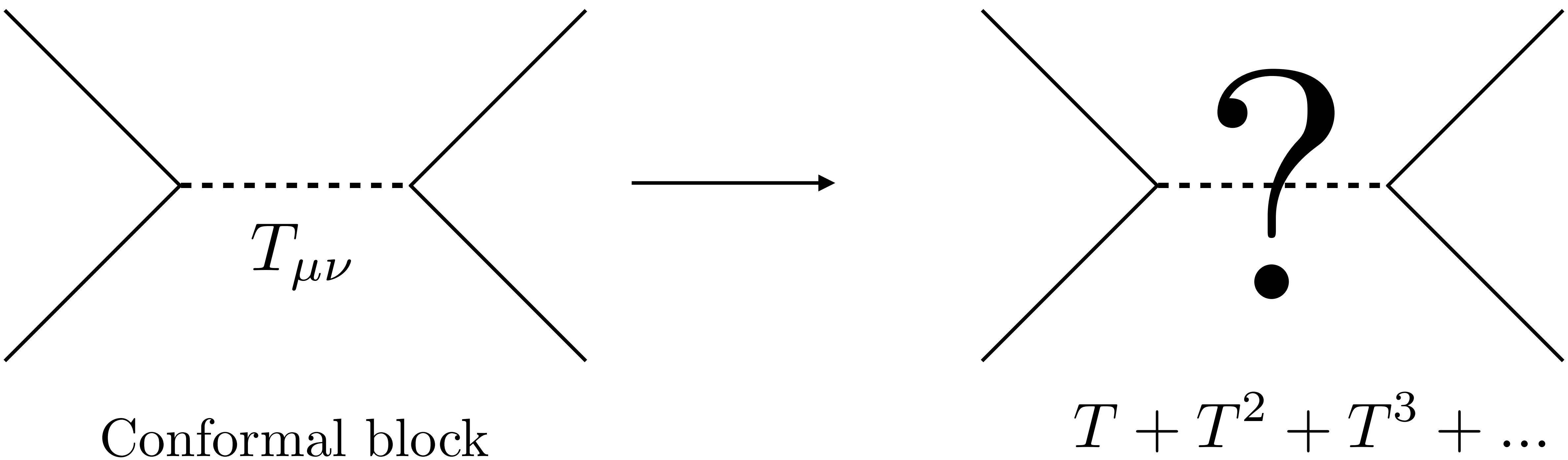


Conformal block

$T + T^2 + T^3 + ...$
Virasoro block

Multi-stress tensor dynamics

In d>2 there is no such powerful symmetry but the progress still can be made in special cases.
(gravity waves, nontrivial curvature)



In this talk I will discuss recent work on this problem in four-dimensional CFTs with a simple gravity dual.

Current themes

1. Black hole perturbations and supersymmetric instantons.

[Aminov, Grassi, Hatsuda, Bonelli, Iossa, Lichtig, Tanzini, Bianchi, Consoli, Grillo, Morales, Amado, Carneiro de Cunha, Pallante, Console, Fucito, Morales, Poghossian '20-22]

2. Light-cone bootstrap and many-body scars.

[Kulaxizi, Ng, Parnachev, Fitzpatrick, Huang, Karlsson, Tadic, D. Li, Y.-Z. Li, Zhang, Perlmutter, Simmons-Duffin, Antunes, Costa, Goncalves, Vilas Boas, Dodelson, AZ '18-22] [Bernien et al. '17]

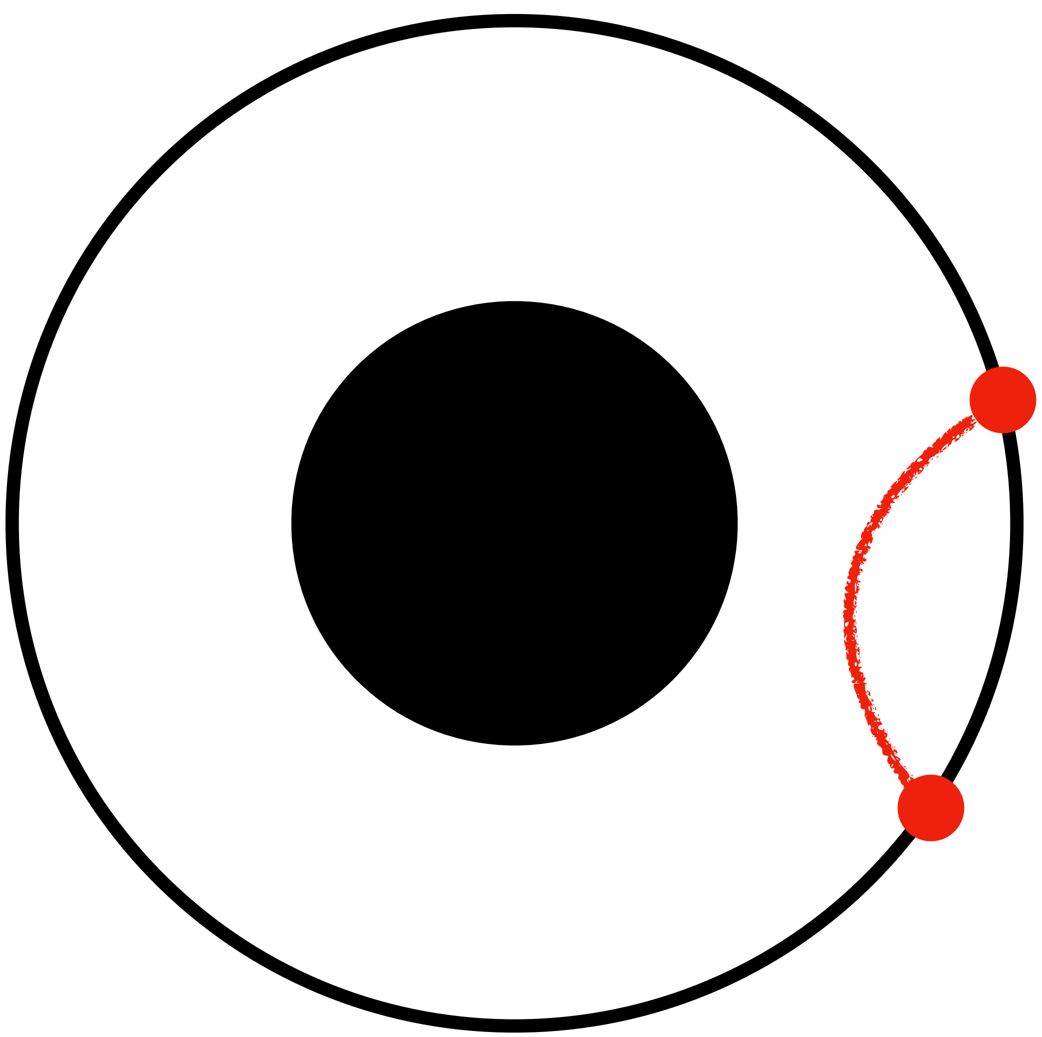
3. Connect the two.

Holographic thermal two-point function

Thermal two-point function

A particularly simple observable that receives the contribution from all the multi-stress operators is the thermal two-point function

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle_\beta \qquad \mathcal{O}\mathcal{O} \sim T^n$$



In a theory with a simple gravity dual this is described by propagation on the black hole background

$$(\Box_{\mathrm{BH}} - m^2)\phi = 0$$

Wave equation

The BH metric takes the form

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Omega_{d-2}^2, \qquad f(r) = \frac{r^2}{R_{AdS}^2} + 1 - \frac{GM}{r^{d-2}}$$

Using spherical symmetry we write

$$\phi(t, r, \Omega) = \int d\omega \sum_{\ell, \vec{m}} e^{-i\omega t} Y_{\ell\vec{m}}(\Omega)\psi_{\omega\ell}(r).$$

we get the ordinary differential equation

$$(-\partial_z^2 + V_\ell(r) - \omega^2)\psi_{\omega\ell}(r) = 0$$

Response function

A physically interesting quantity is the response function. We choose purely ingoing boundary conditions at the horizon

$$\psi^{\text{in}}_{\omega\ell}(r) = (r - R_+)^{-i\omega\#} + \ldots$$

close to the boundary

$$\psi^{\text{in}}_{\omega\ell}(r) = \mathcal{A}(\omega,\ell)(r^{\Delta-d} + \ldots) + \mathcal{B}(\omega,\ell)(r^{-\Delta} + \ldots).$$

source **response**

$$\boxed{G_R(\omega,\ell) = \frac{\mathcal{B}(\omega,\ell)}{\mathcal{A}(\omega,\ell)}.}$$

$i\theta(t)\langle[\mathcal{O}(t,\vec{n}),\mathcal{O}(0,\vec{n}')]\rangle_\beta$

[thousands of papers]

Type of ODE

We get the Fuchsian differential equation (rational coefficients, regular singularities) with the number of singular points:

$$(-\partial_z^2 + V_\ell(r) - \omega^2)\psi_{\omega\ell}(r) = 0$$

- $\frac{d}{2} + 2$ for even d
- $d + 2$ for odd d

$$CFT_d$$

AdS_3/CFT_2	AdS_5/CFT_4	AdS_4/CFT_3
3 singularities	**4 singularities**	**5 singularities**
Hypergeometric function	**Heun function**	

Problem

Holographic thermal response function for holographic CFT_4 = connection coefficient of the Heun equation.

https://theheunproject.org

The Heun Project

Heun functions, their generalizations and applications

Particularly, the connection problem for the Heun functions is not solved - one cannot connect two local solutions at different singular points using known constant coefficients.

Bad news.

Good news is that this is not true!

Liouville 4-point function

[Bonelli, Grassi, Iossa, Panea Lichtig, Tanzini, '22]

We consider the Liouville theory

$$c = 1 + 6Q^2 \qquad\qquad Q = b + \frac{1}{b}$$

and start with the 4-point function of chiral primaries, where one of the fields is degenerate

$$\langle \Delta_\infty | V_1(1) \Phi_{2,1}(z) | \Delta_0 \rangle \qquad\qquad \Delta_\Phi = -\frac{1}{2} - \frac{3b^2}{4}$$

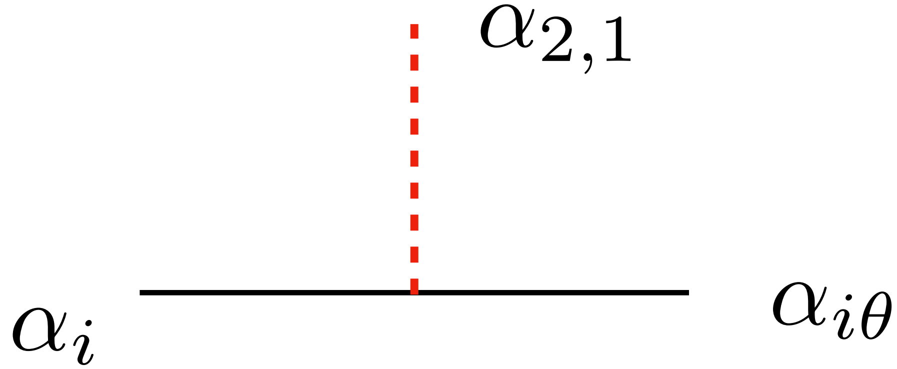
It admits a level-2 null state which leads to the BPZ equation

$$\left(b^{-2}\partial_z^2 - \left(\frac{1}{z-1} + \frac{1}{z} \right) \partial_z + \frac{\Delta_1}{(z-1)^2} + \frac{\Delta_0}{z^2} + \frac{\Delta_\infty - \Delta_1 - \Delta_{2,1} - \Delta_0}{z(z-1)} \right) \langle \Delta_\infty | V_1(1) \Phi(z) | \Delta_0 \rangle = 0 \,.$$

The solution is a familiar hypergeometric function.

Liouville 4-point function

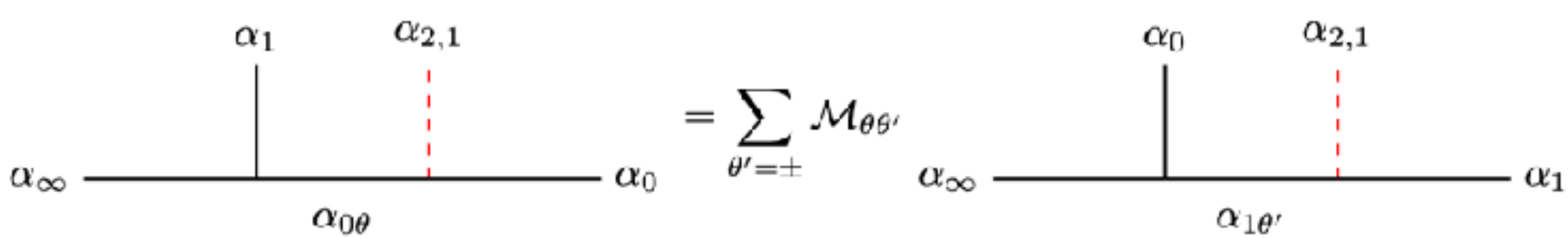
Only two Virasoro primary appear in the OPE of the degenerate field with another primary



$$\theta = \pm$$

(there are two solutions)

Crossing symmetry of the four-point function leads to connection formulas for the hypergeometric functions



$$\mathcal{M}_{\theta\theta'}(\alpha_0, \alpha_1; \alpha_2) = \frac{\Gamma(-2\theta'\alpha_1)}{\Gamma\left(\frac{1}{2} + \theta\alpha_0 - \theta'\alpha_1 + \alpha_2\right)} \frac{\Gamma(1 + 2\theta\alpha_0)}{\Gamma\left(\frac{1}{2} + \theta\alpha_0 - \theta'\alpha_1 - \alpha_2\right)}$$

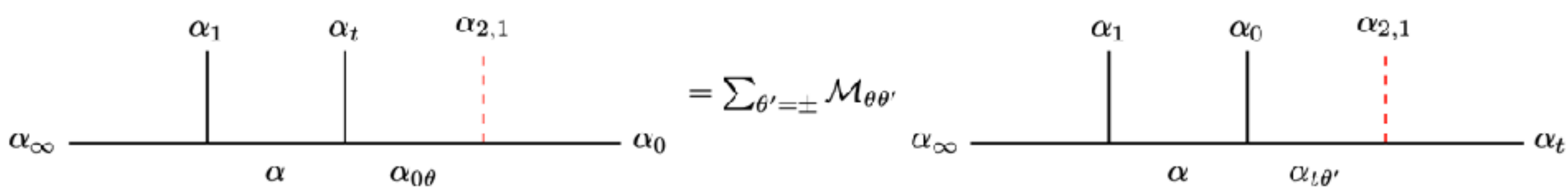
(can be read off from the DOZZ formula)

Liouville 5-point function

Consider next the five-point function. The BPZ equation takes the form

$$\left(b^{-2}\partial_z^2 + \frac{\Delta_1}{(z-1)^2} - \frac{\Delta_1 + t\partial_t + \Delta_t + z\partial_z + \Delta_{2,1} + \Delta_0 - \Delta_\infty}{z(z-1)} + \frac{\Delta_t}{(z-t)^2} + \frac{t}{z(z-t)}\partial_t - \frac{1}{z}\partial_z + \frac{\Delta_0}{z^2}\right)\langle\Delta_\infty|V_1(1)V_t(t)\Phi(z)|\Delta_0\rangle = 0$$

The crossing relation works exactly as before

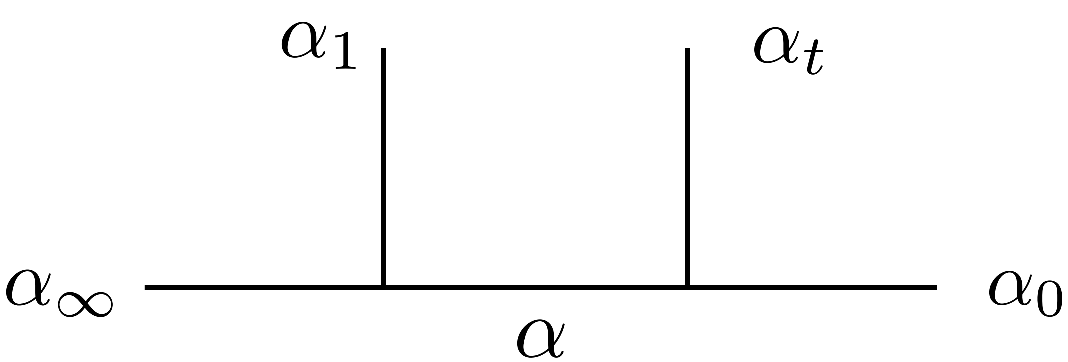


This is a connection formula for Virasoro blocks. The same construction generalizes to an arbitrary number of points.

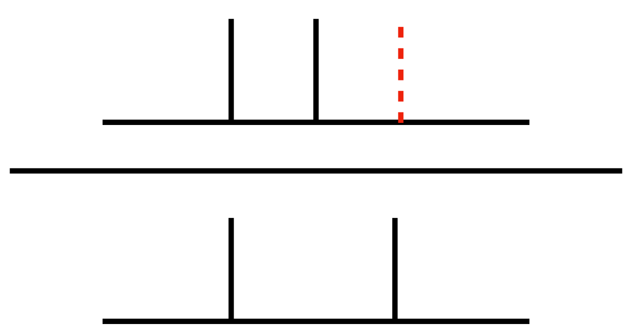
Semi-classical limit

To go from the BPZ PDE to the Heun ODE we consider the semi-classical limit

$$b \to 0, \quad \alpha_i \to \infty, \quad b\alpha_i = a_i - \text{fixed}$$



$$= t^{\Delta - \Delta_t - \Delta_0} e^{b^{-2} F(t, a_j) + \dots}$$

$\mathcal{F} =$ 

$$u^{(0)} \equiv \lim_{b \to 0} b^2 t \partial_t \mathcal{F}$$

$$\left(\partial_z^2 + \frac{\frac{1}{4} - a_1^2}{(z-1)^2} - \frac{\frac{1}{2} - a_1^2 - a_t^2 - a_0^2 + a_\infty^2 + u^{(0)}}{z(z-1)} + \frac{\frac{1}{4} - a_t^2}{(z-t)^2} + \frac{u^{(0)}}{z(z-t)} + \frac{\frac{1}{4} - a_0^2}{z^2} \right) \mathcal{F} = 0$$

(Heun equation)

Relation to gauge theory

At this point we expressed the solution to our problem in terms of the classical Virasoro block.

We next can use the AGT correspondence to express it in terms of the partition function it the supersymmetric gauge theory in the so-called Nekrasov-Shatashvili limit

$$F = \lim_{\epsilon_2 \to 0} \epsilon_2 \log \left((1-t)^{-2\epsilon_2^{-1}(\frac{1}{2}+a_1)(\frac{1}{2}+a_t)} \sum_{\vec{Y}} t^{|\vec{Y}|} z_{\mathrm{v}}\left(\vec{a}, \vec{Y}\right) \prod_{\theta=\pm} z_{\mathrm{h}}\left(\vec{a}, \vec{Y}, a_t + \theta a_0\right) z_{\mathrm{h}}\left(\vec{a}, \vec{Y}, a_1 + \theta a_\infty\right) \right)$$

$$z_{\mathrm{h}}\left(\vec{a}, \vec{Y}, \mu\right) = \prod_{I=1,2} \prod_{s \in Y_I} \left(a_I + \mu + \epsilon_1 \left(i - \frac{1}{2} \right) + \epsilon_2 \left(j - \frac{1}{2} \right) \right) ,$$

$$z_{\mathrm{v}}\left(\vec{a}, \vec{Y}\right) = \prod_{I,J=1}^{2} \prod_{s \in Y_I} \frac{1}{a_I - a_J - \epsilon_1 v_{Y_J}(s) + \epsilon_2 (h_{Y_I}(s)+1)} \prod_{s \in Y_J} \frac{1}{a_I - a_J + \epsilon_1 (v_{Y_I}(s)+1) - \epsilon_2 h_{Y_I}(s)} .$$

(explicit function given by a convergent series)

Exact result

The final result is

[Dodelson, Grassi, Iossa, Panea Lichtig, AZ '22]

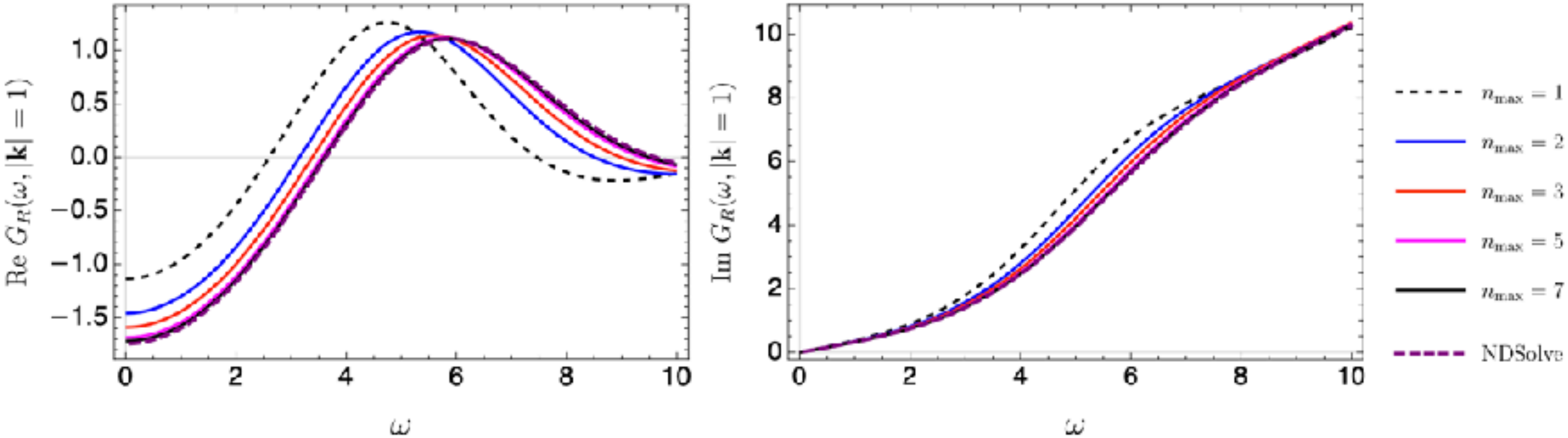
$$G_R(\omega,\ell) = e^{-\partial_{a_1}F}\frac{\sum_{\sigma'=\pm}\mathcal{M}_{-\sigma'}(a_t,a;a_0)\mathcal{M}_{(-\sigma')+}(a,a_1;a_\infty)t^{\sigma' a}e^{-\frac{\sigma'}{2}\partial_a F}}{\sum_{\sigma=\pm}\mathcal{M}_{-\sigma}(a_t,a;a_0)\mathcal{M}_{(-\sigma)-}(a,a_1;a_\infty)t^{\sigma a}e^{-\frac{\sigma}{2}\partial_a F}}$$

Gauge theory	t	a_0	a_t	a_1	a_∞
Black hole	$\frac{R_+^2}{2R_+^2+1}$	0	$\frac{i\omega}{2}\frac{R_+}{2R_+^2+1}$	$\frac{\Delta-2}{2}$	$\frac{\omega}{2}\frac{\sqrt{R_+^2+1}}{2R_+^2+1}$

+ $a(\omega,\ell)$

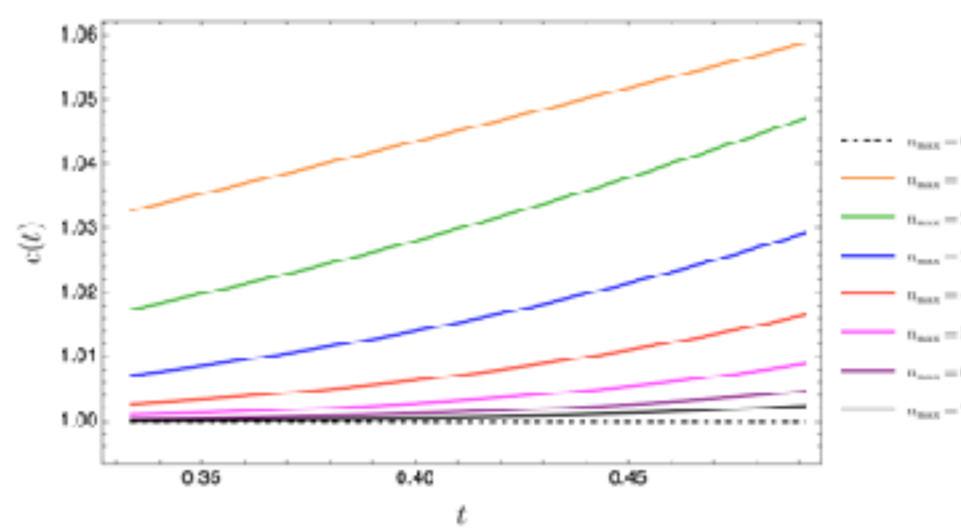
Gauge theory	t	a_0	a_t	a_1	a_∞
Black brane	$\frac{1}{2}$	0	$\frac{i\omega}{4\pi}$	$\frac{\Delta-2}{2}$	$\frac{\omega}{4\pi}$

+ $a(\omega,\mathbf{k})$



- generalizes to any d, flat, dS (quiver gauge theory)

- charged and rotating (?) black holes

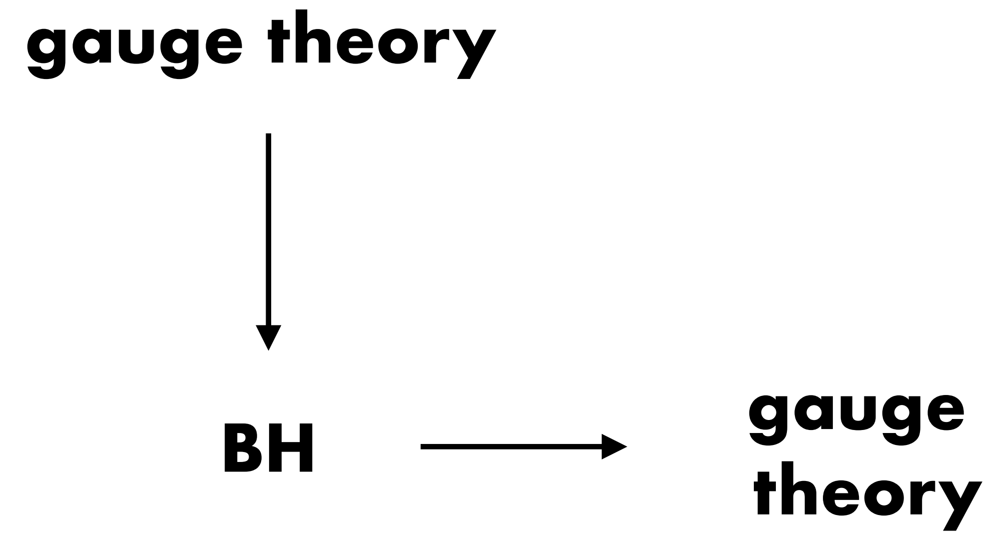


- the OPE limit $\omega \gg T$

- large momentum limit $|\mathbf{k}| \gg T$

$$G_R^{\text{brane}}(\omega, |\mathbf{k}|) \approx \frac{\Gamma(2-\Delta)}{\Gamma(\Delta-2)} \left(\frac{|\mathbf{k}|}{2}\right)^{2(\Delta-2)} + i \frac{2\pi \sinh\left(\frac{\omega}{2}\right)}{\Gamma(\Delta-1)\Gamma(\Delta-2)} \left(\frac{|\mathbf{k}|}{2}\right)^{2(\Delta-2)} \exp\left(-\sqrt{\frac{\pi}{2}} \frac{|\mathbf{k}|}{\Gamma\left(\frac{3}{4}\right)^2}\right)$$

[Son, Starinets '02]



hydrodynamics?

black hole singularity?

higher-point functions?

large d?

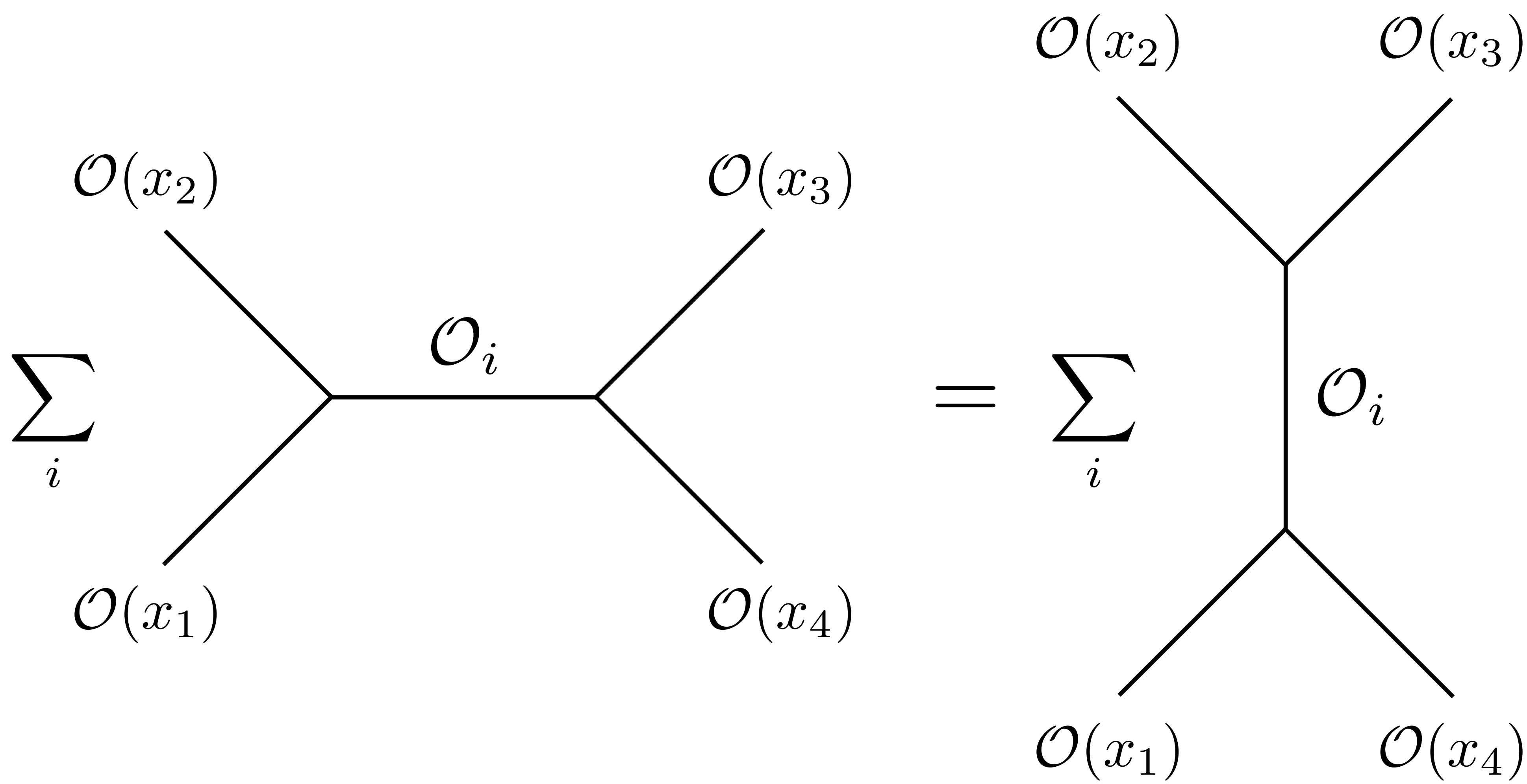
extreme limits?

what does the instanton expansion correspond to in the CFT dual?

Heavy-light conformal bootstrap

Conformal bootstrap

Associativity of the OPE + unitarity



CFT data: $\Delta(J)$ λ_{ijk}

ETH

The eigenstate thermalization hypothesis applied to CFTs states that

$$\langle \mathcal{O}\mathcal{O}\rangle_\beta \quad \longleftrightarrow \quad \langle H|\mathcal{O}\mathcal{O}|H\rangle$$

$$c_T \to \infty, \quad \frac{\Delta_H}{c_T} - \text{fixed} \qquad \beta = \frac{\partial S(\Delta_H)}{\partial \Delta_H}$$

$$G(z,\overline{z}) \equiv \langle \mathcal{O}_H(0)\mathcal{O}_L(z,\overline{z})\mathcal{O}_L(1,1)\mathcal{O}_H(\infty)\rangle \quad = \frac{1}{(1-z)^{\Delta_L}(1-\overline{z})^{\Delta_L}} + \ldots$$

OPE in the **heavy-light** channel takes the form

$$G(z,\overline{z}) = \sum_{\ell=0}^{\infty}\int_{-\infty}^{\infty} d\omega\, g_{\omega,\ell}(z\overline{z})^{\frac{\omega-\Delta-\ell}{2}}\frac{z^{\ell+1}-\overline{z}^{\ell+1}}{z-\overline{z}}$$

$$\omega \equiv \Delta'_H - \Delta_H$$

2pt vs 4pt

The precise relationship between the thermal two-point function and the vacuum four-point function takes the form

$$g_{\omega,\ell} = \frac{\ell+1}{2\pi(\Delta_L-1)(\Delta_L-2)} \frac{\text{Im } G_R(\omega,\ell)}{1-e^{-\beta\omega}}$$

In a CFT

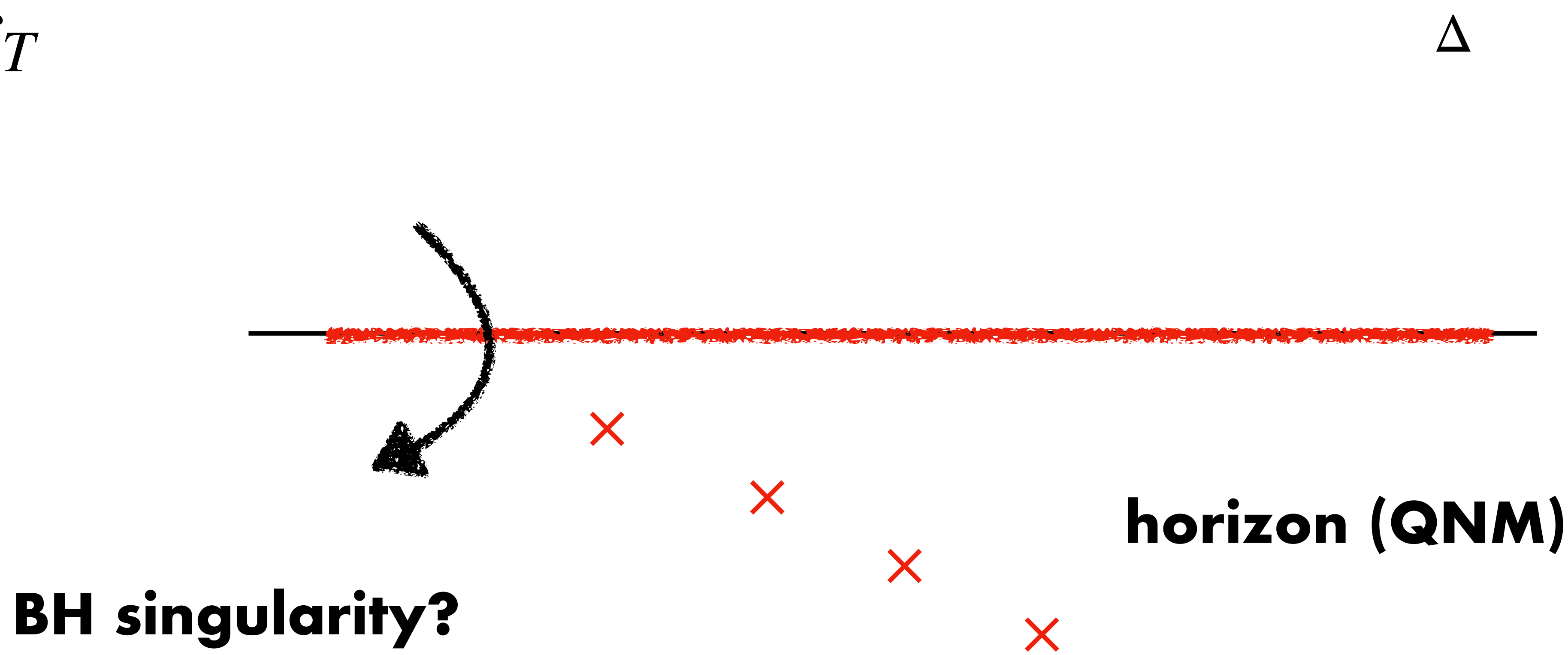
$$g_{\omega,\ell} = \sum_i \lambda_i \delta(\omega-\omega_i) \qquad\qquad \omega_i - \omega_j \sim e^{-c_T}$$

in the thermodynamic limit $c_T \to \infty$ the spectrum becomes effectively continuous. Discreteness is nonperturbative in G_N.

Comment on emergent geometry

Finite c_T

Large c_T



tauberian analytic continuation?

[Liu, Festuccia]

Light-light OPE: Stars vs BHs

In the light-light OPE channel we have

$$\mathcal{O}\mathcal{O} \sim \underline{1 + T + T^2} + \ldots + [\mathcal{O}, \mathcal{O}]_{n,J}$$

(Schwarzschild geometry) (BH horizon)

How do we distinguish stars from black holes?
In the **bulk** this is related to the boundary condition at the horizon.

On the **boundary**, the multi-stress tensor sector will be the same (Birkhoff's theorem), but the structure of the double-twist operators will be different.

[Kulaxizi, Ng, Parnachev, Fitzpatrick, Huang, Karlsson, Tadic, D. Li, Y.-Z. Li, Zhang, Perlmutter, Simmons-Duffin '18-'22]

Dispersive iterations

[Fitzpatrick, Kaplan, Poland, Simmons-Duffin, Komargodski, AZ]
[Caron-Huot]
[Y.-Z. Li]

We can solve crossing iteratively (numerically?)

$$c_{LL} = \int K_{LL} \langle [H,L][H,L] \rangle$$

$$c_{HL} = \int K_{HL} (\langle [H,H][L,L] \rangle + \langle [H,L][H,L] \rangle)$$

This can be solved iteratively starting with Newton's potential seed

$$LL \sim 1 + \mu T \quad \longrightarrow \quad H \Box^n \partial^\ell L$$

$$\mu \sim \frac{\Delta_H}{c_T}$$

$$\Delta_{[H,L]} = \Delta_H + \Delta_L + 2n + \ell + \mu \gamma_{n,\ell}$$

Dispersive iterations

[Y.-Z. Li]

We then plug this in the inversion formula

$$c_{LL} = \int K_{LL} \langle [H,L][H,L] \rangle \qquad \sim \gamma_{n,\ell}^2 \sim \mu^2$$

T^2

$$c_{HL} = \int K_{HL} (\langle [H,H][L,L] \rangle + \langle [H,L][H,L] \rangle)$$

$$\gamma_{n,\ell} = \mu \gamma_{n,\ell}^{(1)} + \mu^2 \gamma_{n,\ell}^{(2)} + \ldots$$

$$c_{n,\ell} = c_{n,\ell}^{(0)} + \mu c_{n,\ell}^{(1)} + \mu^2 c_{n,\ell}^{(2)} + \ldots$$

and so it goes. There are various degrees of freedom here (stringy modes, higher derivative corrections, star vs BH).

Expansion parameter

What about the Hawking-Page transition which requires that $\mu > \mu_0$?

The actual expansion parameter is $\dfrac{\mu}{\ell}$

Small mass expansion $\simeq$ Large spin expansion

This is also related to the large radius expansion of the metric

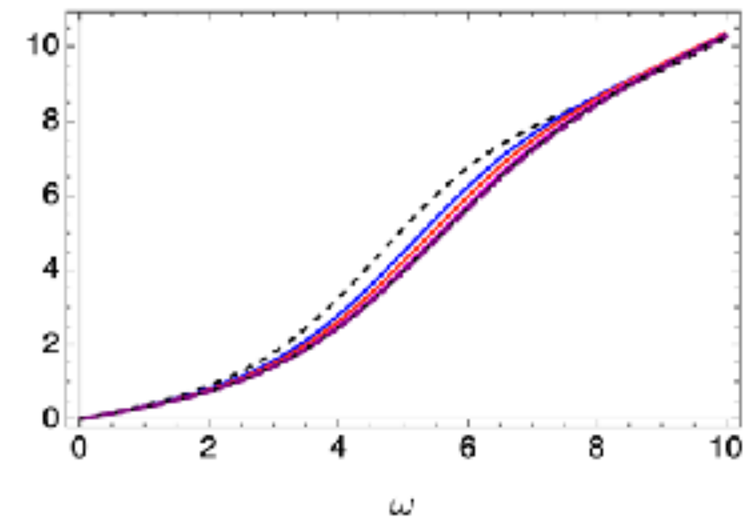
$$f(r) = r^2 + 1 - \frac{\mu}{r^2} \qquad\qquad r \sim \sqrt{\ell}$$

Let’s now connect the two approaches

Puzzle

Exact solution (finite μ):

$$g_{-\omega,\ell} = e^{-\beta\omega} g_{\omega,\ell}$$



KMS

LC bootstrap (small μ expansion):

$$H\Box^n\partial^\ell L$$

$$g_{\omega,\ell} = \theta(\omega)\sum_n \lambda_{n,\ell}\delta(\omega - \omega_{n,\ell}) \qquad \omega = \Delta'_H - \Delta_H$$

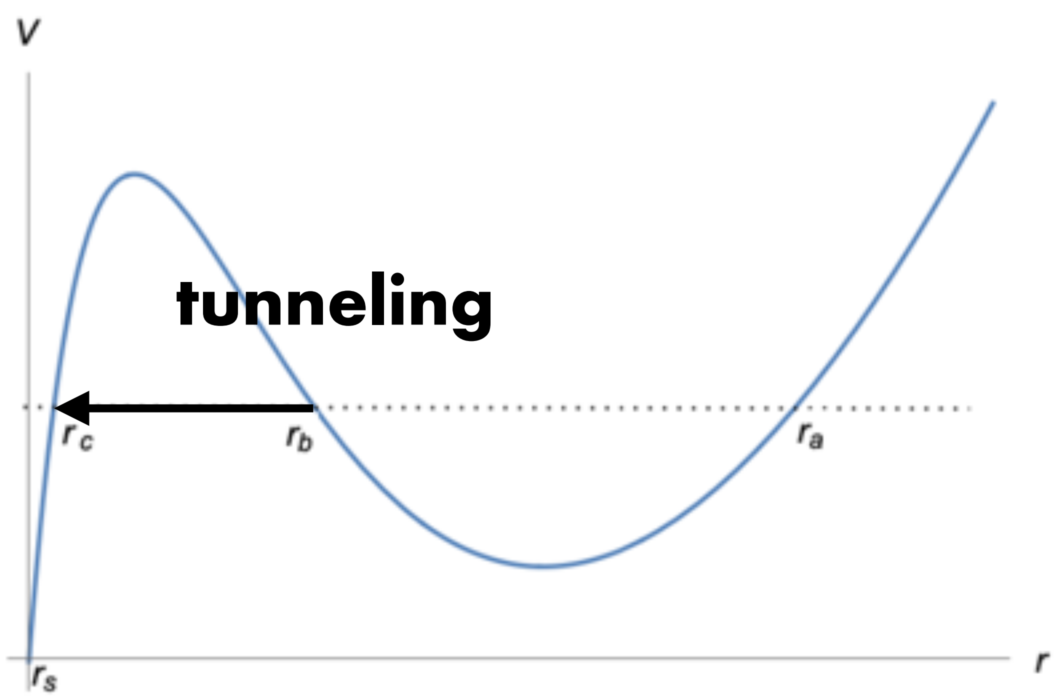
This puzzle has a very clear physical origin.

Double twists and orbits

[Berenstein, Li, Simon '20]

Double-twist operators = Orbit states

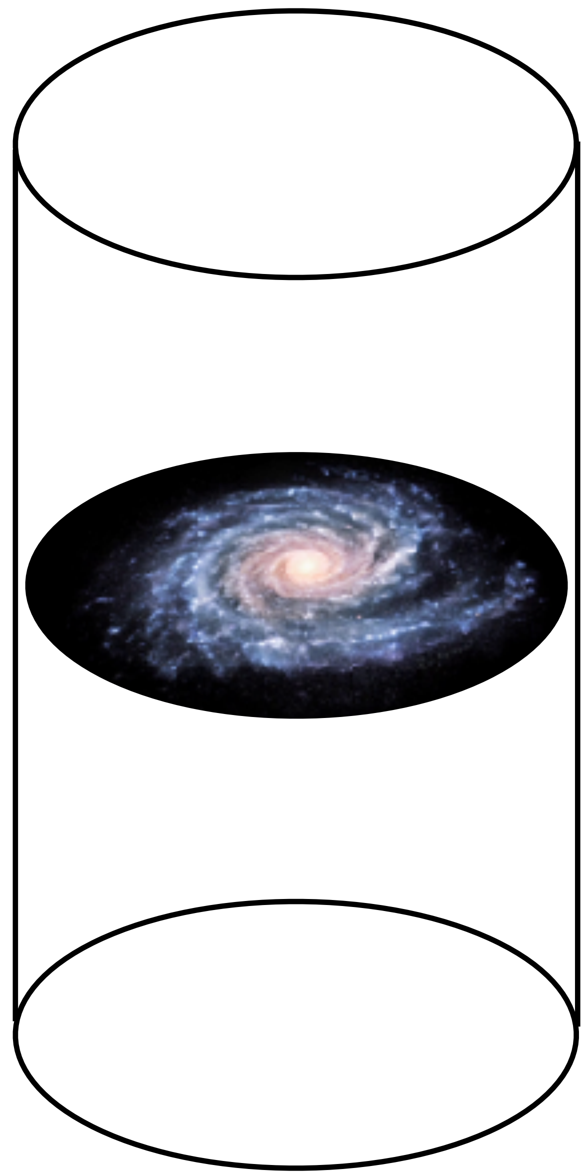
- Tunneling through the potential



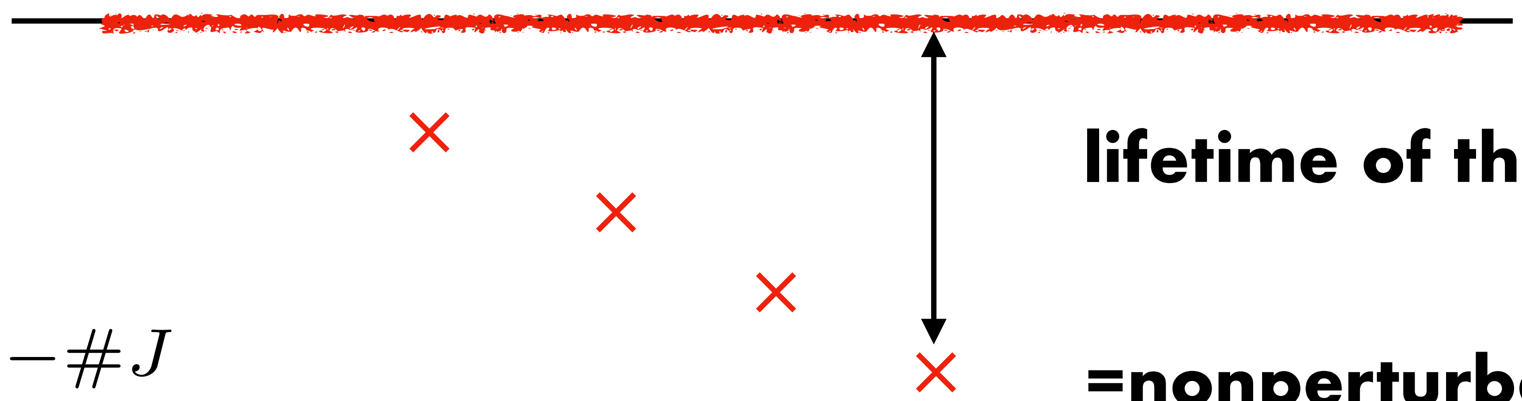
$$t \sim e^{-Jf(\mu)}$$

$$f(\mu) \sim \log \mu, \quad \mu \ll 1$$

$$f(\mu) \sim \frac{1}{\mu^2}, \quad \mu \gg 1$$



- Emission of gravity waves



$$\Gamma \sim e^{-\# J}$$

=nonperturbative effect in the light-cone bootstrap

$$\omega_{\rm QNM} = \underbrace{{\rm Re}[\omega_{\rm QNM}]}_{\text{orbit}} - i\underbrace{{\rm Im}[\omega_{\rm QNM}]}_{\text{decay}}$$

Orbit decay effects are **non-perturbative** in spin.
Light-cone bootstrap computations are **perturbative** in spin.

Imagine a continuum spectrum with the following density

$$\frac{1}{2\pi i}\Bigg(\overbrace{\frac{1}{\underbrace{\Delta-\Delta_n(J)}_{\text{stable orbit}}-ie^{-c_0(\mu)J}}-\frac{1}{\Delta-\Delta_n(J)+i\underbrace{e^{-c_0(\mu)J}}_{\text{BH tunneling}}}}^{\text{BH microstates}}\Bigg)\overset{\rm PT}{\simeq}\underbrace{\delta(\Delta-\Delta_n(J))}_{\text{double-twist}},$$

Perturbative in μ/ℓ expansion still captures the contribution of multi-stress tensor operators. Nonperturbative effects are related to the double trace operators/tunneling through the horizon.

Neglecting the nonperturbative effects our exact formula simplifies

$$G_\ell^R(\omega) = \frac{e^{-\partial_{a_1} F}\Gamma\left(\frac{\Delta}{2} + a(\omega,\ell) - i\alpha(\mu)\omega\right)\Gamma\left(\frac{\Delta}{2} - a(\omega,\ell) + i\alpha(\mu)\omega\right)}{\Gamma\left(-\frac{\Delta}{2} + a(\omega,\ell) - i\alpha(\mu)\omega + 1\right)\Gamma\left(-\frac{\Delta}{2} - a(\omega,\ell) + i\alpha(\mu)\omega + 1\right)}$$

Expanding it in μ/ℓ indeed reproduces the known LC bootstrap results and generates many more.

$$\begin{aligned} g^{\text{pert}}_{\omega,\ell} &= \theta(\omega)\frac{\ell+1}{2\pi(\Delta_L-1)(\Delta_L-2)}\text{Im } G_R^{\text{pert}}(\omega,\ell) \\ &= \sum_{n=0}^{\infty} c_{n\ell}\delta(\omega-\omega_{n\ell})\ , \end{aligned}$$

$O(\mu^2)$

$$\gamma^{(2)}_{n\ell} = -\frac{\left((\Delta-1)\Delta+6(\Delta-1)n+6n^2\right)^2}{8(\ell+1)^3} - \frac{n(\Delta+n-2)(\Delta+2n-2)^2}{2(\ell+2)} - \frac{(n+1)(\Delta+n-1)(\Delta+2n)^2}{2\ell}$$
$$+ \frac{(\Delta-1)\Delta(8\Delta+1)+65n^4+130(\Delta-1)n^3+(3\Delta(27\Delta-43)+133)n^2+(\Delta-1)\left(16\Delta^2+\Delta+68\right)n}{16(\ell+1)}$$
$$- \frac{(n-1)n(\Delta+n-3)(\Delta+n-2)}{32(\ell+3)} - \frac{(n+1)(n+2)(\Delta+n-1)(\Delta+n)}{32(\ell-1)}, \tag{D1}$$

$$c^{(2)}_{n\ell} = \frac{1}{8}(\Delta-2)(9\Delta-44) - \frac{(2n+3)(\Delta+n-1)(\Delta+n)}{32(\ell-1)} - \frac{3(\Delta+2n-1)\left((\Delta-1)\Delta+6n^2+6(\Delta-1)n\right)}{4(\ell+1)^3}$$
$$+ \frac{(\Delta+2n-1)\left(\Delta(16\Delta-71)+130n^2+130(\Delta-1)n+212\right)}{32(\ell+1)} - \frac{(\Delta+n-1)(\Delta+2n)(\Delta+4n+2)}{2\ell}$$
$$- \frac{(n-1)n(2\Delta+2n-5)}{32(\ell+3)} - \frac{n(\Delta+2n-2)(3\Delta+4n-6)}{2(\ell+2)} + \frac{1}{4}(\psi^{(0)}(n+\ell+2)-\psi^{(0)}(n+\ell+\Delta))$$
$$\times\Bigg(9\Delta^2 - \frac{89\Delta}{2} + \frac{\left((\Delta-1)\Delta+6n^2+6(\Delta-1)n\right)^2}{2(\ell+1)^3} - \frac{3(\Delta+2n-1)\left((\Delta-1)\Delta+6n^2+6(\Delta-1)n\right)}{(\ell+1)^2}$$
$$+ \frac{(\Delta-1)(\Delta(4\Delta-73)+36)-65n^4-130(\Delta-1)n^3+(3(67-27\Delta)\Delta-493)n^2-(\Delta-1)(\Delta(16\Delta-71)+428)n}{4(\ell+1)}$$
$$+(18\Delta-89)n + \frac{2n(\Delta+n-2)(\Delta+2n-2)^2}{\ell+2} + \frac{2(n+1)(\Delta+n-1)(\Delta+2n)^2}{\ell} + \frac{(n-1)n(\Delta+n-3)(\Delta+n-2)}{8(\ell+3)}$$
$$+ \frac{(n+1)(n+2)(\Delta+n-1)(\Delta+n)}{8(\ell-1)} + \left(9\Delta-\frac{71}{2}\right)\ell+9\Bigg) + \left(\Delta(\Delta+2)+6n^2+6n(\Delta+\ell)+3\ell^2+3(\Delta+1)\ell\right)^2$$
$$\times \frac{(\psi^{(0)}(n+\ell+2)-\psi^{(0)}(n+\ell+\Delta))^2+\psi^{(1)}(n+\ell+\Delta)-\psi^{(1)}(n+\ell+2)}{8(\ell+1)^2}\,. \tag{D2}$$

Virasoro block

2d:

$$G_\ell^R(\omega) = \frac{\Gamma\left(\frac{\Delta}{2} + \frac{i(\ell-\omega)}{2\sqrt{\mu-1}}\right)\Gamma\left(\frac{\Delta}{2} - \frac{i(\ell+\omega)}{2\sqrt{\mu-1}}\right)}{\Gamma\left(-\frac{\Delta}{2} + \frac{i(\ell-\omega)}{2\sqrt{\mu-1}} + 1\right)\Gamma\left(-\frac{\Delta}{2} - \frac{i(\ell+\omega)}{2\sqrt{\mu-1}} + 1\right)}$$

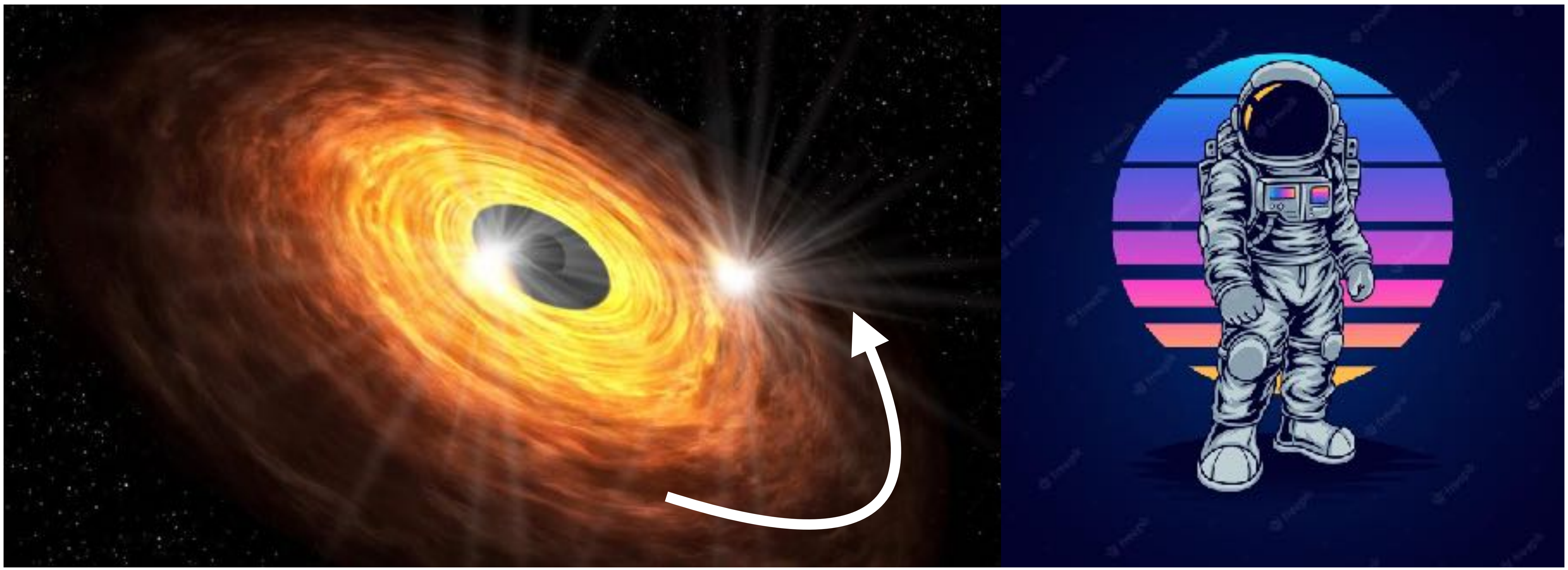
4d:

$$G_\ell^R(\omega) = \frac{e^{-\partial_{a_1}F}\Gamma\left(\frac{\Delta}{2} + a(\omega,\ell) - i\alpha(\mu)\omega\right)\Gamma\left(\frac{\Delta}{2} - a(\omega,\ell) + i\alpha(\mu)\omega\right)}{\Gamma\left(-\frac{\Delta}{2} + a(\omega,\ell) - i\alpha(\mu)\omega + 1\right)\Gamma\left(-\frac{\Delta}{2} - a(\omega,\ell) + i\alpha(\mu)\omega + 1\right)}$$

(exact solution to the LC bootstrap)

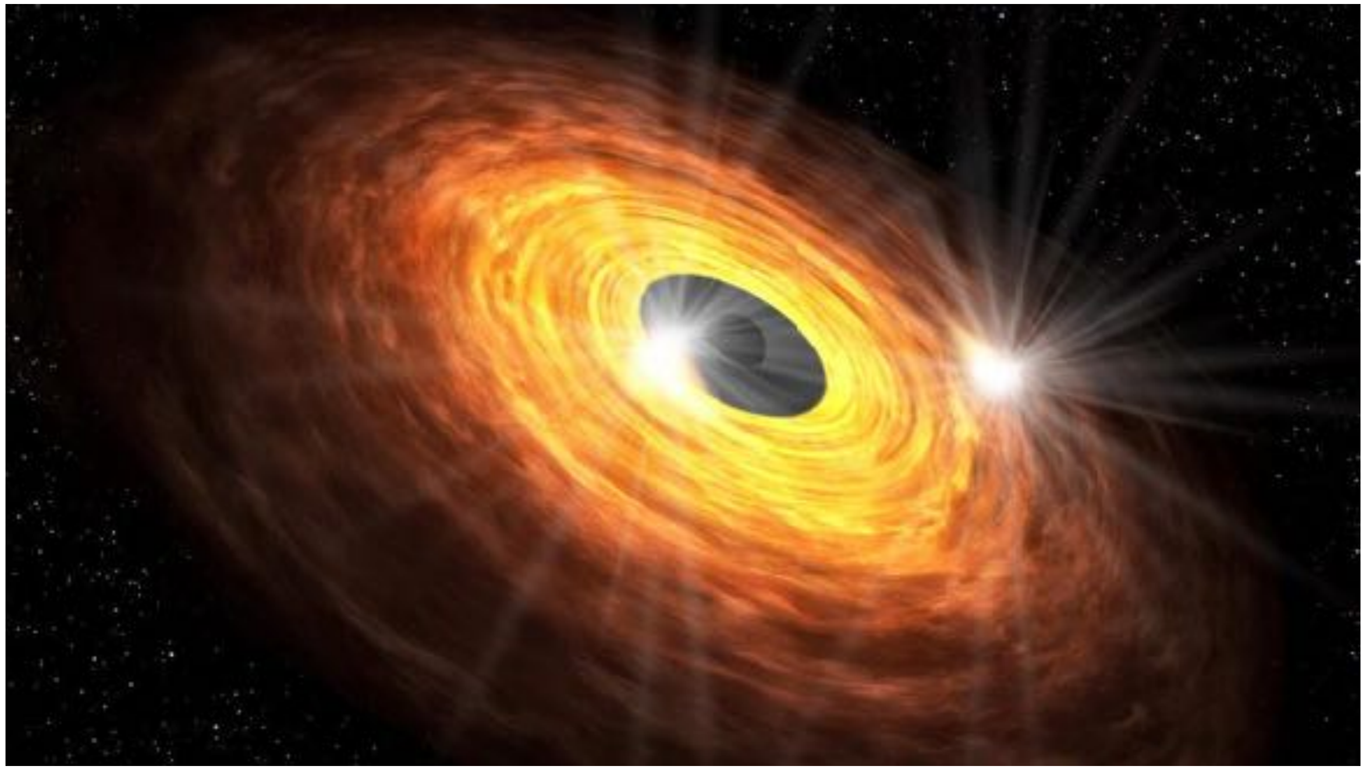
Scattering close to the BH horizon: maximally chaotic!

[MSS]

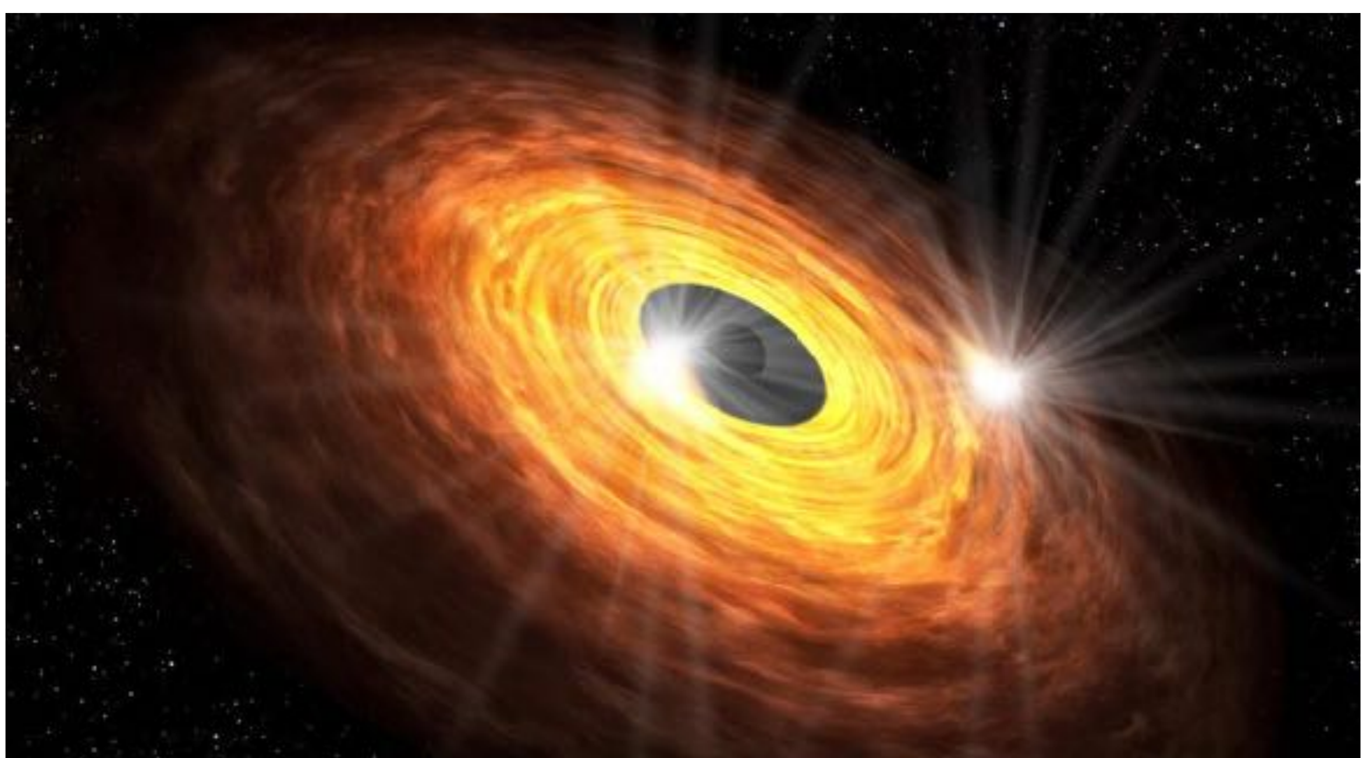


Orbiting around the black hole: maximally integrable?

Thermalization

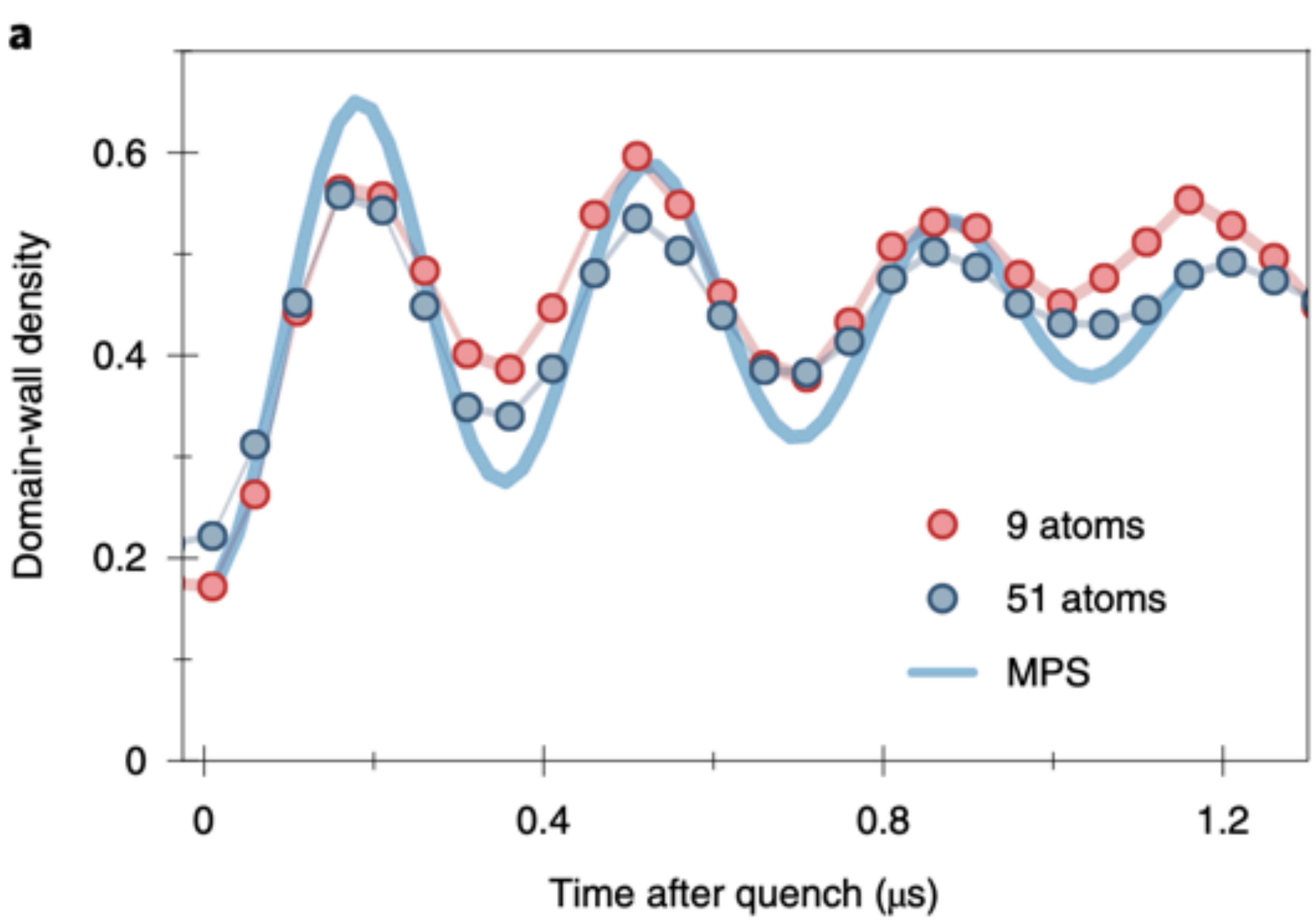


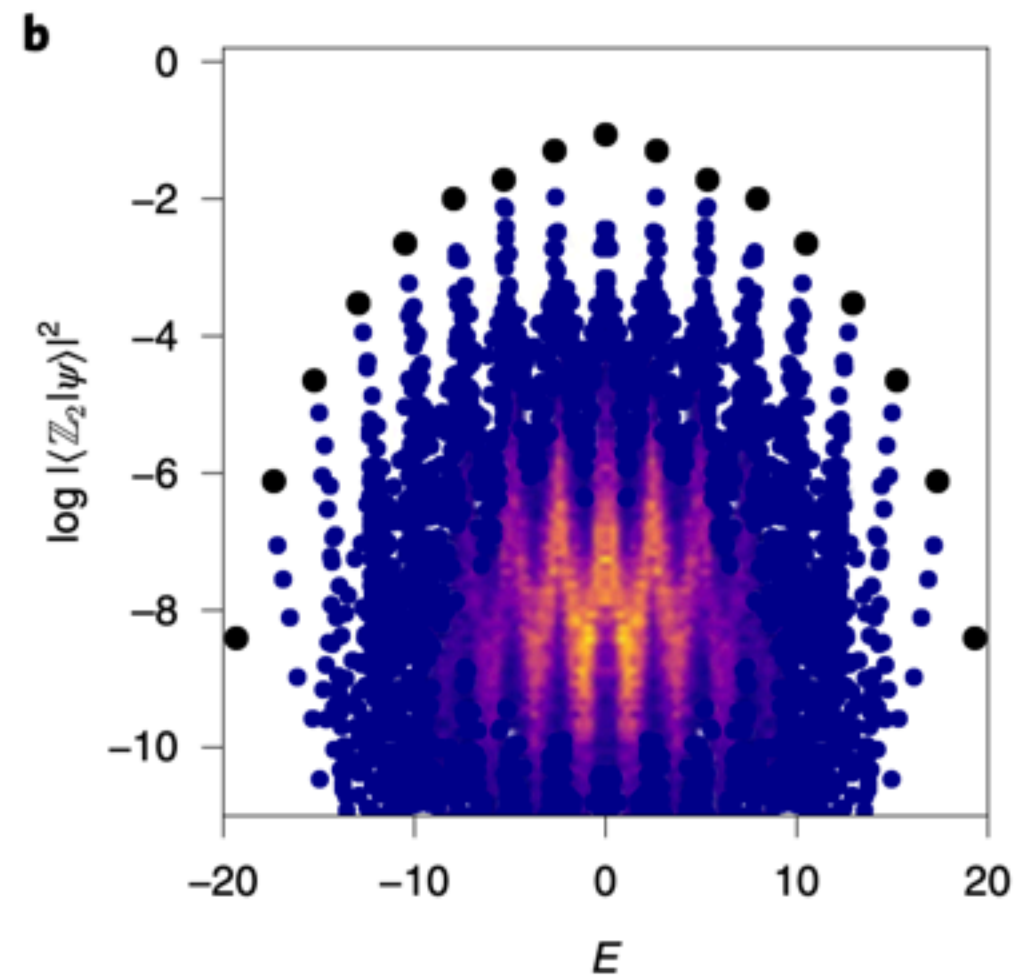
Non-thermalization



Many-body quantum scars = atypical non-thermalizing energy eigenstates in the high-energy part of the spectrum

They were discovered experimentally in 2017 in the chain of Rydberg atoms and triggered our work





[Bernien et al. ’17]

[Abanin, Papic, Serbyn]

$$\langle E|\mathcal{O}|E\rangle \neq \text{thermal}$$

Quantum many-body scars

- Recently many examples of such systems were found (PXP, AKLT, Hubbard,…)

$$H = H_0^G + \sum_a T^a \mathcal{O}_a \qquad [T^a, H_0^G] = 0$$

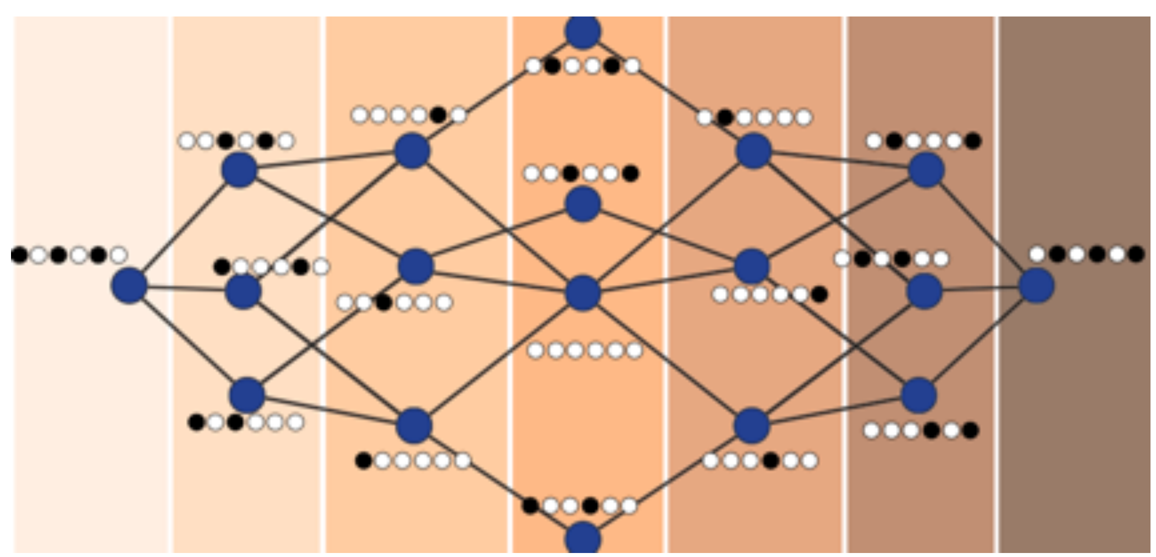
$$T^a |\mathrm{scar}\rangle = 0$$

[Pakrouski, Pallegar, Popov, Klebanov '20]

$$H_{\mathrm{PXP}} = \sum_i P_{i-1} \sigma_i^x P_{i+1}.$$

$$\sigma_i^x = |\circ\rangle_i \langle \bullet|_i + |\bullet\rangle_i \langle \circ|_i$$

$$P_i = |\circ\rangle_i \langle \circ|_i$$



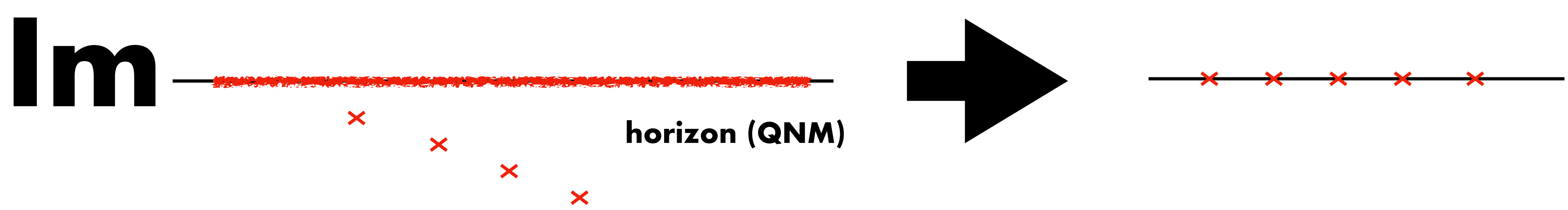
- Periodic orbits in many-body system (time-dependent variational principle TDVP).

[Ho, Choi, Pichler, Lukin, '19]

- Occupy a small fraction of Hilbert space.

thermal two-point function **4pt function**

$$\frac{1}{2\pi i}\left(\overbrace{\frac{1}{\underbrace{\Delta-\Delta_n(J)}_{\text{stable orbit}}-ie^{-c_0(\mu)J}}-\frac{1}{\Delta-\Delta_n(J)+i\underbrace{e^{-c_0(\mu)J}}_{\text{BH tunneling}}}}^{\text{BH microstates}}\right)\overset{\text{PT}}{\simeq}\overbrace{\underbrace{\delta(\Delta-\Delta_n(J))}_{\text{double-twist}}}^{\text{scar}},$$



Violation of ETH?

$$\langle \mathcal{O}_H(x_1)\mathcal{O}_L(x_2)\mathcal{O}_L(x_3)\mathcal{O}_L(x_5)\mathcal{O}_H(x_4)\rangle \overset{x_{14}^2\to 0}{=} \langle \mathcal{O}_H(x_1)\mathcal{O}_H(x_4)\rangle\langle \mathcal{O}_L(x_2)\mathcal{O}_L(x_3)\mathcal{O}_L(x_5)\rangle + \ldots$$
$$= \frac{1}{x_{14}^{2\Delta_H}} \frac{g}{(x_{23}^2 x_{25}^2 x_{35}^2)^{\Delta_L/2}} + \ldots$$

In the dual channel this behavior is reproduced by orbit states

$$\langle \mathcal{O}_H(x_1)\mathcal{O}_L(x_2)\mathcal{O}_L(x_3)\mathcal{O}_L(x_5)\mathcal{O}_H(x_4)\rangle$$
$$\simeq \frac{1}{(x_{12}^2 x_{34}^2)^{\frac{\Delta_L+\Delta_H}{2}}} \left(\frac{x_{13}^2}{x_{15}^2 x_{35}^2}\right)^{\frac{\Delta_L}{2}} \left(\frac{x_{23}^2}{x_{14}^2}\right)^{\frac{\Delta_H-\Delta_L}{2}} \sum_{k_1,k_2,\ell} P_{k_1 k_2 \ell}\, \mathcal{G}_{k_1 k_2 \ell}(u_i)$$

$$P_{k_1 k_2 \ell} = C_{H,L,k_1} C_{H,L,k_2} C_{k_1,k_2,\ell}$$

By matching the leading singularity we get for $\langle orbit \,|\, \mathcal{O} \,|\, orbit' \rangle$

$$C_{j_1,j_2,\ell} = \frac{g}{\ell^{\frac{\Delta_L}{2}}} \frac{\Gamma(\Delta_L)}{\Gamma(\frac{\Delta_L}{2})^3} \frac{\Gamma(j_1+\frac{\Delta_L}{2})\Gamma(j_2+\frac{\Delta_L}{2})}{\Gamma(j_1+1)\Gamma(j_2+1)}, \quad J_i = j_i + \ell, \quad \ell \gg 1.$$

[see also Delacretaz, Fitzpatrick, Katz, Walters '22]

Violation of ETH?

The orbit states are particular superpositions of BH micro states

$$|\text{orbit}\rangle = \sum_{E_i = E_{\text{orbit}} - \Gamma}^{E_{\text{orbit}} + \Gamma} c_i |E_i\rangle, \qquad \sum_{i=1}^{N} |c_i|^2 = 1.$$

$$c_i = |c_i| e^{i\phi_i}$$

Using the ETH hypothesis we get

$$\langle \text{orbit}|\mathcal{O}_L|\text{orbit}\rangle = \sum_{i,j} c_j^* c_i \langle E_j|\mathcal{O}_L|E_i\rangle$$

$$= \sum_{i,j} e^{-S\left(\frac{E_i+E_j}{2}\right)/2} c_j^* R_{j,i} c_i f_{\mathcal{O}_L}\left(\frac{E_i+E_j}{2}, E_i - E_j\right)$$

$$\phi_i = \tilde{\phi}_i, \quad i = 1, ..., N \qquad \sum_{j=1}^{N} R_{j,i} c_j^* \equiv O(1) \times e^{-i\tilde{\phi}_i}$$

$$\sum_{i,j} e^{-S\left(\frac{E_i+E_j}{2}\right)/2} c_j^* R_{j,i} c_i \simeq e^{-S(E_{\rm orbit})/2} \sum_{i,j=1}^{N} c_j^* R_{j,i} c_i$$

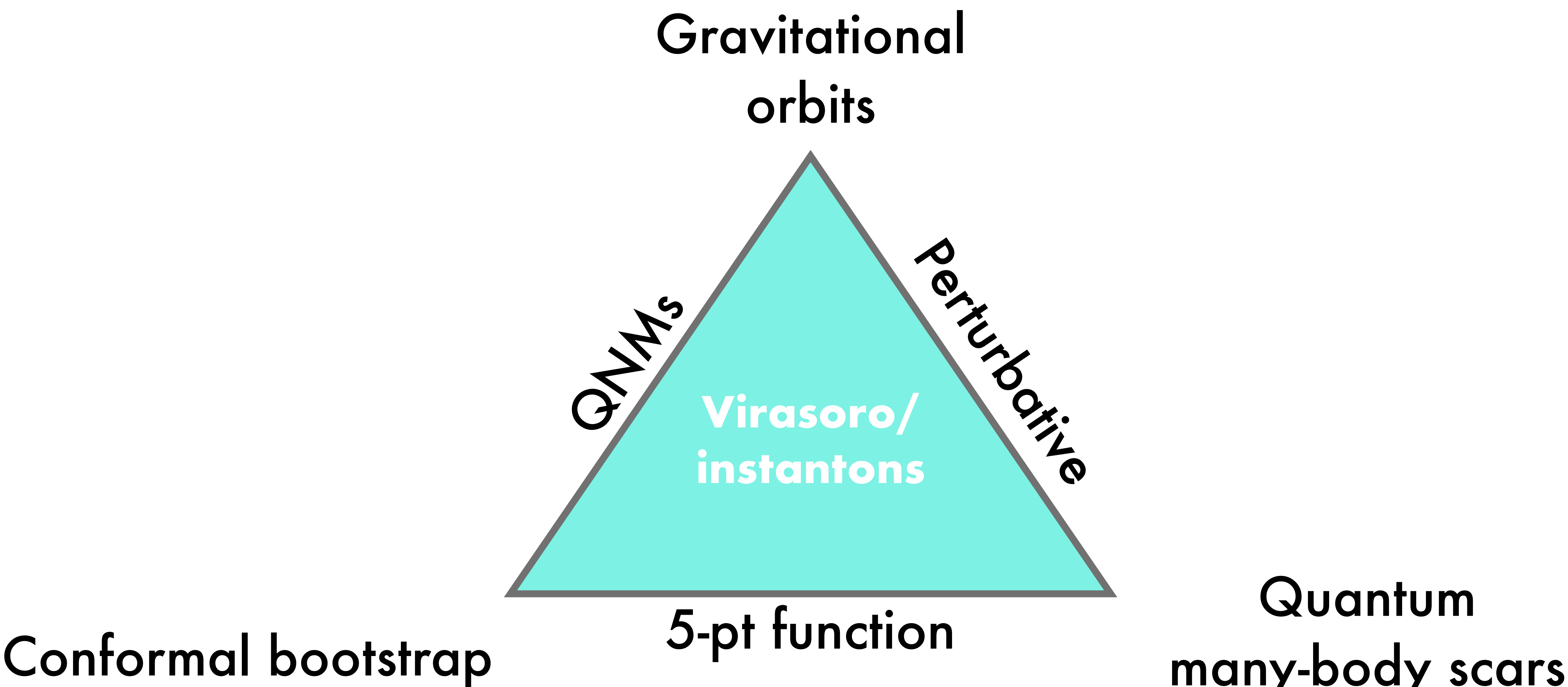
$$\sim e^{-S(E_{\rm orbit})/2} \sum_{i=1}^{N} c_i e^{-i\tilde{\phi}_i}$$

$$\sim O(N^{1/2}) e^{-S(E_{\rm orbit})/2} \sim O(1)$$

The orbit states are encoded in the off-diagonal elements of the black hole microstates.

Current themes

Is it just a mathematical curiosity or there is some deeper hidden symmetry to be uncovered in higher-dimensional gravity/CFTs?



thank you for your attention!