Bohr-100, NBI, Aug. 4 2022

Gravitational Scattering: from Particles, to Strings, to Black Holes

Gabriele Veneziano

Introduction

- The spectacular direct detection of gravitational waves (GW) from coalescing binaries has pushed through a rich timeline for upgrades and future GW observatories (KAGRA, ET, Cos.Exp....LISA).
- In parallel, it has also boosted theoretical work on the two-body problem in GR.
- While the traditional methods for computing the expected waveforms (and interpret the signals):
  - Numerical Relativity (Pretorius, ...)
  - Post-Newtonian expansions (PN) (Blanchet, ...)
  - Effective one body (EOB) (Buonanno-Damour, ...)

  are essentially classical, new avenues based on taking the classical limit of quantum-mechanical scattering amplitudes have also been vigorously pursued.

- This brought together two theory communities:
  1. from Classical General Relativity;
  2. from High-Energy Particle Physics,

  generating a lot of synergy (GGI/KITP workshops, "Amplitudes" , "QCD meets gravity", talks here).
- Actually, the HE community has been interested in the gravitational 2-body problem since the late eighties ('t Hooft, Amati-Ciafaloni-GV, Muzinich & Soldate,...) albeit w/ completely different motivations (see below)
- In that context transplanckian energies are crucial in order to make gravity relevant/dominant in the collision of two light objects (=> UR limit unavoidable)
- Ultra high energy is also needed in that case to justify a semiclassical approximation (see below).

- What was missed at the time is that, at large enough distance, massive black holes can also be thought of as elementary particles (no hair => just mass and spin). If so, those gedanken experiments become all but gedanken.
- Of course, for BHs the NR regime is the most relevant one. Should we then forget about that earlier work?
- I'll try to convince you that the answer is NO!
- In 1710.10599, Damour argued that useful input to the EOB can be obtained from the high-energy/UR regime of gravitational scattering and gave an example (see below).
- Other examples of useful connections between the UR limit of light particle/strings collisions and the classical two-body problem in GR will be the leitmotif of this talk (with some overlap with Paolo's talk on Monday).

Outline

- Particles & strings ('87-'07)
  - Weak gravity
  - String gravity
  - Strong gravity
- Gravitational radiation and an energy crisis ('08-'18)
- Strings on branes ('10-'15, no time, sorry)
- Black holes up to 3PM ('19-today)
  - A deflection-angle puzzle and its resolution.
  - A second energy crisis?
  - Towards a unitary semiclassical S-matrix
- Outlook

(Light) particles and strings:
a quick reminder of ACV (1987-2007)

Motivations at the time were purely theoretical:

- Recovering perturbative unitarity, emergence of classical and quantum/string gravity from amplitude calculations in flat spacetime (quite successful)

- Checking unitarity even in regimes where the process is expected to lead, classically, to black-hole formation (not quite as successful so far)

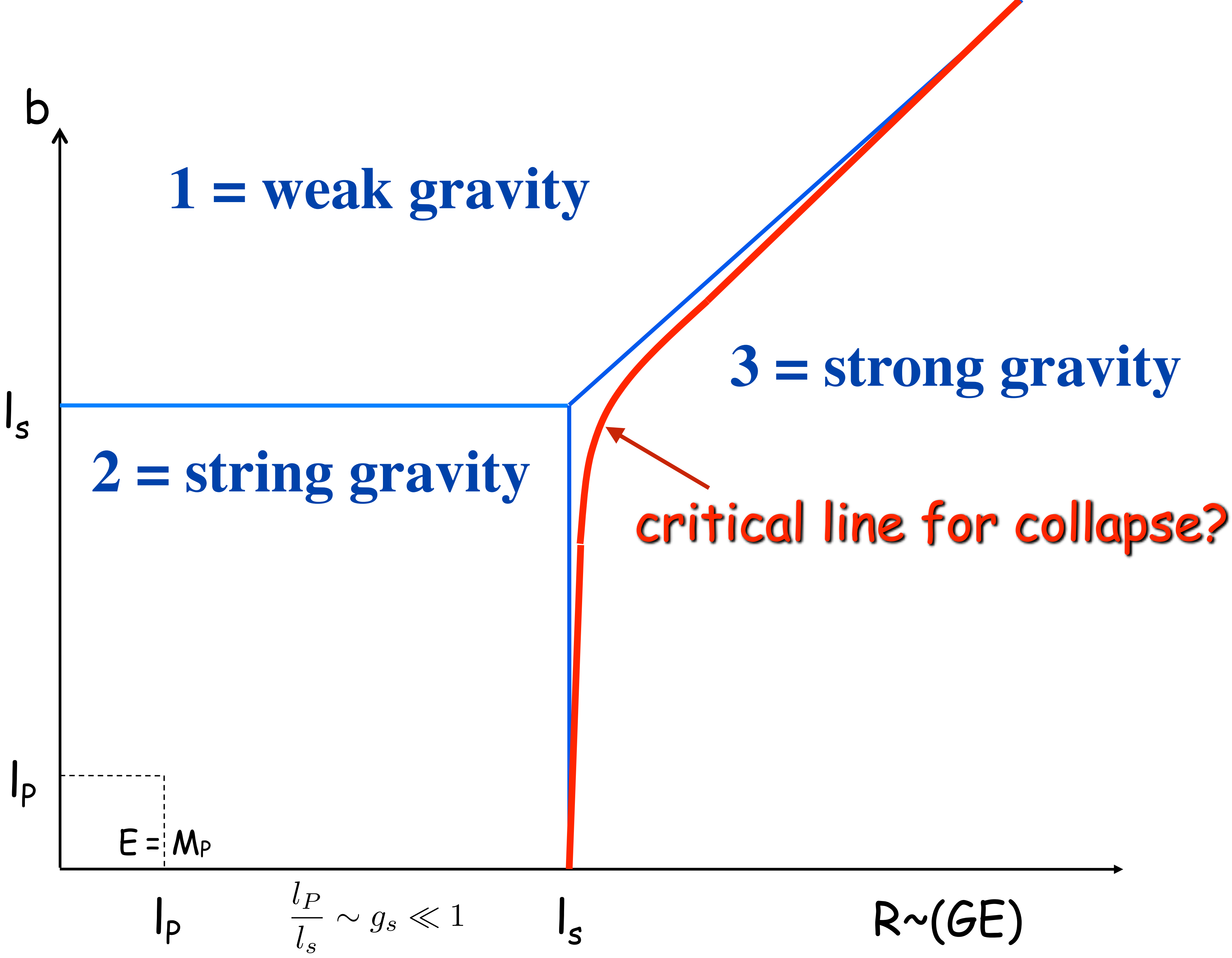
Parameter-space (D=4, c=1), regimes

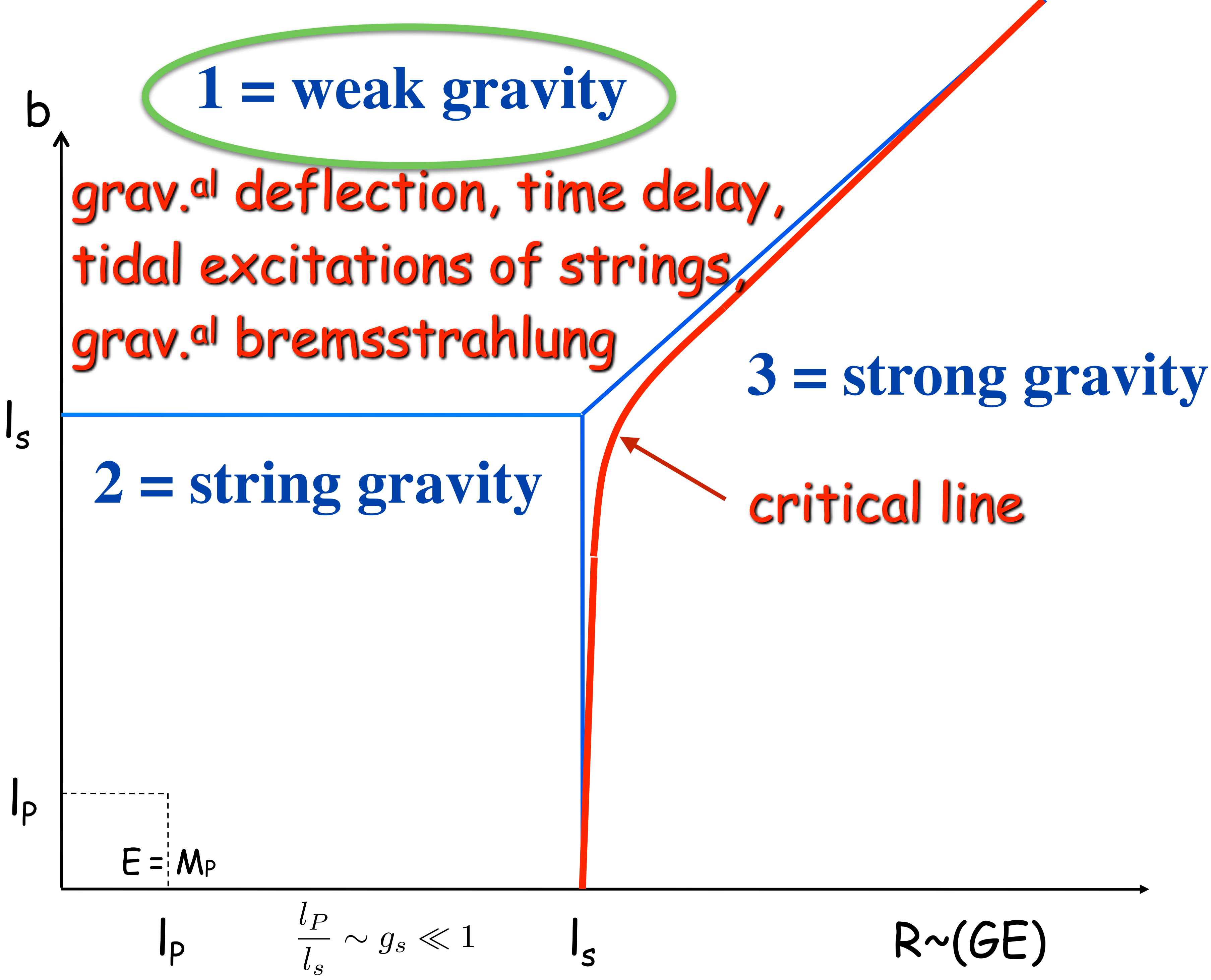
$$b \sim \frac{2J}{\sqrt{s}} \ \ ; \ \ R \sim G\sqrt{s} \ \ ; \ \ l_s \sim \sqrt{\alpha' \hbar} \ \ ; \ \ G\hbar = l_P^2 \sim g_s^2 l_s^2$$

- 3 relevant length scales (neglecting l_P @ g_s << 1) i.e. 2 relevant ratios, 2-dimensional phase diagram.

- Different regimes emerge.

Basic technique: eikonal resummation in b-space plus saddle-point approx. for h -> 0


b
1 = weak gravity
l_s
2 = string gravity
3 = strong gravity
critical line for collapse?
l_P
E = M_P
l_P
$\frac{l_P}{l_s} \sim g_s \ll 1$
l_s
R~(GE)



1 = weak gravity
grav.al deflection, time delay,
tidal excitations of strings,
grav.al bremsstrahlung
b
ls
lP
E = MP
2 = string gravity
3 = strong gravity
critical line
lP
$\frac{l_P}{l_s} \sim g_s \ll 1$
ls
R~(GE)

- Restoration of (elastic) unitarity via eikonal resummation of s-channel ladders.
- Gravitational deflection & time delay: emerging shock-wave metric at O(G) (Cf. 'tHooft 1987); extension up to O(G³) (ACV90, see below)
- t-channel "fractionation": hard scattering (large Q) from large-distance (large b) physics

$$S(E,b) \sim exp(-i\frac{Gs}{\hbar}\log b^2)$$

$$\Downarrow$$

$$Q = \frac{\hbar}{b}\frac{Gs}{\hbar} = \sqrt{s}\frac{R}{b} = \theta_s\sqrt{s}$$

Deflection angle @ 2&3PM

Reminder: the elastic eikonal "phase" defined by

$$S(E,b) = \exp(2i\delta) \ ; \ \delta = \delta_0 + \delta_1 + \delta_2 + \dots \ ; \ \delta_n = \mathcal{O}(G^{n+1})$$

gives the scattering angle and time delay as derivatives of Re 2δ w.r.t. impact parameter and energy, respectively.

On the other hand, Im 2δ > 0 is related to the opening of inelastic channels and to the consequent suppression of the elastic one (two examples below).

ACV90 results up to 3PM (D=4, GR, m=0)

1PM

$$2\delta_0 = -\frac{Gs}{\hbar}\log b^2$$

classical

****

2PM

$$2Re\delta_1 = \frac{12G^2 s}{\pi b^2}\log s \; ; \; Im\delta_1 = 0$$

quantum and non-universal

Damour's use of URL: URL -> 0 @ 2PM in classical limit

****

3PM

deflection

$$2Re\delta_2 = \frac{4G^3 s^2}{\hbar b^2}$$

classical, finite

radiation

$$Im\delta_2 \sim \frac{G^3 s^2}{\hbar b^2}\log s \log \frac{b^2}{\lambda^2}$$

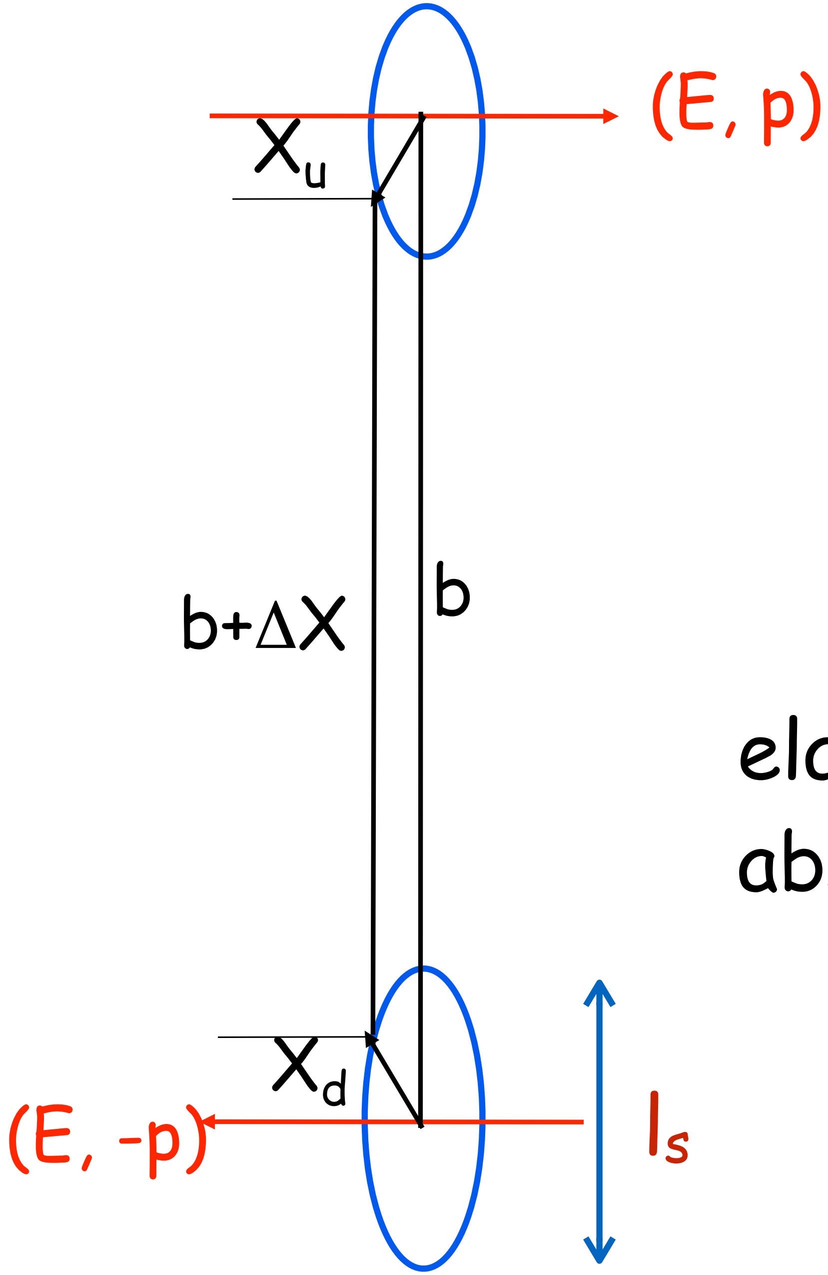
classical, divergent

- Tidal excitation of colliding strings, inelastic unitarity via unitary eikonal operator

$$S(b,\dots) = \exp\left(2i\delta(b,\dots)\right) \Rightarrow \hat{S}(b,\dots) = \exp\left(2i\hat{\delta}(b,\dots)\right)$$

$$\hat{\delta}(b,\dots) = \frac{1}{4\pi^2} : \int_0^{2\pi} d\sigma_u d\sigma_d \; \delta(b + \hat{X}_u(\sigma_u, 0) - \hat{X}_d(\sigma_d, 0), \dots) := \hat{\delta}^\dagger$$

... **with a nice physical interpretation**



kicks in @ b = b_t

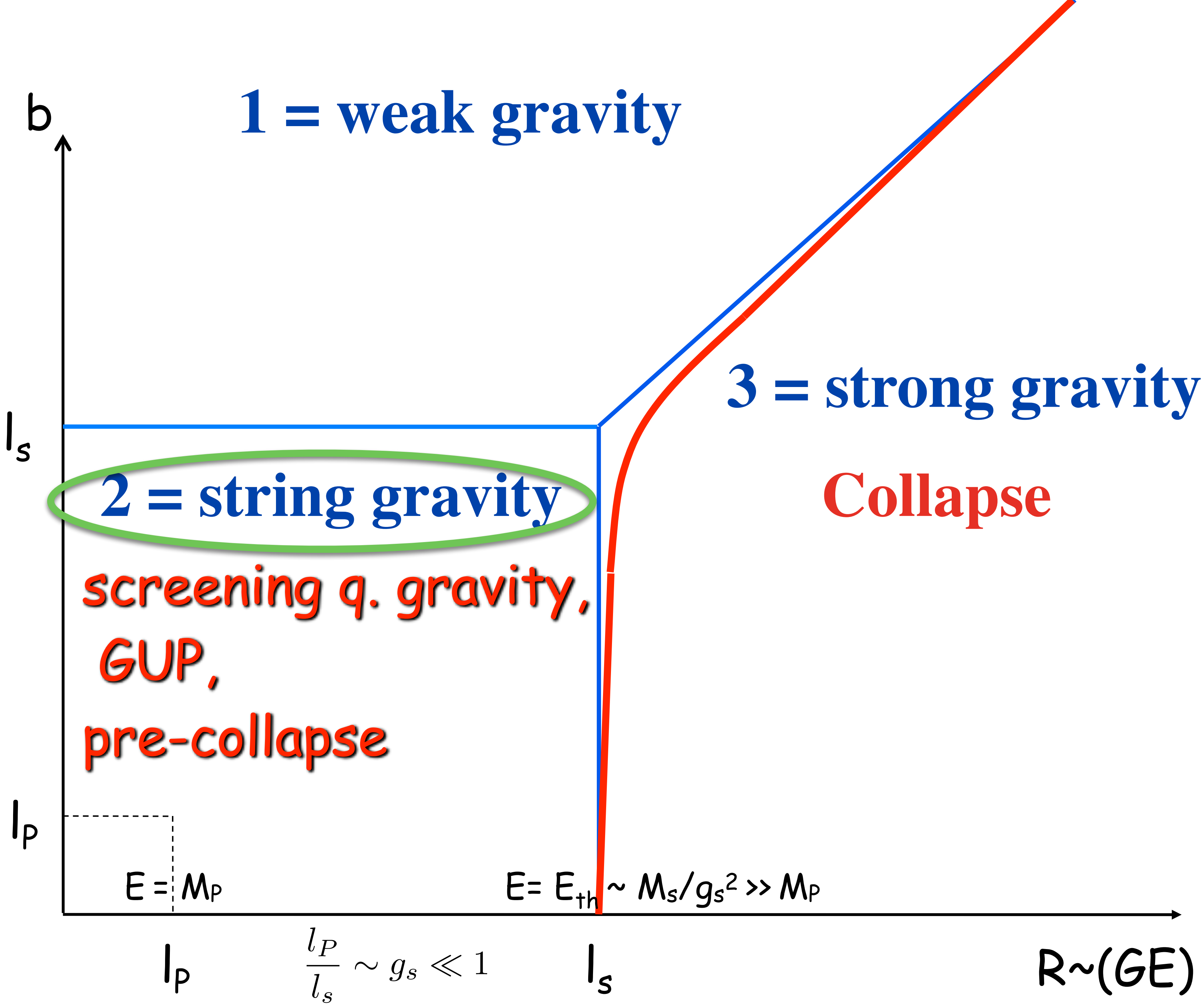
w/ $b_t^2 \sim (Gs/h)\, l_s^2 \gg l_s^2$

elastic amplitude strongly absorbed below b_t

$$S(b,\ldots) = \exp\left(2i\delta(b,\ldots)\right) \Rightarrow \hat{S}(b,\ldots) = \exp\left(2i\hat{\delta}(b,\ldots)\right)$$

$$\hat{\delta}(b,\ldots) = \frac{1}{4\pi^2} : \int_0^{2\pi} d\sigma_u d\sigma_d \; \delta(b + \hat{X}_u(\sigma_u, 0) - \hat{X}_d(\sigma_d, 0), \ldots) :$$

A first go at gravitational bremsstrahlung and an energy crisis (see below)



1 = weak gravity
b
l_s
2 = string gravity
screening q. gravity,
GUP,
pre-collapse
3 = strong gravity
Collapse
l_P
E = M_P
E= E_{th} ~ M_s/g_s^2 >> M_P
l_P
$\frac{l_P}{l_s} \sim g_s \ll 1$
l_s
R~(GE)

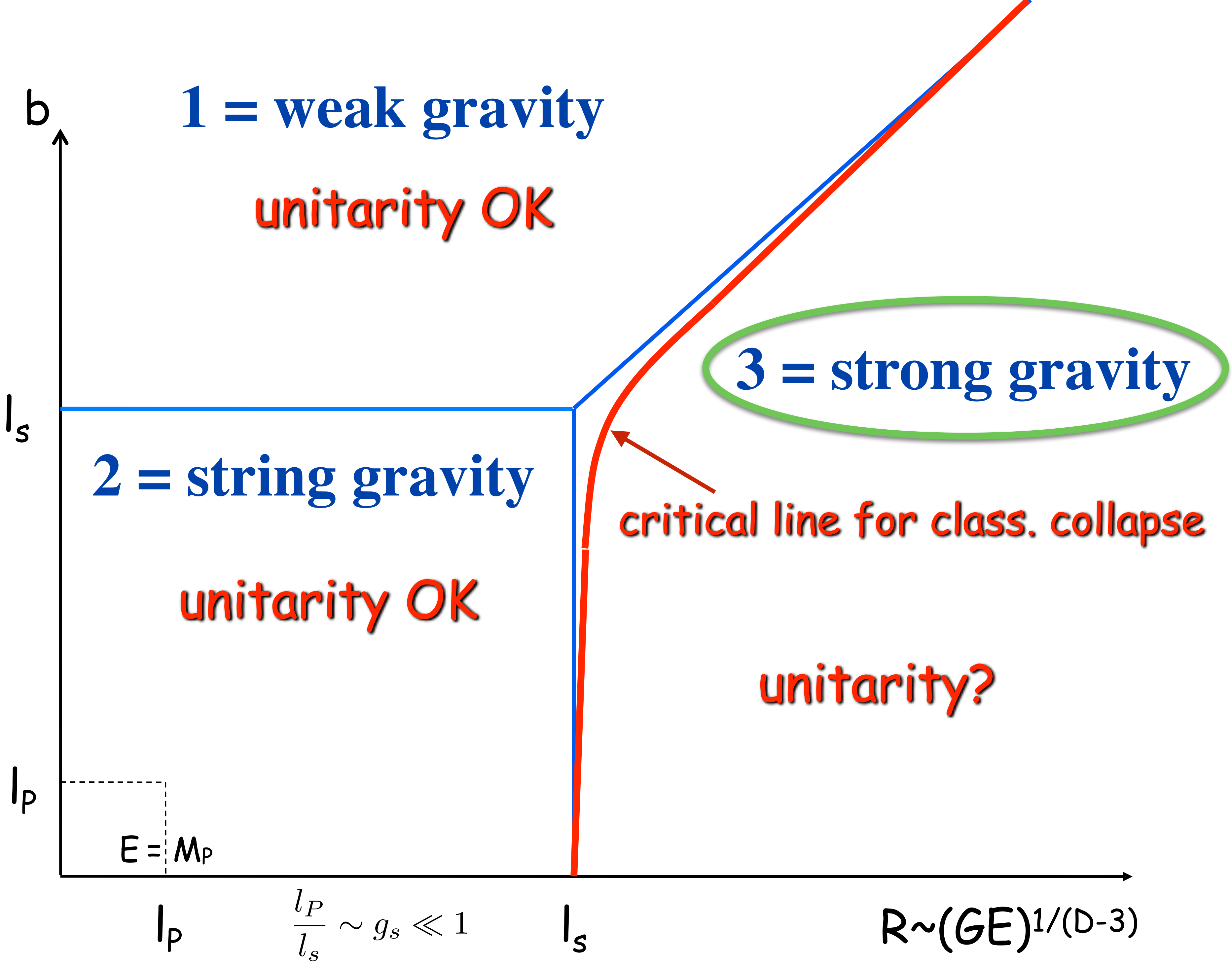
- String softening of quantum gravity @ small b: solving a causality problem via Regge-behavior (DDRV in string-brane collisions, works here too)

- Maximal classical deflection, comparison/ agreement w/ Gross-Mende-Ooguri above

- Generalized uncertainty principle (ACV, GM)

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \Delta p \geq l_s$$

- s-channel "fractionation", anti-scaling, and precocious black-hole-like behavior


b
1 = weak gravity
unitarity OK
3 = strong gravity
l_s
2 = string gravity
critical line for class. collapse
unitarity OK
unitarity?
l_P
E = M_P
l_P
$\frac{l_P}{l_s} \sim g_s \ll 1$
l_s
R~(GE)$^{1/(D-3)}$

- Identifying (semi) classical contributions as connected trees

- An effective 2D field theory to resum them after some “truncation“

- Emergence of critical surfaces in good agreement with CGR collapse criteria. But…

- Unitarity beyond critical surface still problematic…(work by Ciafaloni and Colferai)

We(I) switched to simpler problems

- Gravitational radiation (Gruzinov & GV, CC(C)V)

- String-brane collisions (DDRV*, 2010-'15)

NB. A second form of absorption when the closed string is captured by the brane system leading to a closed -> open transition. Not for today, sorry!

* D'Appollonio, Di Vecchia, Russo, GV

Gravitational Radiation
and a first “energy crisis”

ACV 0712.1209, J.Wosiek & GV 0805.2973 had found

Graviton spectrum @ $\frac{Gs}{\hbar}\frac{R^2}{b^2} \sim \langle n_{gr} \rangle \gg 1$

$$R \equiv 2G\sqrt{s}\ ,\ \ \theta_s \sim \frac{2R}{b}$$

$$\frac{dE_{gr}}{d^2k\ d\omega} = Gs\ R^2\ exp\left(-|k||b| - \omega\frac{R^3}{b^2}\right)\ ;\quad \Rightarrow \frac{E_{gr}}{\sqrt{s}} \sim 1$$

even @ small θ_s => E-crisis.

Two approaches

1. A classical GR approach
   (A. Gruzinov & GV, 1409.4555)

2. An amplitude-based (quantum) approach
   (CCColadeschi & GV, 1512.00281, Ciafaloni, Colferai & GV, 1812.08137)

NB: 2. goes over to 1. in the classical limit in spite of their completely different methodologies!
Both limited to small θ_s and θ.

The classical limit (NB: a resummation in G!)

Frequency + angular spectrum ($s = 4E^2$, $R = 4GE$)

$$\frac{dE^{GW}}{d\omega\ d^2\tilde{\theta}} = \frac{GE^2}{\pi^4}|c|^2\ ;\ \tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \boldsymbol{\theta}_s\ ;\ \boldsymbol{\theta}_s = 2R\frac{\boldsymbol{b}}{b^2}$$

$$c(\omega, \tilde{\boldsymbol{\theta}}) = \int \frac{d^2x\ \zeta^2}{|\zeta|^4}\ e^{-i\omega\mathbf{x}\cdot\tilde{\boldsymbol{\theta}}}\left[e^{-2iR\omega\Phi(\mathbf{x})} - 1\right]$$

$$\zeta = x + iy \qquad \Phi(\mathbf{x}) = \frac{1}{2}\ln\frac{(\mathbf{x}-\mathbf{b})^2}{b^2} + \frac{\mathbf{b}\cdot\mathbf{x}}{b^2}$$

Re ζ^2 and Im ζ^2 correspond to the usual (+, x) GW polarizations, ζ^2, ζ^{*2} to the two circular ones.

Analytic results: a Hawking knee

& (not for today) an unexpected

bump @ $\omega\, b \sim 0.5$

For $b^{-1} < \omega < R^{-1}$ the GW-spectrum is almost flat in ω

$$\frac{dE^{GW}}{d\omega} \sim \frac{4G}{\pi}\theta_s^2 E^2 \log(\omega R)^{-2}$$

Below $\omega = b^{-1}$ it "freezes", giving the expected zero-frequency limit (ZFL) (Smarr 1977)

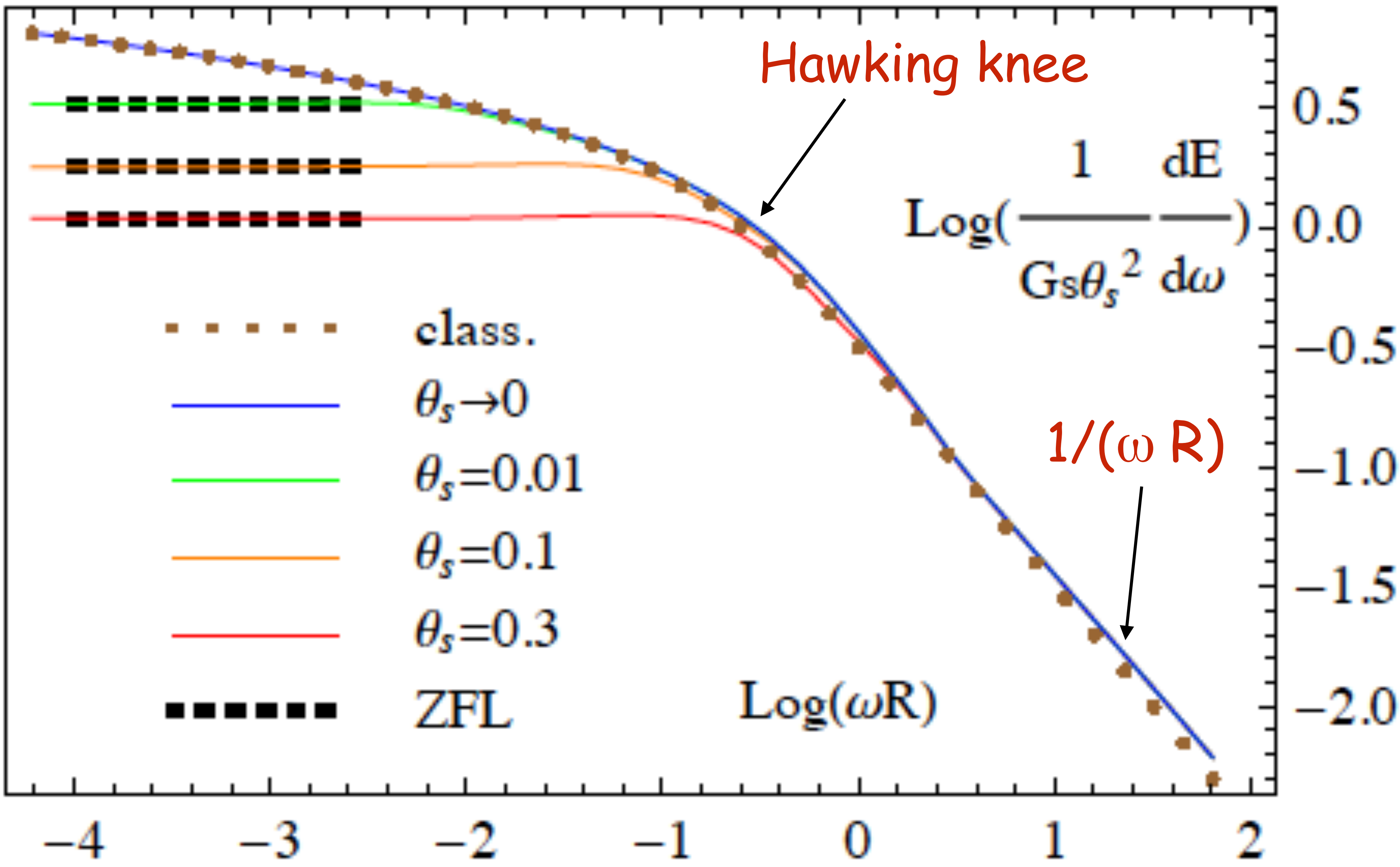
$$\frac{dE^{GW}}{d\omega} \to \frac{4G}{\pi}\ \theta_s^2 E^2\ \log(\theta_s^{-2})$$

Above $\omega = R^{-1}$ drops, becomes "scale-invariant"

Hawking knee!

$$\frac{dE^{GW}}{d\omega} \sim \theta_s^2 \frac{E}{\omega}$$

(CCCV 1512.00281)



The "scale-invariant" spectrum gives a log ω^* sensitivity in the total radiated energy for a cutoff at $\omega = \omega^*$

Using, with some motivations, $\omega^* \sim R^{-1}\,\theta_s^{-2}$ we find (to leading-log accuracy):

$$\frac{E^{GW}}{\sqrt{s}} = \frac{1}{2\pi}\,\theta_s^2\,\log(\theta_s^{-2})$$

The massless E-crisis is thus only partly solved: we need to go beyond some approximations made in G&V or CCCV, find the actual value of ω^*, and also extend the method to arbitrary θ.
We will come back to this at the end of the talk.

The D'Eath (Kovacs-Thorne) bound

- Before embarking in those non-trivial calculations of the URL we (G&V) checked the literature and asked some experts, including NR guys.
- Each time, after some initial optimism, the feedback was disappointing...
- Instead, we found Kovacs & Thorne's warning on the limit of validity of their 1977 result, and decided to try go beyond it.

THE GENERATION OF GRAVITATIONAL WAVES. IV. BREMSSTRAHLUNG*†‡

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Received 1977 October 21; accepted 1978 February 28

ABSTRACT

This paper attempts a definitive treatment of "classical gravitational bremsstrahlung"—i.e., of the gravitational waves produced when two stars of arbitrary relative mass fly past each other with arbitrary relative velocity v, but with large enough impact parameter that

$$(\text{angle of gravitational deflection of stars' orbits}) \ll (1 - v^2/c^2)^{1/2} .$$

$\theta_S\, \sigma^{1/2} \ll 1$ in our notations

I will refer to $\theta_S\, \sigma^{1/2} = 1$ as the KT bound

L REVIEW D VOLUME 18, NUMBER 4 15 AUG

High-speed black-hole encounters and gravitational radiation

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(Received 15 March 1977)

Encounters between black holes are considered in the limit that the approach velocity tends to the speed of light. At high speeds, the incoming gravitational fields are concentrated in two plane-fronted shock regions, which become distorted and deflected as they pass through each other. The structure of the resulting curved shocks is analyzed in some detail, using perturbation methods. This leads to calculations of the gravitational radiation emitted near the forward and backward directions. These methods can be applied when the impact parameter is comparable to $Gc^{-2}M\gamma^2$, where M is a typical black-hole mass and γ is a typical Lorentz factor (measured in a center-of-mass frame) of an incoming black hole. Then the radiation carries power/solid angle of the characteristic strong-field magnitude c^5G^{-1} within two beams occupying a solid angle of order γ^{-2}. But the methods are still valid when the black holes undergo a collision or close encounter, where the impact parameter is comparable to $Gc^{-2}M\gamma$. In this case the radiation is apparently not beamed, and the calculations describe detailed structure in the radiation pattern close to the forward and backward directions. The analytic expressions for strong-field gravitational radiation indicate that a significant fraction of the collision energy can be radiated as gravitational waves.

below KT bound $M\gamma \sim \sqrt{s}$; $\gamma \sim \sqrt{\sigma}$ above KT bound!

(urgent) Questions?

Black hole-black hole scattering

(elliptic/bound problem connected by analytic continuation? R. Porto...)

Sharpening and solving a 3PM puzzle

(DHRV 2008.12743, see also Paolo's talk)

- In 1901.04424/1908.01493 an impressive calculation by BCRSSZ led to the first 3PM (i.e. 2-loop) result (in GR for two massive scalars).
- Checked to be consistent up to "6PN" (integer) order but presented a puzzle.

- The high-energy (or just the massless) limit of the BCRSSZ result exhibited a logarithmic divergence in contrast with the finite result by ACV90 shown earlier.

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BCRSSZ = Bern, Cheung, Roiban, Shen, Solon, Zeng

The ACV90 argument (m=0)

- Combining:
  - Real analyticity: $A^*(s^*,t) = A(s,t)$
  - Asymptotics => fixed-t dispersion relations.
  - Xing symmetry: $A(s,t) = A(u,t)$
  - Perturbative Unitarity

an explicit calculation of $Im\delta_2$ from the inelastic (3-particle) cut of the two-loop amplitude gives ACV's result for $Re\delta_2$ from

$$2Re\delta_2 = \frac{\pi}{2\log s}(2Im\delta_2) - \frac{\delta_0}{s}(2\nabla\delta_0)^2$$

$$2Re\delta_2 = \frac{\pi}{2\log s}(2Im\delta_2) - \frac{\delta_0}{s}(2\nabla\delta_0)^2$$

The logarithmically growing term in $Im\delta_2$ has an IR divergence which, however, cancels against the δ_0 term. This yields the finite ACV result for $Re\delta_2$

By contrast, in BCRSSZ $Im\delta_2$ grows like $\log^2 s$ and this implies their (in)famous log in $Re\delta_2$.

- In 2008.12743 DHRV considered the massive UR limit and confirmed both the ACV arguments and the HE behavior of the 3-particle cut (see Paolo's talk for the latter).
- ACV90 was then definitely confirmed by computing the full amplitude in (massive) N=8 SUGRA at arbitrary energy including contributions from the full soft (rather than just the potential) integration region.
- The final outcome is amazingly simple.

3PM eikonal in N=8 SUGRA

$$\mathrm{Re}(\delta_2) = \frac{2G^3(2m_1m_2\sigma)^2}{\hbar b^2}$$

$$\left[\frac{\sigma^4}{(\sigma^2-1)^2} - \cosh^{-1}(\sigma)\left(\frac{\sigma^2}{\sigma^2-1} - \frac{\sigma^3(\sigma^2-2)}{(\sigma^2-1)^{5/2}}\right)\right]$$

P-MRZ/BCRSSZ

ACV-limit

New

cancel @ large σ

$$2m_1m_2\sigma = s - m_1^2 - m_2^2$$

$$\cosh^{-1}(\sigma) \sim \log\sigma \text{ as } \sigma \to \infty$$

NB: old and new terms behave quite differently in the NR limit, $\sigma \to 1$ ($s \to (m_1+m_2)^2$): even vs odd powers of v

- When we presented this result at a workshop in Aug. 2020, Damour immediately grasped its physical meaning:
- Our half-integer PN terms (odd powers of v) meant that we had added to the conservative dynamics of Bern et al's calculation the effect of radiation reaction.
- A couple of months later, using a smart shortcut (based on a linear-response formula by Bini-Damour), Damour extended the result to GR.

Damour's result for GR (2010.01641)

1PN

2PN(BCRSSZ)

$$2Re\delta_2 = \frac{2G^3 m_1 m_2 s}{\hbar b^2(\sigma^2-1)^{3/2}}(12\sigma^4 - 10\sigma^2 + 1)$$
$$-\frac{4G^3 m_1^2 m_2^2}{\hbar b^2(\sigma^2-1)^{1/2}}\left(\frac{\sigma(14\sigma^2+25)}{3} + (4\sigma^4 - 12\sigma^2 - 3)\frac{cosh^{-1}(\sigma)}{(\sigma^2-1)^{1/2}}\right)$$
$$+\frac{2G^3 m_1^2 m_2^2(2\sigma^2-1)^2}{\hbar b^2(\sigma^2-1)^2}\left(\frac{8-5\sigma^2}{3} + \sigma(2\sigma^2-3)\frac{cosh^{-1}(\sigma)}{(\sigma^2-1)^{1/2}}\right)$$

2.5PN

UR-limit: log s terms become again subleading &

$$(48 - 56/3 - 40/3)\sigma^2 = 16\sigma^2$$ => ACV90!

UNIVERSALITY OF THE MASSLESS LIMIT!

(cf. PdV's talk)

- A bit later, using a different shortcut, DHRV gave another simple derivation of both the N=0 and the N=8 result from analyticity, crossing & Weinberg's soft theorems. We found (2101.05772)

$$Re\ 2\delta_2^{RR} = -\pi\epsilon\ Im\ 2\delta_2(\epsilon \to 0) = \frac{\pi}{4\hbar}\frac{dE^{rad}}{d\omega}(\omega \to 0)$$

Zero-Frequency-Limit (Smarr 1977)
can be derived from Weinberg's soft theorem

- Several confirmations were given later through full-fledged two-loop calculations*.

* DHRV 2104.03256; Herrmann Parra-Martinez Ruf Zeng 2104.03957; Bjerrum-Bohr Damgaard Planté Vanhove 2105.05218; Brandhuber Chen Travaglini Wen, 2108.04216

- In N=8 SUGRA the total radiated energy comes from the graviton, the dilaton, 2 vectors, and 2 scalars.
- Their ZFLs add up to reproduce the correct RR term in Re δ_2

$$1/3\left[(8-5\sigma^2+3\sigma(2\sigma^2-3)F(\sigma))+(\sigma^2+2-3\sigma F(\sigma))+8(\sigma^2-1)+2(\sigma^2-1)\right]$$

$$\boxed{=2(\sigma^2+\sigma(\sigma^2-2)F(\sigma))}\;;\quad F(\sigma)=\frac{\cosh^{-1}(\sigma)}{\sqrt{\sigma^2-1}}$$

i.e. the combination we saw in

$$\mathrm{Re}\delta_2^{RR}\sim\left[\frac{\sigma^4}{(\sigma^2-1)^2}+\cosh^{-1}(\sigma)\frac{\sigma^3\left(\sigma^2-2\right)}{(\sigma^2-1)^{5/2}}\right]$$

Radiation and energy loss: a second "energy crisis" ?

- An $O(G^3)$ calculation of the total E^{rad} (HP-MRZ*, 2101.07255) has confirmed KT's result leading to an "energy crisis" similar to the one we have discussed (and partially solved).

- Indeed E^{rad}/E grows like $\theta_s^3\, \sigma^{1/2}$ violating E-cons. as σ goes to infinity @ fixed θ_s

- Remember KT's warning on limit of validity of their result: $\theta_s\, \sigma^{1/2} < 1$. In that situation $E^{rad}/E < \theta_s^2$ and there is no crisis.

HP-MRZ= Hermann, Parra-Martinez, Ruf, Zeng

HP-MRZ, 2101.07255 (confirmed in DHRV, 2104.03256)

$$E^{rad} = \frac{\pi G^3 m_1^2 m_2^2 (m_1 + m_2)}{b^3 \sqrt{s}} \left[f_1(\sigma) + f_2(\sigma) \log \frac{\sigma + 1}{2} + f_3(\sigma) \frac{\sigma \cosh^{-1} \sigma}{2\sqrt{\sigma^2 - 1}} \right]$$

where in $\mathcal{N} = 8$

$$f_1 = \frac{8\sigma^6}{(\sigma^2 - 1)^{\frac{3}{2}}}, \qquad f_2 = -\frac{8\sigma^4}{\sqrt{\sigma^2 - 1}}, \qquad f_3 = \frac{16\sigma^4(\sigma^2 - 2)}{(\sigma^2 - 1)^{\frac{3}{2}}},$$

while in GR

$$f_1 = \frac{210\sigma^6 - 552\sigma^5 + 339\sigma^4 - 912\sigma^3 + 3148\sigma^2 - 3336\sigma + 1151}{48(\sigma^2 - 1)^{\frac{3}{2}}},$$

$$f_2 = -\frac{35\sigma^4 + 60\sigma^3 - 150\sigma^2 + 76\sigma - 5}{8\sqrt{\sigma^2 - 1}},$$

$$f_3 = \frac{(2\sigma^2 - 3)(35\sigma^4 - 30\sigma^2 + 11)}{8(\sigma^2 - 1)^{\frac{3}{2}}},$$

$$\nu \equiv \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$\frac{E^{rad}}{\sqrt{s}} \sim \theta_s^3 \sqrt{\frac{\sigma}{\nu}} \; ; \; \text{for } \sigma \to \infty$$

Another "energy crisis" above KT

- Amusingly, a warning can already be found in the ZFL (for the URL). Integrating the ZFL up to 1/b:

$$\frac{dE^{rad}}{d\omega} \to \frac{Gs}{\pi}\theta_s^2 \log(\sigma) \Rightarrow \frac{E^{rad}}{\sqrt{s}} \sim \theta_s^3 \log(\sigma)$$

- In this case, however, Weinberg tells us how to cure the problem.

- One can directly study the ZFL for massless scattering and the result is quite different (and finite!):

$$\frac{dE^{rad}}{d\omega} \to \frac{Gs}{\pi}\theta_s^2 \log(\theta_s^{-2})$$

- The price to pay is that the result is non-polynomial in G (as in the (G+V) and CCCV "solution")! We'll come back to this at the end of the talk.

URL, radiation & eikonal

A better (actually the correct) framework to discuss both conservative and dissipative phenomena (and their interplay) is to upgrade the eikonal "phase" to an hermitian eikonal operator. This was recognized long ago by ACV (starting w/ tidal excitations), CC(C)V (for grav. rad.), etc.

Actually, the much appreciated KMOC* formalism is already using in a crucial way the existence of a unitary S-matrix(operator).

* Kosower, Maybee, O'Connell (1811.10950)

From $S^{+}S = 1$ and $|\psi, \mathrm{out}\rangle = S|\psi, \mathrm{in}\rangle$

$$\langle \mathcal{O} \rangle_{\mathrm{out}} = \frac{\langle \psi, \mathrm{out}|\hat{\mathcal{O}}|\psi, \mathrm{out}\rangle}{\langle \psi, \mathrm{out}|\psi, \mathrm{out}\rangle} = \frac{\langle \psi, \mathrm{in}|S^{\dagger}\hat{\mathcal{O}}S|\psi, \mathrm{in}\rangle}{\langle \psi, \mathrm{in}|\psi, \mathrm{in}\rangle}$$

as well as

$$\langle \mathcal{O} \rangle_{\mathrm{out}} - \langle \mathcal{O} \rangle_{\mathrm{in}} = \frac{\langle \psi, \mathrm{in}|S^{\dagger}[\hat{\mathcal{O}}, S]|\psi, \mathrm{in}\rangle}{\langle \psi, \mathrm{in}|\psi, \mathrm{in}\rangle}$$

Can we actually compute such observables in the classical limit? We can always write $S = \exp(i\,\chi)$ with an hermitian χ (Damgaard-Planté-Vanhove, 2107.12891)

The classical limit will then be determined by χ at the leading order in h (to be carefully defined). That implies, BTW, that "superclassical" terms in S ($\sim h^{-n}$ w/ $n > 1$) should exponentiate as $\exp(i/h\, I_{cl})$.

One further simplification occurs if χ itself is already fixed by the exponentiation of some low order calculation.

This leads naturally to a coherent state (the closest possible to a classical field) representation with χ a linear function of creation and destruction operators

- Such an approximation is known to be valid in the soft-graviton limit (cf. exponentiation of soft divergences) but how soft is soft?
- Individual gravitons are expected to be very soft. They typically carry a classical frequency related to some classical length in the problem (b, GE), leading one to hope that exponentiation works at all wavelengths (since they carry energy $h\,\omega$) .
- However, the radiated gravitons are known to carry, all together, a classical fraction of the energy of the process, hence, at least, energy conservation has to be implemented in the coherent state formalism.

- A challenge is to reconcile energy-momentum conservation with unitarity.
- Hints that this may work were found (~ 2016) in unpublished work by Ciafaloni and myself.
- Recently, Cristofoli et al. (2112.07556) have addressed and (partially?) answered this question.
- Our group (DHRV) is about to finish paper on this. Claim: leading-order (in h) inelastic unitarity is OK.
- We are then able to reliably compute various radiative observables: waveforms, memory, radiated-energy and angular momentum spectra, linear/angular momentum loss by each particle.
- Some already known (e.g. in Mougiakakos Riva Vernizzi 2102.08339, RV 2110.10140) others are new.

In the rest of my time, if any, I will discuss what we did for the ZFL of radiation at arbitrary σ and end up with some unpublished results and/or speculations on what may happen, at arbitrary σ, beyond the ZFL.

An improved eikonal operator in the soft-graviton limit

(DHRV 2204.02378)

- We start from Weinberg's soft theorem in momentum space (a multiplication!)

$$S^{(M)}_{s.r.,N} = \prod_{r=1}^{N} f_{j_r}(k_r)\, S^{(M)}(\sigma, Q)$$

$$f_j(k) = \varepsilon_j^{*\mu\nu}(k) F_{\mu\nu}(k)\,, \qquad F^{\mu\nu}(k) = \sum_n \frac{\kappa\, p_n^\mu p_n^\nu}{p_n \cdot k}$$

- We then go over to b-space by FT (=> a convolution) and arrive at following operator eikonal:

$$S_{s.r.}(\sigma, b; a, a^\dagger) = \exp\left(\frac{1}{\hbar}\int_{\vec{k}}\sum_j\left[\tilde{f}_j(k)a_j^\dagger(k) - \tilde{f}_j^*(k)a_j(k)\right]\right) e^{i\,\mathrm{Re}\,2\delta(\sigma,b)}$$

where in the f^{tilde} one is supposed to use the replacement:

$$q \to -i\hbar\frac{\partial}{\partial b}$$

- Since Reδ is $O(h^{-1})$ the classical limit is obtained by replacing Q by its classical value:

$$q \to Q = \hbar\,\frac{\partial(Re 2\delta)}{\partial b} = Q^{\mathrm{class}}(\sigma, b)$$

Features of the ZFL in the URL

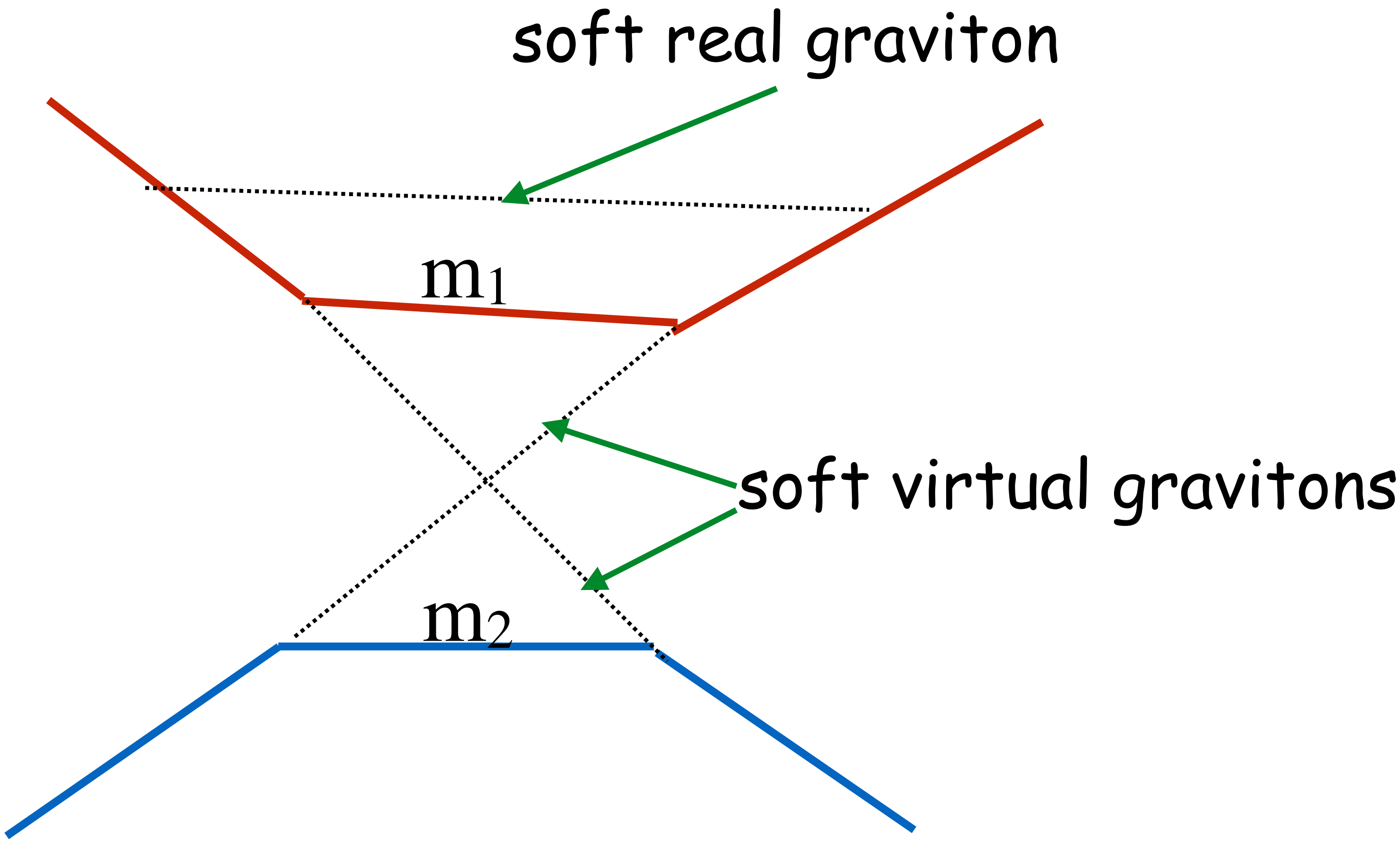
- Rich structure of UR limit emerging
- In URL the ZFL depends non trivially on two "scaling variables": $x_i = Q/2m_i$. One combination is of course related to ν, the other is new, e.g. $\theta_s^2 \gamma_1 \gamma_2 \sim \theta_s^2 \sigma$
- Dependence is non-analytic in G and a PM expansion in powers of G (or in the x_i) has a finite radius of convergence, given precisely by $x_1=1$ and $x_2=1$.
- Reason: a singularity at the unphysical points

$$x_i^2=-1 : Q^2 = - 4 m_i^2$$

corresponding to t-channel thresholds

- This defines quantitatively the KT bound!
- Only the truly massless limit ($m_i \ll Q$) is universal!

Diagram with branch point at $t = 4\ m_1^2$



An interesting use of QFT's crossing symmetry!

Explicit results in the ZFL

GR

$$\sigma_Q = \sigma - \frac{Q^2}{2m_1 m_2} = -\frac{u - m_1^2 - m_2^2}{2m_1 m_2}$$

$$\lim_{\omega\to 0}\frac{dE^{\rm gr}}{d\omega} = \frac{4G}{\pi}\Bigg[2m_1m_2\left(\sigma^2-\tfrac{1}{2}\right)\frac{\operatorname{arccosh}\sigma}{\sqrt{\sigma^2-1}} - 2m_1m_2\left(\sigma_Q^2-\tfrac{1}{2}\right)\frac{\operatorname{arccosh}\sigma_Q}{\sqrt{\sigma_Q^2-1}}$$

$$+\frac{m_1^2}{2} - m_1^2\left(\left(1+\tfrac{Q^2}{2m_1^2}\right)^2-\tfrac{1}{2}\right)\frac{\operatorname{arccosh}\left(1+\frac{Q^2}{2m_1^2}\right)}{\sqrt{\left(1+\frac{Q^2}{2m_1^2}\right)^2-1}}$$

$$+\frac{m_2^2}{2} - m_2^2\left(\left(1+\frac{Q^2}{2m_2^2}\right)^2-\tfrac{1}{2}\right)\frac{\operatorname{arccosh}\left(1+\frac{Q^2}{2m_2^2}\right)}{\sqrt{\left(1+\frac{Q^2}{2m_2^2}\right)^2-1}}\Bigg]_{Q=2p\sin\frac{\Theta_s}{2}}$$

in URL

$$\frac{4G}{\pi}\Bigg[1+\frac{1}{8x_1^2}+\frac{1}{8x_2^2}+\log(\theta_s^{-2})+\log(16x_1x_2)$$

$$-\frac{\left(1+x_1^2+\frac{1}{8x_1^2}\right)\cosh^{-1}(1+2x_1^2)}{\sqrt{(1+2x_1^2)^2-1}} - \frac{\left(1+x_2^2+\frac{1}{8x_2^2}\right)\cosh^{-1}(1+2x_2^2)}{\sqrt{(1+2x_2^2)^2-1}}\Bigg]$$

and in N=8-SUGRA

$$\lim_{\omega\to 0}\frac{dE^{\mathcal{N}=8}}{d\omega}=\frac{4G}{\pi}\Bigg[2m_1m_2\sigma^2\frac{\operatorname{arccosh}\sigma}{\sqrt{\sigma^2-1}}-2m_1m_2\sigma_Q^2\frac{\operatorname{arccosh}\sigma_Q}{\sqrt{\sigma_Q^2-1}}$$
$$-\frac{(Q^2)^2}{4m_1^2}\frac{\operatorname{arccosh}\left(1+\frac{Q^2}{2m_1^2}\right)}{\sqrt{\left(1+\frac{Q^2}{2m_1^2}\right)^2-1}}-\frac{(Q^2)^2}{4m_2^2}\frac{\operatorname{arccosh}\left(1+\frac{Q^2}{2m_2^2}\right)}{\sqrt{\left(1+\frac{Q^2}{2m_2^2}\right)^2-1}}\Bigg]_{Q=2p\sin\frac{\Theta_s}{2}}$$

becoming in URL

$$\frac{4G}{\pi}\left[1+\log(\theta_s^{-2})+\log(16x_1x_2)-x_1^2\frac{\cosh^{-1}(1+2x_1^2)}{\sqrt{(1+2x_1^2)^2-1}}-x_2^2\frac{\cosh^{-1}(1+2x_2^2)}{\sqrt{(1+2x_2^2)^2-1}}\right]$$

Universality broken at finite x_i, recovered only for x_i going to infinity

URL beyond the ZFL (DHRV, in preparation)

- We have only considered the leading order in $\theta_s \ll 1$.
- Up to $\omega \sim b^{-1}\,\sigma^{1/2}$ (resp. $b^{-1}\,\theta_s^{-1}$) one goes qualitatively from below to above KT simply by $\sigma^{1/2} \to \theta_s^{-1}$
- Exponential fall-off above $\omega^* \sim b^{-1}\,\sigma^{3/2}$ (resp. $\sim b^{-1}\,\theta_s^{-3}$ as in G+V, CC(C)V) confirmed.
- Below KT, and in the intermediate regime $b^{-1}\,\sigma^{1/2} < \omega < b^{-1}\,\sigma^{3/2}$, we confirm a power law behavior with an exponent definitely larger than 1 (~ 2.1).
- If that is still the case above KT (in corresponding window) the log ω^* in (E^{rad}/E) by G&V and CC(C)V would be due to some unjustified approximation.
- A preliminary table summarizing the situation is given below.

UR limits @ different ω (to appear)

	soft ($\omega b < 1$)	interm. ($1 < \omega b < \sigma^{1/2}$) ($1 < \omega b < 1/\theta_s$)	hard ($\sigma^{1/2} < \omega b < \sigma^{3/2}$) ($\theta_s^{-1} < \omega b < \theta_s^{-3}$)
below KT	$\theta_s^3 \log \sigma$ (same)	$\theta_s^3 \log\left(\frac{\sigma}{\omega^2 b^2}\right)$ $(\Delta E/\sqrt{s} = \theta_s^3 \sqrt{\sigma})$	$\theta_s^3 \sqrt{\sigma}\, (\omega b)^{-1-\Delta}$ $\Delta E/\sqrt{s} = \theta_s^3 \sqrt{\sigma}$ confirmed w/ Δ ~ 1
above KT	$\theta_s^3 \log \theta_s^{-2}$ (same)	$\theta_s^3 \log\left(\frac{\theta_s^{-2}}{\omega^2 b^2}\right)$ $(\Delta E/\sqrt{s} = \theta_s^2)$	$\theta_s^2\, (\omega b)^{-1}$ $\Delta E/\sqrt{s} = \theta_s^2 \log \theta_s^{-2}$ G&V/CCCV to be checked

$$\frac{1}{\sqrt{s}} \frac{dE^{rad}}{d\omega b} \quad ; \quad \frac{\Delta E^{\rm rad}}{\sqrt{s}}$$

Final comments, outlook

- Gravitational scattering/inspiralling/merger is a new exciting field for applying theoretical particle physics techniques outside its traditional arena.
- Many puzzles and challenges lie ahead calling both for new tools and for new ideas.
- On-shell methods & double copy techniques can facilitate computations, particularly of integrands.
- But, at the level of the integrals, I believe that the connection is more subtle as in the gauge-gravity correspondence (e.g. IR <->UV).

- An important challenge is finding the most efficient way to extract classical observables starting from a solid, well defined quantum framework. Constructing a reliable eikonal operator looks like a very promising way.
- Like with the SM vis-à-vis accelerator physics one important goal is to provide a solid theoretical basis for interpreting the forthcoming GW observations and perhaps for detecting some beyond-GR physics.
- But it's also possible that synergy with the GW community will generate new ideas on how to solve more fundamental questions in quantum/string gravity like the ones we asked ourselves some 35 years ago.

Thank you!