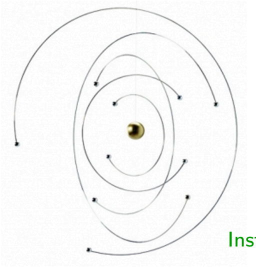
Quantum Clones Inside Black Holes

or

Turning a Black Hole Inside Out

arXiv: 2206.04608 v3



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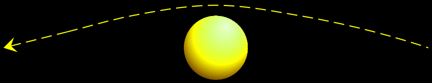
Version 1

Standard gravity is based on Newton's Law:

$$F = \frac{G\, m_1 m_2}{r^2}$$

It leads to General Relativity, with curved space-time everywhere.

But an easier start may be
to base gravity



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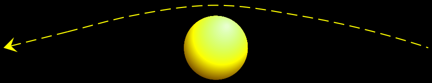
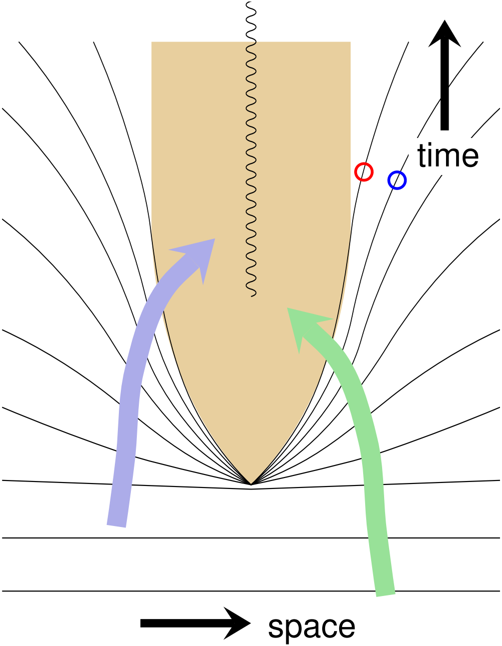


on the Shapiro effect, or

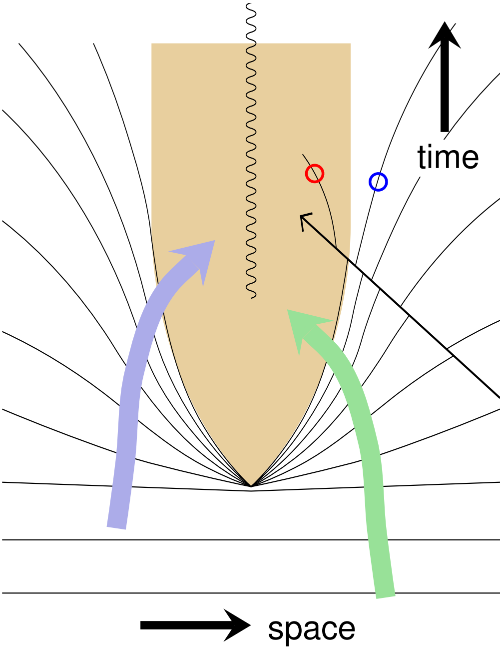
Gravitational lensing :

$$\delta u^- = -8\pi G\, p^- \log(\tilde{x}_1 - \tilde{x}_2)\,.$$

It leads to curvature only on
2-dimensional subspaces.



Shapiro shift



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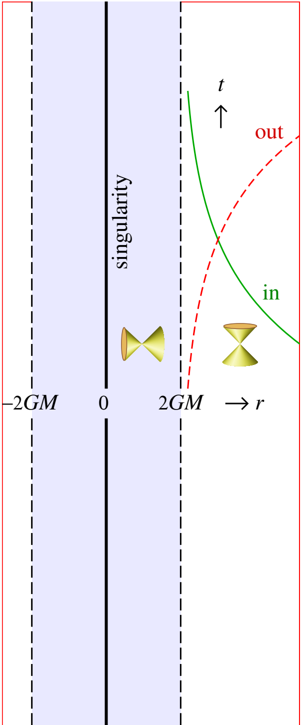

We must use the *eternal* (or *stationary*) black hole as background.

This will be justified a posteriori: quantised particles give divergent Shapiro effects, and therefore we can only consider classical time segments not longer than $M_{\rm BH}\log(M_{\rm BH}/M_{\rm Planck})$, during which the stationary Schwarzschild solution is fine.

Position operators for late particles will not commute with those of the early particles.

Schwarzschild Metric in a spacetime (r, t, θ, φ):

$$\begin{aligned} \mathrm{d}s^2 &= \frac{1}{1-\frac{2GM}{r}}\mathrm{d}r^2 - \left(1-\tfrac{2GM}{r}\right)\mathrm{d}t^2 + r^2\mathrm{d}\Omega^2 \ ; \\ \Omega &\equiv (\theta,\, \varphi)\ , \\ \mathrm{d}\Omega &\equiv (\mathrm{d}\theta,\, \sin\theta\, \mathrm{d}\varphi)\ . \end{aligned}$$

Horizon singularity at $r \to 2GM$.

An essential role is played by the Kruskal-Szekeres (or 'tortoise') coordinates x, y, defined by

$$
\begin{aligned}
x\,y &= \left(\frac{r}{2GM}-1\right)e^{r/2GM} \qquad ; \\
y/x &= e^{t/2GM} \qquad . \\
\mathrm{d}s^2 &= \frac{32(GM)^3}{r}\,e^{-r/2GM}\,\mathrm{d}x\,\mathrm{d}y + r^2\mathrm{d}\Omega^2 \qquad .
\end{aligned}
$$

No horizon singularity , but *two* horizons.

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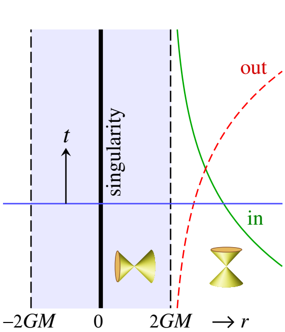
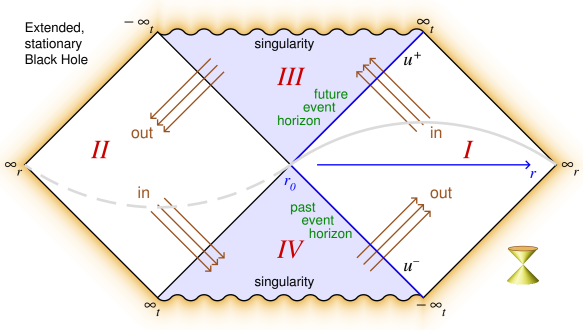
No horizon singularity , but *two* horizons.

At $r\rightarrow 2GM$, we have $x=0$: future event horizon, and
$y=0$: past event horizon.

For every point (r,t,θ,φ), there are **two points** in these new coordinates: with every (x,y,θ,φ) there is also $(-x,-y,\theta,\varphi)$.





In Schwarzschild coordinates (left) you see one outside region (white) and one inside region (blue)

In the more regular Kruskal-Szekeres (KS) coordinates(right) you see two outside regions and two inside regions (blue)

The physics of unbounded Lorentz boosts, in two frames:

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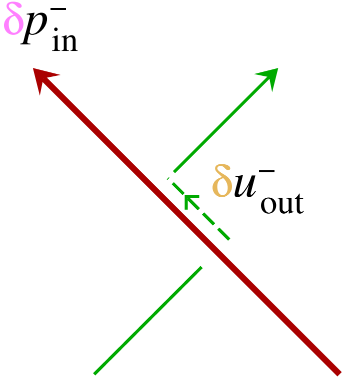

in the out-frame,

The physics of unbounded Lorentz boosts, in two frames:

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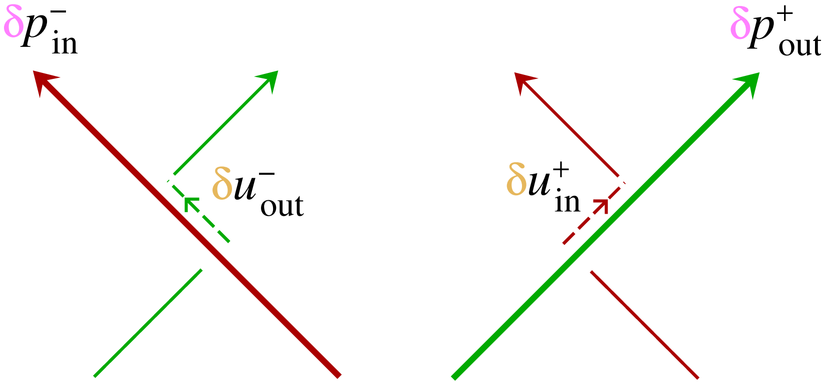


in the out-frame, in the in-frame.

The Shapiro effect.

$$\delta u^- = G\, p^- f(\Omega,\, \Omega') \quad \text{with} \quad (1 - \Delta_\Omega)\, f(\Omega,\, \Omega') = 8\pi G\, \delta^2(\Omega,\, \Omega')\ .$$

Here, Ω is the solid angle at which the out-particle leaves, and Ω' is where the in-particle entered the black hole horizon.

$u^\pm$ are the positions at the horizon, *i.e.*, the time at which the particles enter or leave the horizon.

$p^\pm$ are their momenta.

In case of many particles: $p^\pm \to p^\pm(\Omega)\,, \quad u^\pm \to u^\pm(\Omega)\,.$

Spherical Harmonics: $$\delta u^-_{\ell m} = \frac{8\pi G}{\ell^2+\ell+1}\, p^-_{\ell m}\ .$$

And now comes quantum mechanics:

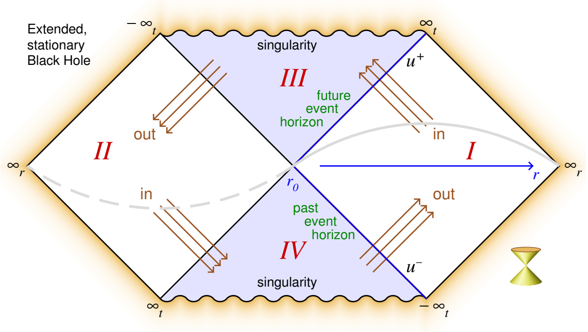
This gives algebra: $$[u^+,\, u^-] = i\lambda\ , \qquad \lambda \equiv \frac{8\pi G}{\ell^2+\ell+1}\ .$$

Therefore, wave functions in u^- are the Fourier transforms of those in u^+.

The different (ℓ, m) modes all decouple.

These are one-dimensional (highly trivial) equations.

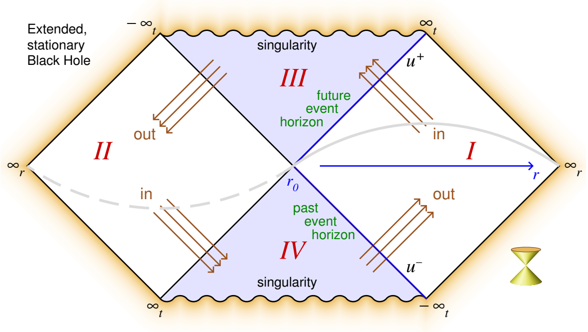
The Fourier transform is unitary! And therefore information is preserved !!



The Fourier transform is unitary, but only if

$$-\infty < u^+ < \infty$$
$$-\infty < u^- < \infty$$

working with only positive $u^\pm$ would give information loss!



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We need a unitary mapping from the interval $[0, \infty]$ to $[0, \infty]$.

But this is easy. Use: Fourier transform maps even functions onto even functions and odd onto odd !

The fact that we may use only even functions means that:

The data in region II are identical to the data in region I.

They are *clones* of one another.

But our message does not end here. Problem: in region *II* *time runs backwards!*.

Therefore, region *II* is only clone of region *I*, if the wave function obeys:

$$\boxed{\psi_{II} = \psi_I^{\,*}}$$

Solve the equations with the *constraint* that the wave functions in *I* and *II* are the same !

Calculations: this does give a unitary evolution law!

But, we must pull $Re(\psi)$ and $Im(\psi)$ apart. They form the quantum vector of a binary clock! The clock must be part of the dynamics: at given (ℓ, m), we may allow ony one quantum variable.

This may mean that we may have to give up the conservation law associated with rotations in the complex plane. *There should not be additively conserved charges*.

This should have been expected: black holes cannot obey any global conservation law.

N. Gaddam, S. Kumar, C. Ripken

The complex conjugation, $\psi \leftrightarrow \psi^*$, requires further discussion.

In my interpretation of QM, the fact that the wave functons in region *I* and *II* are clones, implies that exactly the same phenomena occur in region *I* and *II*,

> *as if they were simply described as in the Schwarzshild coordinates, not the KS coordinates!*

This suggests that our result can easily be generalised to more complicated events such as the coalescence of two black holes, or other complex phenomena.

According to Hawking,
a stationary black hole emits particles with radiation temperature

$$(?) \qquad kT_{\mathrm{Hawking}} = \tfrac{\hbar c^3}{8\pi G}\frac{1}{M_{\mathrm{BH}}}$$

In present theory, we are not dealing with thermal states (Grand Canonical Ensemble), but we an use a microcanonical ensemble: total energy of all states is fixed. In standard statistical mechanics; the results will be as usual: the probability of a state is

$\mathcal{P} \propto e^{-\beta E}$, which for a quantum system is periodic when

time $\to$ time $+i\beta$.

$\langle E_i \rangle = kT = 1/\beta$; β is periodicity in imaginary time τ

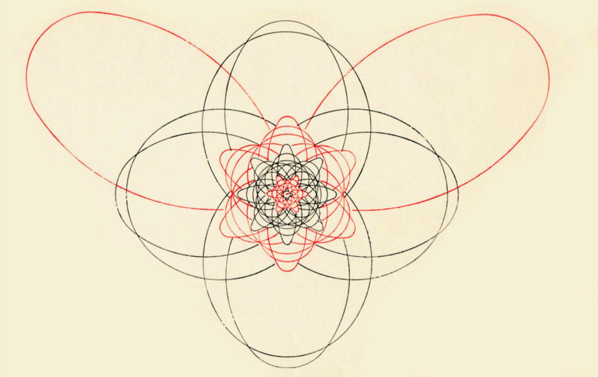
$\tau = t/4GM_{\rm BH}$; $(x, y) \to (x\, e^{\tau},\, y\, e^{-\tau})$,   so that
$\tau \to \tau + 2\pi i$ is a symmetry for all solutions in KS coordinates.

Hawking derives that $\beta = 1/T_{\rm Hawking} = 2\pi$ in natural units. But now we have only the cloned states, symmetric under
$(x, y) \leftrightarrow (-x, -y)$,   so that $\beta \overset{?}{=} 2\pi$ is to be replaced by

$$\beta = \pi \ , \qquad T = 2\,T_{\rm Hawking} = \frac{\hbar c^3}{4\pi G}\,\frac{1}{M_{\rm BH}} \ .$$

An Ambiguity of the equivalence principle and Hawking's temperature,
J. of Geometry and Physics 1 (1984) 45-52.

Happy Centuriversary to the



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